Sequential Games & Decision Analysis

Microeconomics (SPI 615b)

Princeton University

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Proposer/Responder Ultimatum Bargaining, SPNE & Game Trees, Surplus Sharing / Deals, Multi-Case Entry Deterrence with Unknown Payoffs

PROPOSER/RESPONDER GAME

Suppose I stop you (proposer) and a random stranger (responder) on the street and explain a game to both of you: I give you, the proposer, $100 to split with the other person. You have **one opportunity** to offer some amount ($X) of the $100 to the responder, who can either accept or reject your proposal.

If the responder accepts your offer, then you each keep the amounts you proposed. ($100-X for you and $X for the responder)

If the responder rejects your offer, then you each get zero and I move on to another pair of strangers to continue the experiment.

Assuming you are both rational and self-interested with monotonic increasing utility over money, what will you do and what will be the result ?

PROPOSER/RESPONDER GAME: Units Modification

- Now suppose the units are scaled up from dollars to **billions of dollars**.

[*But the exact fraction of the amount offered (0.01 out of 100) cannot be reduced]*

Think about what changes.

Would this alter your behavior as responder?

PROPOSER/RESPONDER GAME

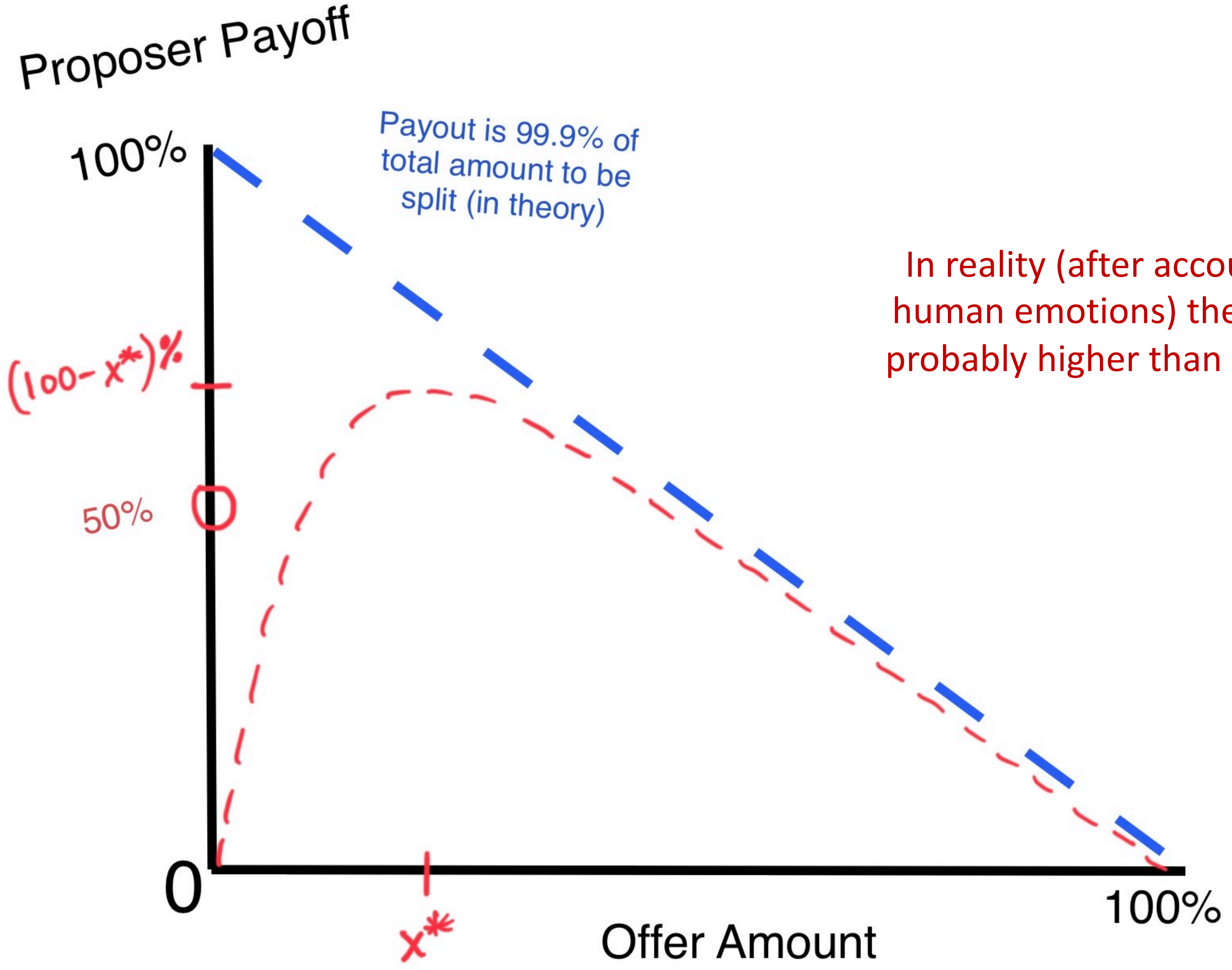
- The unique Subgame Perfect NE outcome here is, once again, where neither player has any incentive to deviate while taking the other players' behaviors into account: in a Nash Equilibrium the players are "stuck" since they are all best-responding to each other.

- The optimal offer is the smallest amount possible, which is $0.01 (since a penny is the smallest denomination of US currency) and this offer will always be accepted by any rational responder because it is more than zero. This is the unique SPNE outcome of the game.

  - *Whether people really do behave rationally is a different question…*

DISCUSSION - PROPOSER/RESPONDER GAME

- Is the unique theoretical SPNE (offering \$0.01) a realistic assumption?
- What happens in the real world with this game?
- What factors might affect how different people may respond differently?



In reality (after accounting for the irrationality of human emotions) the optimal offer amount (x*) is probably higher than one penny but less than 50%.

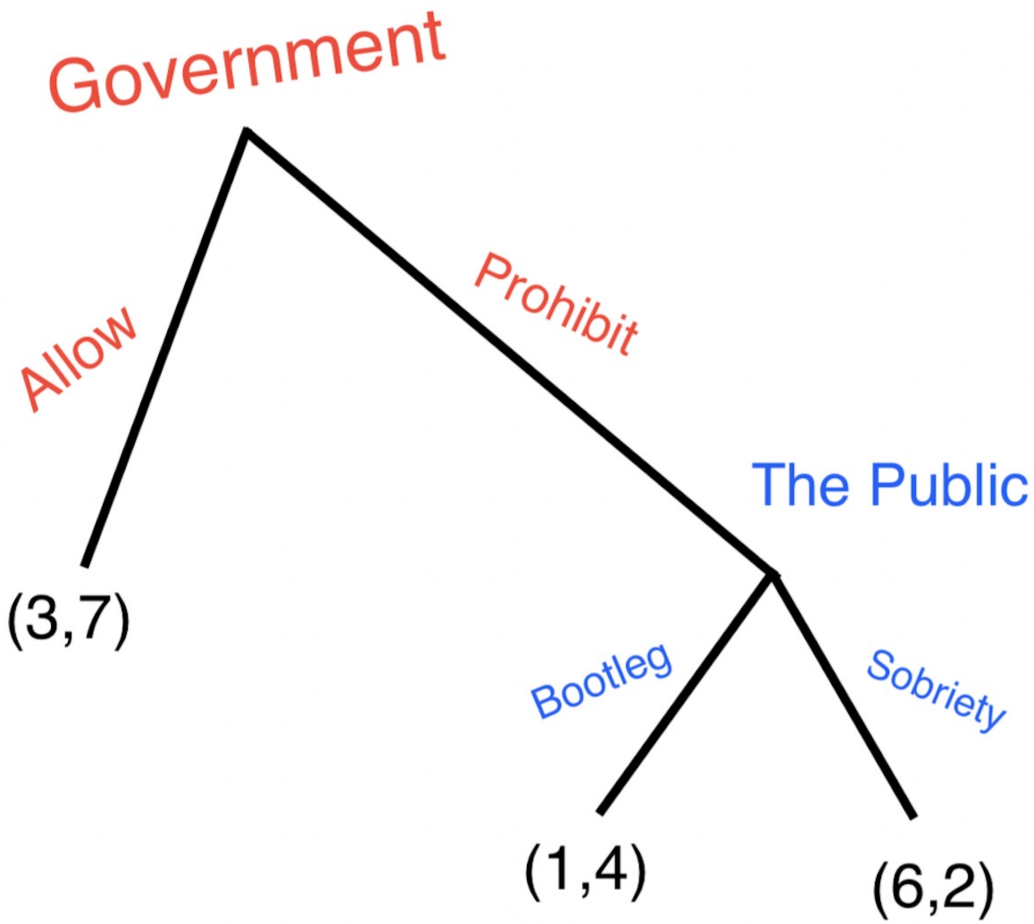
Sequential Games - Basics

- The order of payoffs is given in the order of the players' ability to move, so the first number listed in the payoffs always corresponds to the player that is first to make a choice.

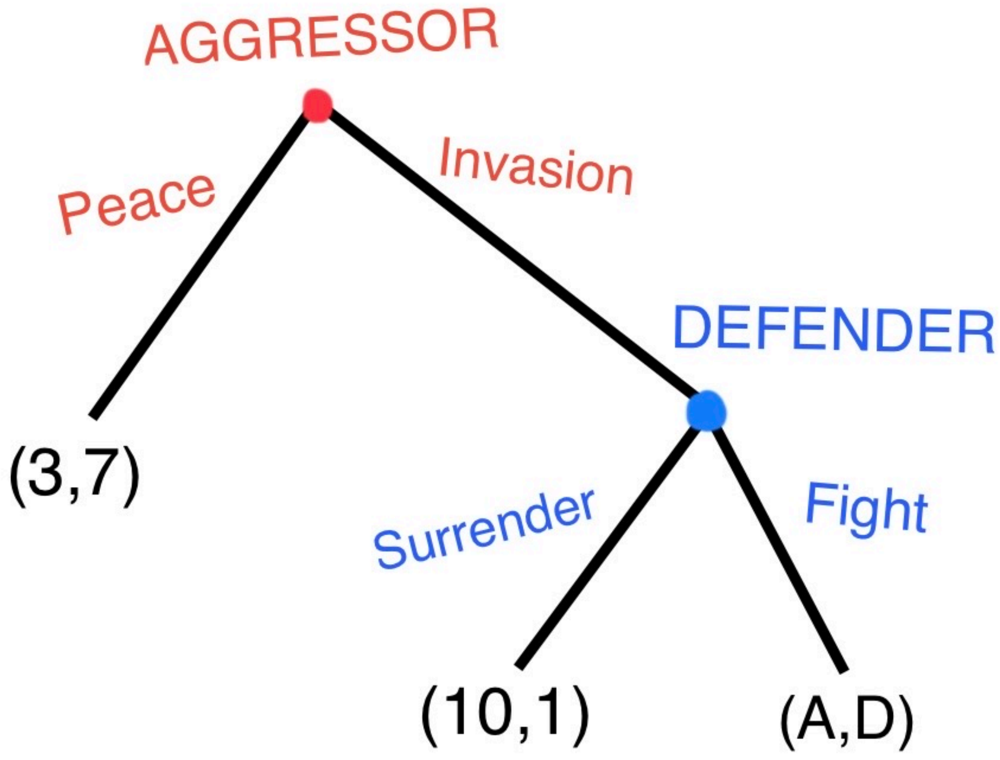
- Now that *order matters* we cannot accurately represent these games in a matrix, since matrix setup games are always completely simultaneous.

- *Backwards Induction* is the process of starting at the bottom of the game tree to decide at each level (decision "node") what the optimal action will be: any actions chosen in a proper SPNE must take all possible subsequent actions by all players into account, including what will optimally be chosen to maximize attainable payoffs.

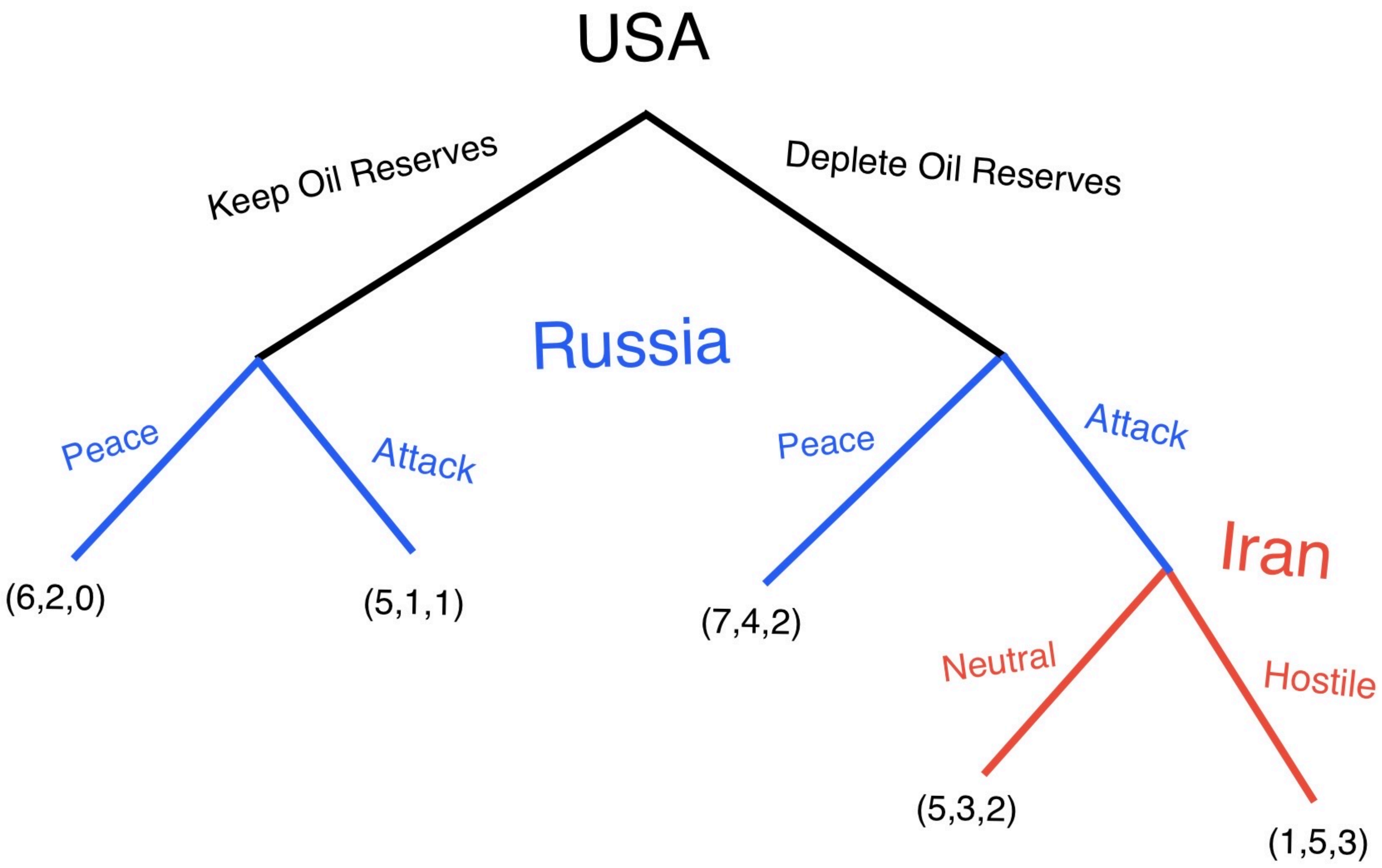
Prohibition Game Tree: USA 18th Amendment in 1920



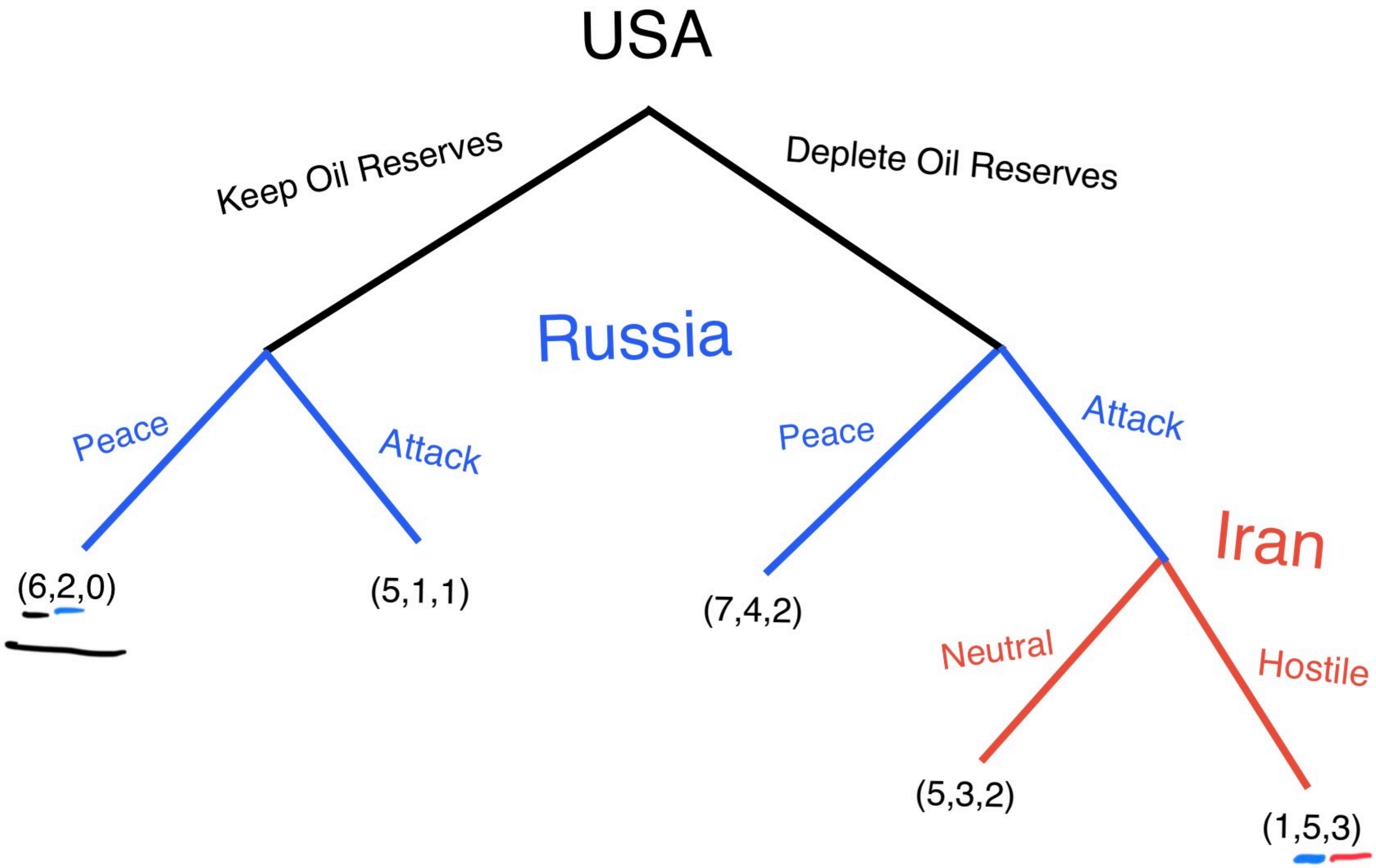
Under what conditions can Peace remain the outcome?



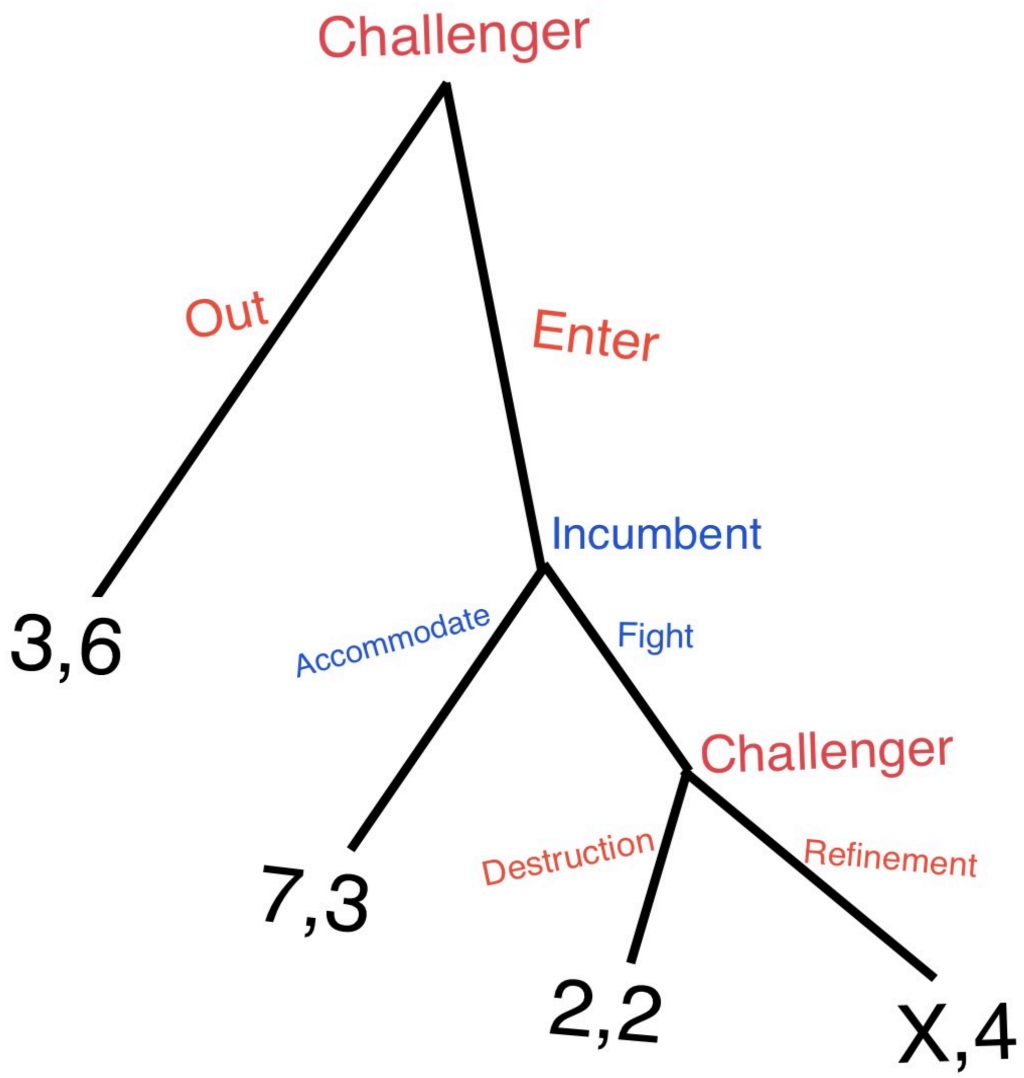
Three Player Sequential Game:



Best Responses - Three Player Sequential Game:



ENTRY DETERRENCE WITH AN UNKNOWN PAYOFF



SOLVING CONDITIONALLY VIA BACKWARD INDUCTION

- **We need to consider different cases of X values again**

Where to start?

- If $X < 2$ then Challenger chooses Destruction if Entry and Fight are chosen.
- If $X > 2$ then Challenger chooses Refinement if Entry and Fight are chosen.

- *This time, with a larger game tree, there may be additional X values to consider…*

SOLVING CONDITIONALLY VIA BACKWARD INDUCTION

- If $X < 2$ then Challenger chooses Destruction if Entry and Fight are chosen.
  - Payoffs would be (2,2) in this case, so the Incumbent would choose Accommodate instead of Fight.
  - Knowing this, the Challenger will choose Entry.
    - Thus, if $X < 2$ we have **{Entry; (D if F) , Accommodate}** as our SPNE. **Payoffs = (7,3)**

- *Note there are also off-path (non-optimal) outcomes to specify if Out was chosen, even though it will not be chosen when X<2 …*

SOLVING CONDITIONALLY VIA BACKWARD INDUCTION

- If $X < 2$ we have **{Entry; (D if F) , Accommodate}** as our SPNE. **Payoffs = (7,3)**

- If $X > 2$ then Challenger chooses Refinement if Entry & Fight are both chosen: payoffs = (X,4).
  - The Incumbent would therefore choose Fight over Accommodate in this case…

So what will be our outcome if $X > 2$?

SOLVING CONDITIONALLY VIA BACKWARD INDUCTION

- If $X < 2$ we have **{Entry; (D if F) , Accommodate}** as our SPNE. **Payoffs = (7,3)**

- If $X > 2$ then Challenger chooses Refinement if Entry & Fight are both chosen.
  - The Incumbent would therefore choose Fight over Accommodate in this case.
  - Knowing this, if $X < 3$ then the Challenger would choose Out instead of Enter.
    - Thus, if $2 < X < 3$ we have **{Out; (R if F) , Fight if E}** as our SPNE. **Payoffs = (3,6)**

SOLVING CONDITIONALLY VIA BACKWARD INDUCTION

- If X < 2 then Challenger chooses Destruction if Entry and Fight are chosen.
  - Payoffs would be (2,2) in this case, so the Incumbent would choose Accommodate instead of Fight.
  - Knowing this, the Challenger will choose Entry.
    - Thus, if X < 2 we have **{Entry; (D if F) , Accommodate}** as our SPNE. **Payoffs = (7,3)**
- If X > 2 then Challenger chooses Refinement if Entry & Fight are both chosen: payoffs = (X,4).
  - The Incumbent would therefore choose Fight over Accommodate in this case.
  - Knowing this, if X < 3 then the Challenger would choose Out instead of Enter.
    - Thus, if 2 < X < 3 we have **{Out; (R if F) , Fight}** as our SPNE. **Payoffs = (3,6)**

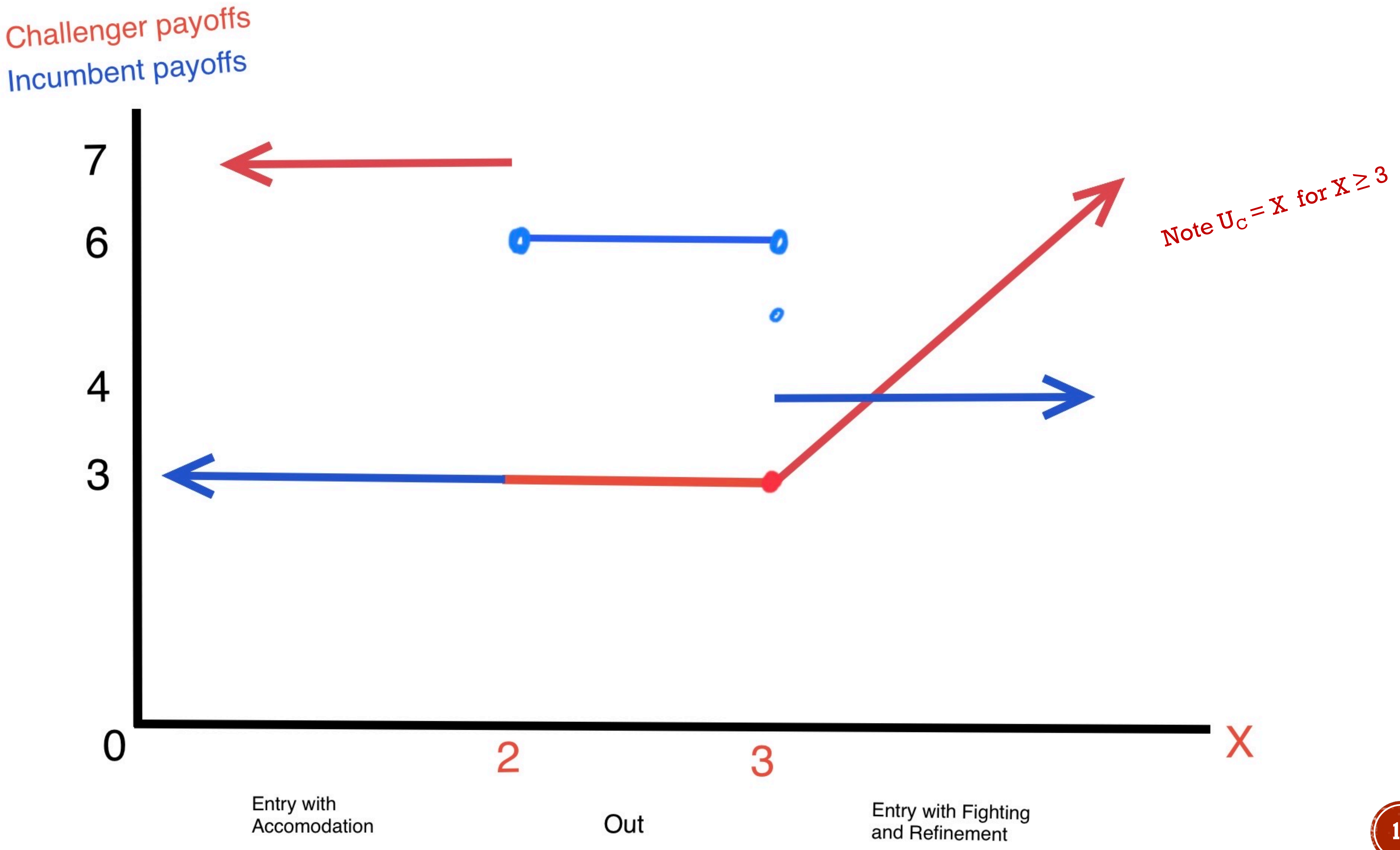
SOLVING CONDITIONALLY VIA BACKWARD INDUCTION

- If $X < 2$ we have **{Entry; (D if F) , Accommodate}** as our SPNE. **Payoffs = (7,3)**
- If $2 < X < 3$ we have **{Out; (R if F) , Fight if E}** as our SPNE. **Payoffs = (3,6)**

- If $X > 3$ then the Challenger chooses Refinement if Entry & Fight are both chosen.
  - The Incumbent would therefore choose Fight over Accommodate if there is entry.
  - Knowing this, the Challenger will choose Entry.
    - Thus, if $X > 3$ we have **{Enter; (R if F) , Fight}** as our SPNE. **Payoffs = (X,4)**

- Note some of the non-optimal off-path outcomes:

{Enter; R if A, Fight}
{Enter; D if A, Fight}
{Enter; R if A, A if Out}
{Enter; D if A, A if Out}


Challenger payoffs
Incumbent payoffs
7
6
4
3
0
2
3
X
Note $U_C = X$ for $X \geq 3$
Entry with Accomodation
Out
Entry with Fighting and Refinement

DISCUSSION: SOLVING ENTRY DETERRENCE WITH CASES

- What is the "best possible scenario" for the Challenger?
- What is paradoxical about this game if we think of the unknown payoff X as representing challenger "adaptation ability" in the case of Entry & Fight & Refinement being the path that is chosen ?

EXTRA NOTES ON INDIFFERENCE CASES:

- Note that if X = 2 then Challenger is indifferent between D and R in the Fight subgame, so expected utility for Incumbent is 3 from Fight and also 3 from Accommodate, so the Incumbent is also indifferent: so if X=2 … Risk-neutral Incumbent would want to choose Fight (EV=3) instead of Accommodate (certain payoff 3) so that Challenger chooses Out for 3 instead of certain payoff 2 from Fight... BUT: Challenger can choose Fight credibly (payoff is 2 in either case if X=2), thus lowering Incumbent's payoff from Fight to 2 and therefore inducing Incumbent to choose Accommodate: the Incumbent obviously prefers Out but cannot credibly threaten while the Challenger can credibly threaten Destruction if Fight… So Accommodate is the result if X=2.

- Note that if X = 3 then Challenger is indifferent between Out and {Entry, Refinement} if Incumbent chooses Fight: in both cases Uc = 3. Incumbent would obviously prefer Out but has no control here because Accommodate is not a credible threat. If we assume a 50% chance for each of Challenger's possible moves, then Incumbent's expected payoff is 0.5(6) + 0.5(4) = 5 on average.

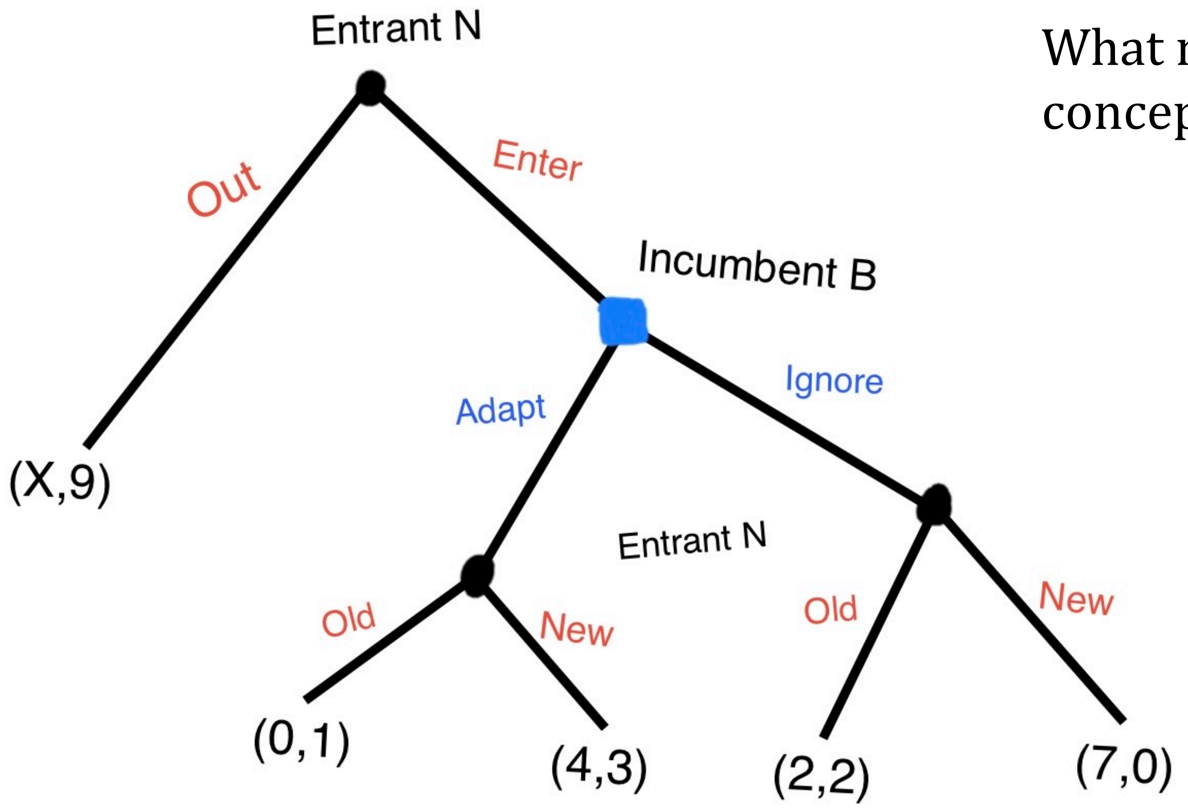
ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME INVOLVING BARGAINING LOGIC

There is a new media streaming company "N" which is considering entry to a new market where incumbent company "B" is the only major firm in the business.

- If N chooses OUT, then payoffs are (**X**,9) where **X** is some unknown payoff value for company N.
- If N chooses ENTER, then B must choose IGNORE or ADAPT, and N will then choose OLD or NEW in response to B's choice.
- With IGNORE chosen by B, the payoffs from OLD are (2,2) and payoffs from NEW are (7,0).
- With ADAPT chosen by B, the payoffs from OLD are (0,1) and the payoffs from NEW are (4,3).

Also note that this is a "single shot" game, meaning it is not repeated: equivalently, you can think of these payoff values as a total discounted stream of all future profits if that is more intuitive than "utility".

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME TREE

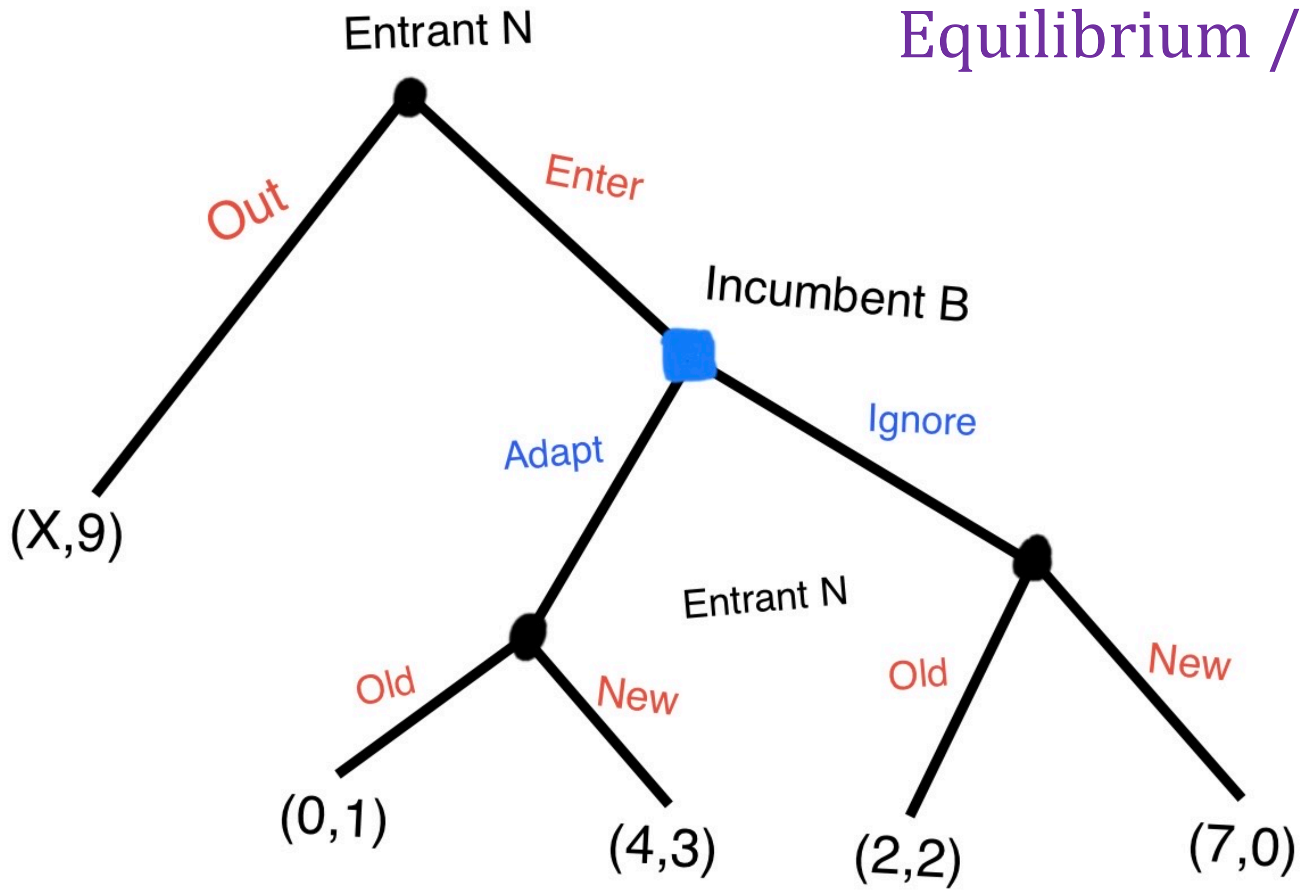


What major economic concept does X represent?

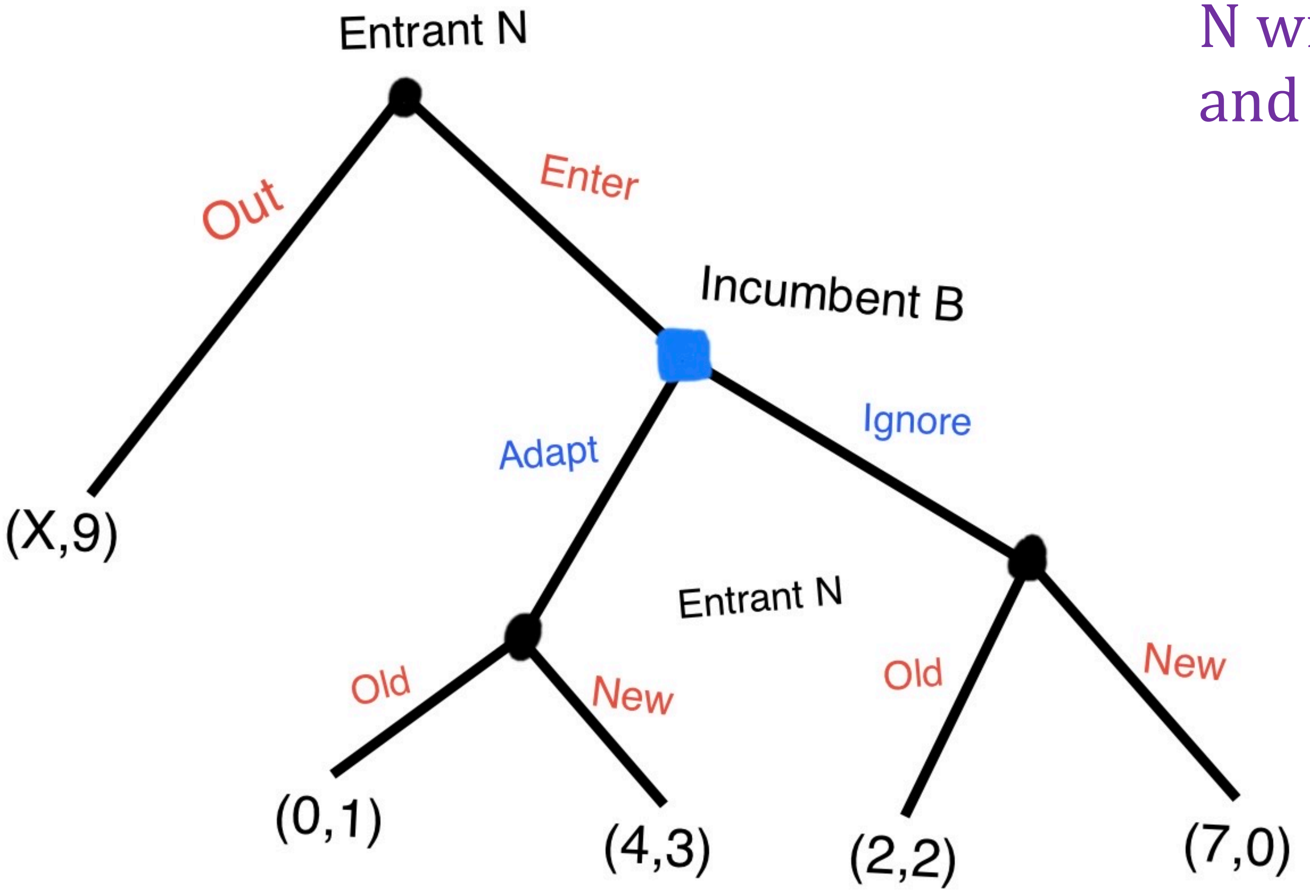
X is the ***opportunity cost of entry*** for N, which is the value of the alternative to entry: this represents what X could get by choosing Out, so naturally it would be determined by the quality of alternative options like N's ability to compete in other markets.

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SOLVING FOR SPNE

What is the Subgame Perfect Nash Equilibrium / outcome of this game?



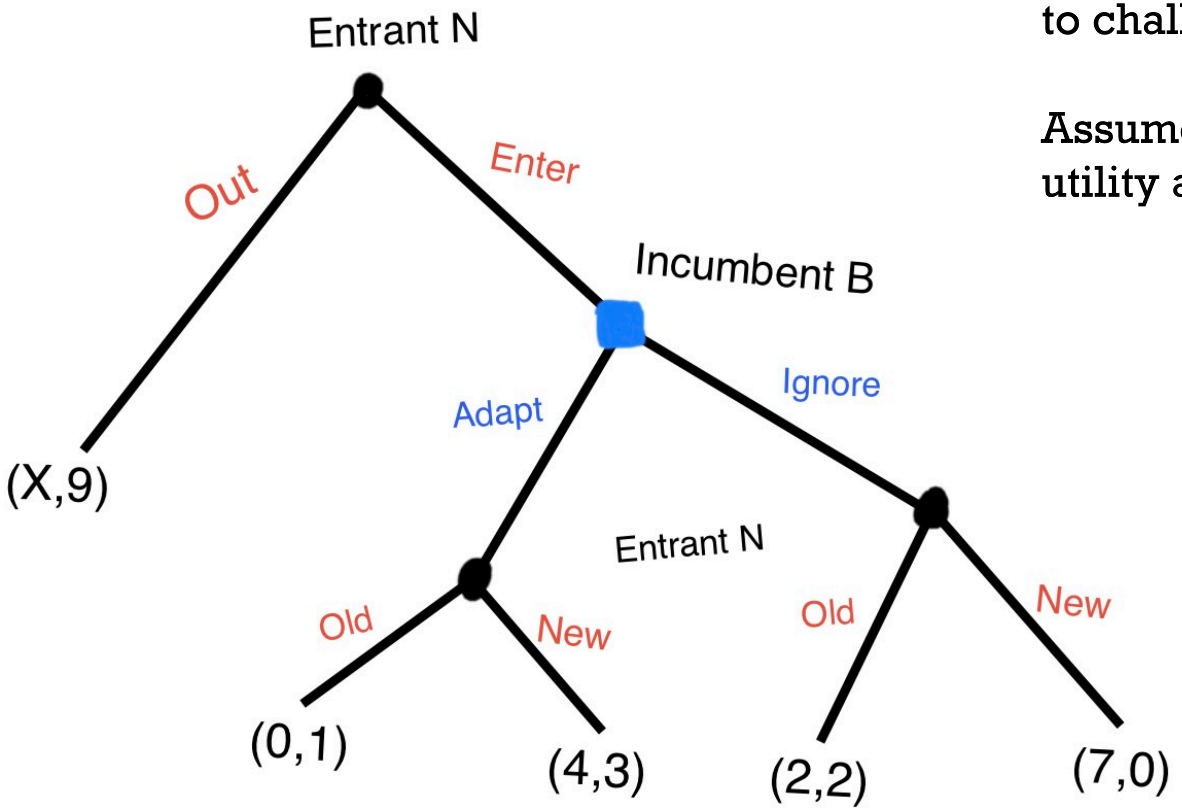
ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SOLVING FOR SPNE



X < 4:
N will Enter, B will Adapt, and N will choose New.

X > 4:
N will choose Out.

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME

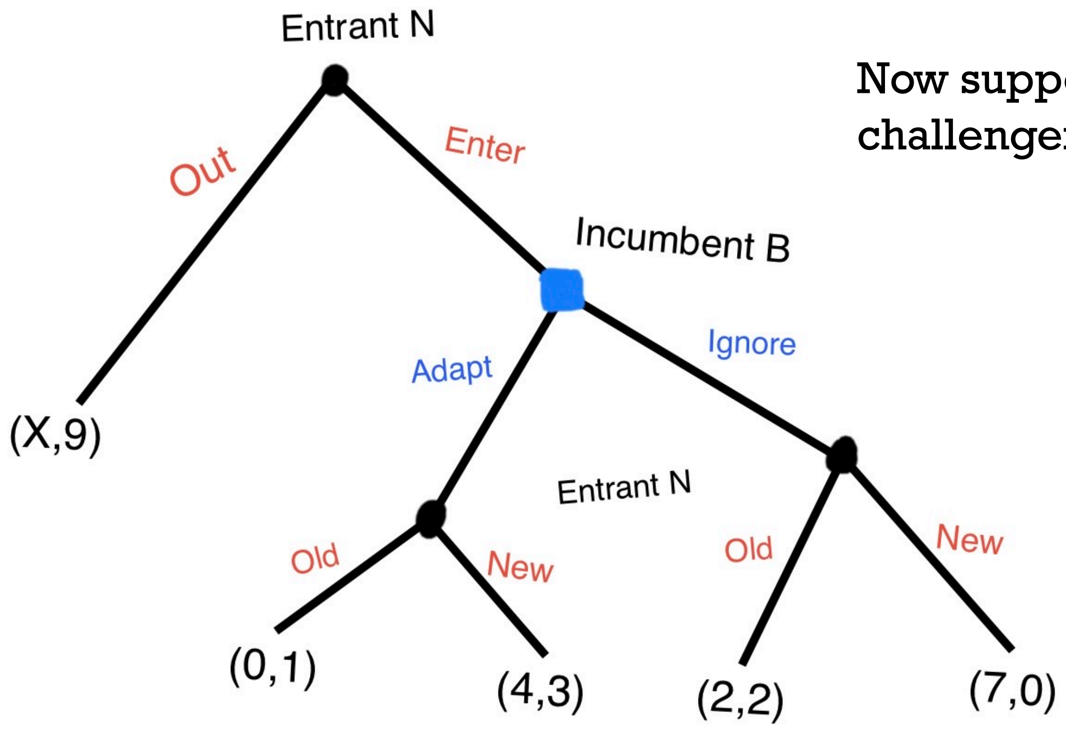


Now suppose incumbent company B can pay ***$b*** to challenger company N to stay out of the market.

Assume U(m) = m , which means that money and utility are interchangeable at a 1:1 rate.

What is the most that B would be willing to pay N to stay out of the market?

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME

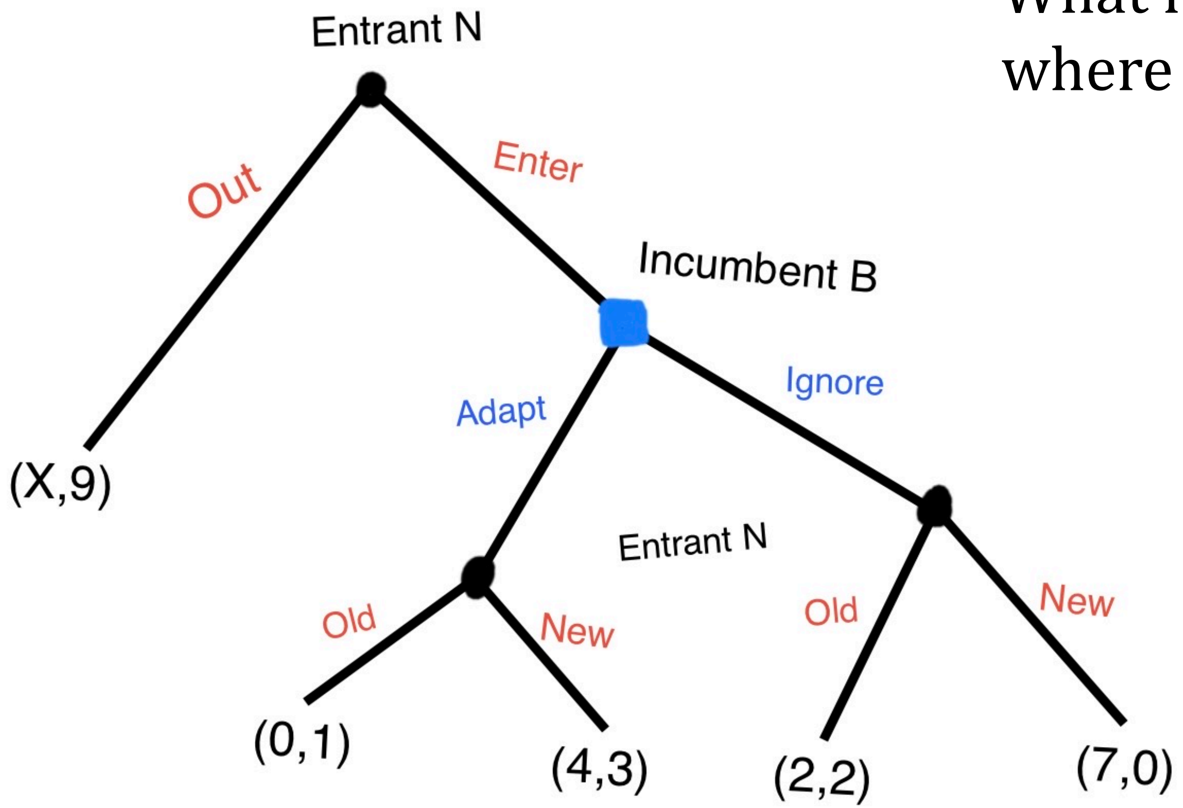


Now suppose incumbent company B can pay ***$b*** to challenger company N to stay out of the market…

What is the most that B would be willing to pay N to stay out of the market?

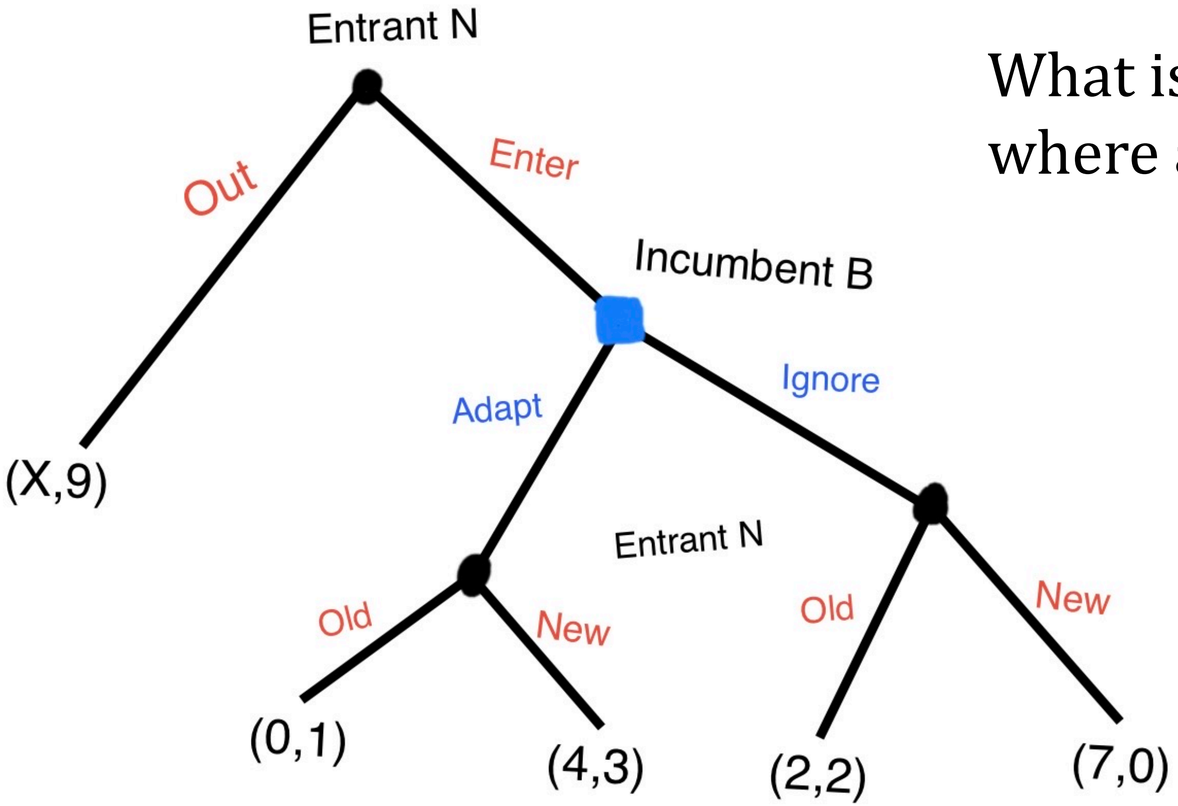
The maximum value is **b_{max} = $6** , which is what B would gain since B's payoff from Out is 9 and B's payoff from the Entry NE outcome is 3. Any bribe above $6 would result in a lower net payoff for B.

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME



What is the lowest possible value for **X** where a payment can be made to stay out?

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME

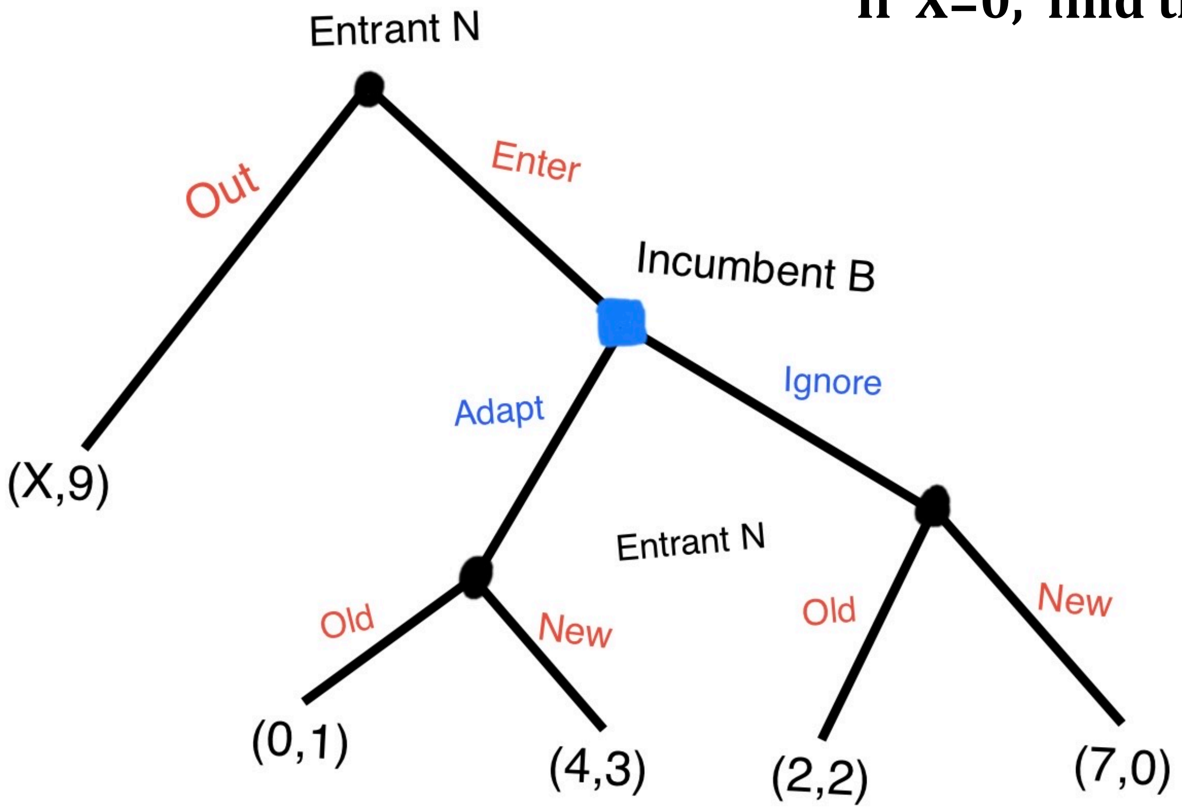


What is the lowest possible value for **X** where a payment can be made to stay out?

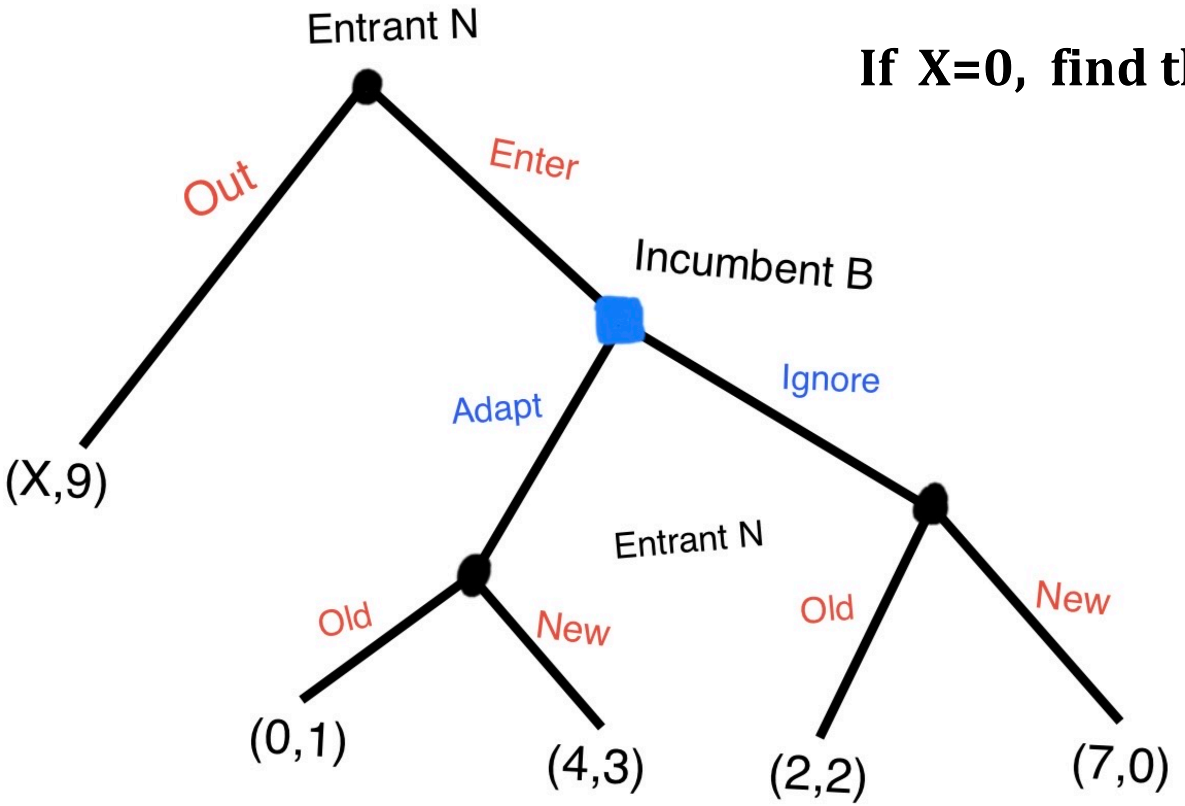
Since B can only gain from offering at most \$6, N would need to have an X value large enough to make $X+6 \geq 4$ in order to accept the bribe, so this requires $\mathbf{X \geq -2}$. For any X value smaller than -2, B would need to bribe N more than 6 in order for N to be better off staying out, and offering more than 6 would result in B actually being worse off than in the NE outcome from entry.

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME

If X=0, find the exact range of possible values for *b*



ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME

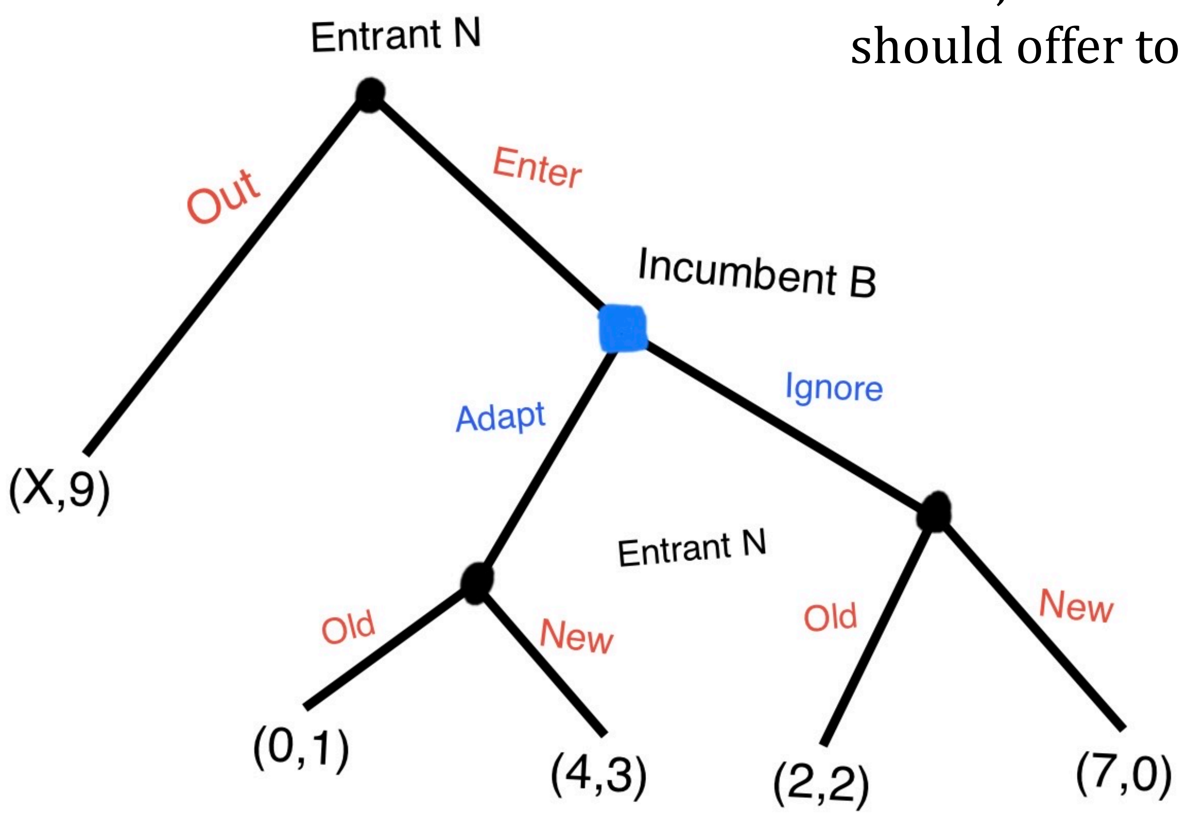


If X=0, find the exact range of possible values for *b*

With X=0 there must be a bribe of at least $4 in order for N to have a higher payoff from choosing Out.
$6 is the maximum bribe that B could offer which would increase B's net payoff.

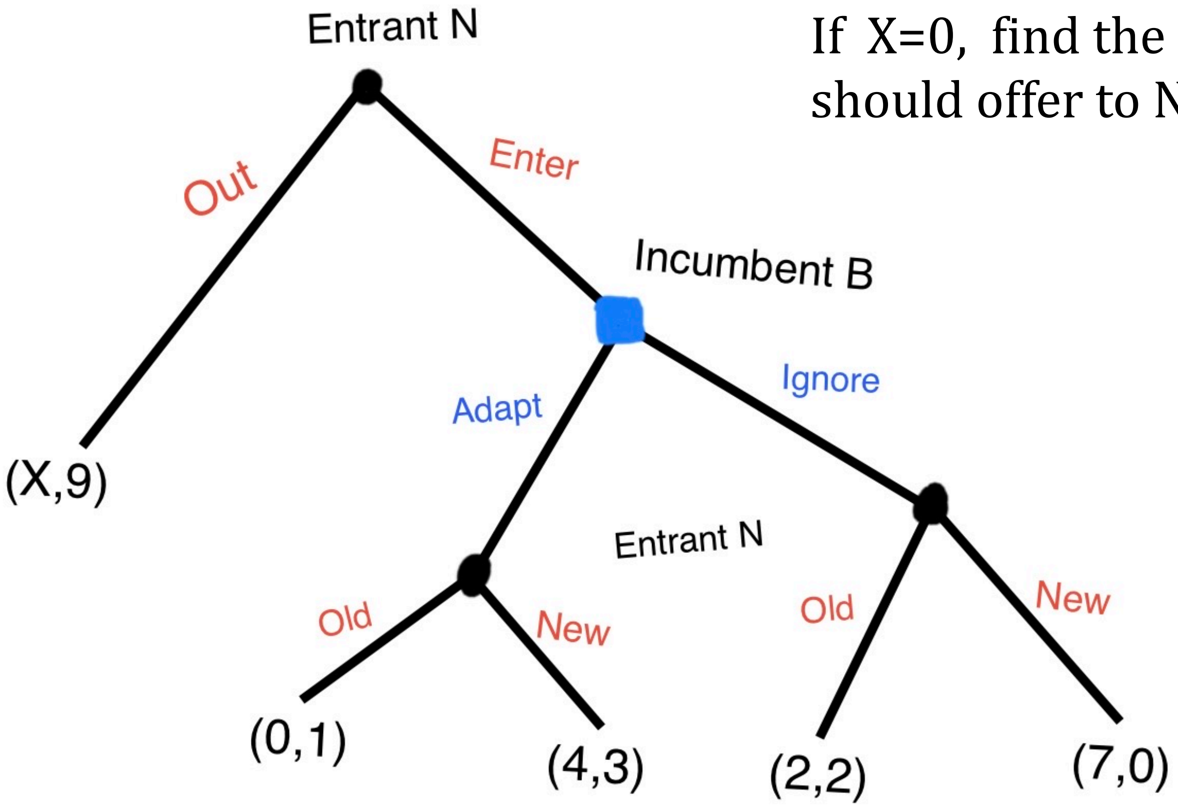
Therefore the range of possible values is everything between 4 and 6, or formally we can write **b** ∈ **(4,6)**.

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME



If X=0, find the optimal SPNE value $b*$ that Incumbent B should offer to N. *(Hint: think about Proposer/Responder)*

ENTRY DETERRENCE & SURPLUS SHARING OFFERS: SEQUENTIAL GAME



If X=0, find the optimal SPNE value b^* that Incumbent B should offer to N. *(Hint: think about Proposer/Responder)*

Using simple "bargaining logic" (the core reasoning of the proposer/responder game) we know that **$4.01** is the optimal offer which maximizes B's net payoff (4.99) and induces N to stay out to achieve a net gain of 0.01 compared to the payoff of 4 from entry.