Firms and Production

Typical Cobb-Douglass production function: $\mathbf{f\ (K,L) = A * K^{\alpha} * L^{\beta}}$

Output ($\mathbf{f}$) is usually concave over inputs capital (**K**) and labor (**L**) in normal circumstances, reflecting assumptions of ***monotonicity*** and ***diminishing marginal returns*** from both: more of either is always better for production but the additional benefits are declining as you use more inputs. The technological efficiency parameter **A** represents how advanced the technology and skill levels are, while the exponent parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, which are sometimes called output elasticity factors, represent the relative contribution values of capital and labor for production output. Alpha and Beta are normally values between zero and one since production is generally concave. If Beta was greater than one, for example, that would indicate increasing marginal returns from labor. Intuitively, if you have a factory where workers use machines to make something, adding more workers will increase total production, but at a declining rate since you are increasingly limited by available space for each person to work. Adding more machines will increase production, but also at a declining rate because workers will get more tired and eventually there will not be enough workers to use all of the machines. There are of course examples of linear or increasing returns to production inputs, but these are not often used in basic microeconomics.

The simplest example of a Cobb-Douglass concave production function representing the quantity of output (**Z**) for a pizza factory could be $\mathbf{Z = 8}\ \sqrt{(\boldsymbol{K * L})}$. This is a special case where alpha and beta are each equal to 0.5, reflecting equal elasticity production factors with symmetric contributions to output as well as diminishing marginal returns. In this case, if input prices were equal, the optimal production configuration would be a 1:1 ratio of inputs.

Profit functions:

Firms maximize profits exactly the way consumers maximize utility with a cost included, and we can analyze these equations using identical approaches.

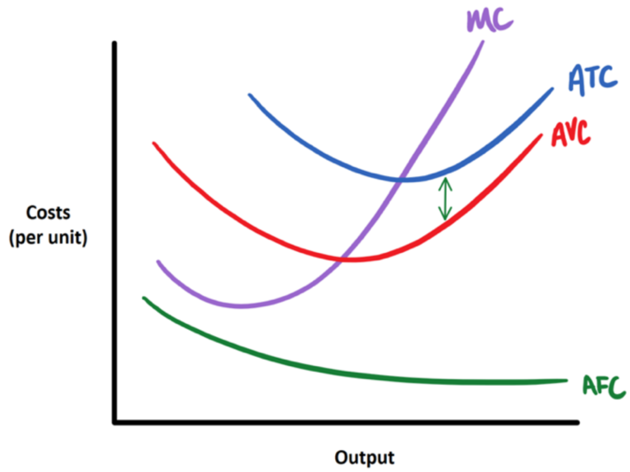
Profit equals total revenues minus total costs, which is the same as price times quantity minus all costs.

$$\Pi = TR - TC$$

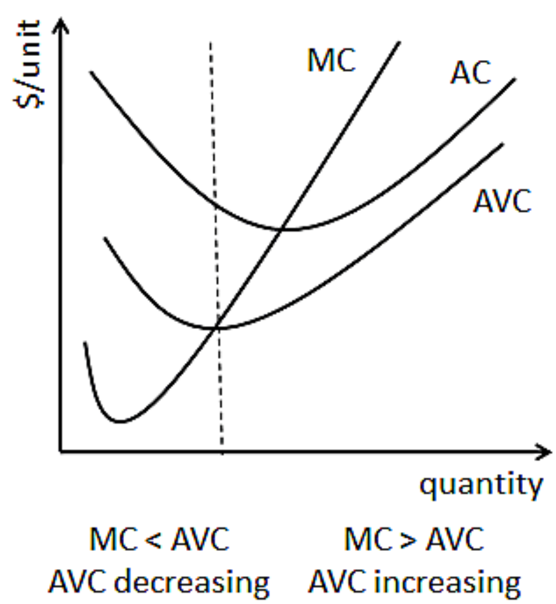
$$= p*q - c(q) = \text{[market price * quantity]} - \{\text{sum of all costs for that output quantity}\}$$

Total costs are the sum of all variable costs and all fixed costs: both of these are normally included in a cost function **c(q)**. Variable costs change as the production quantity (**q**) changes. Fixed costs do not change based on the output level. For a pizza factory some variable costs would include ingredients, cardboard boxes, labor, electricity, water… so variable costs usually increase as production quantity increases. The average variable cost per unit, however, generally decreases as output goes up to a certain extent but then increases eventually at high levels of output. Fixed costs would include property, machines, the recipe, the name/trademark of the brand (Intellectual Property or "IP"), an operating license from the government, and safety inspections / certifications.

Cost Curves for a Typical Firm



Notice that the ***average total cost*** curve (total cost divided by total number of units produced) and the ***average variable cost*** curve (total variable cost divided by number of units produced) each intersect the ***marginal cost curve*** (individual cost for each specific unit, which is the change in total overall costs) at their respective minimum values. ***Average fixed costs*** decline with a convex shape (never actually reaching zero) as output increases since this represents "up-front" non-varying costs being distributed over more and more units.



Profit Maximization: Startup Brewery Example

A small brewery requires **$600** in startup costs for equipment and then has a (slightly unrealistic) variable cost function **$c(b) = 10b^2 - 100b$** over producing kegs of beer (**b**) which sell for **$100** each on a huge competitive market with many sellers and is therefore not sensitive to small supply changes from one brewer's production choices. This assumption establishes that equilibrium price is determined by supply and demand in a large free market, so each brewery must optimize over production output instead of optimizing over price. With fewer firms in any specific market, the ability of each individual firm to influence the market conditions increases. A monopolist would have total control over both market quantity and the price.

To find the brewery's production choices, we must first construct a *total cost function* and then a *profit function*. Firm profit maximization is almost identical to the process for individual utility maximization. The level of output which maximizes profits is the one where there is the largest difference between total revenues and total costs. The two approaches to profit maximization via optimizing production levels are:

(1) to directly find the level of output which corresponds with the largest value for profit;

(2) to find the largest level of output where marginal cost (MC) is less than or equal to marginal revenue (MR) with a positive corresponding value for profit.

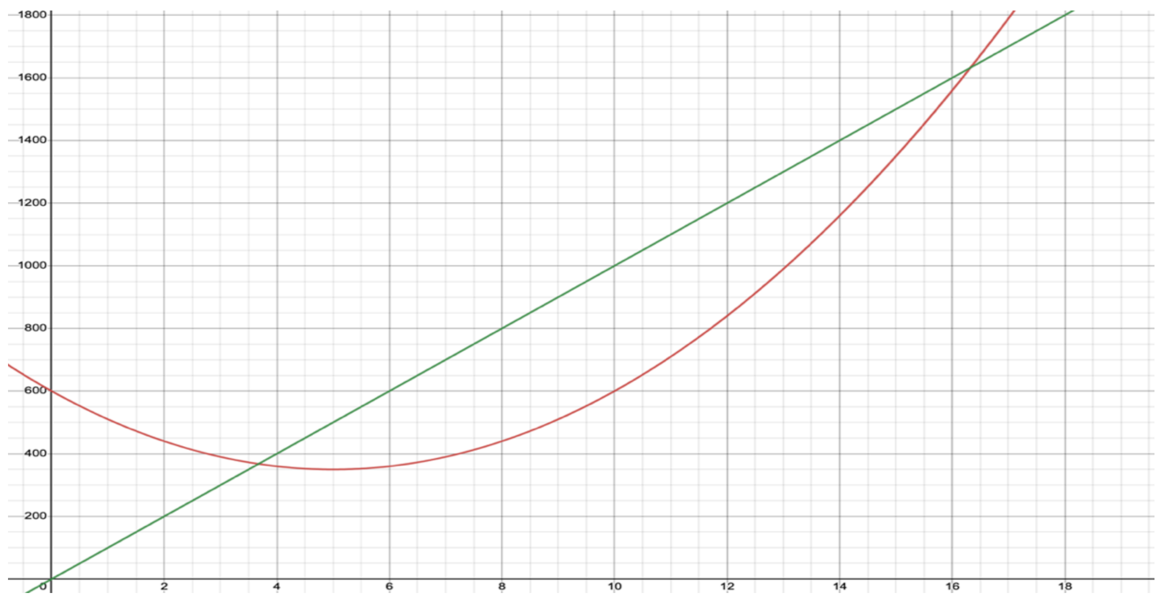
Both approaches will always obtain the same optimal level of production, which must be either a positive value or zero if there is no way to make positive profit.

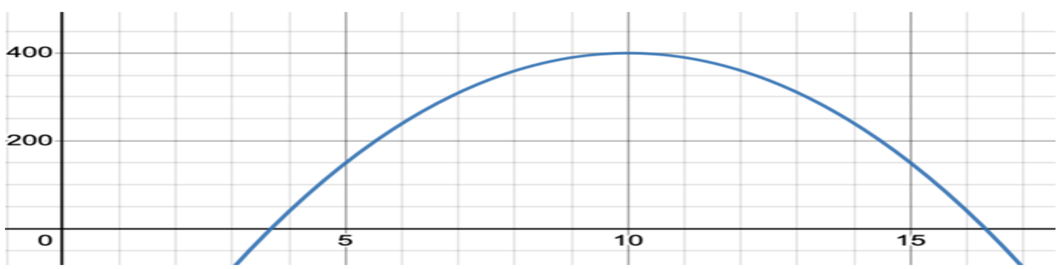
kegs produced	Fixed Cost	$VC = -100b + 10b^2$	$TC = 600 - 100b + 10b^2$	MC	MR	Revenue	Profit	Avg Fixed Cost
0	*600*	0	600			0	**-600**	*0*
				-90	100			
1	*600*	-90	510			100	**-410**	*600.00*
				-70	100			
2	*600*	-160	440			200	**-240**	*300.00*
				-50	100			
3	*600*	-210	390			300	**-90**	*200.00*
				-30	100			
4	*600*	-240	360			400	**40**	*150.00*
				-10	100			
5	*600*	-250	350			500	**150**	*120.00*
				10	100			
6	*600*	-240	360			600	**240**	*100.00*
				30	100			
7	*600*	-210	390			700	**310**	*85.71*
				50	100			
8	*600*	-160	440			800	**360**	*75.00*
				70	100			
9	*600*	-90	510			900	**390**	*66.67*
				90	100			
10	*600*	0	600			1000	**400**	*60.00*
				110	100			
11	*600*	110	710			1100	**390**	*54.55*
				130	100			
12	*600*	240	840			1200	**360**	*50.00*
				150	100			
13	*600*	390	990			1300	**310**	*46.15*

Total cost is the sum of all *variable costs* and *fixed costs*, and ***marginal cost*** is the change in total cost per unit of production output. In this case variable costs (and therefore also total costs) initially decrease as output increases (which is unusual but keeps the numbers simple in this example)… in reality this might reflect the fact that brewing very small batches of beer can be much more tedious and expensive than brewing medium sized batches. Profits are negative for output levels lower than 4 kegs, so this business would not attempt to open or operate unless it was confident that there was sufficient demand to sell at least 4 kegs at the market price of $100.

Revenues: $R(b)= 100b$

Total Cost Function: $C(b)= 600 – 100b +10b^2$





Profit Function: $U(b) = 200b – 600 – 10b^2$

This particular example has an optimal level of production at $b^* = 10$ kegs with resulting profit of $400. We can see this by graphing the functions for total cost and total revenue with dollars on the vertical axis and the output level (quantity produced) on the horizontal axis. Our optimal level of production is the point where the difference between these two curves is the largest. This point (output quantity b=10) is also exactly where the slopes of the two curves are the same: the curves are exactly parallel here, indicating that this is the one and only output quantity level where the marginal revenue equals marginal cost. Beyond this optimal point, we can see from the table that marginal cost would be larger than marginal revenue, so producing an 11th keg would reduce profit since the cost of doing so would be $110 but the gain from selling it would only be $100. Marginal revenue is always $100 because that is the additional revenue from one more unit sold when they are all sold at the same price on a competitive market. Marginal revenue is the slope of the revenue curve and marginal cost is the slope of the cost curve.

Sunk Costs & Economic Decision Logic

If a firm invests money in research which does not produce a viable product and it cannot take any action to recover this "wasted" investment, then that loss is called a ***sunk cost***. The phrase references the idea of something that has fallen into the middle of the ocean and cannot be recovered, and the reasoning behind this concept is that all decision-makers should ignore sunk costs when making logical economic decisions. As long as any present or future action has a positive expected marginal utility (marginal benefit greater than marginal cost) then that action should be taken. It is not logical to incorporate any costs or benefits from the past into a present or future decision unless the past is directly relevant to estimating the value of present or future variables. If a tornado destroys a profitable business but the value of future revenues is projected to be larger than its future costs, then the business should rebuild and continue to operate.

Suppose a pharma company lost $55 million on failed research last year and cannot recover that money, but it now believes that it will invent a new drug if it invests another $80 million into new research. They estimate that demand for the new drug will be 2.3 million units per year, market price will be $10 per unit, variable cost per unit will be $6, and the patent to legally control this market (maintaining all of these economic conditions) lasts for 12 years. After that, profits will be extremely small or zero as other competitors are allowed to enter and other formulas may be used instead which could be superior and make the product obsolete. For the next 12 years, therefore, the firm's projected revenues are 2.3m * $10 = $23 million per year and projected variable costs are 2.3m * $6 = $13.8 million per year. Profit equals total revenues minus total costs, so over the whole 12 year period this firm projects the following financials:

Total Revenues = 2.3m * $10 * 12 = $276m

Variable Costs = 2.3m * $6 * 12 = $165.6m
Fixed Costs = $80m

Profit = TR – TC = $276 – 165.6 – 80 = $30.4 million

The firm should invest in this new research because it will result in future profit despite the fact that $55 million in sunk costs was already lost on the failed research from earlier. The firm's overall profit, when including the past failed research, is actually ($30.4m – $55m) = **-$24.6m**, but it would instead be a worse overall profit of **-$55m** if they choose to not invest the $80m in new research because of prior losses. Incorrectly including the $55m sunk cost from the past mistake in the mathematical calculations of a current/future decision would result in making the wrong choice here. Obviously the firm is still losing money overall, so if it had known this prior to making the first investment then its logical choice would be to stay out of the market and do nothing. Information is valuable and it is not always correct, but a rational decision-maker must choose whatever option is best based on current information and the variables which are currently within the decision-maker's control. It is not possible to go back in time and change the decision to invest in the failed research, so the firm must act rationally by ignoring this sunk cost.