

Gains from Comparative Advantage, Specialization, & Trade

Consider two countries, **A** and **B**, which are both operating under an *autarky* system - each being totally economically independent and self-sustaining with no trade. Suppose there are two goods in the world, called **x** and **y**, and consumers in both countries have identical utility functions over both of these goods. Everyone's preferences in this world can be described by the symmetric, monotonic, convex utility function $U_E(\mathbf{x}, \mathbf{y}) = \mathbf{x} * \mathbf{y}$. This detail establishes that people in both countries have demand for both goods (which are in fact complements here) and prefer a mixture of them, which is one of several necessary conditions for trade to have potential gains for both sides, which is required for two self-interested countries to agree to engage in trade.

Suppose there are 20 people in country A who can make 6 of good **x** or 4 of good **y** in one day. There are 6 people in country B who can make 2 of good **x** or 3 of good **y**. Country A has an *absolute advantage* over country B because workers in country A are superior at producing all goods. "A-workers" are more efficient than "B-workers" at producing both types of products. Each country has a *comparative advantage* - a higher relative efficiency (lower opportunity cost) at production of one of the goods. Country A has a comparative advantage in producing **x** and country B has a comparative advantage in producing **y**. The opportunity cost for country A of one A-worker making 6 units of **x** is the 4 units of **y** that she could otherwise produce.

With no trade and the established production possibilities frontiers from these workers' abilities and symmetric convex preferences, each country is best off allocating half of its resources towards making each good. Country A obtains the highest possible utility level with 5 workers producing 30 units of **x** and the other 5 workers producing 20 units of **y**, obtaining $U_A^* = 30 * 20 = \mathbf{600 \text{ utils}}$ as its maximum under autarky. Country B obtains its highest possible utility when it has 3 workers produce 6 units of **x** and the other 3 workers produce 9 units of **y**, obtaining $U_B^* = 6 * 9 = \mathbf{54 \text{ utils}}$ as country B's maximum under autarky. It seems paradoxical for neither of these countries to specialize in the good where its workers have naturally superior abilities, but we can observe this and prove that allocating half of workers towards each good is in fact the best feasible option for both countries without trade. We can mathematically verify these optimal sets of allocations by making tables the same way that we do for consumer utility maximization with two goods:

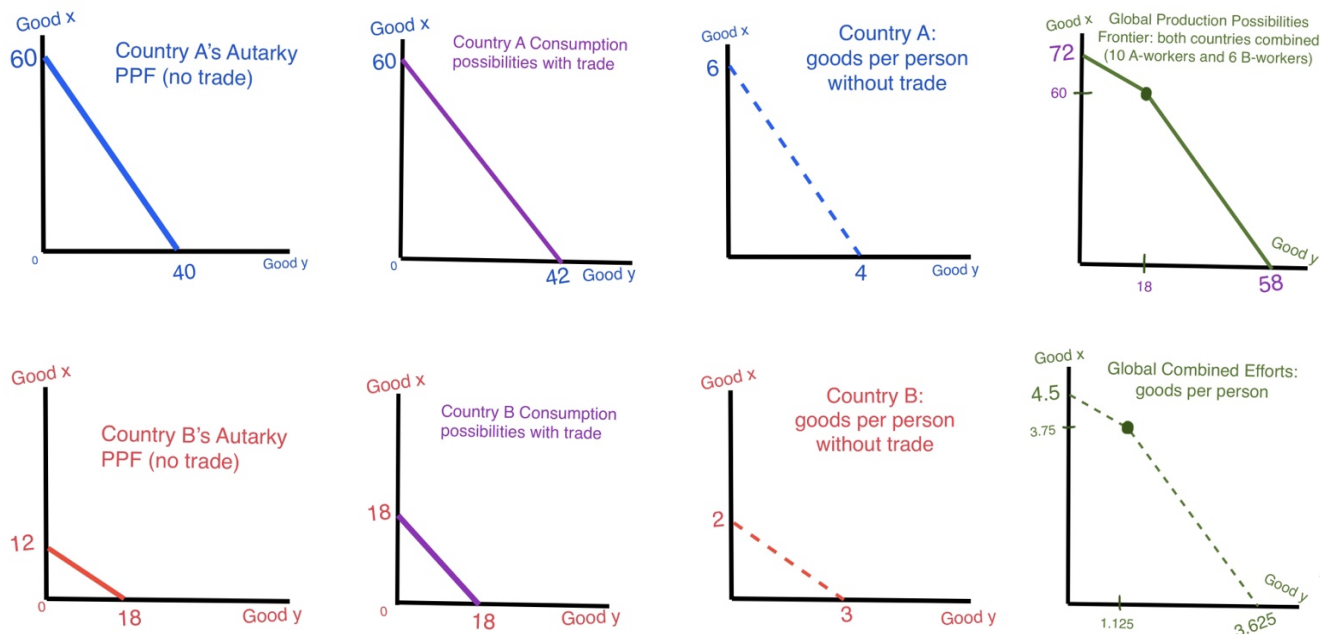
Country A				
workers making x	workers making y	total x produced	total y produced	utility
0	10	0	40	0
1	9	6	36	216
2	8	12	32	384
3	7	18	28	504
4	6	24	24	576
5	5	30	20	600
6	4	36	16	576
7	3	42	12	504
8	2	48	8	384
9	1	54	4	216
10	0	60	0	0

Country B				
workers making x	workers making y	total x produced	total y produced	utility
0	6	0	18	0
1	5	2	15	30
2	4	4	12	48
3	3	6	9	54
4	2	8	6	48
5	1	10	3	30
6	0	12	0	0

Since consumers in both countries enjoy both goods with monotonic and convex preferences, and since each country has a comparative advantage in a different good, there are potential gains from trade for both nations.

Country A is willing to trade 1 unit of good x for at least 0.67 units of good y because that is country A's opportunity cost established by the 6:4 ratio of A-worker production capabilities. If one of these A-workers switched from making x to making y then A would lose 6 of x and gain 4 of y with no trading. If country A was able to trade these 6 units of x for 5 units of y , then one A-worker could switch from making y to making x and the outcome after trade would be A having the same amount of x and consuming 1 more unit of y .

Identical reasoning shows that Country B is willing to trade 1 unit of good y for at least 0.67 units of good x because that opportunity cost is the economic trade-off inside country B established by the 2:3 ratio of production capabilities for B-workers. Ignoring potential complications from unequal leverage, negotiations, strategic externalities, shipping costs, etc, we can assume that the countries will agree to engage in trade with a mutually beneficial **1:1 exchange rate**, which is the midpoint of the interval (range) of exchange rates where both countries would be able to gain. If either country is unable to benefit from trade, then trade will not occur.



If country B specializes - producing more of the good where it has a comparative advantage in production - it can make a total of 18 units of good y when all 6 workers focus only on producing good y . With the symmetric convex consumer utility function in this example, country B would be willing to trade up to half of the 18 units of y it produces in a 1:1 exchange for good x . Trading 9 of the 18 units of y for 9 units of x would give country B a total utility of $U_B^* = 9 \cdot 9 = 81$ utils, which is more than the maximum 54 utils optimally obtained under autarky.

If country A does not re-allocate its workers, then it makes 30 of good x and 20 of good y , and after trading 9 units of good x for 9 units of good y , country A will have utility $U_A(21,29) = 21 \cdot 29 = 609$ utils. This is an improvement from the 600 utils it could optimally obtain with autarky, but with trade, country A can do even better by re-allocating its workers. Country A can shift two of its workers towards making good x so that 6 A-workers produce 36 units of good x and 4 A-workers produce 16 units of good y . Utility from consuming this would be $U_A(36,16) = 36 \cdot 16 = 576$ utils, which is worse than before... but with trade they can exchange 9 units of good x for 9 units of good y from country B. This results in 27 units of good x and 25 units of good y , obtaining a total utility of $U_A^* = 27 \cdot 25 = 675$ utils for country A.

Both countries are better off after trade. Each country specializes in producing the good where it has a comparative advantage, and the result of this is more overall global production of both goods. People in both countries consume at least as much of each product after trading is completed, and everyone is able to have more things overall and obtain a higher level of utility. Both countries actually choose to produce in a way that is suboptimal under autarky (because it is skewed towards the good where there are relative production advantages instead of being based on its consumer preferences) if both countries are confident that they will be able to successfully trade and end up with superior allocations for all of their consumers afterwards. This simple concept is the single biggest reason why humanity has been able to produce and consume so much more over time and exponentially increase the "standard of living" for most people in the world.

Note that the 42 maximum possible units of good y for country A results from 1 worker making 6 units of x to trade for 6 units of y , with the other 9 workers producing 36 total units of y .