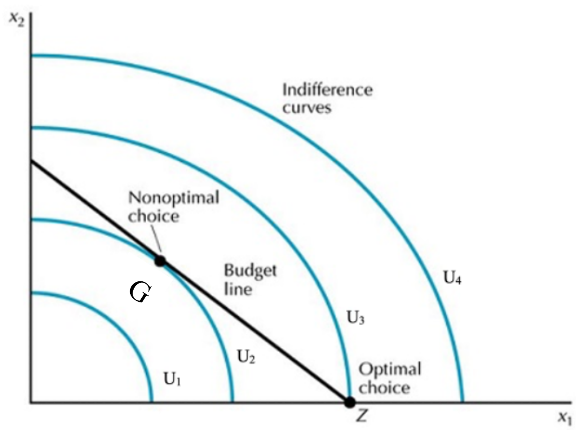
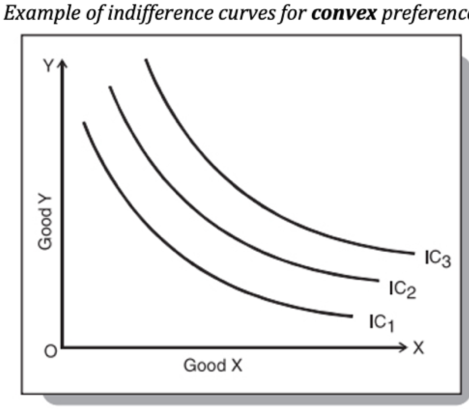
Consumer Preferences: Complements and Substitutes

While every point on indifference curve U_1 and point G on indifference curve U_2 are all feasible (affordable, attainable, achievable… since they are inside of the budget line representing the limits of feasibility for different combinations of x_1 and x_2) the highest feasible utility level is obtained by choosing point Z on indifference curve U_3. Any rational person with these ***concave preferences*** will choose Z as the optimal *consumption bundle*, which we can formally write as $(x^*_1, x^*_2) = (z, 0)$ to show that this means z units of x_1 and zero units of x_2. One example of a function describing the concave indifference curves below could be $\mathbf{U(x_1, x_2) = 3x_1^2 + 4x_2^2 - x_1x_2}$

Example of concave indifference curves representing concave preferences
where "extremes" are preferred over a "mixture" of x_1 and x_2
(with monotonic increasing utility: $U_4 > U_3 > U_2 > U_1$)



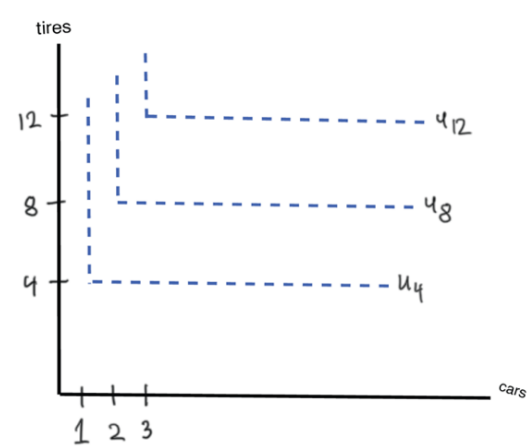
*Example of indifference curves for **convex** preferences:*



Convex preferences are the opposite of concave preferences and the most common way to realistically represent value. With convex preferences, some mixture of the two utility sources is always preferable over any allocation with only one utility source and none of the other. In this case, the two things will be multiplied together somewhere in the utility equation, which indicates that more of either one also has utility benefits through the other. One example of convex indifference curves in a consumer choice problem might be coffee and bagels for Jay's utility function $\mathbf{U_J(c,b) = c * b}$. Here the optimal choice is allocating an equal share of available resources towards each of the two goods since the exponents are equal. Note the symmetry here: Jay's exponent is 1 on **c** and on **b**. His friend Kim with utility $\mathbf{U_K(c,b) = c^3 * b^2}$ would prefer more coffee. In each case, utility is obviously equal to zero whenever there is zero of either good.

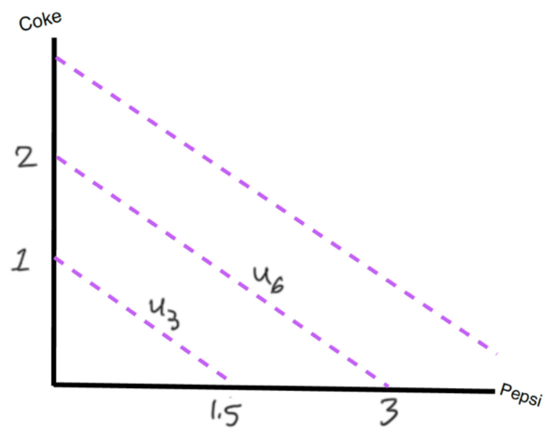
A specific case of convex preferences called ***perfect complements*** has "L-shaped" indifference curves. In these situations, goods must be consumed together in a specific ratio to increase utility. Some examples are pancakes & maple syrup, pencil & paper, or **U(L,R)= min { L , R }** for left shoe & right shoe. These can be considered "pessimistic preferences" since the utility is completely determined by the lesser input. In other words, the well-being of someone with these preferences is based entirely on whichever utility source is the most limiting.

Consider the perfect complements case of cars and tires with utility function **U(c,t)= min { 4c , t }**. This function returns the value of whichever argument is smaller: if you have 1 car and 4 tires then your utility is 4; if you have 1 car and 9 tires your utility is still 4; if you have 0 of either cars or tires then your utility is 0 regardless of the other quantity; if you have 2 cars and 7 tires your utility is 7; if you have 2 cars and 8 tires your utility is 8; if you have 3 cars and at least 12 tires your utility is 12… this should make sense using integer values.



The ***monotonicity*** of utility in most of these consumer choice examples represents the fact that both x_1 and x_2 increase utility: more of either one is always better. Utility is monotonically increasing over both x_1 and x_2. If more of either one of these was always worse instead, then we would say utility is "monotonic decreasing". With concave indifference curves, for some level of feasibility, a mixture of the two is not as good as prioritizing one. One example of these concave utility preferences could be a factory which is better off specializing in the production of one output instead of trying to produce a mixture of two different products and as a result being less efficient overall with lower profits. Monotonic curves indicate in this example that both products would always be profitable, but given the feasibility constraints of the factory, the highest profit would be obtained by choosing just one thing to do extremely well instead of trying to do both given the same set of conditions and inputs defining the budget constraint.

The case of ***perfect substitutes*** occurs when the sources of utility are added together and not directly mathematically linked in any term. An example would be someone who prefers Coke (**c**) over Pepsi (**p**) but monotonically benefits from both, with utility function **U(c,p) = 3c + 2p** .



In this case, and in the case of all perfect substitutes, the indifference curves are straight lines. The consumer will choose to drink only Coke if the price ratio $\mathbf{P_C / P_P}$ is less than the corresponding ratio of the utilities ("***Marginal Rate of Substitution***") which in this case is 3/2. If both drinks cost the same, any person with this utility function will choose to allocate all money towards Coke to maximize utility over these preferences. If Coke is more than 50% more expensive than Pepsi, however, the opposite will be true and the consumer will choose to allocate all money towards Pepsi. Suppose, for example, that this person has budget **m = $12** : if both cost $2 then utility is highest (18 utils) with 6 Cokes; if Coke costs $2 and Pepsi only costs $1, then utility is now highest (24 utils) with 12 Pepsis; and if Coke is $3 and Pepsi is $2 then this consumer is actually *indifferent* across all choices which spend all of the money since any combination of spending $12 on Coke and Pepsi results in exactly 12 utils. In this case, the price ratio has a slope (*Marginal Rate of Transformation*) which is exactly equal to the *Marginal Rate of Substitution* and therefore the highest attainable indifference curve will be a line that is identical to the budget constraint, intersecting at every point instead of only intersecting at one optimal point.