

Neil Tyson Math Problem

PROBLEM: Show that the maximum speed a car can have without sliding when taking a curve with a bank angle of 24 degrees is 163 miles per hour.

SOLUTION: Derived while in Apple Bees. The best place to do your homework

$\cos \theta = \frac{F_1}{F_c}$ $F_1 = F_c \cos(\theta)$ $F_2 = \mu_s F_N$ $F_2 = m_s \sin \theta$
 $F_N = W \cos \theta + F_c \sin \theta$
 $F_2 = \mu_s W \cos \theta + \mu_s F_c \sin \theta$
 $F_1 = F_2 + F_3 = \mu_s W \cos \theta + \mu_s F_c \sin \theta + m g \sin \theta = F_c \cos \theta$
 $F_c \cos \theta - \mu_s F_c \sin \theta = \mu_s W \cos \theta + m_s \sin \theta$
 $F_c (\cos \theta - \mu_s \sin \theta) = \mu_s m g \cos \theta + m_s \sin \theta$
 $F_c = \frac{\mu_s m g \cos \theta + m g \sin \theta}{\cos \theta - \mu_s \sin \theta} = \frac{\mu_s v^2}{r}$

$$v = \sqrt{\frac{r \cdot g \cdot (\mu_s \cos \theta + \sin \theta)}{\cos \theta - \mu_s \sin \theta}}$$

F_c = Centripetal Force
 μ_s = Static Friction = 0.10 ($\mu_s = \tan 45^\circ$) the car slides for $\theta > 45^\circ$
 F_N = Normal Force
 F_2 = Friction Force = $F_N \cdot \mu_s$
 r = Turn Radius = 685 feet
 W = Weight of Vehicle
 m = mass of Vehicle
 g = gravity = 9.81 m/s^2
 $F_3 = m g \sin \theta$
 θ = Bank Angle = 24°

$W = mg$
 $v = \sqrt{\frac{(685)(.3048)(9.81) [1.4 \cos(24) + \sin(24)]}{\cos(24) - 1.4 \sin(24)}} \cdot \frac{3600}{1609} = 163.43 \text{ miles/hour}$