THEOREMS CH#09

10th class Math Science (English medium)

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Merging man and math

by
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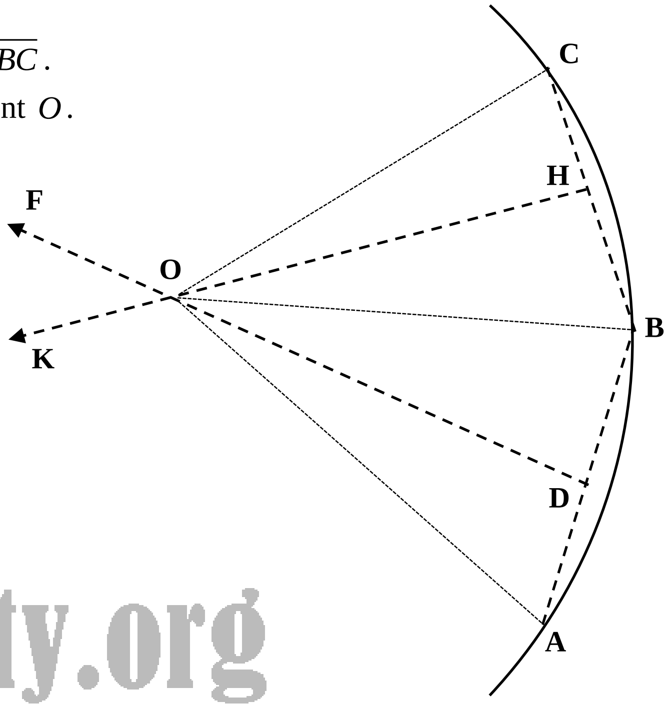
Theorem 01: One and only one circle can pass through three non-collinear points.

Given: A, B and C are three non collinear points in a plane.

To Prove: One and only one circle can pass through three non-collinear points A, B and C.

Construction: Join A with B and B with C.
Draw $\overrightarrow{DF}$ $\perp$ bisector to $\overline{AB}$ and $\overrightarrow{HK}$ $\perp$ bisector to $\overline{BC}$.
So, $\overrightarrow{DF}$ and $\overrightarrow{HK}$ are not parallel and intersect at point O.
Also Join A, B and C with point O.

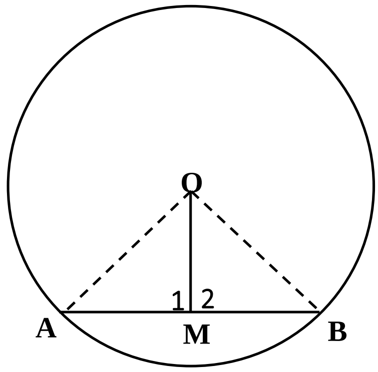


Pr*oof* :

Statements	Reasons
Every point on $\overrightarrow{DF}$ is equidistant from A and B $m\overline{OA} = m\overline{OB}$ …….(i)	$\overrightarrow{DF}$ is $\perp$ bisector to $\overline{AB}$
Every point on $\overrightarrow{HK}$ is equidistant from B and C $m\overline{OB} = m\overline{OC}$ ………(ii)	$\overrightarrow{HK}$ is $\perp$ bisector to $\overline{BC}$
Now O is the only point common to $\overrightarrow{DF}$ and $\overrightarrow{HK}$ which is equidistant from A, B and C $m\overline{OA} = m\overline{OB} = m\overline{OC}$ There is no such point except O Hence, there is only one circle with center O and radius $\overline{OA}$ passes through A, B and C.	Using (i) and (ii)

Theorem 02: A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.

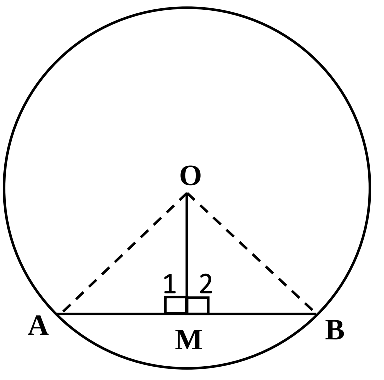
Given: A circle whose centre is O.
M is the mid point of any chord $\overline{AB}$ of the circle Where chord $\overline{AB}$ is not diameter of the circle.
To prove: $\overline{OM} \perp \overline{AB}$
Construction: Join A and B with centre O. Write $\angle 1$ and $\angle 2$.



Proof :

Statements	Reasons
In $\Delta OAM \leftrightarrow \Delta OBM$	
$m\overline{OA} = m\overline{OB}$	Radii of the same circle
$m\overline{AM} = m\overline{BM}$	Given
$m\overline{OM} = m\overline{OM}$	Common
$\Delta OAM \cong \Delta OBM$	$S.S.S \cong S.S.S$
$\Rightarrow \quad m\angle 1 = m\angle 2$ ----(i)	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 2 = 180^o$ …..(ii)	Adjacent supplementary angles
$m\angle 1 = m\angle 2 = 90^o$	From (i) and (ii)
Hence, $\overline{OM} \perp \overline{AB}$	

Theorem 03: Perpendicular from the centre of a circle on a chord bisects it.



Given: A circle whose centre is O and chord is $\overline{AB}$.
And $\overline{OM} \perp \overline{AB}$
To Prove: $m\overline{AM} = m\overline{BM}$
Construction: Join A and B with centre O.

Proof :

Statements	Reasons
In $\Delta OAM \leftrightarrow \Delta OBM$	
$m\angle 1 = m\angle 2 = 90^o$	Given
$m\overline{OA} = m\overline{OB}$	Radii of same circle
$m\overline{OM} = m\overline{OM}$	Common
$\Delta OAM \cong \Delta OBM$	$(S.A.S \cong S.A.S)$ or $H.S \cong H.S$
$m\overline{AM} = m\overline{BM}$	Corresponding sides of congruent triangles
Hence $\overline{OM}$ bisects the chord $\overline{AB}$	

Theorem 04: If two chords of a circle are congruent then they will be equidistant from the centre.

Given: A circle with centre O have two equal chords $\overline{AB}$ and $\overline{CD}$.

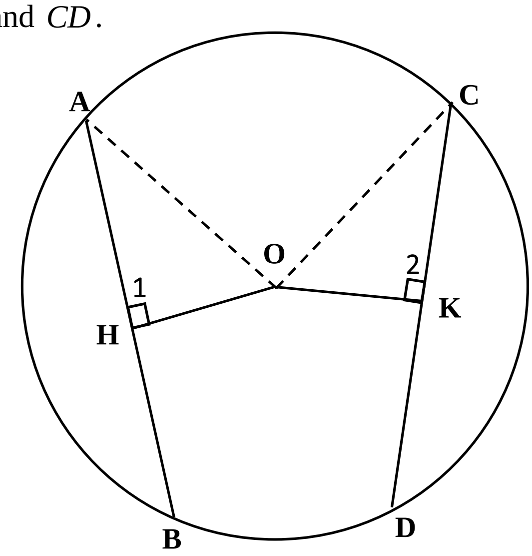
$m\overline{AB} = m\overline{CD}$

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$

To prove: $m\overline{OH} = m\overline{OK}$

Construction: Join O with A and C.

So that we get two right triangles OAH and OCK.



Proof

Statements	Reasons
In $\Delta OAH \leftrightarrow \Delta OCK$	
$m\overline{OA} = m\overline{OC}$	Radii of same Circle
$m\angle 1 = m\angle 2 = 90^o$	Given
$m\overline{AH} = \frac{1}{2} m\overline{AB}$ ……(1)	$\overline{OH} \perp \overline{AB}$ (Perpendicular from the centre of a circle on a chord bisects it)
$m\overline{CK} = \frac{1}{2} m\overline{CD}$ ……(2)	$\overline{OK} \perp \overline{CD}$ (Perpendicular from the centre of a circle on a chord bisects it)
$m\overline{AB} = m\overline{CD}$ ………..(3)	Given
$m\overline{AH} = m\overline{CK}$	From (1), (2) and (3)
$\Delta OAH \cong \Delta OCK$	$(S.A.S \cong S.A.S)$ or $H.S \cong H.S$
Hence, $m\overline{OH} = m\overline{OK}$	Corresponding sides of congruent triangles

Theorem 05: Two chords of a circle which are equidistat from the centre, are congruent.

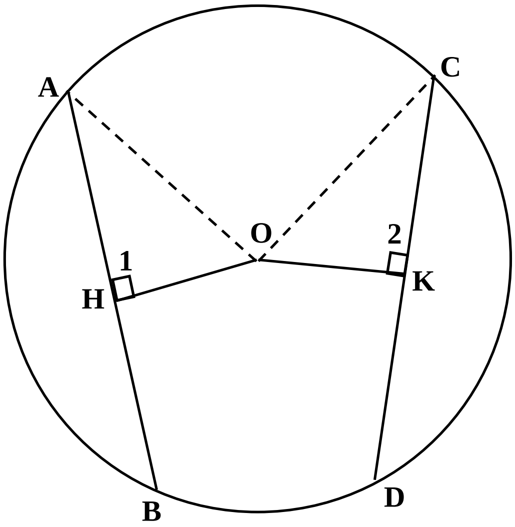
Given: A circle with centre O have two chords $\overline{AB}$ and $\overline{CD}$.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$

To Prove: $m\overline{AB} = m\overline{CD}$

Construction: Join O with A and C.

So that we get two right triangles OAH and OCK.



Pr*oof* :

Statements	Reasons
In $\Delta OAH \cong \Delta OCK$	
$m\overline{OA} = m\overline{OC}$	Radii of same circle
$m\overline{OH} = m\overline{OK}$	Given
$m\angle 1 = m\angle 2 = 90^o$	Given
$\Delta OAH \cong \Delta OCK$	$(S.A.S \cong S.A.S)$ or $H.S \cong H.S$
$m\overline{AH} = m\overline{CK}$ ....(1)	Corresponding sides of congruent triangles
$m\overline{AH} = \frac{1}{2} m\overline{AB}$ ......(2)	$\overline{OH} \perp \overline{AB}$ (Perpendicular from the centre of a circle on a chord bisects it)
$m\overline{CK} = \frac{1}{2} m\overline{CD}$ ......(3)	$\overline{OK} \perp \overline{CD}$ (Perpendicular from the centre of a circle on a chord bisects it)
$m\overline{AH} = m\overline{CK}$	Already proved
$\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	From (2) and (3)
$m\overline{AB} = m\overline{CD}$	