Geometric Parameters, Governing Equations and Essential Stresses

geometric half-width b   geometric half-thickness h   dimensionless coordinate Y=y/b dimensionless coordinate Z=z/h thickness-to-width ratio $\boldsymbol{\varepsilon}$=h/b   boundary layer coordinate $\boldsymbol{\eta}$=(b-y)/h=(1-Y)/$\boldsymbol{\varepsilon}$ relation between coordinates y=b-h$\boldsymbol{\eta}$ partial derivative relation ∂/∂y= -(1/h) $\partial/\partial\boldsymbol{\eta}$ applied uniaxial strain $\boldsymbol{\varepsilon}_x$ superscript **1** denotes outer layers (h/2<z<h and -h<z<-h/2); superscript **2** denotes inner layers (-h/2<z<0 and 0<z<h/2) lamination coefficient $D^{(1)} = D^{(2)} = [-(C12^{(1)} + C12^{(2)}) + C23^{(1)}\, C13^{(1)}/C33^{(1)} + C23^{(2)}\, C13^{(2)}/C33^{(2)})]\ (b\, \boldsymbol{\varepsilon}_x)/\ [C22^{(1)} + C22^{(2)} - C23^{(1)}\, C23^{(1)}/C33^{(1)} - C23^{(2)}\, C23^{(2)}/C33^{(2)}]$ lamination coefficient $E^{(1)} = -C13^{(1)} h\, \boldsymbol{\varepsilon}_x / C33^{(1)} - [C23^{(1)}\, \boldsymbol{\varepsilon} / C33^{(1)}]\, D^{(1)}$ lamination coefficient $E^{(2)} = -C13^{(2)} h\, \boldsymbol{\varepsilon}_x / C33^{(2)} - [C23^{(2)}\, \boldsymbol{\varepsilon} / C33^{(2)}]\, D^{(2)}$

Layer-wise reduced equilibrium differential equations with negligible body forces (plane strain): $\partial\sigma_y/\partial y + \partial\tau_{yz}/\partial z = 0$, $\partial\tau_{yz}/\partial y + \partial\sigma_z/\partial z = 0$

Boundary conditions: $\sigma_y = \tau_{yz} = 0$ on free edges y=+b and y=-b for all z $\tau_{yz} = \sigma_z = 0$ on top and bottom surfaces z=+h and z=-h for all y

Derivation of normal stresses from shear stress By substituting τ_{yz} (predetermined from strain-displacement relations and constitutive equation while satisfying exact boundary conditions) into the reduced equilibrium differential equation, σy and σz are derived by simple integration.

Essential Stress Components for the uniaxially loaded symmetric bidirectional laminate that are closely related to interlaminar interactions.

$\tau_{yz}^{(k)} = \{\sum_1^{\infty}\{[2(k1b - k1a)\sin(n\boldsymbol{\pi}/\mathbf{2})(e^{\lambda 1\eta} - e^{\lambda 2\eta})] / (n\boldsymbol{\pi}(1-\beta_1/\beta_2)\sin(n\boldsymbol{\pi}Z)\}^{(k)}$

$\sigma y^{(k)} = [\ \{\sum_1^{\infty} -2(k1b - k1a)\sin(n\boldsymbol{\pi}/\mathbf{2})\ (e^{\lambda 1\eta} - (\beta_1/\beta_2)\, e^{\lambda 2\eta}) / (n\boldsymbol{\pi}(1-\beta_1/\beta_2)] +$

$2(k1b-k1a)\sin(n\boldsymbol{\pi}/\mathbf{2})/(n\boldsymbol{\pi})\}\cos(n\boldsymbol{\pi}Z)]^{(k)}$

$\sigma z^{(k)} = \{\sum_1^{\infty} [-2(k1b - k1a)\sin(n\boldsymbol{\pi}/\mathbf{2})\ (\beta 1\, \boldsymbol{e}^{\lambda 1\eta} - \beta_2\, \boldsymbol{e}^{\lambda 2\eta}) / (n\boldsymbol{\pi}(1-\beta_1/\beta_2)]\cos(n\boldsymbol{\pi}Z)\}^{(k)} + G(\boldsymbol{\eta})^{(k)}$ where k=1,2

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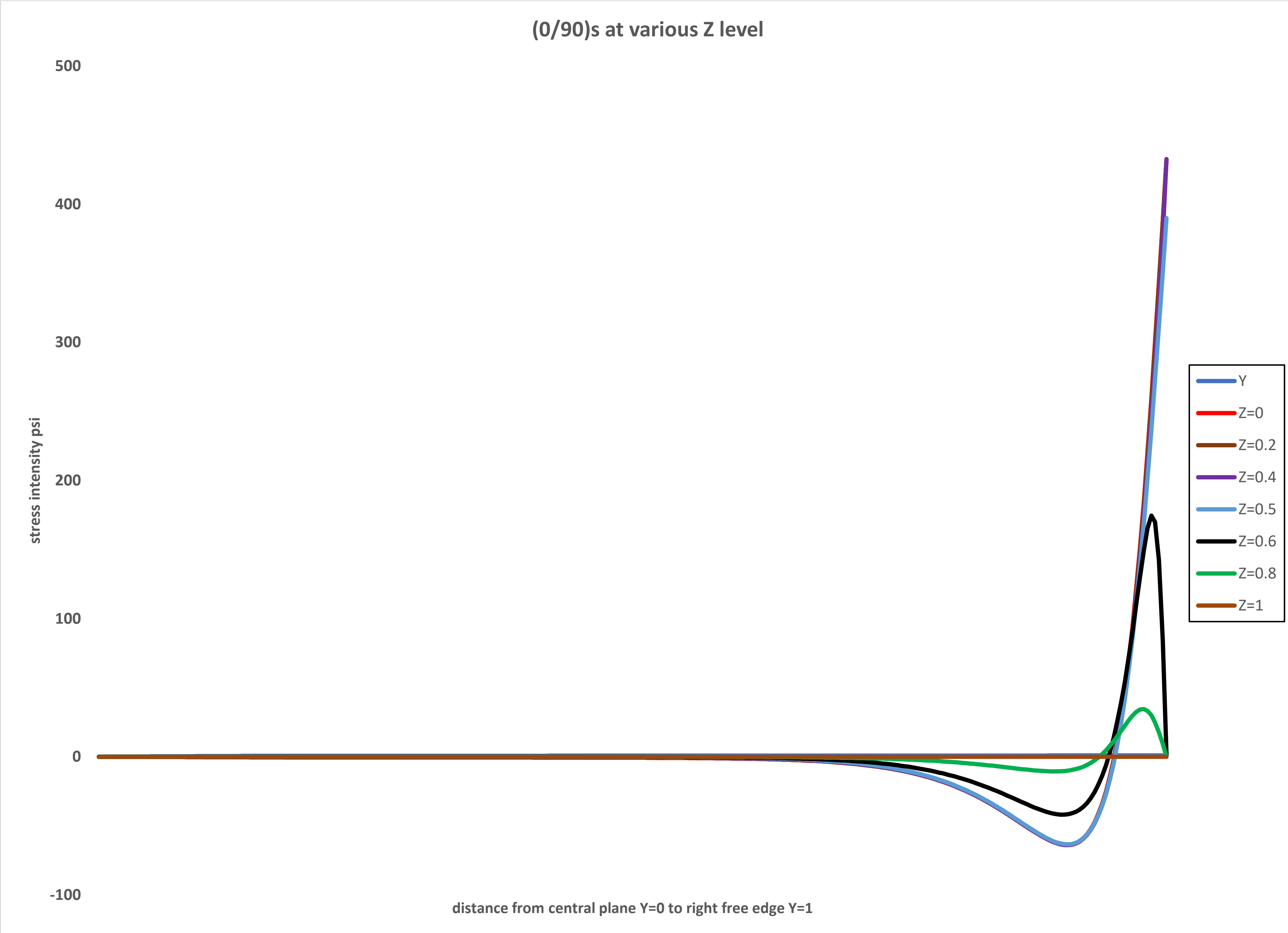
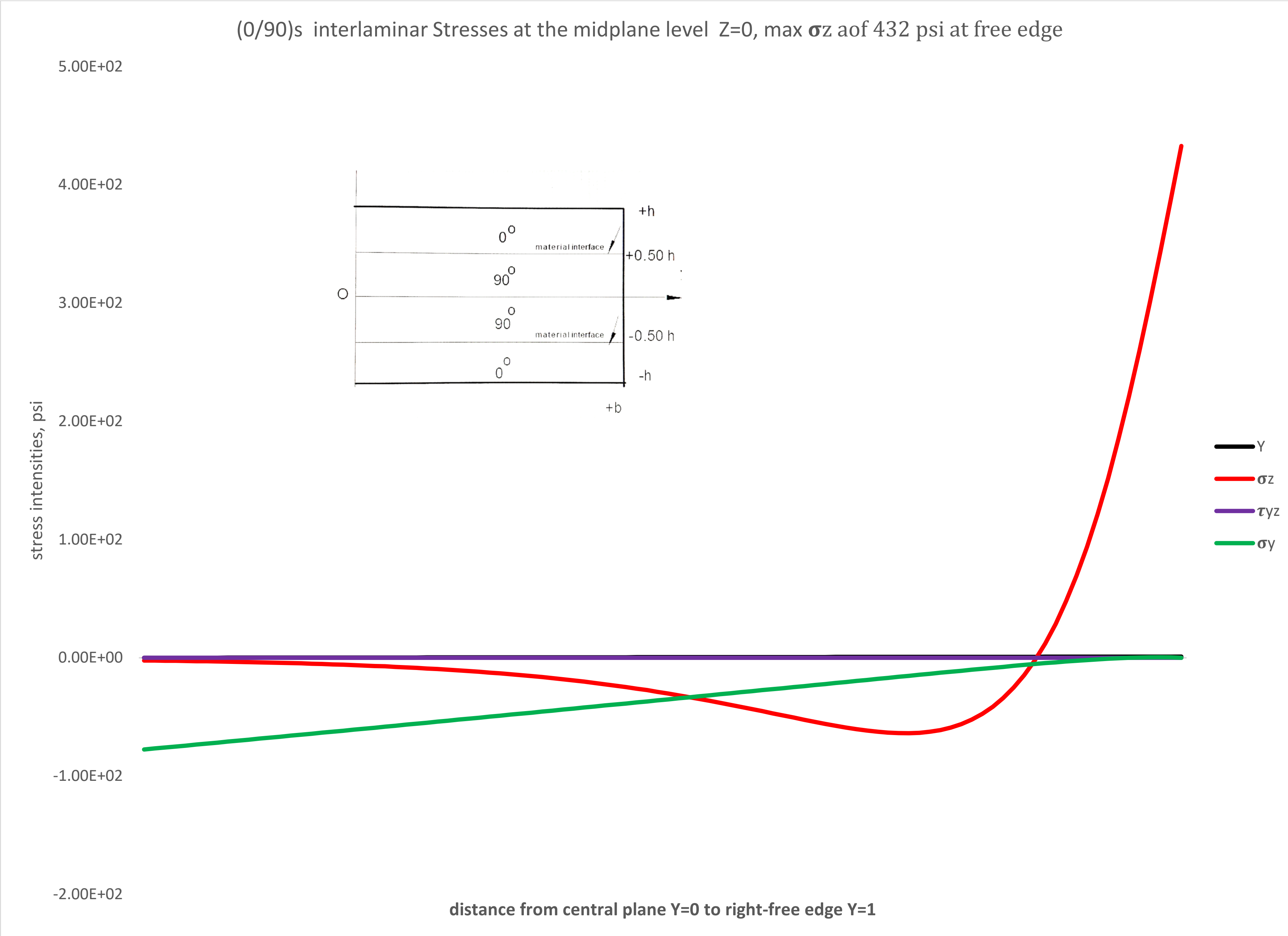
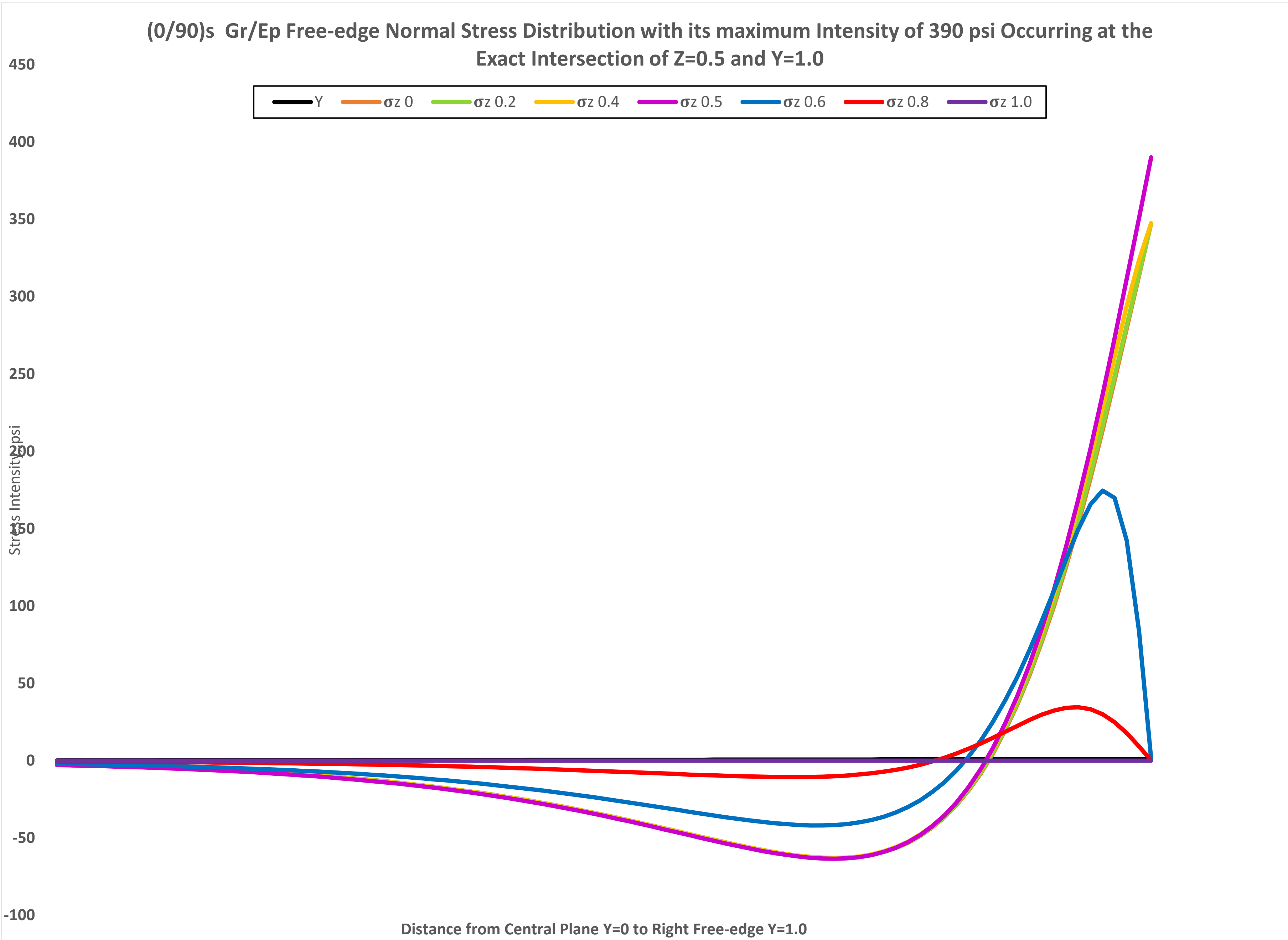
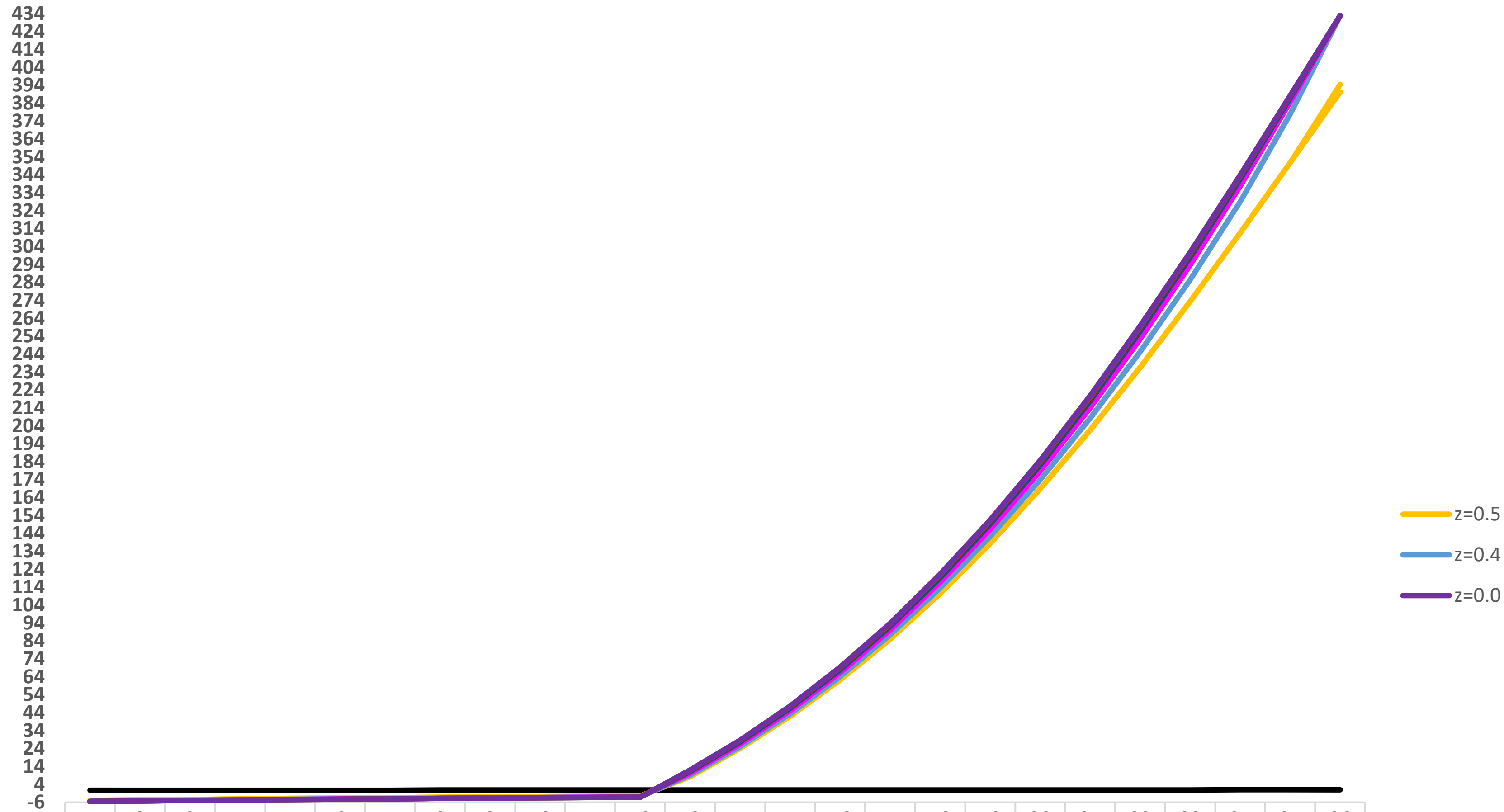

(0/90)s at various Z level
500
400
300
200
100
0
-100
stress intensity psi
distance from central plane Y=0 to right free edge Y=1
Y
Z=0
Z=0.2
Z=0.4
Z=0.5
Z=0.6
Z=0.8
Z=1



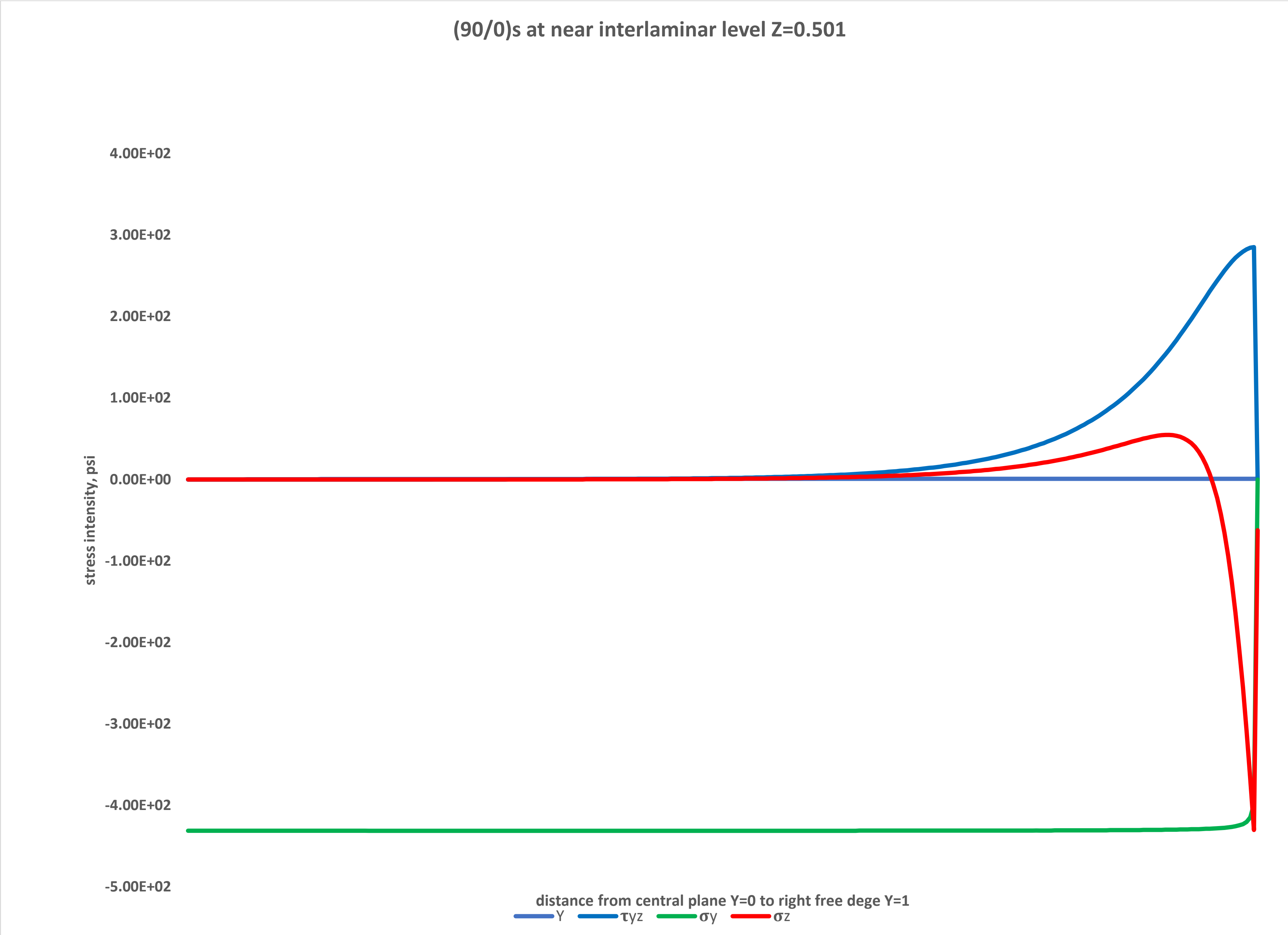
(0/90)s interlaminar Stresses at the midplane level Z=0, max σz aof 432 psi at free edge
5.00E+02
4.00E+02
3.00E+02
2.00E+02
1.00E+02
0.00E+00
-1.00E+02
-2.00E+02
stress intensities, psi
0°
material interface
90°
O
90°
material interface
0°
+h
+0.50 h
-0.50 h
-h
+b
Y
σz
τyz
σy
distance from central plane Y=0 to right-free edge Y=1


(0/90)s Gr/Ep Free-edge Normal Stress Distribution with its maximum Intensity of 390 psi Occurring at the Exact Intersection of Z=0.5 and Y=1.0
Y
σz 0
σz 0.2
σz 0.4
σz 0.5
σz 0.6
σz 0.8
σz 1.0
450
400
350
300
250
200
150
100
50
0
-50
-100
Stress Intensity psi
Distance from Central Plane Y=0 to Right Free-edge Y=1.0

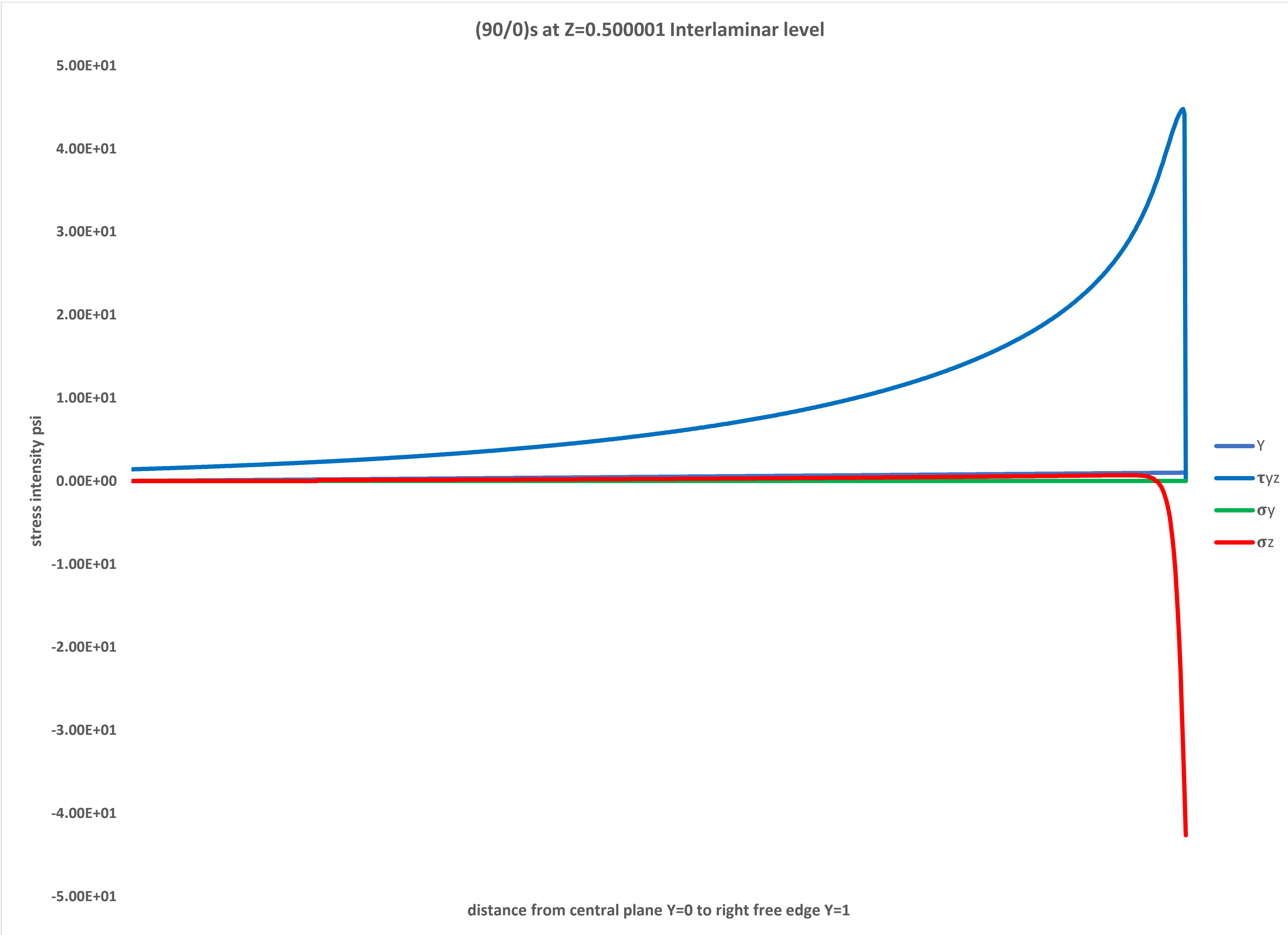
(0/90)s GR/Ep σz from Y=0.75 to Right-Free Edge Y=1.0, peaking at Midplane Z=0.0



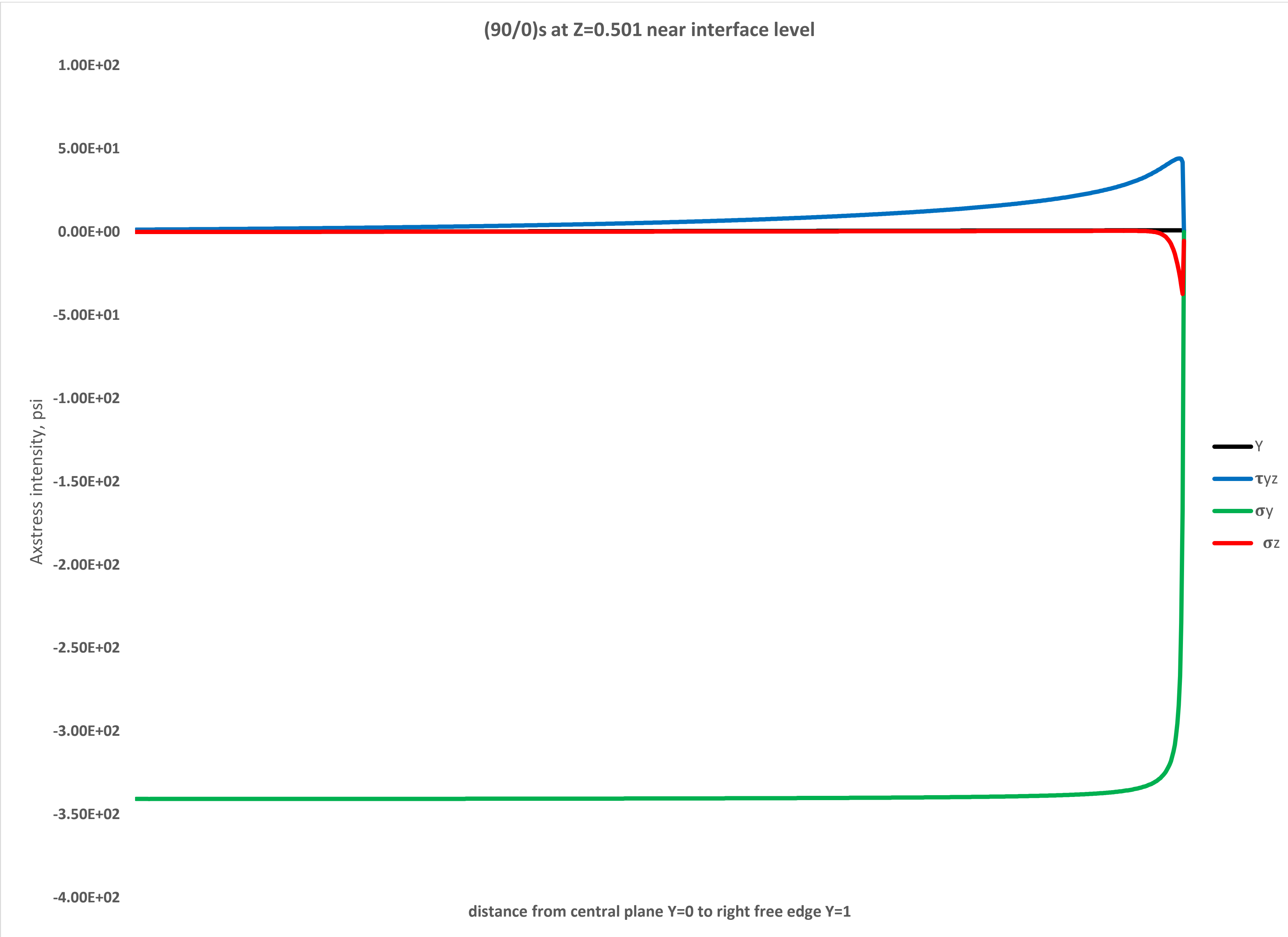
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Y	0.75	0.76	0.77	0.78	0.79	0.8	0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.9	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1
z=0.5	-4.9	-4.6	-4.4	-4.1	-3.9	-3.6	-3.4	-3.2	-3	-2.8	-2.7	-2.5	8.39	24.1	42.1	62.4	85.2	111	138	169	202	236	273	311	351	390
z=0.4999	-4.9	-4.6	-4.4	-4.1	-3.9	-3.6	-3.4	-3.2	-3	-2.9	-2.6	-2.5	8.39	24.1	42.1	62.4	85.2	111	138	169	202	236	273	311	351	394
z=0.4	-5.3	-5	-4.8	-4.5	-4.3	-4	-3.8	-3.6	-3.4	-3.2	-3	-2.9	9.05	25.1	43.4	64.1	87.5	114	142	174	208	245	285	329	378	433
z=0.3	-5.5	-5.2	-4.9	-4.6	-4.4	-4.2	-4	-3.8	-3.6	-3.4	-3.2	-3	10.2	26.5	45.2	66.3	90.2	117	146	179	214	252	294	338	385	433
z=0.2	-5.5	-5.2	-5	-4.7	-4.5	-4.2	-4	-3.8	-3.6	-3.4	-3.3	-3.1	11.2	27.7	46.6	68	92.2	119	149	182	218	257	298	342	387	433
z=0.1	-5.5	-5.3	-5	-4.7	-4.5	-4.3	-4	-3.8	-3.6	-3.5	-3.3	-3.1	11.8	28.4	47.5	69.1	93.5	121	151	184	220	259	300	343	388	433
z=0.0	-5.5	-5.3	-5	-4.7	-4.5	-4.3	-4	-3.8	-3.7	-3.5	-3.3	-3.1	12	28.7	47.8	69.5	93.9	121	151	185	221	260	301	344	388	433

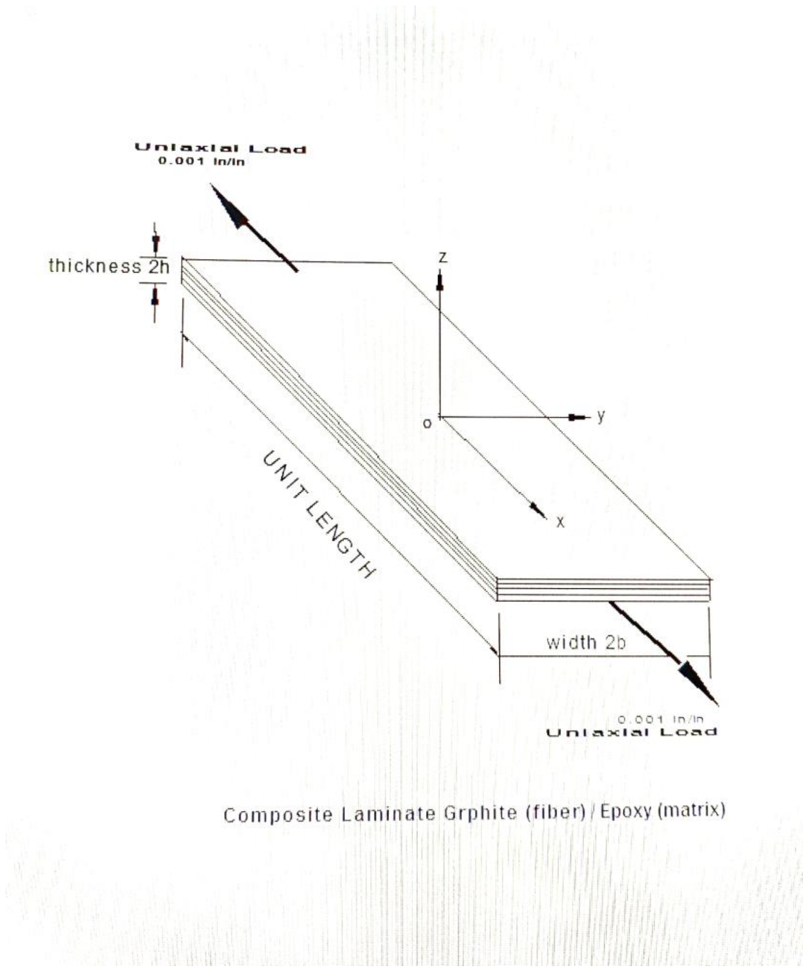


(90/0)s at near interlaminar level Z=0.501
4.00E+02
3.00E+02
2.00E+02
1.00E+02
0.00E+00
-1.00E+02
-2.00E+02
-3.00E+02
-4.00E+02
-5.00E+02
stress intensity, psi
distance from central plane Y=0 to right free dege Y=1
Y
τyz
σy
σz

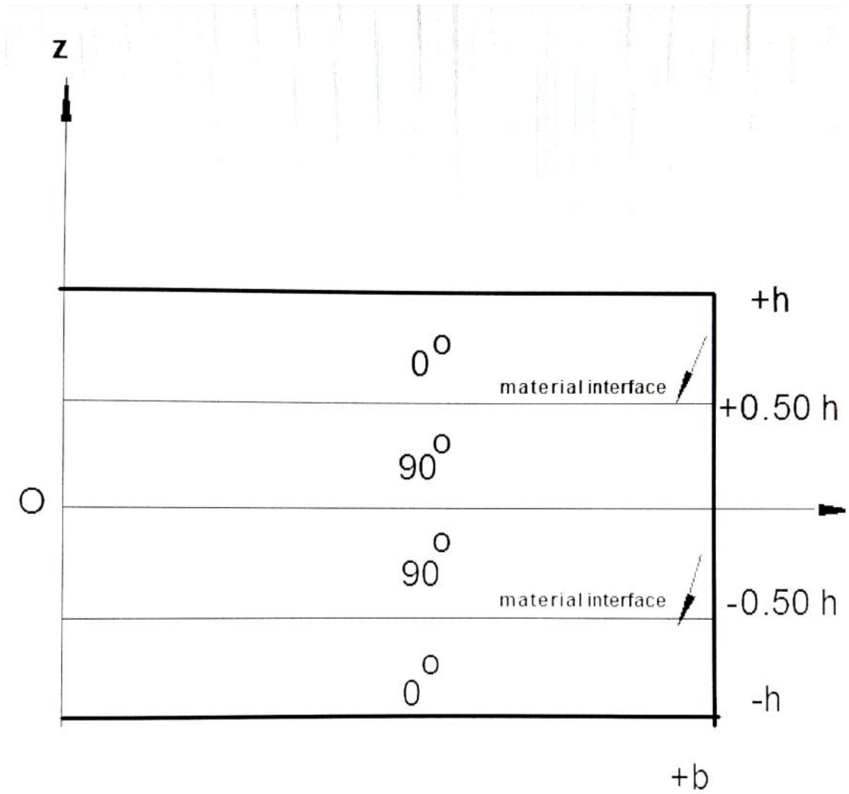


(90/0)s at Z=0.500001 Interlaminar level
5.00E+01
4.00E+01
3.00E+01
2.00E+01
1.00E+01
0.00E+00
-1.00E+01
-2.00E+01
-3.00E+01
-4.00E+01
-5.00E+01
stress intensity psi
distance from central plane Y=0 to right free edge Y=1
Y
τyz
σy
σz

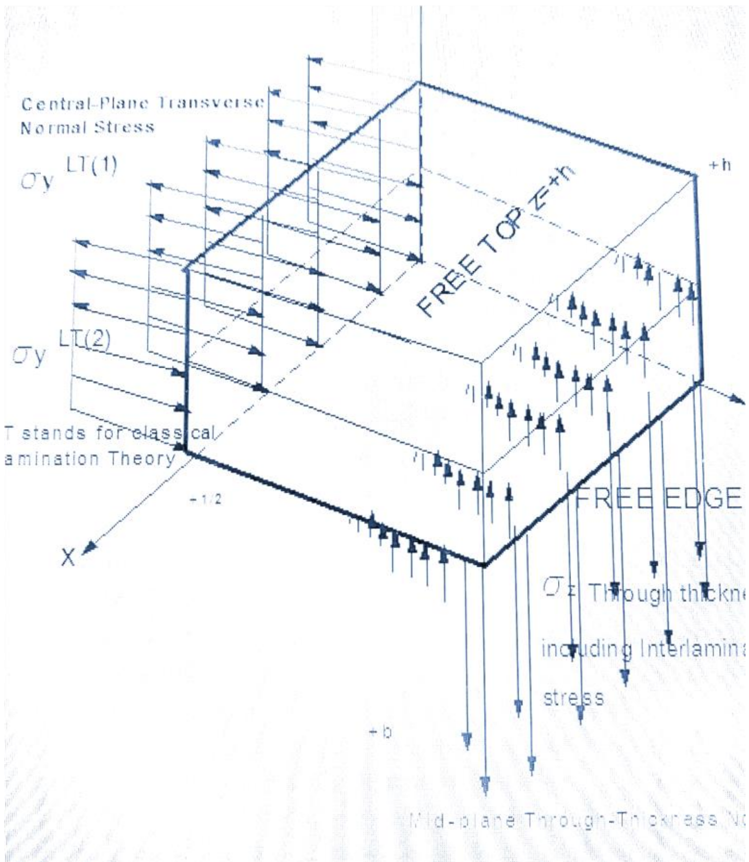

(90/0)s at Z=0.501 near interface level
1.00E+02
5.00E+01
0.00E+00
-5.00E+01
-1.00E+02
-1.50E+02
-2.00E+02
-2.50E+02
-3.00E+02
-3.50E+02
-4.00E+02
Axstress intensity, psi
distance from central plane Y=0 to right free edge Y=1
Y
τyz
σy
σz



This is the figure for geometry and loading of the four-layer bidirectional laminate



This is the figure for the right-half of the 4-layer symmetric laminate



This is qualitative insight into interlaminar stresses around an octant of a unit-length of the laminate