Fuel-Efficient Jet Flights by Minimization of Thrust-to-Velocity Ratio

by

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ABSTRACT

A. Climb and Descent

This paper presents a mathematical analysis for jet airplanes based on a flight mechanics approach to thrust required, range, and endurance in a level, un-accelerated flight [1]. We aim to develop explicit expressions for climbing and descending actions. The objective of this work is to analytically explore how the most fuel-efficient actions can be achieved. For math simplicity, we are now making an idealizing assumption that there are neither sudden gusts nor inclement weather change, i.e., environmental conditions remain unchanged except for temperature and air density. The first step now is to develop useful mathematical expressions for the maximum range and endurance: the basic equations of motion with the inclusion of the flight path angle. The important thrust required term is now practically recognized as a function of four variables: the climbing angle, the changing velocity, the elevating altitude, and the changing weight of the airplane. By assuming the weight remains essentially constant and the climb or descending angle stay unchanged, the thrust-to-velocity ratio T_R/V_∞ is an equation shown below, and the relation between the velocity and the altitude can be derived as follows.

Applicable equilibrium equations developed by the author in 2022 can be summarized below: First, we have the ambient density ρ expressed in terms of the gravity coefficient g_o, temperature gradient λ, gas constant R, sea level temperature To, and the altitude h.

$$\phi = -\{g_0/(\lambda R) + 1\}$$

$$\rho = \rho_0 (1+\lambda h/T_0)^\wedge\phi$$

On the ascent, the thrust required can be expressed as follows [1]:

$$T_R = (½)(\rho V^2_\infty) SC_{D,o} + W^2\cos^2\theta/(½\rho V^2_\infty S\pi eAR) + W\sin\theta$$

On the descent, it is

$$T_R = (½)(\rho V^2_\infty) SC_{D,o} + W^2\cos^2\theta/(½\rho V^2_\infty S\pi eAR) - W\sin\theta$$

Since T_R is a function of speed V_∞, and the altitude h, the maximum range can be derived by minimizing T_R/V_∞ (which implies using the least amount of jet fuel to reach the expected altitude during the climb). So, we have $\{\partial (T_R/V_\infty)/\partial V_\infty\}(dV_\infty/dh) + \partial(T_R/V_\infty)/\partial h = 0$. This equation, an ordinary differential equation of (d V∞/dh), is now solved by the 4^{th}-order Runge-Kutta method and the results have been generated.

B. Cruising Action

To do a detailed theoretical analysis of the cruising action, we start by assuming that the airplane is finishing its climb action at an altitude of c where the fuel content is Wc and the velocity of the airplane is Vc, and the airplane is at a ground distance Xc from the starting point of take-off. Before descending, the airplane is now carrying a fuel weight Wd and located at ground distance Xd.

A detailed derivation process gives the following meticulous steps:

By assuming weight W varies as a linear function of distance x, W=Ax+B, where A and B are expressed in terms of α, β, W_F, xc, and x_d, an approximate relation between V_c and V_d can be derived by minimizing T_R/V_∞, followed by a simplifying step (to be detailed), to arrive at

$$V_d=V_c[(Vd+B/A)/(Vc+B/A)]^{2/3}$$

The maximum range can be derived by the following equation:

$$\int -dW= \int Ct\ (T_R/V_\infty)\ dx$$

where the integration limits are Wc and Wd, Ct is the fuel consumption rate.

After a set of meticulous steps of simplification and solution, a number of figures have been generated for the dynamic equilibrium relations between the airplane velocity and the altitude.

References

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