

Fuel-Efficient Jet Flights by Minimization of Thrust-to-Velocity Ratio

by

Peter Wenhan Hsu*

NQstar.com, Seattle, WA, USA

ABSTRACT

A. Climb and Descent

This paper presents a mathematical analysis for jet airplanes based on a flight mechanics approach to thrust required, range, and endurance in a level, un-accelerated flight [1]. The goal is to develop explicit expressions for climbing and descending actions. The objective of this work is to neglect sudden wind gusts and clement weather changes and to explore how the most fuel-efficient actions can be achieved. The first step is to explore and digitize useful mechanics relations for the maximum range and endurance: the basic equations of motion with the inclusion of the flight path angle. The important thrust required term is now practically recognized as a function of four variables: the climbing angle, the changing velocity, the elevating altitude and the changing weight of the airplane. By assuming the weight remains essentially constant and the climb or descending angle stay unchanged, the thrust-to-velocity ratio T_R/V_∞ is an equation shown below, and the relation between the velocity and the altitude can be derived as follows.

Applicable equilibrium equations can be summarized below:

Ambient air density ρ expressed in terms of the gravity coefficient g_0 , temperature gradient λ , gas constant R , sea level temperature T_0 , and the altitude h .

$$\phi = - \{g_0/(\lambda R) + 1\}$$

$$\rho = \rho_0 (1 + \lambda h/T_0)^\phi$$

On the ascent, the thrust required can be expressed as follows [1]:

$$T_R = \left(\frac{1}{2}\right) (\rho V_\infty^2) S (C_{D,0} + W^2 \cos^2 \theta / ((\frac{1}{2}) \rho V_\infty^2)) S \pi e AR + W \sin \theta$$

$$\text{On the descent, it is } T_R = \left(\frac{1}{2}\right) (\rho V_\infty^2) S (C_{D,0} + W^2 \cos^2 \theta / ((\frac{1}{2}) \rho V_\infty^2)) S \pi e AR - W \sin \theta$$

Since T_R is a function of speed V_∞ , and the altitude h , the maximum range can be derived by minimizing T_R/V_∞ (which implies using the least amount of jet fuel to reach the expected altitude during the climb). The resulting equation, $\{\partial (T_R/V_\infty)/\partial V_\infty\} (dV_\infty/dh) + \partial (T_R/V_\infty)/\partial h = 0$, an ordinary differential equation of (dV_∞/dh) , is solved by the 4th-order Runge-Kutta method [2] and some results have been generated.

B. Cruising Action

To facilitate a detailed theoretical analysis of the cruising action, it is now assumed that the airplane is just finishing its climb action at an altitude of c where the fuel content is W_c , and the velocity of the airplane is V_c , and the airplane is at a ground distance X_c from the starting point of take-off. Before descending, the airplane is now carrying a fuel weight W_d and located at ground distance X_d .

A detailed derivation process gives the following meticulous steps:

By assuming weight W varies as a linear function of distance x , $W = Ax + B$, where A and B are expressed in terms of α , β , W_F , x_c , and x_d , an approximate relation between V_c and V_d can be derived by minimizing T_R/V_∞ , followed by a simplifying step (to be detailed), to arrive at

$$V_d = V_c [(V_d + B/A) / (V_c + B/A)]^{2/3}$$

The maximum range can be derived by the following equation:

$$\int -dW = \int C_t (T_R/V_\infty) dx$$

Where the integration limits are W_c and W_d , C_t is the fuel consumption rate.

After a set of meticulous steps of simplification and solution, some figures have been generated for the dynamic equilibrium relations [3] between the airplane velocity and the altitude.

References

1. Anderson, J. D., *Introduction to Flight*, 4th ed., McGraw-Hill, 2000, pp.355-358.
2. Hilderbrand, F. B., *Advanced Calculus for Applications*, Prentice-Hall, Inc. 1962, p.104.
3. Bertin, J. J. and Smith, M. L., *Aerodynamics for Engineers*, Prentice-Hall, Inc., 1979 p.169-177.