Interlaminar Stresses in Composite Laminates – A Perturbation Analysis for the Solution of a Generalized Plane Strain Problem

Peter Wenhan Hsu*

NQstar.com

Seattle, WA, USA

*Founder & CEO

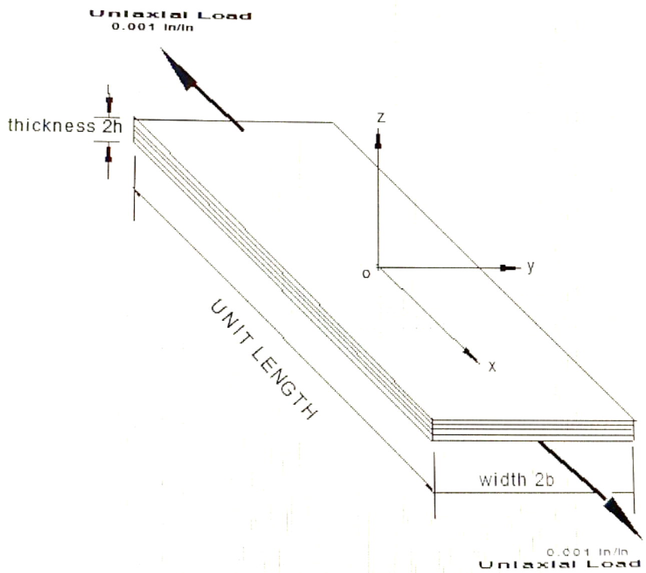


Fig.1 Symmetric, Laminated Composite Plate Geometry and Loading Model

Abstract

This work is a rigorous treatment of a previous analysis by Peter Wenhan Hsu [1] to determine the solution to an anisotropic problem in   the interlaminar stress problem has been a topic of enormous interest due to its significant in the so-called delamination failure in composite laminate s under in-plane tensile loadings.

Introduction

Anisotropic elasticity theory dealing with stress-strain relations in materials has been applied to solve interlaminar stress problems in the area of composite materials and mechanics.

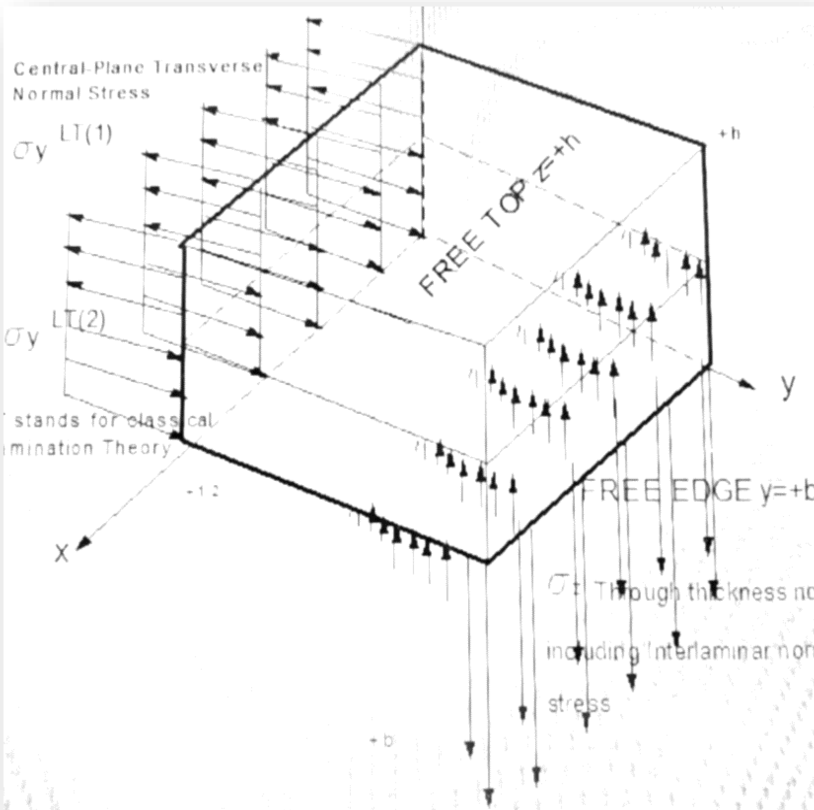


Fig.2 Detailed insight into stress distribution in one-eighth part of section

Geometric Parameters and Equations

geometric half-width b

geometric half-thickness h

dimensionless coordinate Y=y/b

dimensionless coordinate Z=z/h

thickness-to-width ratio $\boldsymbol{\varepsilon}$=h/b boundary layer coordinate $\boldsymbol{\eta}$=(b-y)/h=(1-Y)/$\boldsymbol{\varepsilon}$ relation between coordinates y=b-h$\boldsymbol{\eta}$ partial derivative relation $\partial/\partial y = -(1/h)\, \partial/\partial\eta$ applied uniaxial strain ε_x superscript **1** denotes outer layers (h/2<z<h and -h<z<-h/2); superscript **2** denotes inner layers (-h/2<z<0 and 0<z<h/2) lamination coefficient $D^{(1)} = D^{(2)} = [-(C12^{(1)} + C12^{(2)}) + C23^{(1)}\, C13^{(1)}/C33^{(1)} + C23^{(2)}\, C13^{(2)}/C33^{(2)})]\, (b\, \varepsilon_x)/\, [C22^{(1)} + C22^{(2)} - C23^{(1)}\, C23^{(1)}/C33^{(1)}-$

$C23^{(2)}$ $C23^{(2)}/C33^{(2)}$] lamination coefficient $E^{(1)} = -C13^{(1)} h\, \varepsilon_{x} / C33^{(1)} - [C23^{(1)}\, \varepsilon\, C33^{(1)}]\, D^{(1)}$ lamination coefficient $E^{(2)} = -C13^{(2)} h\, \varepsilon_{x} / C33^{(2)} - [C23^{(2)}\, \varepsilon\, C33^{(2)}]\, D^{(2)}$

Layer-wise reduced equilibrium differential equations with negligible body forces (plane strain): $\partial\sigma y/\partial y + \partial\tau yz/\partial z = 0$, $\partial\tau yz/\partial y + \partial\sigma z/\partial z = 0$

Boundary conditions: $\sigma y = \tau yz = 0$ on free edges y=+b and y=-b for all z $\tau yz = \sigma z = 0$ on top and bottom surfaces z=+h and z=-h for all y

Essential Stress Components for the uniaxially loaded symmetric bidirectional laminate that are closely related to interlaminar interactions.

$\tau_{yz}{}^{(k)} = \{\sum_{1}^{\infty}\{[2(k1b - k1a)\sin(n\,\pi/2)(e^{\lambda 1\eta} - e^{\lambda 2\eta})] / (n\,(1-\beta_1/\beta_2)\sin(n\pi Z)\}^{(k)}$

$\sigma_{y}{}^{(k)} = [\ \{\sum_{1}^{\infty} -2(k1b - k1a)\sin(n\,\pi/2)\,(e^{\lambda 1\eta} - (\beta_1/\beta_2)\, e^{\lambda 2\eta}) / (n\pi(1-\beta_1/\beta_2)] +$

$2(k1b - k1a)\sin(n\pi/2)/(n\pi)\}\cos(n\pi Z)]^{(k)}$

$\sigma_{z}{}^{(k)} = \{\sum_{1}^{\infty} [-2(k1b - k1a)\sin(n\,\pi/2)\,(\beta 1\, e^{\lambda 1\eta} - \beta_2\, e^{\lambda 2\eta}) / (n\pi\,(1-\beta_1/\beta_2)]\cos(n\pi Z)\}^{(k)} + G(\eta)^{(k)}$ where k=1,2

References Hsu, P.W., "Interlaminar Stresses In Composite Laminates – A Perturbation Analysis," Doctor of Philosophy in Engineering Mechanics Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Virginia Jan. 1976.

Pipes, R. B. and Pagano, N. J., "Interlaminar Stresses in Composite Laminates Under Uniform Axial Extensions," J. of Composite Materials, Vol. 4, 1970, p.538.

Hsu, P. W. and Herakovich, C. T., "Edge Effects in Angle-Ply Laminates," J. Composite Materials, vol. 11, Oct. 1977, pp.422-428.

Hsu, P. W. and Herakovich, C. T., "A Perturbation Solution for Interlaminar Stresses in Bidirectional Laminates," Composite Materials: Testing and Design (Fourth Conference, ASTM STP 617, American Society for Testing and Materials, 1977 pp.296-316.

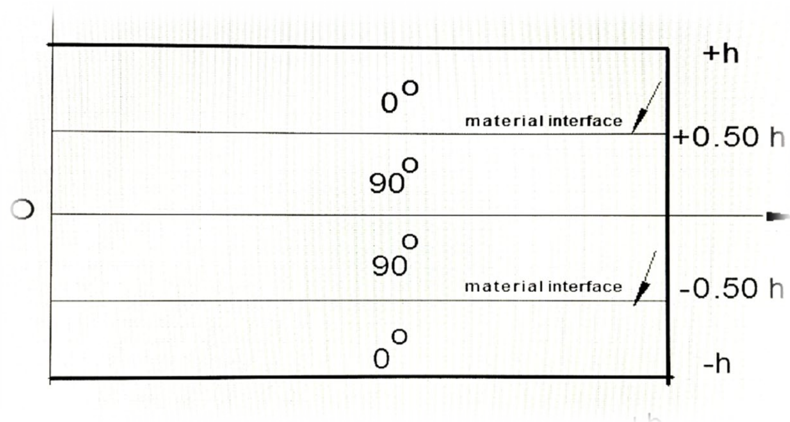
Pagano, N. J. and Pipes, R. B., "Some Observations on The Interlaminar Strength of Composite Laminates," Inter. J. Mech. Sci., Pergamon Press, 1973, vol.15, pp.679-688.

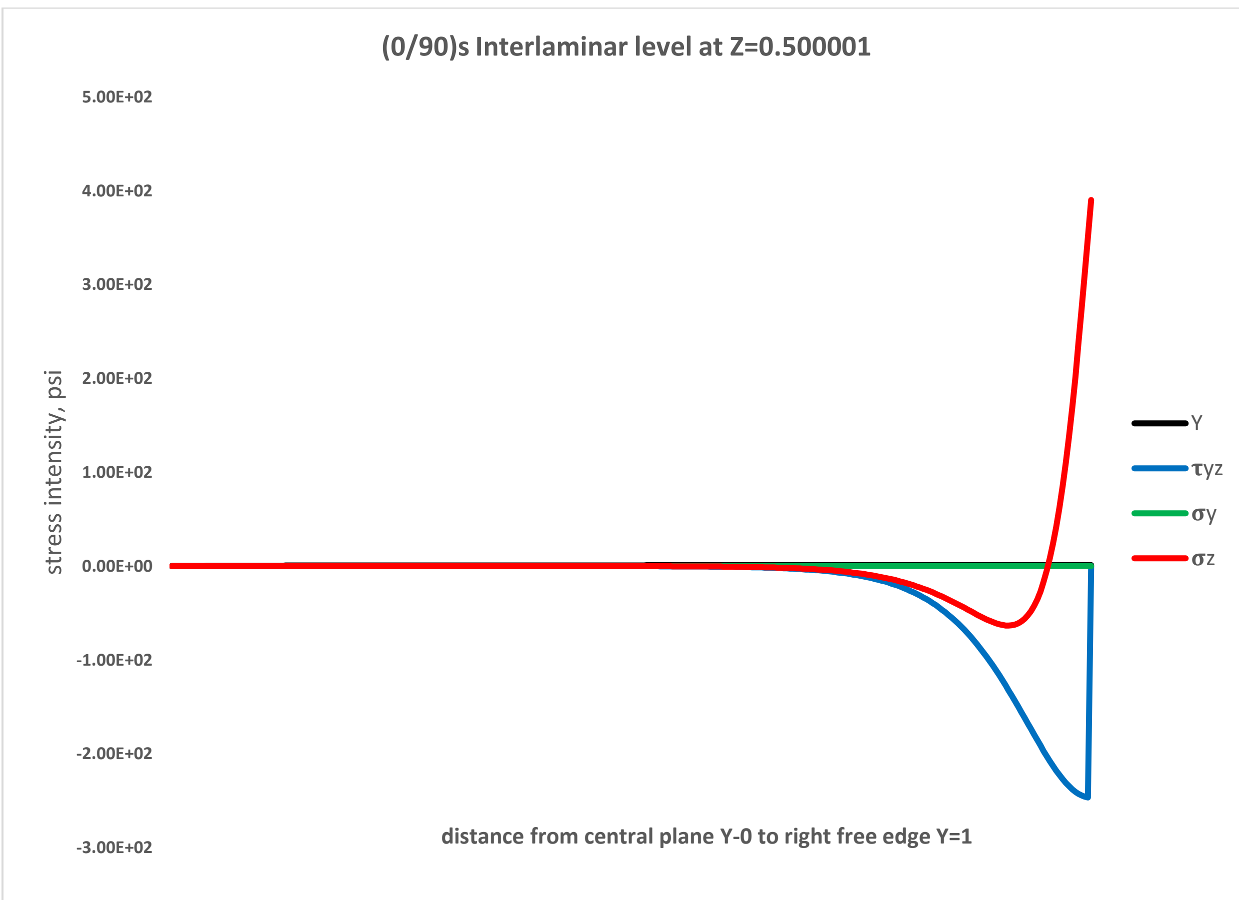
Pagano, N.J. and Pipes, R.B., "The Influence of Stacking Sequence on laminate Strength," J. of Composite Materials, vol. 5, Jan. 1971.

Nayfeh, A. H., *Perturbation Methods*, Wiley - Interscience, New York, 1973.

Timoshenko, S. P. and J. N. Goodier, *Theory of Elasticity*, McGraw-Hill Book Company, 1970.

Hsu, P. W., "Three dimensional stresses in an Adhesively bonded Plate under a Uniaxial Load',' presented at ASME/ASCE/SES MMC conference, June 2001, San Diego, Ca.


+h
0°
material interface
+0.50 h
90°
90°
material interface
-0.50 h
0°
-h


(0/90)s Interlaminar level at Z=0.500001
5.00E+02
4.00E+02
3.00E+02
2.00E+02
1.00E+02
0.00E+00
-1.00E+02
-2.00E+02
-3.00E+02
stress intensity, psi
Y
τyz
σy
σz
distance from central plane Y-0 to right free edge Y=1

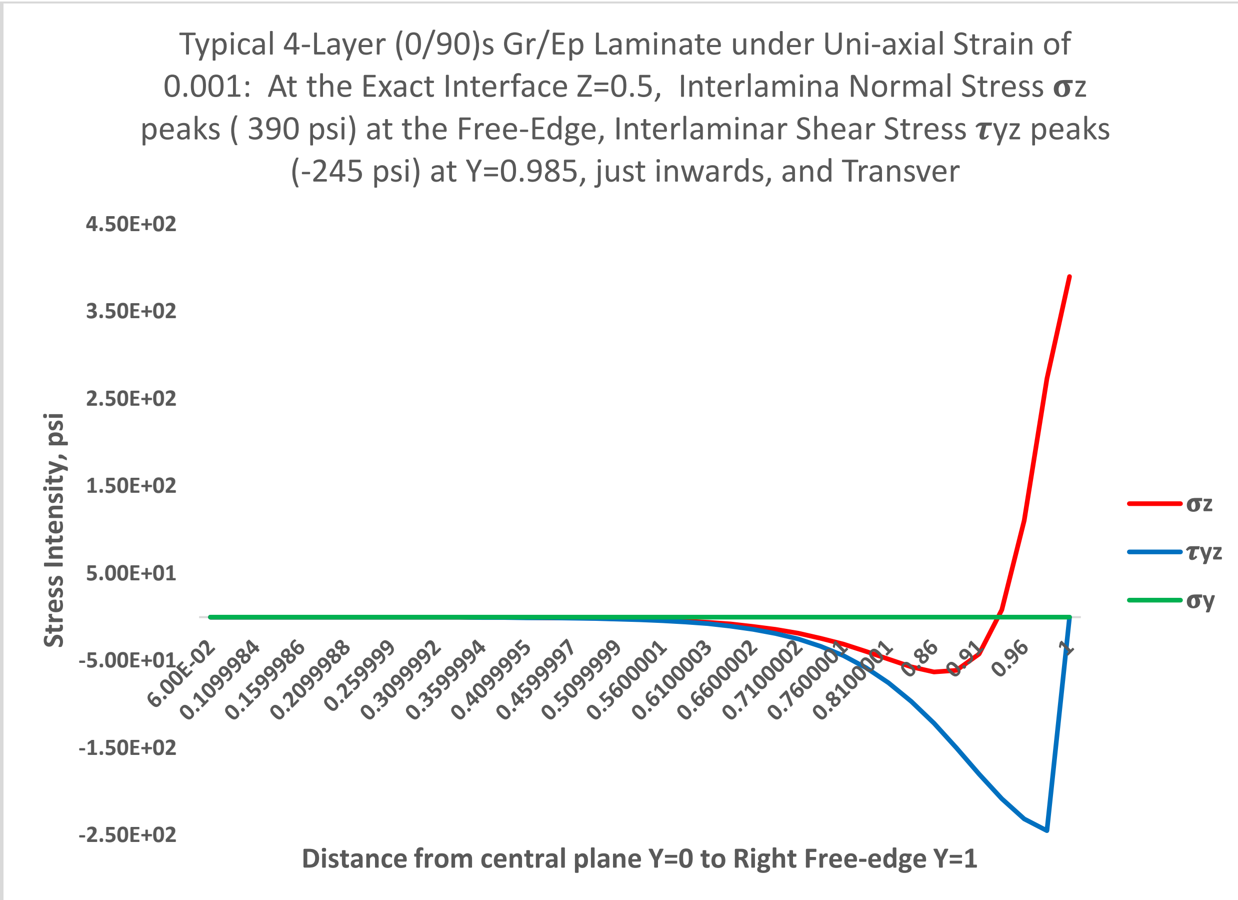
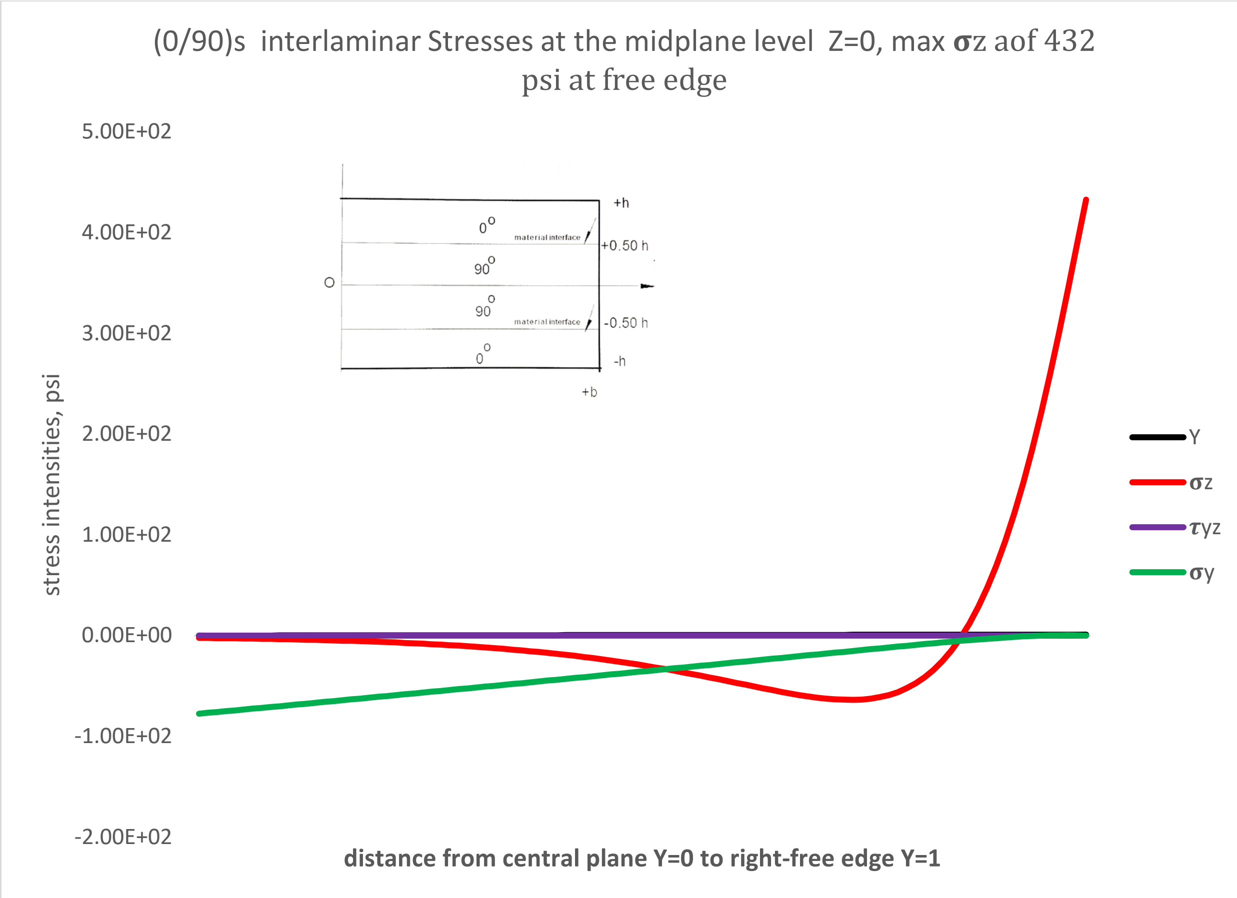
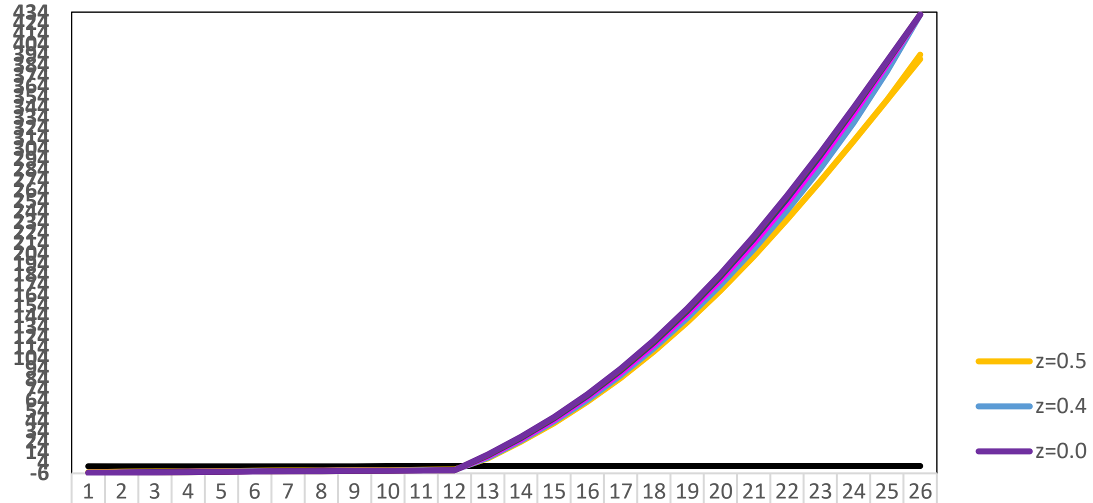


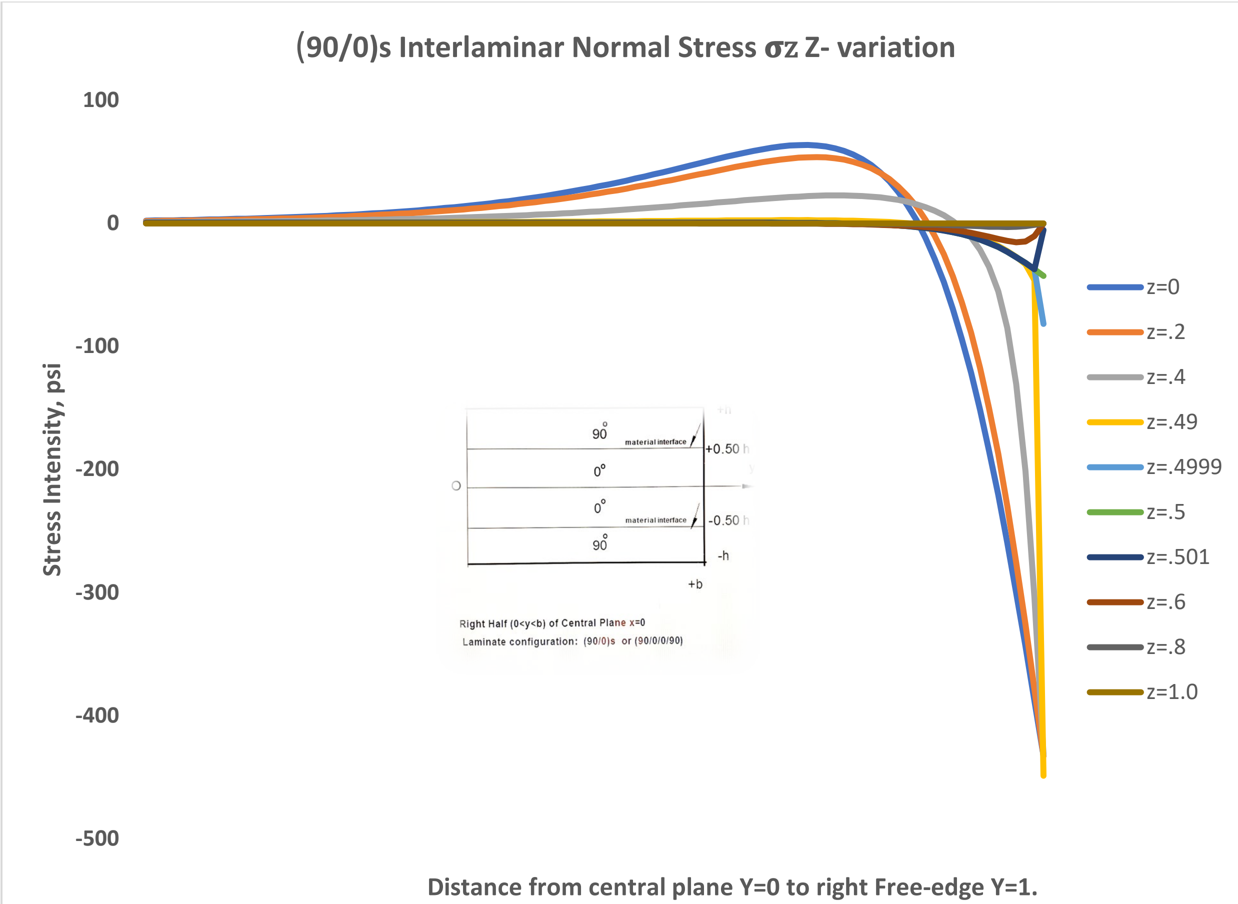
Fig.3 Right-half of central plane (x=0) cross sectional view of 4-layer Laminate


(0/90)s interlaminar Stresses at the midplane level Z=0, max σz aof 432 psi at free edge
5.00E+02
4.00E+02
3.00E+02
2.00E+02
1.00E+02
0.00E+00
-1.00E+02
-2.00E+02
stress intensities, psi
distance from central plane Y=0 to right-free edge Y=1
Y
σz
τyz
σy
+h
0°
material interface
+0.50 h
90°
O
90°
material interface
-0.50 h
0°
-h
+b

σz from Y=0.75 to Right-Free Edge Y=1.0, peaking at Midplane Z=0.0



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Y	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1	1	1	1	1	1
z=0.5	-5	-5	-4	-4	-4	-4	-3	-3	-3	-3	-3	-3	8.4	24	42	62	85	111	138	169	202	236	273	311	351	390
z=0.4999	-5	-5	-4	-4	-4	-4	-3	-3	-3	-3	-2.	-3	8.4	24	42	62	85	111	138	169	202	236	273	311	351	394
z=0.4	-5	-5	-5	-4	-4	-4	-4	-4	-3	-3	-3	-3	9.1	25	43	64	87	114	142	174	208	245	285	329	378	433
z=0.3	-5	-5	-5	-5	-4	-4	-4	-4	-4	-3	-3	-3	10	27	45	66	90	117	146	179	214	252	294	338	385	433
z=0.2	-6	-5	-5	-5	-4	-4	-4	-4	-4	-3	-3	-3	11	28	47	68	92	119	149	182	218	257	298	342	387	433
z=0.1	-6	-5	-5	-5	-4	-4	-4	-4	-4	-3	-3	-3	12	28	47	69	93	121	151	184	220	259	300	343	388	433
z=0.0	-6	-5	-5	-5	-4	-4	-4	-4	-4	-3	-3	-3	12	29	48	69	94	121	151	185	221	260	301	344	388	433


(90/0)s Interlaminar Normal Stress σz Z- variation
100
0
-100
-200
-300
-400
-500
Stress Intensity, psi
Distance from central plane Y=0 to right Free-edge Y=1.
z=0
z=.2
z=.4
z=.49
z=.4999
z=.5
z=.501
z=.6
z=.8
z=1.0
90°
material interface
+0.50 h
0°
0°
material interface
-0.50 h
90°
-h
+b
Right Half (0<y<b) of Central Plane x=0
Laminate configuration: (90/0)s or (90/0/0/90)