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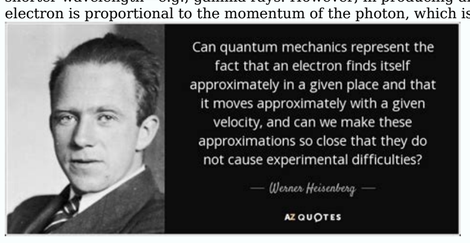
State and explain heisenberg uncertainty principle engineering physics

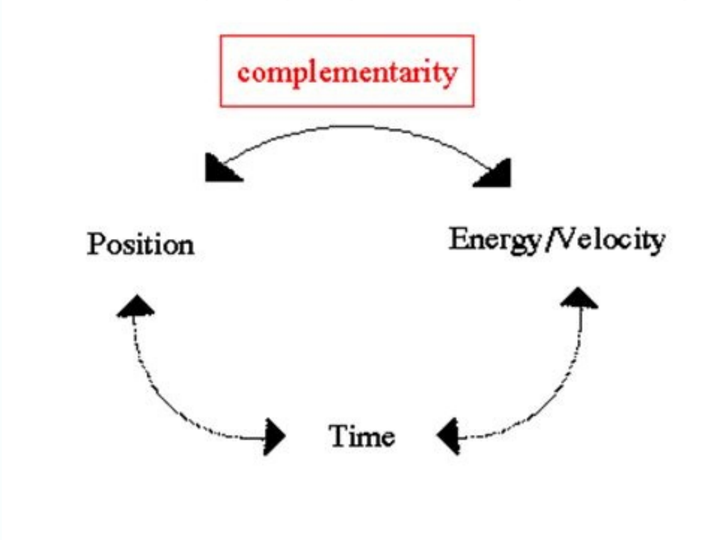
Formulated by the German physicist and Nobel laureate Werner Heisenberg in 1927, the uncertainty principle states that we cannot know both the position and speed of a particle, such as a photon or electron, with perfect accuracy; the more we nail down the particle's position, the less we know about its speed and vice versa.In other words, if we could shrink a tortoise down to the size of an electron, we would only be able to precisely calculate its speed or its location, not both at the same time.Though the Heisenberg uncertainty principle is famously known in quantum physics, a similar uncertainty principle also applies to problems in pure math and classical physics—basically, any object with wave-like properties will be affected by this principle. Quantum objects are special because they all exhibit wave-like properties by the very nature of quantum theory.To understand the general idea behind the uncertainty principle, think of a ripple in a pond. To measure its speed, we would monitor the passage of multiple peaks and troughs. The more peaks and troughs that pass by, the more accurately we would know the speed of a wave—but the less we would be able to say about its position. The location is spread out among the peaks and troughs. Conversely, if we wanted to know the exact position of one peak of a wave, we would have to monitor just one small section of the wave and would lose information about its speed. In short: the uncertainty principle describes a trade-off between two complementary properties, such as speed and position. Home Science Physics The observables discussed so far have had discrete sets of experimental values. For example, the values of the energy of a bound system are always discrete, and angular momentum components have values that take the form mℏ, where m is either an integer or a half-integer, positive or negative. On the other hand, the position of a particle or the linear momentum of a free particle can take continuous values in both quantum and classical theory.

The mathematics of observables with a continuous spectrum of measured values is somewhat more complicated than for the discrete case but presents no problems of principle. An observable with a continuous spectrum of measured values has an infinite number of state functions. The state function Ψ of the system is still regarded as a combination of the state functions of the observable, but the sum in equation (10) must be replaced by an integral. light passing through a slitMeasurements can be made of position x of a particle and the x-component of its linear momentum, denoted by px. These two observables are incompatible because they have different state functions. The phenomenon of diffraction noted above illustrates the impossibility of measuring position and momentum simultaneously and precisely. If a parallel monochromatic light beam passes through a slit (Figure 4A), its intensity varies with direction, as shown in Figure 4B. The light has zero intensity in certain directions. Wave theory shows that the first zero occurs at an angle θ0, given by sin θ0 = λ/b, where λ is the wavelength of the light and b is the width of the slit. If the width of the slit is reduced, θ0 increases—i.e., the diffracted light is more spread out. Thus, θ0 measures the spread of the beam. The experiment can be repeated with a stream of electrons instead of a beam of light. According to de Broglie, electrons have wavelike properties; therefore, the beam of electrons emerging from the slit should widen and spread out like a beam of light waves. This has been observed in experiments. If the electrons have velocity u in the forward direction (i.e., the y-direction in Figure 4A), their (linear) momentum is p = meu. Consider px, the component of momentum in the x-direction. After the electrons have passed through the aperture, the spread in their directions results in an uncertainty in px by an amount where λ is the wavelength of the electrons and, according to the de Broglie formula, equals h/p. Thus, Δpx ≈ h/b. Exactly where an electron passed through the slit is unknown; it is only certain that an electron went through somewhere. Therefore, immediately after an electron goes through, the uncertainty in its x-position is Δx ≈ b/2. Thus, the product of the uncertainties is of the order of ℏ. More exact analysis shows that the product has a lower limit, given by This is the well-known Heisenberg uncertainty principle for position and momentum. It states that there is a limit to the precision with which the position and the momentum of an object can be measured at the same time. Depending on the experimental conditions, either quantity can be measured as precisely as desired (at least in principle), but the more precisely one of the quantities is measured, the less precisely the other is known. The uncertainty principle is significant only on the atomic scale because of the small value of h in everyday units. If the position of a macroscopic object with a mass of, say, one gram is measured with a precision of 10−6 metre, the uncertainty principle states that its velocity cannot be measured to better than about 10−25 metre per second. Such a limitation is hardly worrisome. However, if an electron is located in an atom about 10−10 metre across, the principle gives a minimum uncertainty in the velocity of about 106 metre per second. The above reasoning leading to the uncertainty principle is based on the wave-particle duality of the electron. When Heisenberg first propounded the principle in 1927 his reasoning was based, however, on the wave-particle duality of the photon.

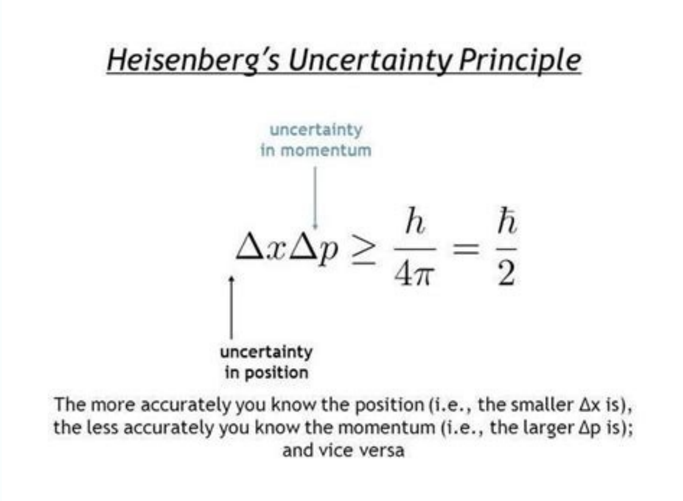
He considered the process of measuring the position of an electron by observing it in a microscope. Diffraction effects due to the wave nature of light result in a blurring of the image; the resulting uncertainty in the position of the electron is approximately equal to the wavelength of the light. To reduce this uncertainty, it is necessary to use light of shorter wavelength—e.g., gamma rays. However, in producing an image of the electron, the gamma-ray photon bounces off the electron, giving the Compton effect (see above Early developments: Scattering of X-rays). As a result of the collision, the electron recoils in a statistically random way. The resulting uncertainty in the momentum of the electron is proportional to the momentum of the photon, which is inversely proportional to the wavelength of the photon. So it is again the case that increased precision in knowledge of the position of the electron is gained only at the expense of decreased precision in knowledge of its momentum.



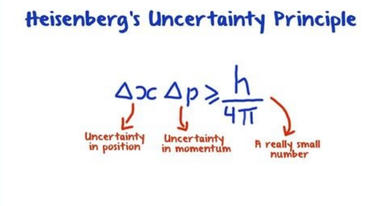
A detailed calculation of the process yields the same result as before (equation [12]). Heisenberg’s reasoning brings out clearly the fact that the smaller the particle being observed, the more significant is the uncertainty principle. When a large body is observed, photons still bounce off it and change its momentum, but, considered as a fraction of the initial momentum of the body, the change is insignificant. The Schrödinger and Dirac theories give a precise value for the energy of each stationary state, but in reality the states do not have a precise energy.



The only exception is in the ground (lowest energy) state. Instead, the energies of the states are spread over a small range.



The spread arises from the fact that, because the electron can make a transition to another state, the initial state has a finite lifetime. The transition is a random process, and so different atoms in the same state have different lifetimes. If the mean lifetime is denoted as τ, the theory shows that the energy of the initial state has a spread of energy ΔE, given by This energy spread is manifested in a spread in the frequencies of emitted radiation. Therefore, the spectral lines are not infinitely sharp. (Some experimental factors can also broaden a line, but their effects can be reduced; however, the present effect, known as natural broadening, is fundamental and cannot be reduced.) Equation (13) is another type of Heisenberg uncertainty relation; generally, if a measurement with duration τ is made of the energy in a system, the measurement disturbs the system, causing the energy to be uncertain by an amount ΔE, the magnitude of which is given by the above equation. The application of quantum theory to the interaction between electrons and radiation requires a quantum treatment of Maxwell’s field equations, which are the foundations of electromagnetism, and the relativistic theory of the electron formulated by Dirac (see above Electron spin and antiparticles). The resulting quantum field theory is known as quantum electrodynamics, or QED. QED accounts for the behaviour and interactions of electrons, positrons, and photons. It deals with processes involving the creation of material particles from electromagnetic energy and with the converse processes in which a material particle and its antiparticle annihilate each other and produce energy.

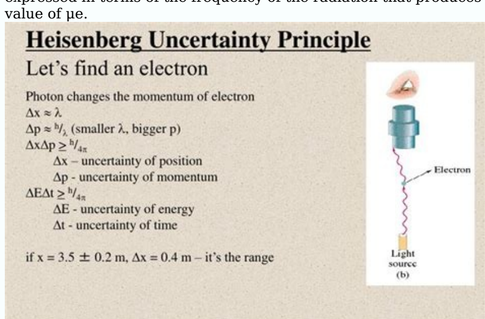


Initially the theory was beset with formidable mathematical difficulties, because the calculated values of quantities such as the charge and mass of the electron proved to be infinite. However, an ingenious set of techniques developed (in the late 1940s) by Hans Bethe, Julian S.

Schwinger, Tomonaga Shin’ichirō, Richard P. Feynman, and others dealt systematically with the infinities to obtain finite values of the physical quantities. Their method is known as renormalization. The theory has provided some remarkably accurate predictions. According to the Dirac theory, two particular states in hydrogen with different quantum numbers have the same energy. QED, however, predicts a small difference in their energies; the difference may be determined by measuring the frequency of the electromagnetic radiation that produces transitions between the two states.

This effect was first measured by Willis E. Lamb, Jr., and Robert Retherford in 1947.

Its physical origin lies in the interaction of the electron with the random fluctuations in the surrounding electromagnetic field. These fluctuations, which exist even in the absence of an applied field, are a quantum phenomenon. The accuracy of experiment and theory in this area may be gauged by two recent values for the separation of the two states, expressed in terms of the frequency of the radiation that produces the transitions: An even more spectacular example of the success of QED is provided by the value for μe, the magnetic dipole moment of the free electron. Because the electron is spinning and has electric charge, it behaves like a tiny magnet, the strength of which is expressed by the value of μe.



According to the Dirac theory, μe is exactly equal to μB = eℏ/2me, a quantity known as the Bohr magneton; however, QED predicts that μe = (1 + a)μB, where a is a small number, approximately 1/860.

Again, the physical origin of the QED correction is the interaction of the electron with random oscillations in the surrounding electromagnetic field. The best experimental determination of μe involves measuring not the quantity itself but the small correction term μe − μB. This greatly enhances the sensitivity of the experiment. The most recent results for the value of a are Since a itself represents a small correction term, the magnetic dipole moment of the electron is measured with an accuracy of about one part in 1011. One of the most precisely determined quantities in physics, the magnetic dipole moment of the electron can be calculated correctly from quantum theory to within about one part in 1010. By the end of this section, you will be able to: Describe the physical meaning of the position-momentum uncertainty relation Explain the origins of the uncertainty principle in quantum theory Describe the physical meaning of the energy-time uncertainty relation Heisenberg’s uncertainty principle is a key principle in quantum mechanics. Very roughly, it states that if we know everything about where a particle is located (the uncertainty of position is small), we know nothing about its momentum (the uncertainty of momentum is large), and vice versa. Versions of the uncertainty principle also exist for other quantities as well, such as energy and time. We discuss the momentum-position and energy-time uncertainty principles separately. To illustrate the momentum-position uncertainty principle, consider a free particle that moves along the x-direction. The particle moves with a constant velocity u and momentum p=mup=mu. According to de Broglie’s relations, p=ℏkp=ℏk and E=ℏωE=ℏω. As discussed in the previous section, the wave function for this particle is given by ψk(x,t)=A[cos(ωt−kx)−isin(ωt−kx)]=Ae−i(ωt−kx)=Ae−iωteikxψk(x,t)=A[cos(ωt−kx)−isin(ωt−kx)]=Ae−i(ωt−kx)=Ae−iωteikx 7.14 and the probability density |ψk(x,t)|2=A2|ψk(x,t)|2=A2 is uniform and independent of time.

The particle is equally likely to be found anywhere along the x-axis but has definite values of wavelength and wave number, and therefore momentum.

The uncertainty of position is infinite (we are completely uncertain about position) and the uncertainty of the momentum is zero (we are completely certain about momentum). This account of a free particle is consistent with Heisenberg’s uncertainty principle. Similar statements can be made of localized particles. In quantum theory, a localized particle is modeled by a linear superposition of free-particle (or plane-wave) states called a wave packet. An example of a wave packet is shown in Figure 7.9. A wave packet contains many wavelengths and therefore by de Broglie’s relations many momenta—possible in quantum mechanics! This particle also has many values of position, although the particle is confined mostly to the interval ΔxΔx. The particle can be better localized (Δx(Δx can be decreased) if more plane-wave states of different wavelengths or momenta are added together in the right way (Δp(Δp is increased). According to Heisenberg, these uncertainties obey the following relation.

The product of the uncertainty in position of a particle and the uncertainty in its momentum can never be less than one-half of the reduced Planck constant: ΔxΔp≥ℏ/2.ΔxΔp≥ℏ/2. This relation expresses Heisenberg’s uncertainty principle. It places limits on what we can know about a particle from simultaneous measurements of position and momentum. If ΔxΔx is large, ΔpΔp is small, and vice versa.

Equation 7.15 can be derived in a more advanced course in modern physics. Reflecting on this relation in his work The Physical Principles of the Quantum Theory, Heisenberg wrote “Any use of the words ‘position’ and ‘velocity’ with accuracy exceeding that given by [the relation] is just as meaningless as the use of words whose sense is not defined.” Figure 7.9 Adding together several plane waves of different wavelengths can produce a wave that is relatively localized. Note that the uncertainty principle has nothing to do with the precision of an experimental apparatus. Even for perfect measuring devices, these uncertainties would remain because they originate in the wave-like nature of matter. The precise value of the product ΔxΔpΔxΔp depends on the specific form of the wave function. Interestingly, the Gaussian function (or bell-curve distribution) gives the minimum value of the uncertainty product: ΔxΔp=ℏ/2.ΔxΔp=ℏ/2. The Uncertainty Principle Large and Small Determine the minimum uncertainties in the positions of the following objects if their speeds are known with a precision of 1.0×10−3m/s1.0×10−3m/s: (a) an electron and (b) a bowling ball of mass 6.0 kg. Strategy Given the uncertainty in speed Δu=1.0×10−3m/sΔu=1.0×10−3m/s, we have to first determine the uncertainty in momentum Δp=mΔuΔp=mΔu and then invert Equation 7.15 to find the uncertainty in position Δx=ℏ/(2Δp)Δx=ℏ/(2Δp).

Solution For the electron: Δp=mΔu=(9.1×10−31kg)(1.0×10−3m/s)=9.1×10−34kg·m/s,Δx=ℏ2Δp=5.8cm.Δp=mΔu=(9.1×10−31kg)(1.0×10−3m/s)=9.1×10−34kg·m/s,Δx=ℏ2Δp=5.8cm. For the bowling ball: Δp=mΔu=(6.0kg)(1.0×10−3m/s)=6.0×10−3kg·m/s,Δx=ℏ2Δp=8.8×10−33m.Δp=mΔu=(6.0kg)(1.0×10−3m/s)=6.0×10−3kg·m/s,Δx=ℏ2Δp=8.8×10−33m. Significance Unlike the position uncertainty for the electron, the position uncertainty for the bowling ball is immeasurably small.

Planck’s constant is very small, so the limitations imposed by the uncertainty principle are not noticeable in macroscopic systems such as a bowling ball. Uncertainty and the Hydrogen Atom Estimate the ground-state energy of a hydrogen atom using Heisenberg’s uncertainty principle. (Hint: According to early experiments, the size of a hydrogen atom is approximately 0.1 nm.) Strategy An electron bound to a hydrogen atom can be modeled by a particle bound to a one-dimensional box of length L=0.1nm.L=0.1nm. The ground-state wave function of this system is a half wave, like that given in Example 7.1. This is the largest wavelength that can “fit” in the box, so the wave function corresponds to the lowest energy state. Note that this function is very similar in shape to a Gaussian (bell curve) function. We can take the average energy of a particle described by this function ⟨E⟩ as a good estimate of the ground state energy ⟨E0⟩⟨E0⟩. This average energy of a particle is related to its average of the momentum squared, which is related to its momentum uncertainty. Solution To solve this problem, we must be specific about what is meant by “uncertainty of position” and “uncertainty of momentum.” We identify the uncertainty of position ⟨Δx⟩(Δx) with the standard deviation of position ⟨σx⟩(σx), and the uncertainty of momentum ⟨Δp⟩(Δp) with the standard deviation of momentum ⟨σp⟩(σp). For the Gaussian function, the uncertainty product is where σx2=x2−x–2andσp2=p2−p2.σx2=x2−x–2andσp2=p2−p2. The particle is equally likely to be moving left as moving right, so p–=0p–=0. Also, the uncertainty of position is comparable to the size of the box, so σx=L.σx=L. The estimated ground state energy is therefore E0=EGaussian=p2–m=σp22m=12m(ℏ2σx)2=12m(ℏ2L)2=ℏ28mL2.E0=EGaussian=p2–m=σp22m=12m(ℏ2σx)2=12m(ℏ2L)2=ℏ28mL2. Multiplying numerator and denominator by c2c2 gives E0=(ℏc)28(mc2)L2=(197.3eV·nm)28(0.511·106eV)(0.1nm)2=0.952eV≈1eV.E0=(ℏc)28(mc2)L2=(197.3eV·nm)28(0.511·106eV)(0.1nm)2=0.952eV≈1eV. Significance Based on early estimates of the size of a hydrogen atom and the uncertainty principle, the ground-state energy of a hydrogen atom is in the eV range. The ionization energy of an electron in the ground-state energy is approximately 10 eV, so this prediction is roughly confirmed. (Note: The product ℏcℏc is often a useful value in performing calculations in quantum mechanics.) Another kind of uncertainty principle concerns uncertainties in simultaneous measurements of the energy of a quantum state and its lifetime, where ΔEΔE is the uncertainty in the energy measurement and ΔtΔt is the uncertainty in the lifetime measurement. The energy-time uncertainty principle does not result from a relation of the type expressed by Equation 7.15 for technical reasons beyond this discussion. Nevertheless, the general meaning of the energy-time principle is that a quantum state that exists for only a short time cannot have a definite energy. The reason is that the frequency of a state is inversely proportional to time and the frequency connects with the energy of the state, so to measure the energy with good precision, the state must be observed for many cycles. To illustrate, consider the excited states of an atom. The finite lifetimes of these states can be deduced from the shapes of spectral lines observed in atomic emission spectra. Each time an excited state decays, the emitted energy is slightly different and, therefore, the emission line is characterized by a distribution of spectral frequencies (or wavelengths) of the emitted photons. As a result, all spectral lines are characterized by spectral widths. The average energy of the emitted photon corresponds to the theoretical energy of the excited state and gives the spectral location of the peak of the emission line. Short-lived states have broad spectral widths and long-lived states have narrow spectral widths. Atomic Transitions An atom typically exists in an excited state for about Δt=10−8sΔt=10−8s. Estimate the uncertainty ΔfΔf in the frequency of emitted photons when an atom makes a transition from an excited state with the simultaneous emission of a photon with an average frequency of f=7.1×1014Hzf=7.1×1014Hz. Is the emitted radiation monochromatic? Strategy We invert Equation 7.16 to obtain the energy uncertainty ΔE≈ℏ/2ΔtΔE≈ℏ/2Δt and combine it with the photon energy E=hfE=hf to obtain ΔfΔf. To estimate whether or not the emission is monochromatic, we evaluate Δf/fΔf/f. Solution The spread in photon energies is ΔE=hΔfΔE=hΔf. Therefore, ΔE≈ℏ2Δt⇒hΔf≈ℏ2Δt⇒Δf≈14πΔt=14π(10−8s)=8.0×106Hz,Δff=8.0×106Hz7.1×1014Hz=1.1×10−8.ΔE≈ℏ2Δt⇒hΔf≈ℏ2Δt⇒Δf≈14πΔt=14π(10−8s)=8.0×106Hz,Δff=8.0×106Hz7.1×1014Hz=1.1×10−8. Significance Because the emitted photons have their frequencies within 1.1×10−61.1×10−6 percent of the average frequency, the emitted radiation can be considered monochromatic. A sodium atom makes a transition from the first excited state to the ground state, emitting a 589.0-nm photon with energy 2.105 eV. If the lifetime of this excited state is 1.6×10−8s1.6×10−8s, what is the uncertainty in energy of this excited state? What is the width of the corresponding spectral line?