Notes: Exponents

The base is the number that is being repeatedly multiplied and the exponent dictates how many times the base is multiplied.

$2^3 = 8$ means that 2 is multiplied repeatedly 3 times, $2 \times 2 \times 2 = 8$

Exponent rules:

Preceding term only:
An exponent only applies to the term that directly precedes it.
For example, in $4x^3$, only the x has the 3 exponent. The 4 is unaffected by the exponent.

Implicit exponent of 1:
Every number or term that does not have an exponent showing, has an implicit exponent of 1.
Examples: $8 = 8^1$ $xy^3 = x^1y^3$

Zero exponent = 1
Any non-zero base with an exponent of 0 is equal to 1
Examples: $5^0 = 1$ $y^0 = 1$

Separation of different exponents when adding terms:
Terms with different exponents cannot be added together, just as terms with different variables can't be added together.
$x + x^2 + x^3$ cannot be simplified further and each of the terms would stay separate.

Multiplying the same base → Add the exponents
When multiplying terms with the same base, the base stays the same, and the exponents are added together.
Examples: $5^3 \times 5^2 = 5^5$ $(x^1)(x^1) = x^2$ $(x^3y^7)(x^2y^4) = x^5y^{11}$

Dividing the same base → Subtract exponents
Write the division problem as a fraction. Compare exponents of similar bases. Cross out the exponent of smaller value (with the base), then subtract the eliminated exponent from the remaining exponent of the same base. If the numerator is left empty, place 1 in the numerator.

Examples: $\frac{x^3}{x^5} = \frac{1}{x^{5-3}} = \frac{1}{x^2}$ $\frac{x^6y^3}{x^2y^9} = \frac{x^{6-2}}{y^{9-3}} = \frac{x^4}{y^6}$

Exponents within an exponent → Multiply exponents
Whenever an exponent is applied to a term in parentheses, multiply the existing exponent of every individual part of the term by the exponent outside the parentheses.

Examples:

$(3x^3)^2 = 3^{1\times2}x^{3\times2} = 3^2x^6 = 9x^6$ $\left(\frac{4x^5y^8}{3z^2}\right)^3 = \frac{4^{1\times3}x^{5\times3}y^{8\times3}}{3^{1\times3}z^{2\times3}} = \frac{4^3x^{15}y^{24}}{3^3z^6} = \frac{64x^{15}y^{24}}{27z^6}$

Negative exponents → Must convert to a positive exponent by switching to other side of fraction
Exponents cannot be left as negatives. Move the exponent with its base to the other side of the fraction. Doing this converts the exponent to a positive value. If the numerator is left empty, place 1 in the numerator.

Examples: $x^{-2} = \frac{x^{-2}}{1} = \frac{1}{x^2}$ $\frac{3}{x^{-3}y^2} = \frac{3x^3}{y^2}$ $x^4y^{-5} = \frac{x^4}{y^5}$ $\frac{x^{-3}}{y^{-2}} = \frac{y^2}{x^3}$

Fractional exponents: The numerator is applied as an exponent and the denominator is applied as a root
Solve fractional exponents as two parts. The numerator applies to the base as an exponent. The denominator applies to the base as a root. These can be applied in either order and will have the same solution.

Examples: $4^{\frac{3}{2}} = \sqrt[2]{4^3} = 2^3 = 8$ or $4^{\frac{3}{2}} = \sqrt[2]{4^3} = \sqrt[2]{64} = 8$

$81^{\frac{5}{4}} = \sqrt[4]{81^5} = 3^5 = 243$