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Smart Grid – Modeling and Control

Economic Despatch & Optimal Power Flow

University of Hawaii's Renewable Energy Design Laboratory (REDLab)
in collaboration with Powersim Inc. and MyWay

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News: Wind Blade Failure

Source: Peoria Journal Star

On Oct. 24, 2008, a 140 foot, 6.5 ton blade fell off from a Suzlon Energy wind turbine near Peoria.

Suzlon Energy is one of the world's largest wind turbine manufacturers.

Thermal Plants Failure:
CWLP Dallman Explosion, Fall 2007

Source: The State Journal-Register

CWLP Dallman Explosion, Fall 2007

Source: ebah

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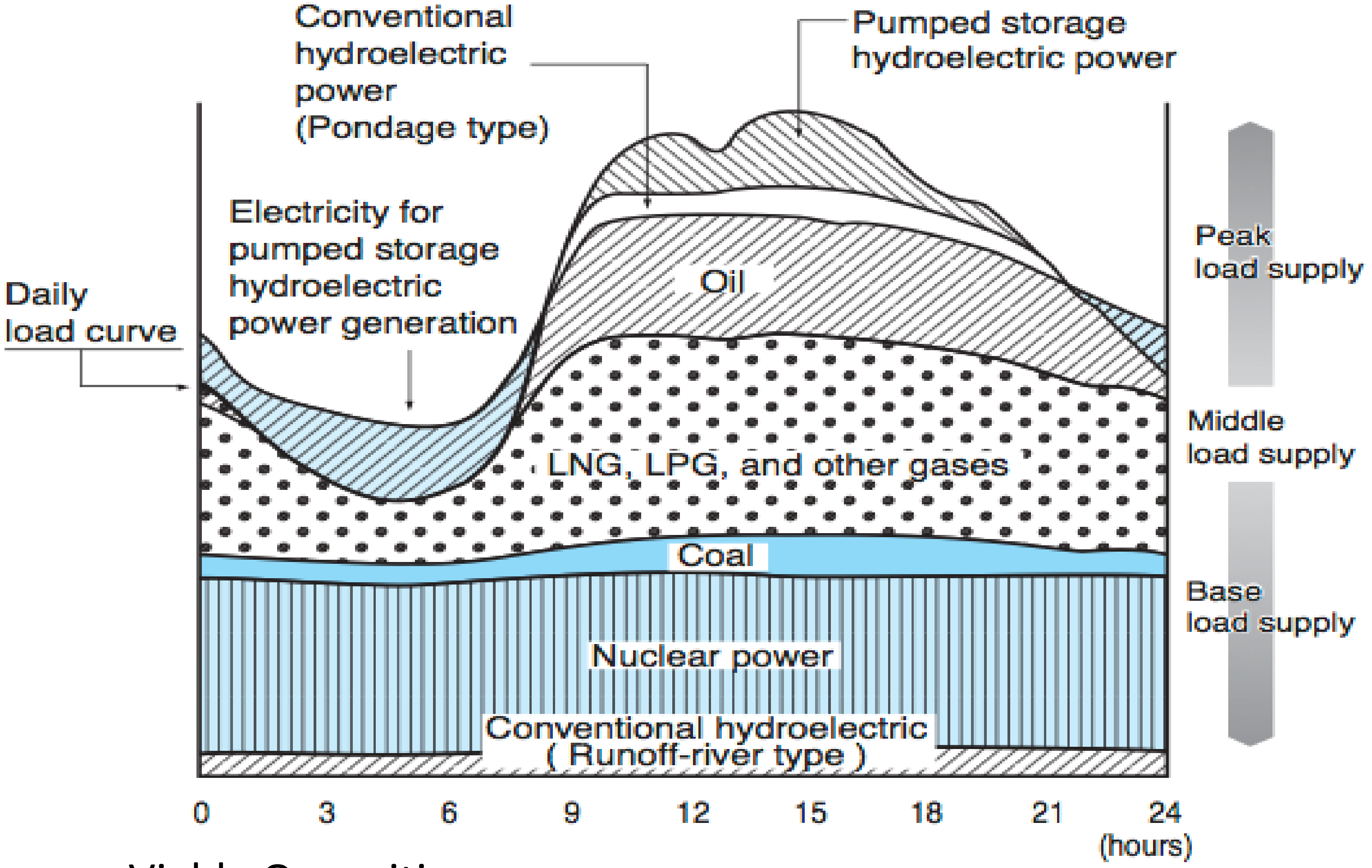
Thermal versus Hydro Generation

- The two main types of generating units:
  - Thermal and Hydro (Renewable picking up slowly)
- For hydro, the fuel (water) is free, but there may be many constraints on operation
  - Fixed amounts of water available
  - Reservoir levels must be managed and coordinated
  - Downstream flow rates for fish and navigation
- Hydro optimization typically requires many months or years

Daily Load Duration Curve



Source: Viable Opposition

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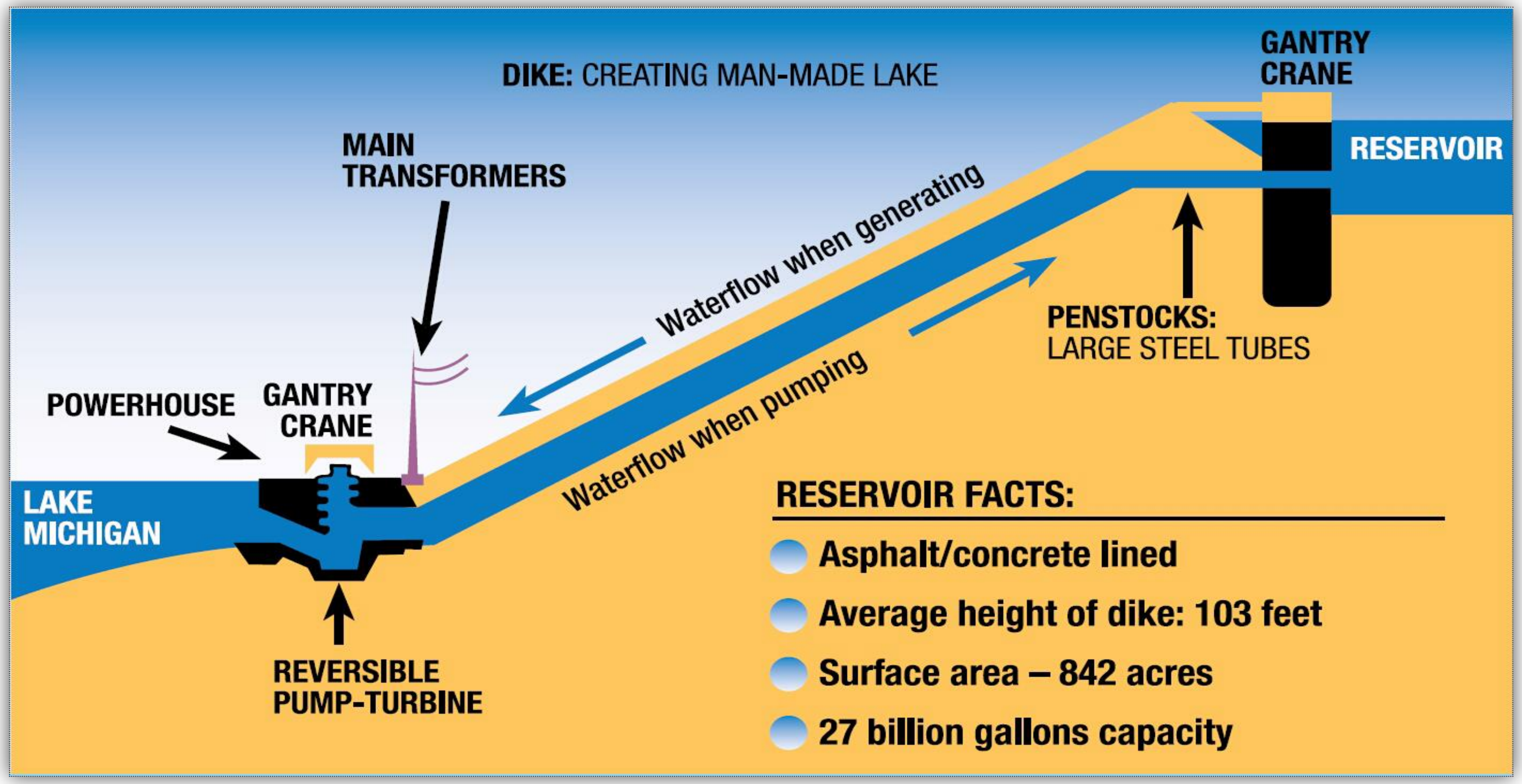
Hydro Power Plant

Source: investphilippines.org

Pumped Storage Hydro Power Plant



Source: Scandia Wind Offshore LLC

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Load Duration Curve: Generator Types

Traditionally utilities have three broad groups of generators:

1. Base load units: Large coal/nuclear; always on at max. capacity
2. Mid load units: Smaller coal that cycle on/off daily
3. Peaker units: Combustion turbines used only for several hours during periods of high demand

Thermal Power Plant Induced Pollution

Source: EPA

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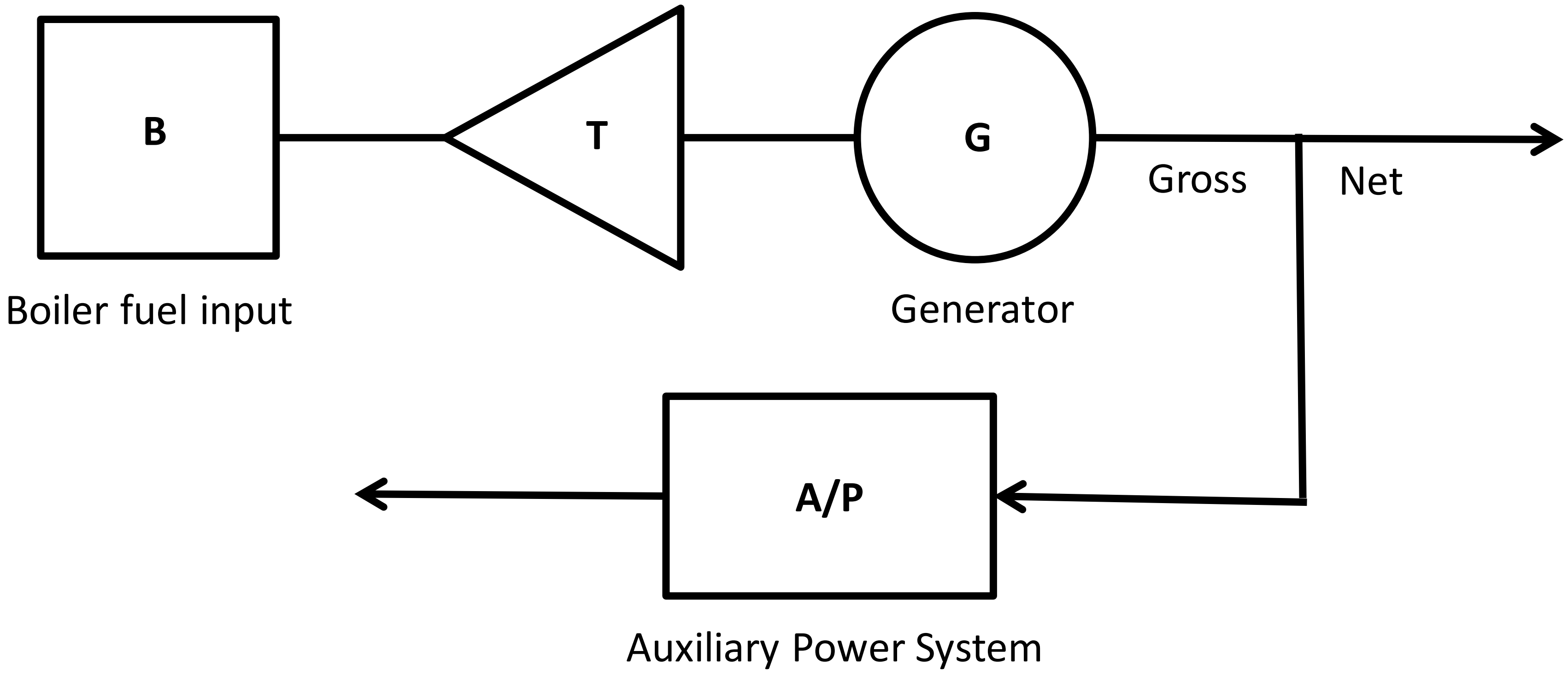
Boiler – Turbine – Generator Unit

Steam Turbine



2 – 6% of power generated is used within the generating plant; this is known as the **auxiliary power**.

Generator Cost Curves

- Generator costs are typically represented by three to four different curves.
  - Input/Output (I/O) Characteristics
  - Incremental Characteristics
  - Net Heat Rate Characteristics
- Reference:
  - 1 Btu (British thermal unit) = 1054 J
  - 1 MBtu = 1×10^6 Btu
  - 1 MBtu = 0.29 MWh

Input/Output Characteristics

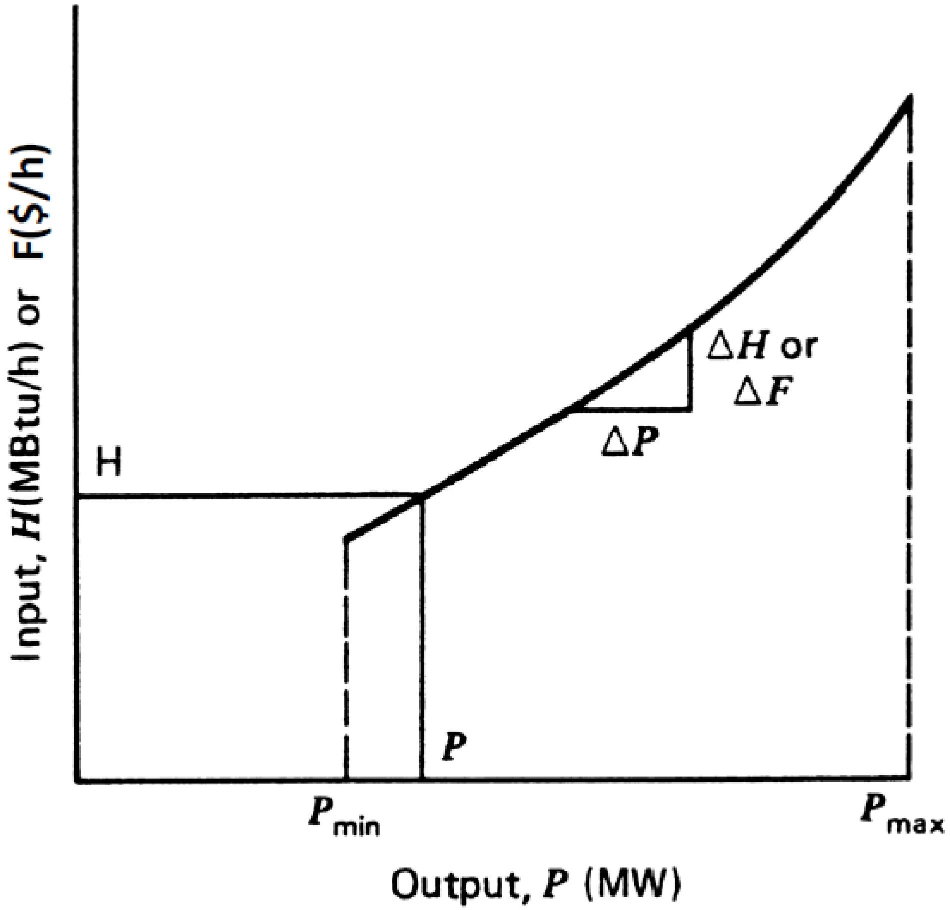
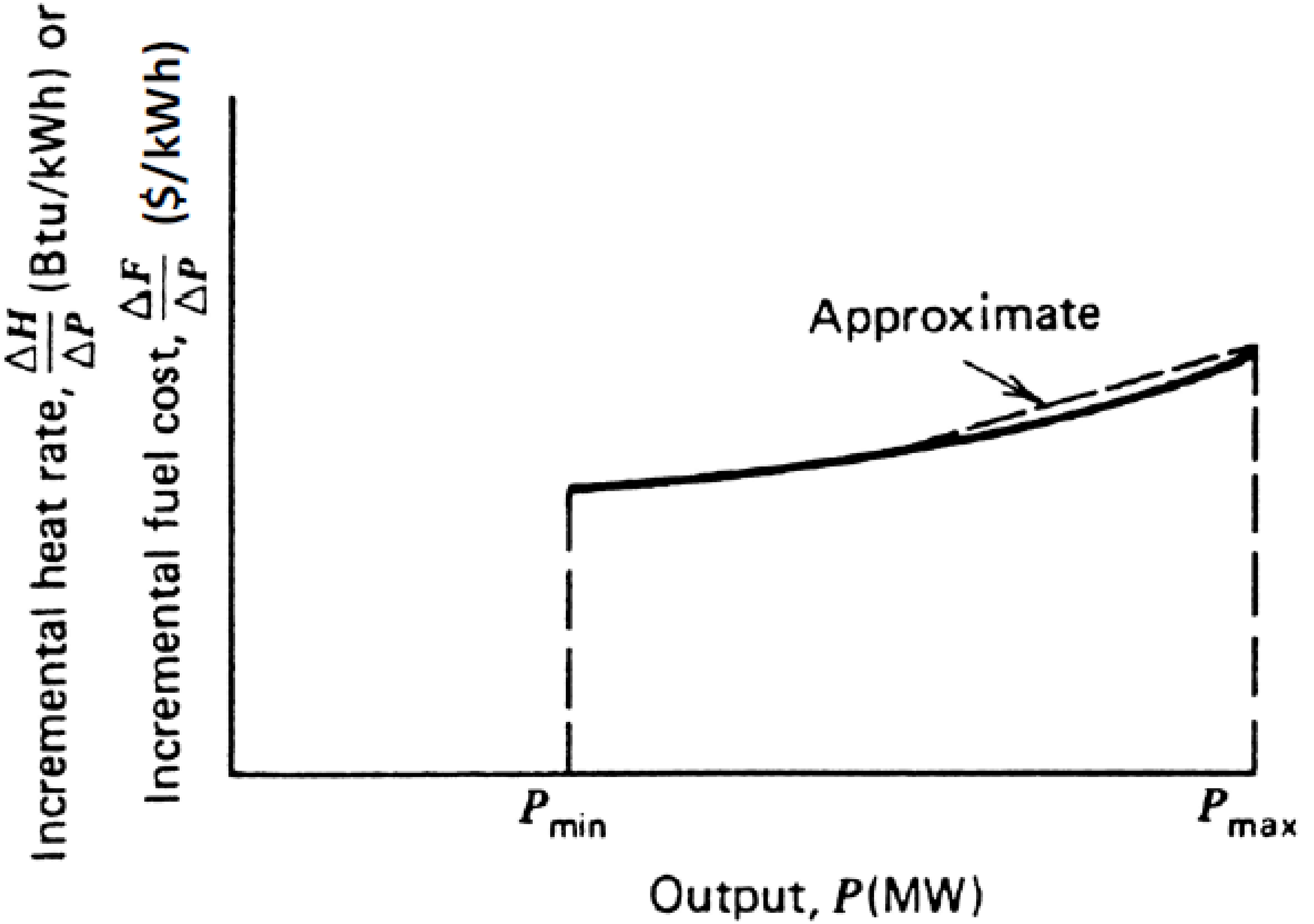
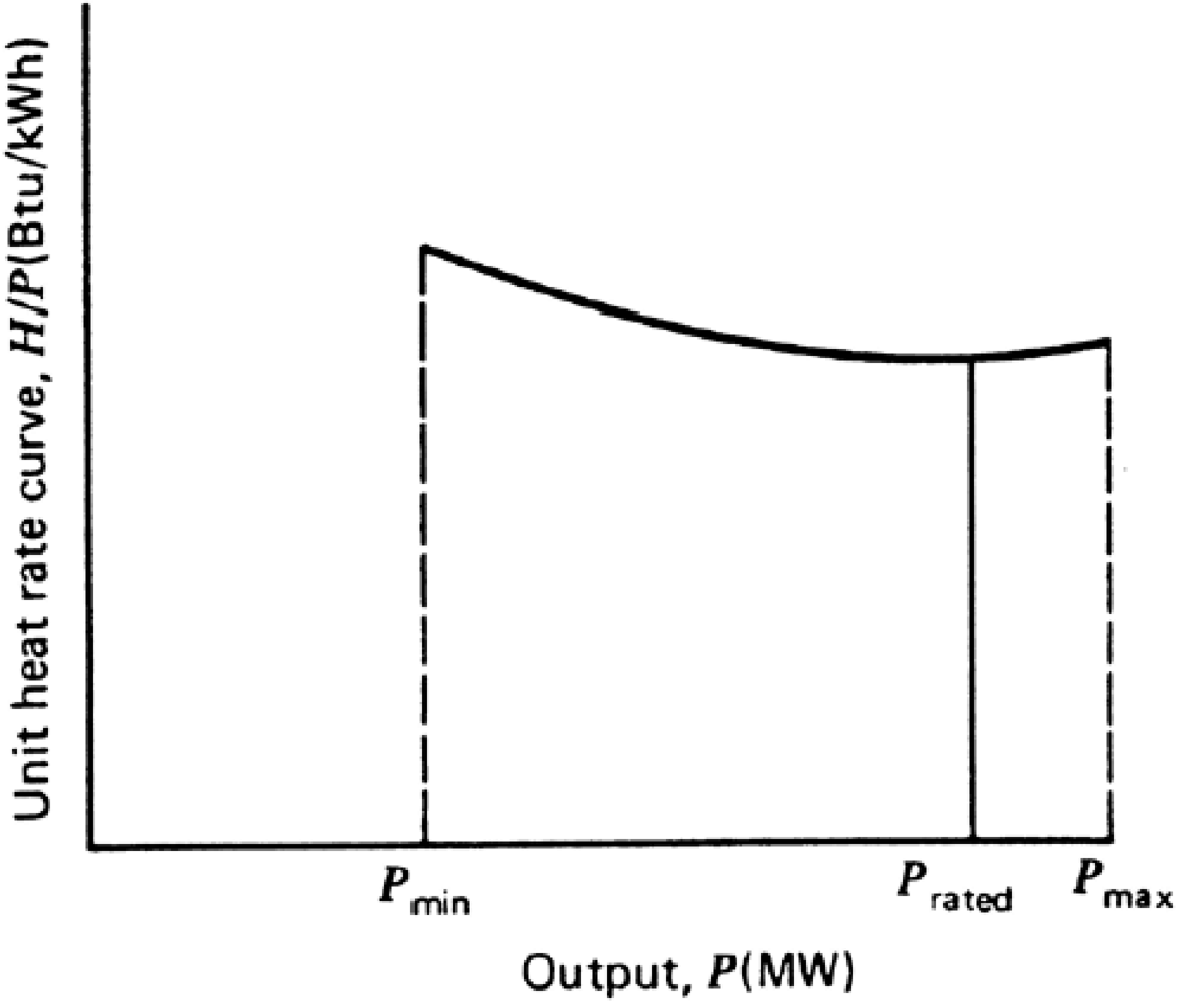


Figure 1.) The I/O curve plots fuel input in MBtu/hr versus net MW output.

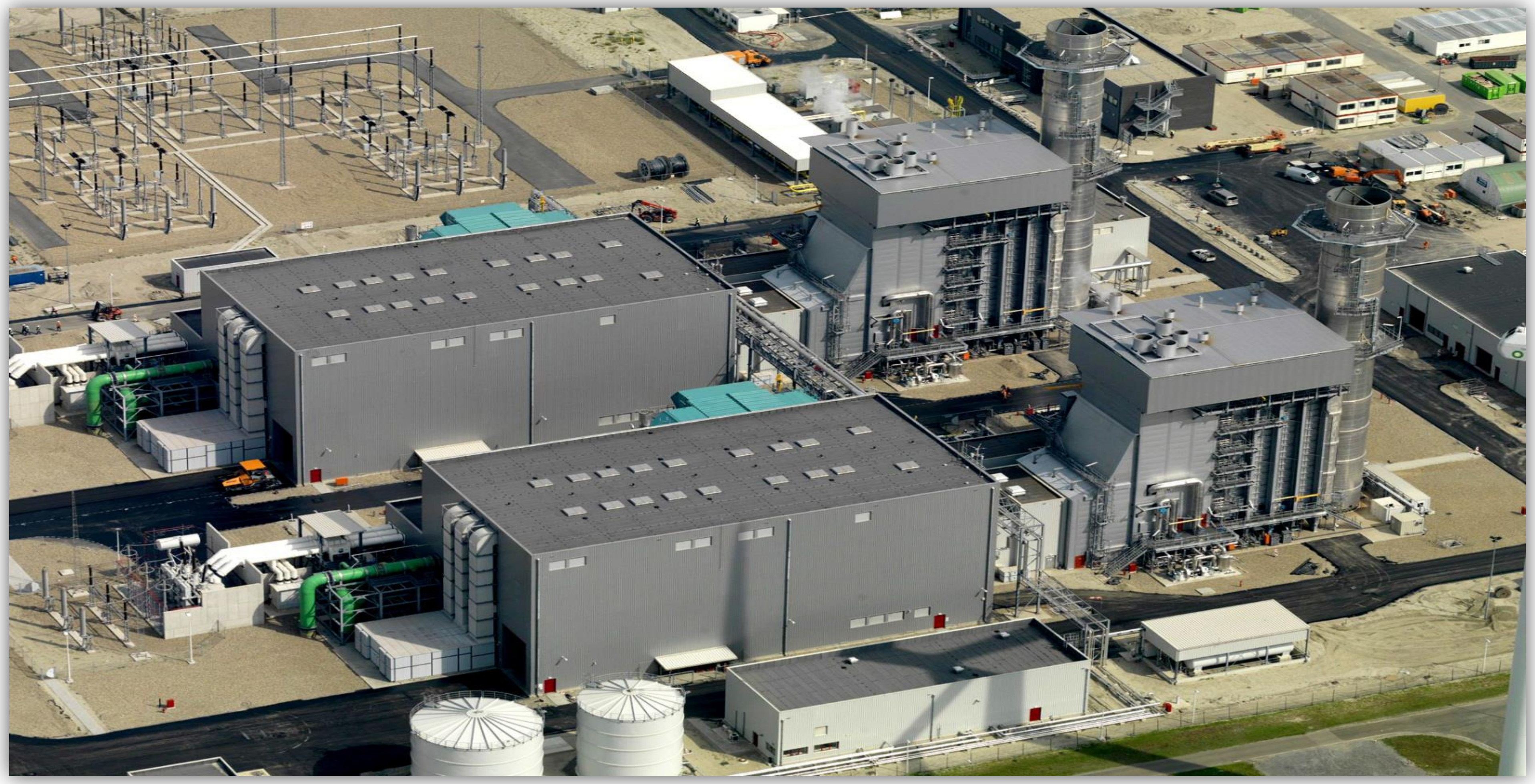
Incremental Heat Rate Characteristics



Net Heat Rate Characteristics



Bird’s Eye View of a Thermal Power Plant



Source: Siemens

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Mathematical Formulation of Costs

- Generator cost curves are usually not smooth.
- The curves, however, can be adequately approximated using piece-wise, smooth functions.
- Two representations dominate:
  - Quadratic or cubic functions
  - Piece-wise linear functions

We can assume a quadratic equation exists, such that…

$$C_i(P_{Gi}) = \alpha_i + \beta P_{Gi} + \gamma P_{Gi}^2 \quad \text{\$/hr (fuel-cost)}$$

$$IC_i(P_{Gi}) = \frac{dC_i(P_{Gi})}{dP_{Gi}} = \beta + 2\gamma P_{Gi} \quad \text{\$/MWh}$$

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Example #1: Coal Usage

A 500 MW (net) generator is 35% efficient. It is being supplied with Western grade coal, which costs \$1.70 per MBtu and has 9000 Btu per pound. What is the coal usage in lbs/hr? What is the cost?

At 35% efficiency required fuel input per hour is

$$\frac{500\text{ MWh}}{\text{hr}\times 0.35}=\frac{1428\text{ MWh}}{\text{hr}}\times\frac{1\text{ MBtu}}{0.29\text{ MWh}}=\frac{4924\text{ MBtu}}{\text{hr}}$$

$$\frac{4924\text{ MBtu}}{\text{hr}}\times\frac{1\text{ lb}}{0.009\text{MBtu}}=\frac{547{,}111\text{ lbs}}{\text{hr}}$$

$$\text{Cost}=\frac{4924\text{ MBtu}}{\text{hr}}\times\frac{\$1.70}{\text{MBtu}}=8370.8\text{ \$/hr or \$16.74/MWh}$$

Example #2: Wasting Coal

Assume a 100W lamp is left on by mistake for 8 hours, and the electricity is supplied by the previous coal plant. Also, the transmission and distribution losses are 20%. How much irreplaceable coal has this person wasted?

With 20% losses, a 100W load on for 8 hrs requires 1 kWh of energy. With 35% gen. efficiency this requires

$$\frac{1 \text{ kWh}}{0.35} \times \frac{1 \text{ MWh}}{1000 \text{ kWh}} \times \frac{1 \text{ MBtu}}{0.29 \text{ MWh}} \times \frac{1 \text{ lb}}{0.009 \text{MBtu}} = 1.09 \text{ lb}$$

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Example #3: Incremental Cost

For a two generator system assume

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \quad \$/hr$$

$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \quad \$/hr$$

Then

$$IC_1(P_{G1}) = \frac{dC_1(P_{G1})}{dP_{G1}} = 20 + 0.02P_{G1} \quad \text{\$/MWh}$$

$$IC_2(P_{G2}) = \frac{dC_2(P_{G2})}{dP_{G2}} = 15 + 0.06P_{G2} \quad \text{\$/MWh}$$

Example #3 cont'd.

If $P_{G1} = 250$ MW and $P_{G2} = 150$ MW Then

$$C_1(250) = 1000 + 20 \times 250 + 0.01 \times 250^2 = \$ \ 6625/\text{hr}$$

$$C_2(150) = 400 + 15 \times 150 + 0.03 \times 150^2 = \$6025/\text{hr}$$

Then

$$IC_1(250) = 20 + 0.02 \times 250 = \$ \ 25/\text{MWh}$$

$$IC_2(150) = 15 + 0.06 \times 150 = \$ \ 24/\text{MWh}$$

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Solution using the LaGrange Function

$$L(x_1, x_2, \lambda) = f(x_1, x_2) + \lambda \omega(x_1, x_2)$$

This problem now has three variables : x_1, x_2, λ

At the optimum :

$$\frac{\partial L}{\partial x_1} = 0$$

$$\frac{\partial L}{\partial x_2} = 0$$

$$\frac{\partial L}{\partial \lambda} = 0$$

Example #4: LaGrange Function

LaGrangian Function :

$$L(x_1, x_2, \lambda) = 0.25x_1^2 + x_2^2 + \lambda(5 - x_1 - x_2)$$

The solution is found by solving these three equations.

$$\frac{\partial L}{\partial x_1} = 0.5x_1 - \lambda = 0$$

$$\frac{\partial L}{\partial x_2} = 2x_2 - \lambda = 0$$

Solution :: $x_1 = 4, x_2 = 1, \lambda = 2$

$$\frac{\partial L}{\partial \lambda} = 5 - x_1 - x_2 = 0$$

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Adding Inequality Constraints

Minimize :

$f(x)$

Subject to :

$\omega_i(x) |= 0 \quad i = 1,2,\ldots,N\omega$

$g_i(x) |\leq 0 \quad i = 1,2,\ldots,Ng$

$x =$ vector of real numbers, dimension $=$ N

By adding the less than or equal to constraint "g" we must follow more elaborate procedures than simply finding where the gradient of the Lagrangian is equal to zero.

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The Karush-Kuhn Tucker Conditions

$$L(x,\lambda,\mu) = f(x) + \sum_{i=1}^{N\omega} \lambda_i \, \omega_i(x) + \sum_{i=1}^{Ng} \mu_i g_i(x)$$

The conditions for the optimum point x^0, λ^0, μ^0 are

1. $\frac{\partial L}{\partial x_i}(x^0,\lambda^0,\mu^0) = 0$ for $i = 1 \dots N$ All partial derivatives equal zero
2. $\omega_i(x^0) = 0$ for $i = 1 \dots N\omega$ Restatement of constraint condition
3. $g_i(x^0) = 0$ for $i = 1 \dots Ng$ Restatement of constraint condition
4. $\left. \begin{matrix} \mu_i^0 g_i(x^0) = 0 \\ \mu_i^0 \geq 0 \end{matrix} \right\rangle$ for $i = 1 \dots Ng$ Complementary slackness condition

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Economic Dispatch: Problem Formulation

The goal of economic dispatch is to determine the generation dispatch that minimizes the instantaneous operating cost, subject to the constraint that total generation = total load + losses.

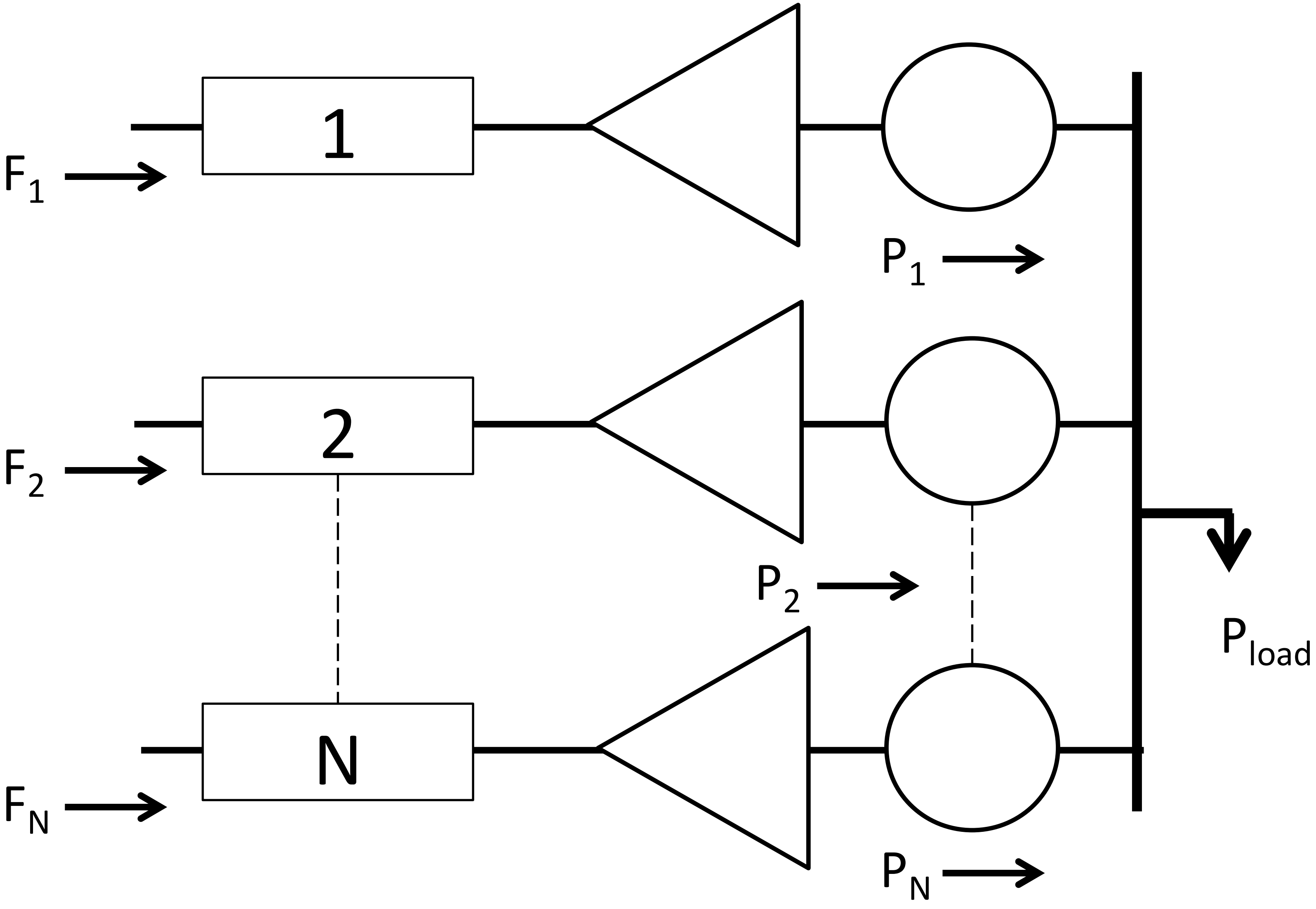
Minimize $$C_T \triangleq \sum_{i=1}^{m} C_i(P_{Gi})$$

Such that

$$\sum_{i=1}^{m} P_{Gi} = P_D + P_{Losses}$$

Initially, we'll ignore generator limits and the losses

Economic Dispatch Problem: Without Losses



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Economic Dispatch LaGrangian

For the economic dispatch we have a minimization constrained with a single equality constraint

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D - \sum_{i=1}^{m} P_{Gi}) \quad \text{(no losses)}$$

The necessary conditions for minimum are

$$\frac{\partial \mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda = 0 \quad (\text{for } i = 1 \text{ to } m)$$

$$P_D - \sum_{i=1}^{m} P_{Gi} = 0$$

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Example #5: Economic Dispatch

What is economic dispatch for a two generator system $P_D = P_{G1} + P_{G2} = 500$ MW and

$$C_1(P_{G1}) \;=\; 1000 + 20P_{G1} + 0.01P_{G1}^2 \qquad \$/\text{h}$$

$$C_2(P_{G2}) \;=\; 400 + 15P_{G2} + 0.03P_{G2}^2 \qquad \$/\text{h}$$

Using the Lagrange multiplier method we know:

$$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda = 20 + 0.02P_{G1} - \lambda = 0$$

$$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda = 15 + 0.06P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

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Example #5 cont'd.

We therefore need to solve three linear equations

$$20 + 0.02 P_{G1} - \lambda = 0$$

$$15 + 0.06 P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

$$\begin{bmatrix} 0.02 & 0 & -1 \\ 0 & 0.06 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ -500 \end{bmatrix}$$

$$\begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 312.5 \text{ MW} \\ 187.5 \text{ MW} \\ 26.2 \text{ \$/MWh} \end{bmatrix}$$

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Example #6: Economic Dispatch of 3 Generators (Load 850 MW)

Unit 1 : Coal - fired steam unit : Max output = 600 MW
Min output = 150 MW

Input–output curve:

$$H_1\left(\frac{\text{MBtu}}{\text{h}}\right) = 510.0 + 7.2P_1 + 0.00142P_1^2$$

Unit2 : Oil – firedsteamunit : Max output = 400 MW
Min output = 100 MW

Input–output curve:

$$H_2\left(\frac{\text{MBtu}}{\text{h}}\right) = 310.0 + 7.85P_2 + 0.00194P_2^2$$

Unit3 : Oil – firedsteamunit : Max output = 200 MW
Min output = 50 MW

Input–output curve:

$$H_3\left(\frac{\text{MBtu}}{\text{h}}\right) = 78.0 + 7.97P_3 + 0.00482P_3^2$$

Unit1 : fuel cost = 1.1 \$ / MBtu
Unit2 : fuel cost = 1.0 \$ / MBtu
Unit3 : fuel cost = 1.0 \$ / MBtu

Example #6: Solution

$$F_1(P_1) = H_1(P_1) \times 1.1 = 561 + 7.92 P_1 + 0.001562 P_1^2 \ \$/h$$

$$F_2(P_2) = H_2(P_2) \times 1.0 = 310 + 7.85 P_2 + 0.00194 P_2^2 \ \$/h$$

$$F_3(P_3) = H_3(P_3) \times 1.0 = 78 + 7.97 P_3 + 0.00482 P_3^2 \ \$/h$$

Applying the conditions of optimum dispatch

$$\frac{dF_1}{dP_1} = 7.92 + 0.003124 P_1 = \lambda$$

$$\frac{dF_2}{dP_2} = 7.85 + 0.00388 P_2 = \lambda$$

Problem #1: Solution

$$\frac{dF_3}{dP_3} = 7.97 + 0.00964 P_3 = \lambda \text{ and } P_1 + P_2 + P_3 = 850 \text{ MW}$$

Solving for λ, one obtains

$$\lambda = 9.148 \,\$ / MWh$$

then solving for P_1, P_2, P_3

$$P_1 = 393.2 \text{ MW}$$

$$P_2 = 334.6 \text{ MW}$$

$$P_3 = 122.2 \text{ MW}$$

Note that all contraints are met; that is,
each unit is within its high and low limit
and the total output when summed over
all three units meet the desired 850 MW total.

Assignment #1

1. If the coal price is reducing to 0.9$/Mbtu, what is the economic dispatch?

2. If Unit 1 is set at maximum output and Unit 2 to the minimum output, what is the economic dispatch?

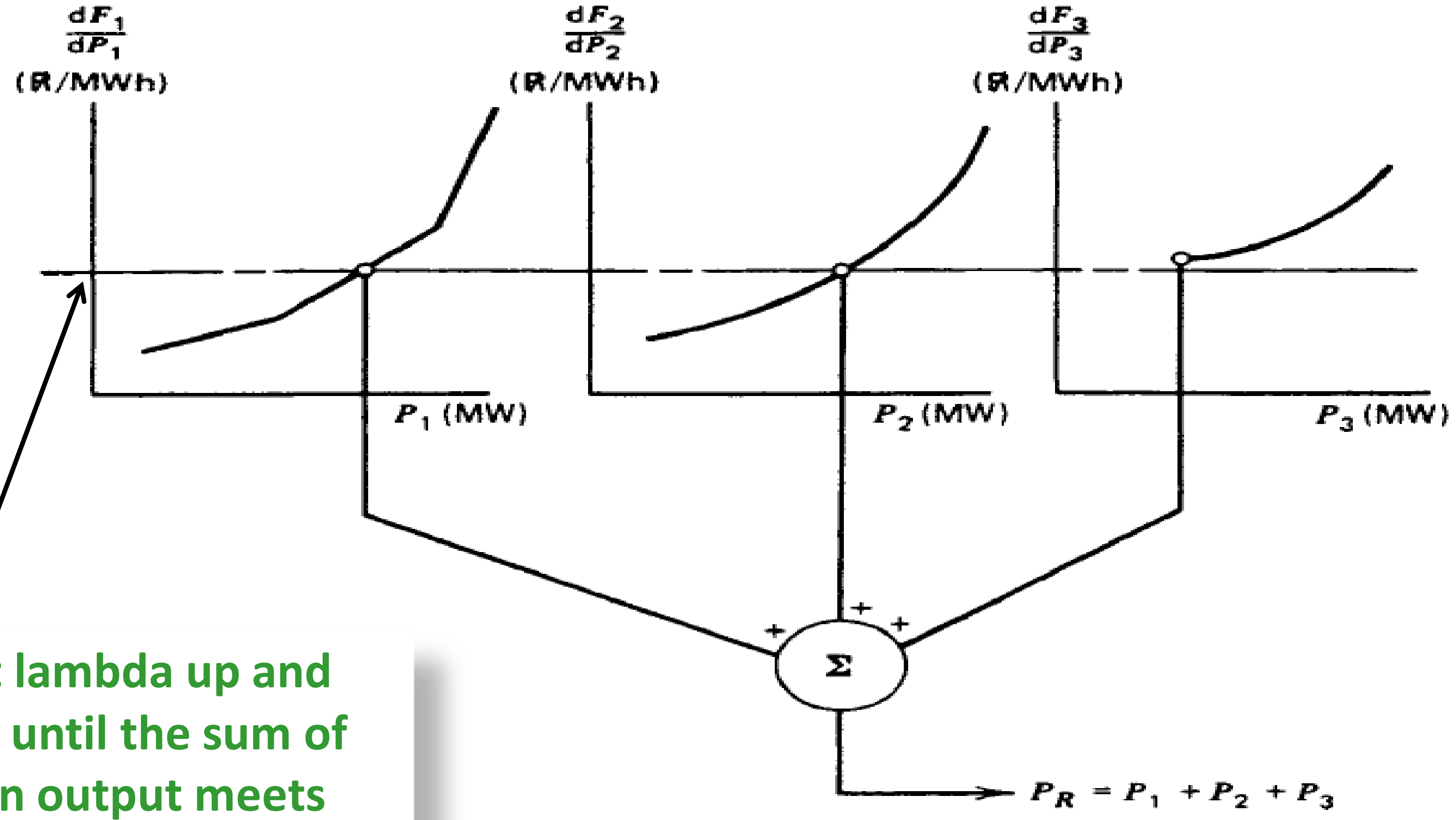
Lambda-Iteration Solution Method

- The direct solution using Lagrange multipliers only works if the generators are **not** at their limits.
- Another method is the Lambda-Iteration Method
- The method requires that there to be a unique mapping from a value of lambda (marginal cost) to each generator's MW output = $P_{Gi}(\lambda)$.

Lambda-Iteration Solution Method

- For any choice of lambda (marginal cost), the generators collectively produce a total MW output.
- The method then starts with values of lambda below and above the optimal value (corresponding to too little and too much total output), then iteratively brackets the optimal value.

Lambda-Iteration: Graphical View



Graphical solution to economic dispatch.

Adjust lambda up and down, until the sum of the gen output meets the load to be supplied.

Explanation

- Draw the individual incremental cost in the same scale.
- On the vertical axis and then line them up as shown so that it will give us a value for power output for each generator assuming we are going to make the lambda the same for each generator.
- If the lambda comes below a generator's minimum we hold it at P_{min}, if above the generator's max we hold it at P_{max}.

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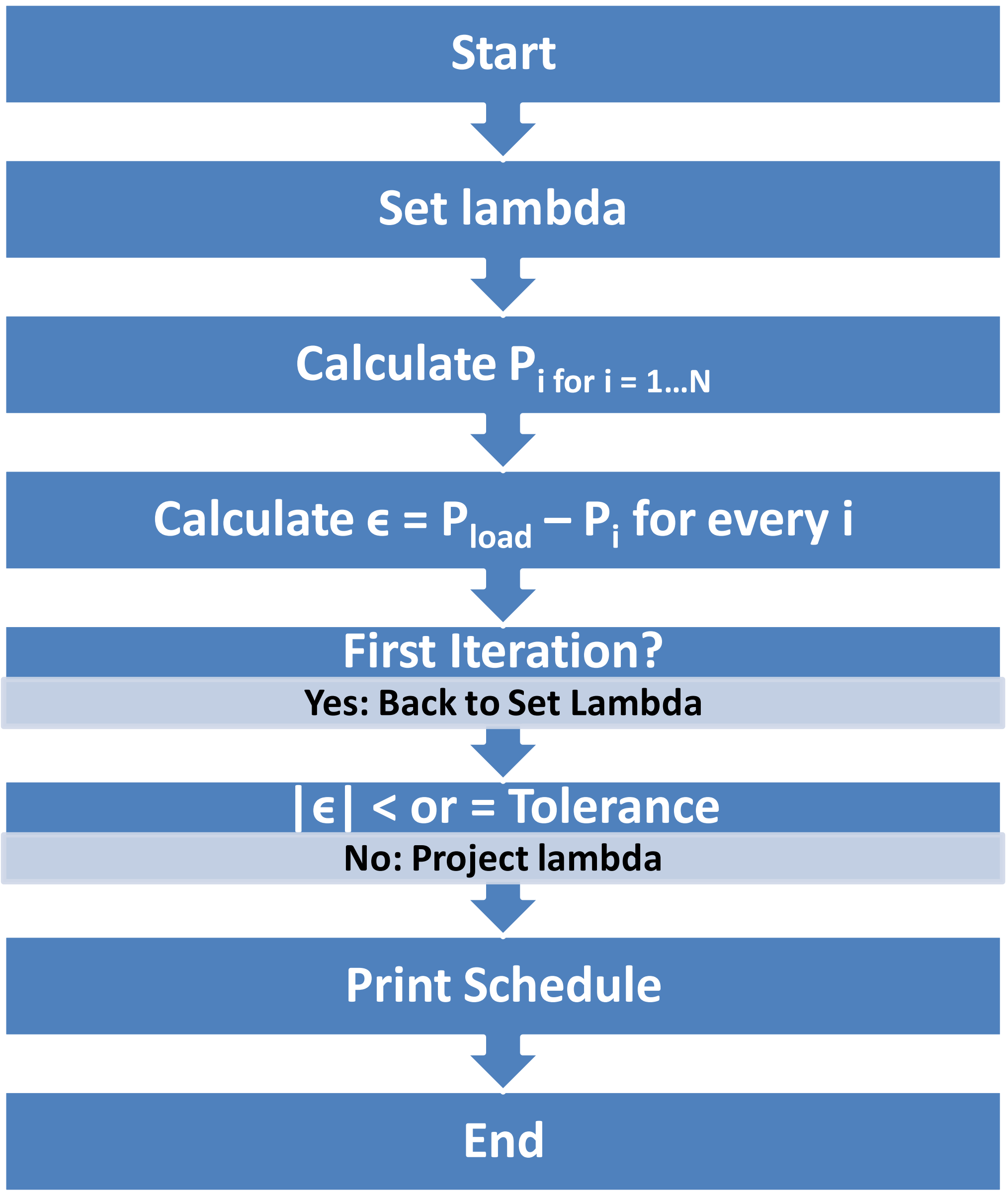
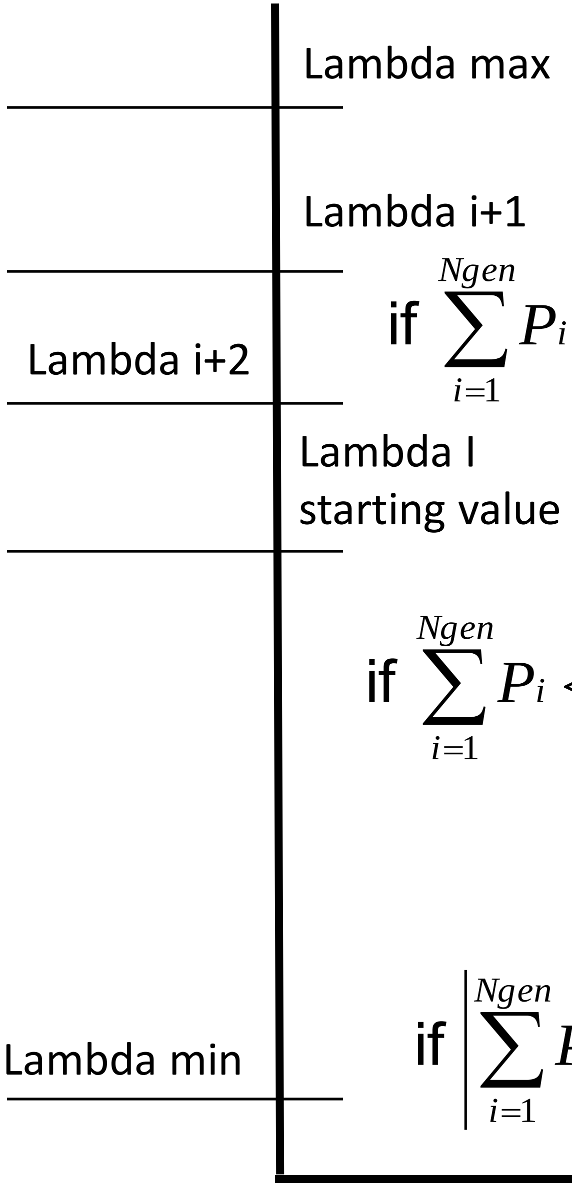
Lambda-Iteration Method- Flow Chart



Determining Lambda Using Binary Search



$$\Delta\lambda = (\lambda_{\min} + \lambda_{\max})/2$$

$$\lambda_i = \lambda_{\min} + \Delta\lambda$$

if $\sum_{i=1}^{Ngen} P_i > P_{load}$ we must reduce lambda so then

$$\Delta\lambda = \lambda/2$$

$$\lambda_{i+1} = \lambda_t - \Delta\lambda$$

if $\sum_{i=1}^{Ngen} P_i < P_{load}$ we must increase lambda so then

$$\Delta\lambda = \lambda/2$$

$$\lambda_{i+1} = \lambda_t + \Delta\lambda$$

if $\left|\sum_{i=1}^{Ngen} P_i - P_{load}\right| \leq$ tolerance alogirthm is done

Lambda-Iteration Algorithm

Pick λ^{L}(min) and λ^{H}(max) such that

$$\sum_{i=1}^{m} P_{Gi}(\lambda^{L}) - P_D < 0 \qquad \sum_{i=1}^{m} P_{Gi}(\lambda^{H}) - P_D > 0$$

While $\left|\lambda^{\mathrm{H}} - \lambda^{\mathrm{L}}\right| > \varepsilon$ Do

$$\lambda^{\mathrm{M}} = (\lambda^{\mathrm{H}} + \lambda^{\mathrm{L}})/2$$

If $\sum_{i=1}^{m} P_{Gi}(\lambda^{M}) - P_D > 0$ Then $\lambda^{\mathrm{H}} = \lambda^{\mathrm{M}}$

Else $\lambda^{\mathrm{L}} = \lambda^{\mathrm{M}}$

End While

Example #7: Lambda-Iteration

Consider a three generator system with

$$IC_1(P_{G1}) = 15 + 0.02 P_{G1} = \lambda \quad \$/\text{MWh}$$
$$IC_2(P_{G2}) = 20 + 0.01 P_{G2} = \lambda \quad \$/\text{MWh}$$
$$IC_3(P_{G3}) = 18 + 0.025 P_{G3} = \lambda \quad \$/\text{MWh}$$

and with constraint $P_{G1} + P_{G2} + P_{G3} = 1000\,\text{MW}$

Rewriting generation as a function of λ, $P_{Gi}(\lambda)$,

we have

$$\text{P}_{\text{G1}}(\lambda) = \frac{\lambda - 15}{0.02} \qquad \text{P}_{\text{G2}}(\lambda) = \frac{\lambda - 20}{0.01}$$

$$\text{P}_{\text{G3}}(\lambda) = \frac{\lambda - 18}{0.025}$$

Lambda-Iteration Example, cont'd

Pick λ^L so $\sum_{i=1}^{m} P_{Gi}(\lambda^L) - 1000 < 0$ and

$$\sum_{i=1}^{m} P_{Gi}(\lambda^H) - 1000 > 0$$

Try $\lambda^L = 20$ then $\sum_{i=1}^{m} P_{Gi}(20) - 1000 =$

$$\frac{\lambda - 15}{0.02} + \frac{\lambda - 20}{0.01} + \frac{\lambda - 18}{0.025} - 1000 = -670 \text{ MW}$$

Try $\lambda^H = 30$ then $\sum_{i=1}^{m} P_{Gi}(30) - 1000 = 1230 \text{ MW}$

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Lambda-Iteration Example, cont'd

Pick convergence tolerance $\varepsilon = 0.05$ \$/MWh

Then iterate since $\left|\lambda^H - \lambda^L\right| > 0.05$

$$\lambda^M = (\lambda^H + \lambda^L)/2 = 25$$

$$\text{Then since } \sum_{i=1}^{m} P_{Gi}(25) - 1000 = 280 \text{ we set } \lambda^H = 25$$

Since $\left|25 - 20\right| > 0.05$

$$\lambda^M = (25 + 20)/2 = 22.5$$

$$\sum_{i=1}^{m} P_{Gi}(22.5) - 1000 = -195 \text{ we set } \lambda^L = 22.5$$

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Lambda-Iteration Example, cont'd

Continue iterating until $\left|\lambda^{H} - \lambda^{L}\right| < 0.05$

The solution value of λ, λ^{*}, is 23.53 \$/MWh

Once λ^{*} is known we can calculate the P_{Gi}

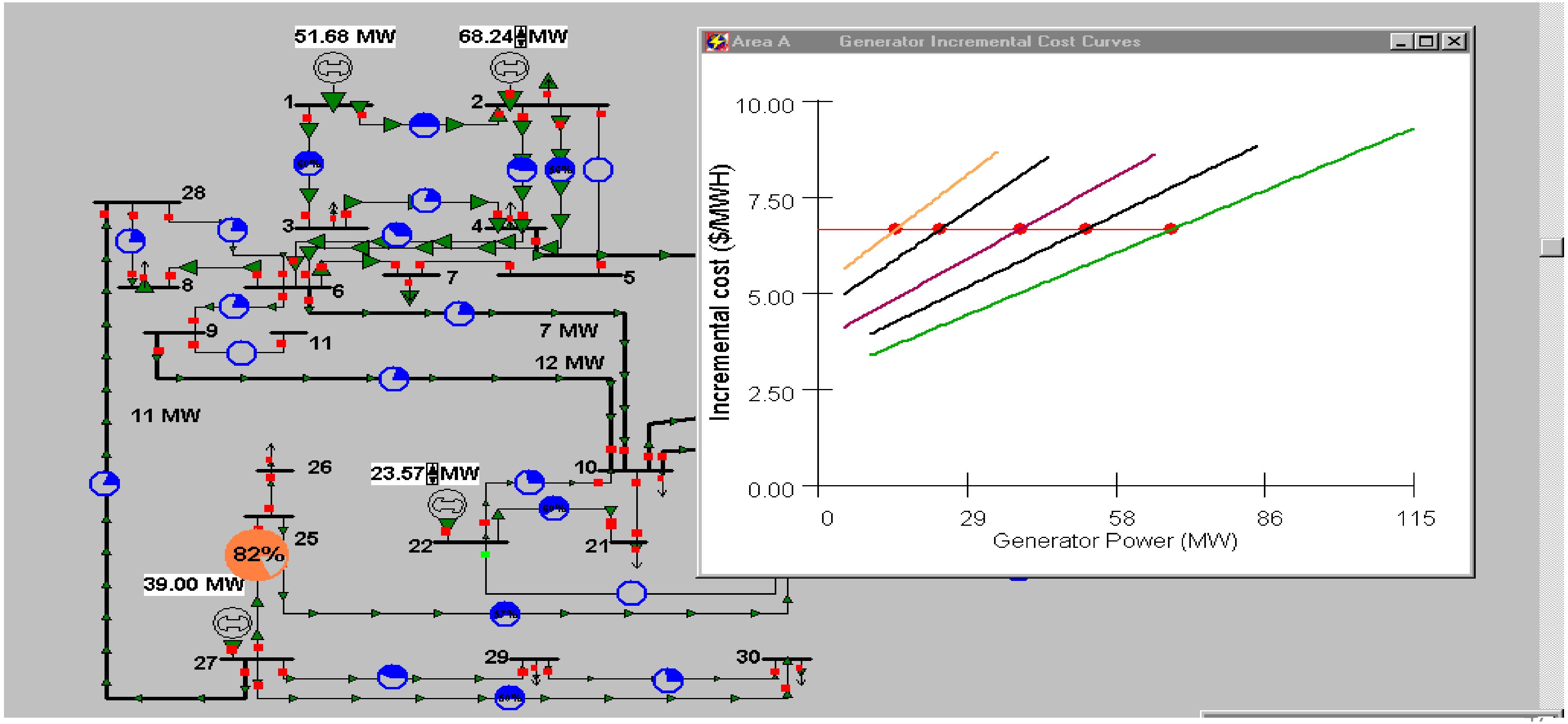
$$P_{G1}(23.5) = \frac{23.53 - 15}{0.02} = 426 \text{ MW}$$

$$P_{G2}(23.5) = \frac{23.53 - 20}{0.01} = 353 \text{ MW}$$

$$P_{G3}(23.5) = \frac{23.53 - 18}{0.025} = 221 \text{ MW}$$

Thirty Bus ED Example

Case is economically dispatched (without considering the incremental impact of the system losses).



Generator MW Limits

- Generators have limits on the minimum and maximum amount of power they can produce.
- Typically, the minimum limit is not zero.
- Because of varying system economics usually many generators in a system are operated at their maximum MW limits:
- Base load generators are at their maximum limits except during the off-peak.

Lambda-Iteration with Gen Limits

In the lambda-iteration method the limits are taken into account when calculating $P_{Gi}(\lambda)$:

$$\text{if calculated production for } P_{Gi} > P_{Gi,\max}$$

$$\text{then set } P_{Gi}(\lambda) = P_{Gi,\max}$$

$$\text{if calculated production for } P_{Gi} < P_{Gi,\min}$$

$$\text{then set } P_{Gi}(\lambda) = P_{Gi,\min}$$

Example #8: Lambda-Iteration with Gen Limit

In the previous three generator example assume the same cost characteristics but also with limits

$$0 \le P_{G1} \le 300 \text{ MW} \qquad 100 \le P_{G2} \le 500 \text{ MW}$$

$$200 \le P_{G3} \le 600 \text{ MW}$$

With limits we get:

$$\sum_{i=1}^{m} P_{Gi}(20) - 1000 = P_{G1}(20) + P_{G2}(20) + P_{G3}(20) - 1000$$

$$= \quad 250 + 100 + 200 - 1000$$

$$= -450 \text{ MW (compared to } -670\text{MW)}$$

$$\sum_{i=1}^{m} P_{Gi}(30) - 1000 = 300 + 500 + 480 - 1000 = 280 \text{ MW}$$

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Example #8: Lambda-Iteration with Gen Limit

Again we continue iterating until the convergence condition is satisfied.

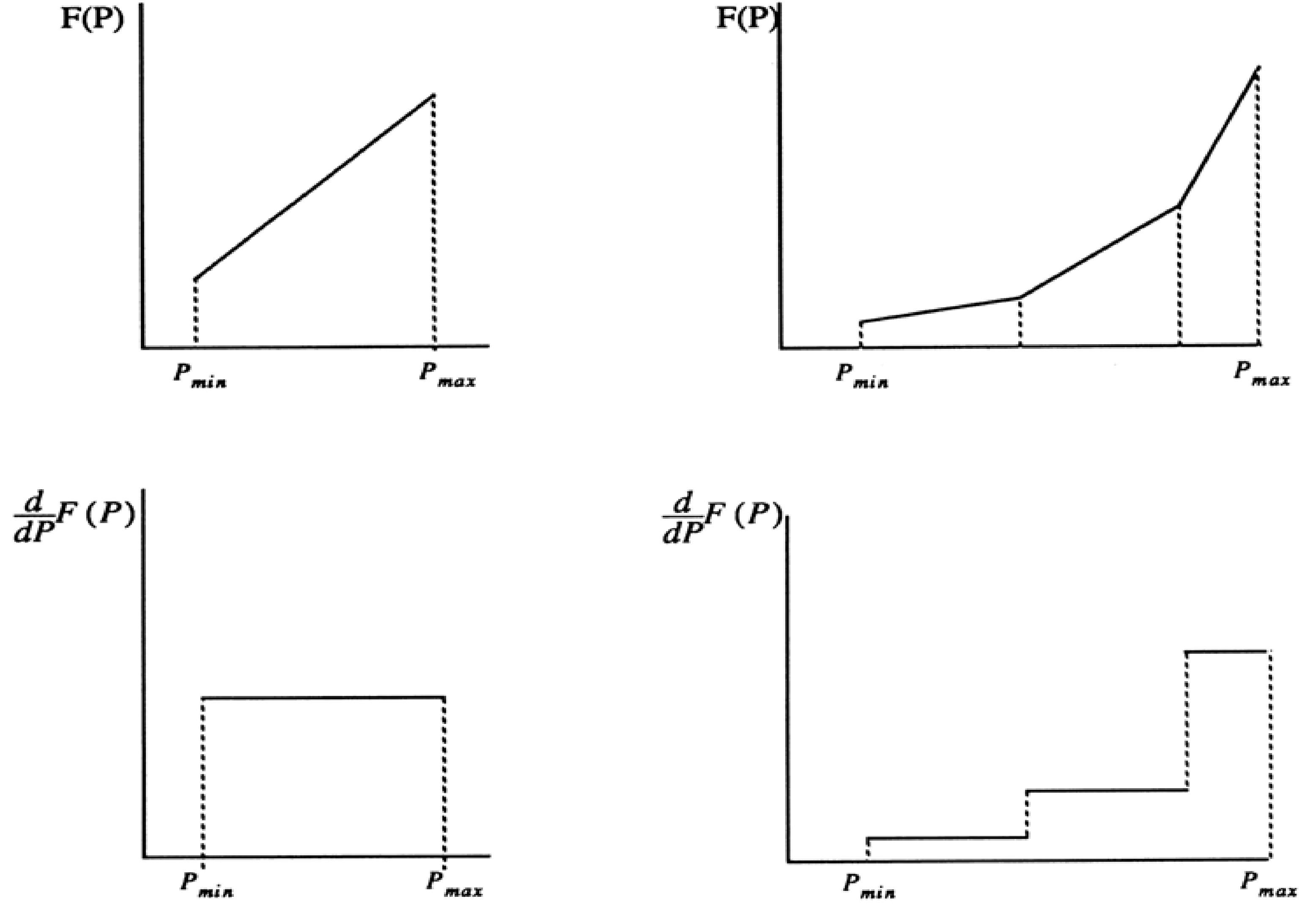
With limits the final solution of λ, is 24.43 \$/MWh (compared to 23.53 \$/MWh without limits).

Maximum limits will always cause λ to either increase or remain the same.

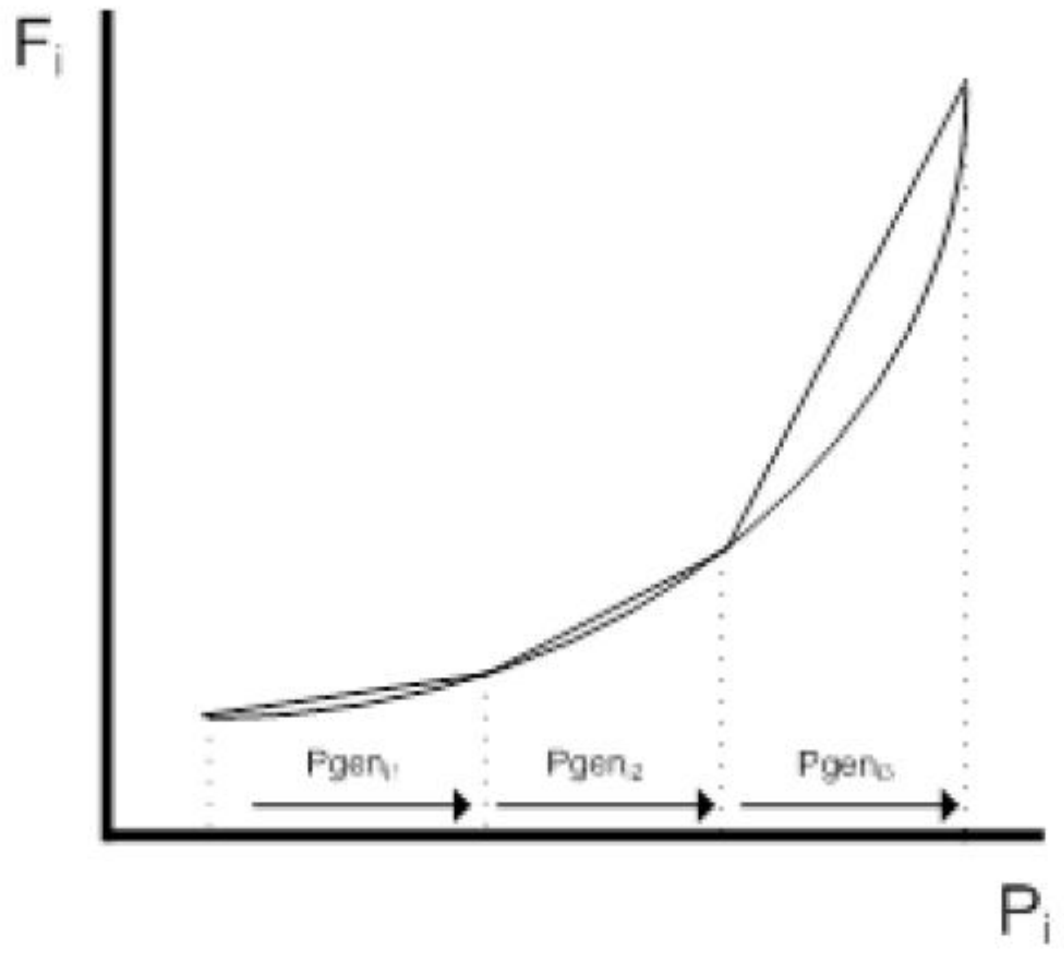
Final solution is:

$$
\begin{aligned}
P_{G1}(24.43) &= 300 \text{ MW (at maximum limit)} \\
P_{G2}(24.43) &= 443 \text{ MW} \\
P_{G3}(24.43) &= 257 \text{ MW}
\end{aligned}
$$

Economic Dispatch: Piece-wise Linear Cost Functions



Economic Dispatch: Linear Programming



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LP formulation

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$$F_i(Pgen_i) = F_i(Pgen_i^{\min}) + s_{i1}Pgen_{i1} + s_{i2}Pgen_{i2} + s_{i3}Pgen_{i3}$$

Where : $0 \le Pgen_{ik} \le Pgen_{ik}^{\max}$ for k = 1, 2, 3

and finally : $Pgen_i = Pgen_i^{\min} + Pgen_{i1} + Pgen_{i2} + Pgen_{i3}$

and $s_{ik} = \dfrac{F_i(Pgen_{ik+1}) - F_i(Pgen_{ik})}{(Pgen_{ik+1} - Pgen_{ik})}$ where k = the index for segments

LP economic dispatch

Minimize

$$\sum_{i=1}^{Ngen}(F_i(Pgen_i^{\min}) + s_{i1}Pgen_{i1} + s_{i2}Pgen_{i2} + s_{i3}Pgen_{i3})$$

$$0 \le Pgen_{ik} \le Pgen_{ik}^{\max} \text{ for k} = 1, 2, 3 \ldots \text{for all generators i} = 1 \ldots \text{Ngen}$$

and finally :

$$P_i = P_i^{\min} + Pgen_{i1} + Pgen_{i2} + Pgen_{i3} \text{ for all generators i} = 1 \ldots \text{Ngen}$$

Subject to :

$$\sum_{i=1}^{Ngen} P_i = P_{load}$$

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Example #5: Solution (Using LP)

Number of Segments	Generator 1 MW	Generator 2 MW	Generator 3 MW	Total cost ($/hr)
1	400	400	50	8227.870
2	375	350	125	8195.369
3	450	300	100	8204.105
5	400	340	110	8195.206
10	385	340	125	8194.554
50	393	335	122	8194.357
Standard solution with Lambda Search	393.2	334.6	122.2	8194.356

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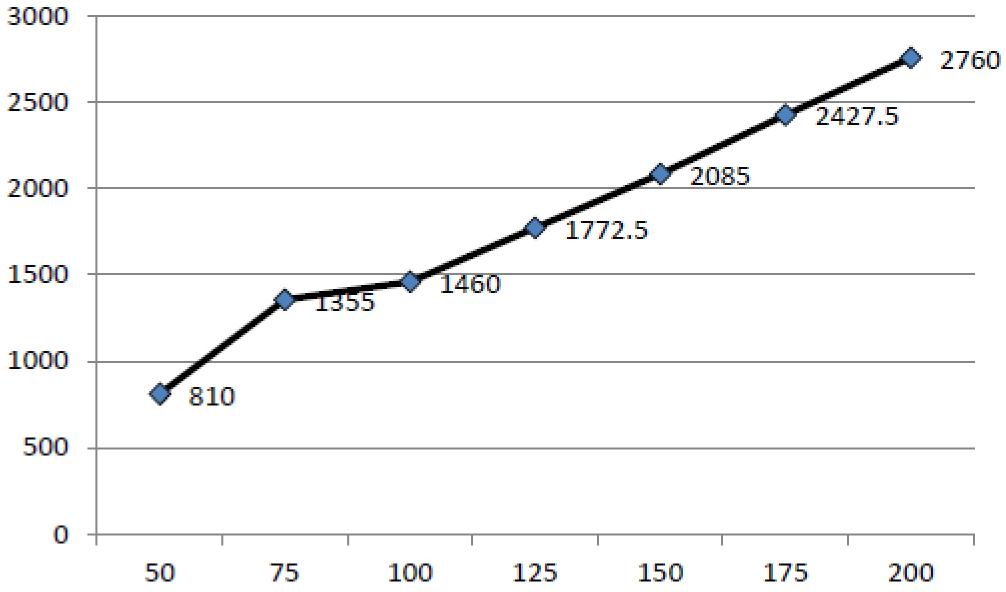
Economic Dispatch: Dynamic Programming

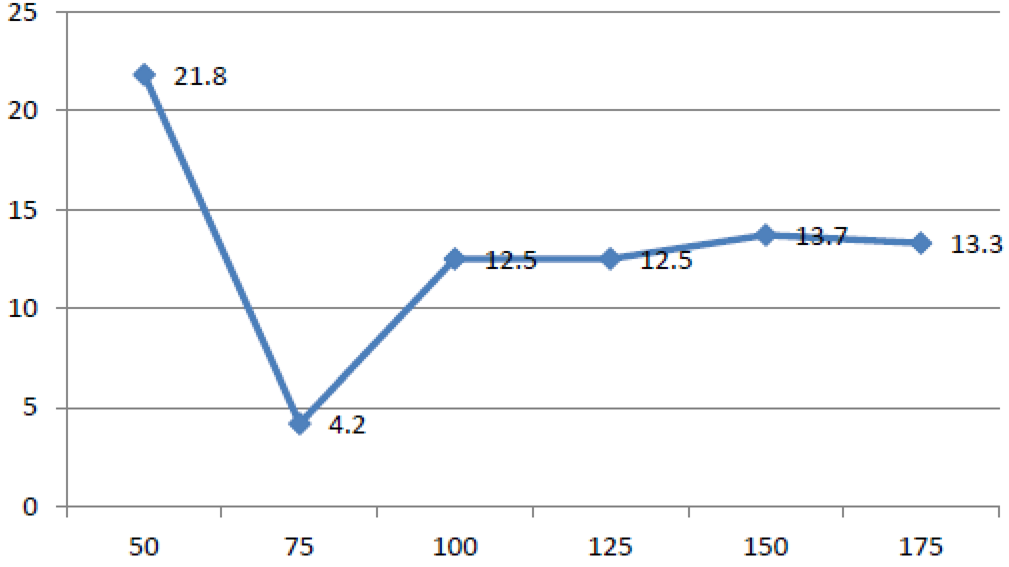
Find the optimum dispatch for a total demand of 310 MW.

	Costs (\$ / hour)		
Power Levels (MW) $P_1 = P_2 = P_3$	F_1	F_2	F_3
0	∞	∞	∞
50	810	750	806
75	1355	1155	1108.5
100	1460	1360	1411
125	1772.5	1655	1704.5
150	2085	1950	1998
175	2427.5	∞	2358
200	2760	∞	∞
225	∞	∞	∞

Plots for Generator - 1



Cost vs. P(MW)



Incremental cost vs. P(MW)

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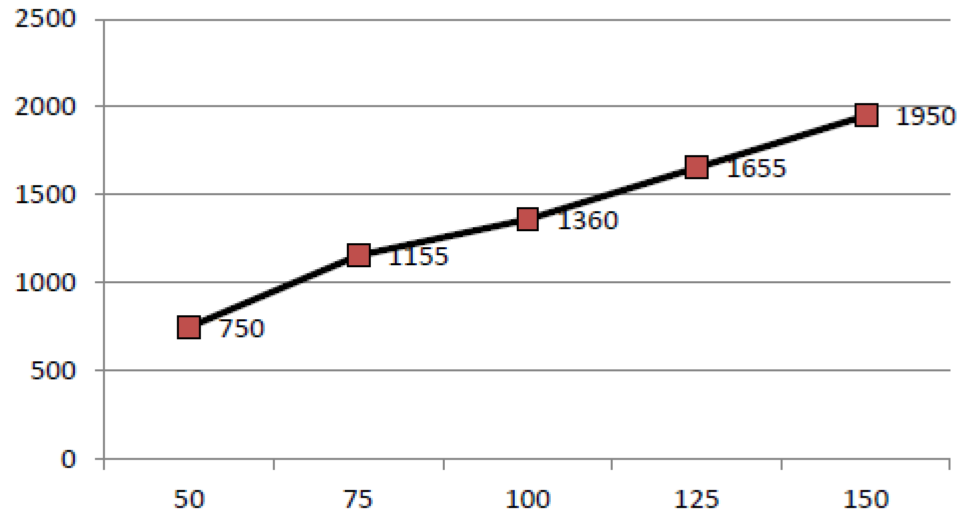
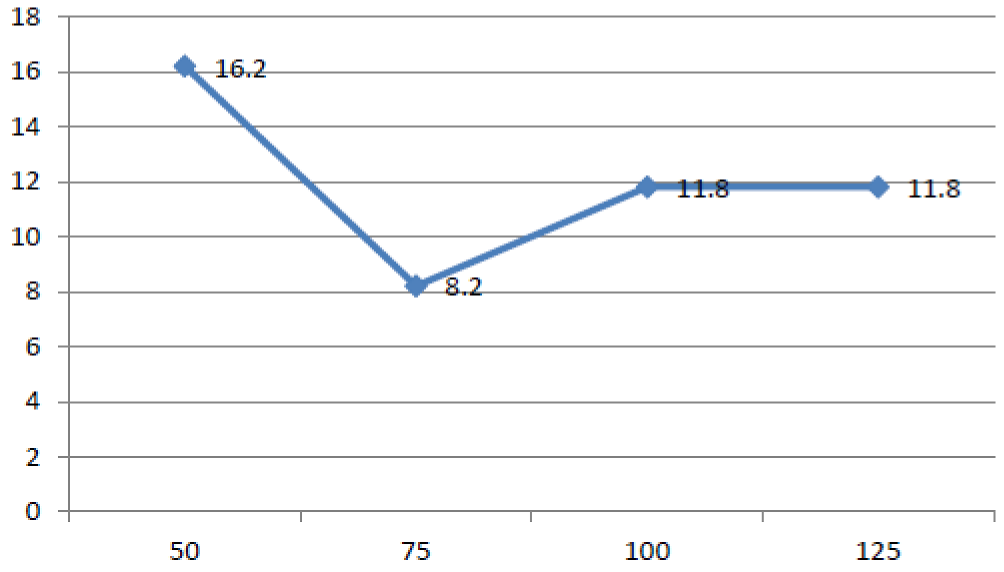
Plots for Generator - 2

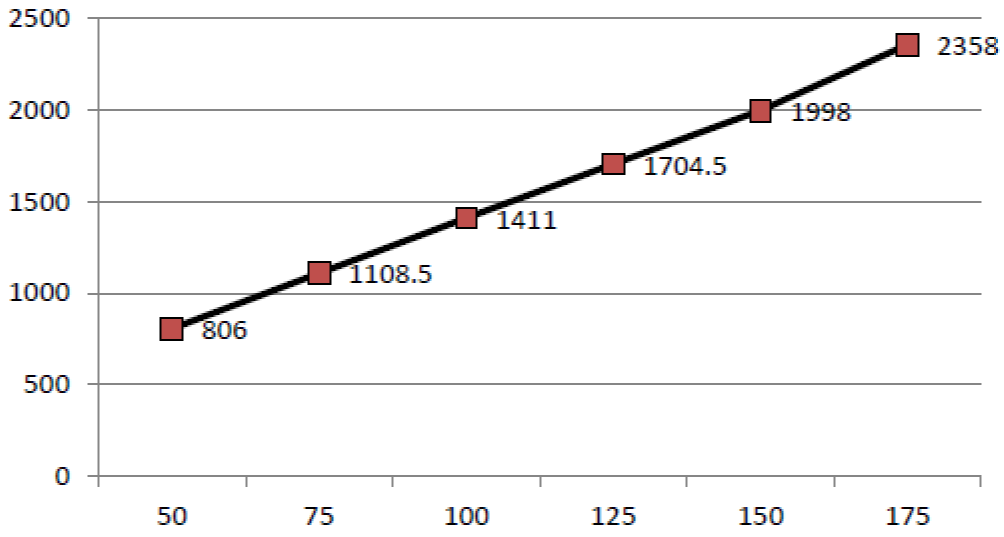


Cost vs. P(MW)

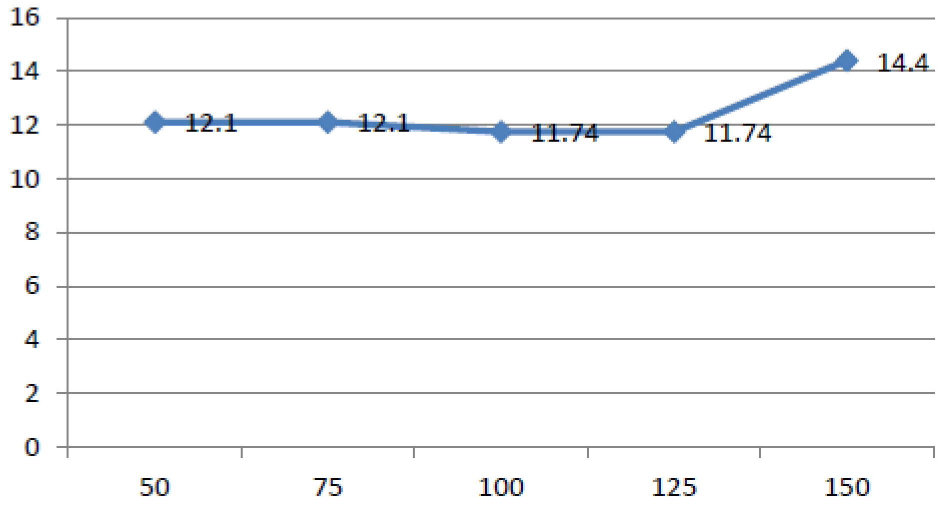


Incremental cost vs. P(MW)

Plots for Generator - 3



Cost vs. P(MW)



Incremental cost vs. P(MW)

DP Method: Dispatch Solution (Generators 1 and 2)

$$f_2 = F_1(D - P_2) + F_2(P_2)$$

Where D is the “Demand” or the total power to be supplied

D (MW)	$F_1(D)$ ($/h)	$P_2 = 0$ $F_2(P_2) = \infty$	50 750	75 1155	100 1360	125 1655	150 (MW) 1950 ($/h)	f_2 ($/h)	P_2^* (MW)
0	∞	∞	∞	∞	∞	∞	∞	∞	
50	810	∞	∞	∞	∞	∞	∞	∞	
75	1355	∞	∞	∞	∞	∞	∞	∞	
100	1460	∞	1560	∞	∞	∞	∞	1560	50
125	1772.5	∞	2105	1965	∞	∞	∞	1965	75
150	2085	∞	2210	2510	2170	∞	∞	2170	100
175	2427.5	∞	3177.5	2615	2715	2465	∞	2465	125
200	2760	∞	2834	2927.5	2820	3010	2760	2760	150
225	∞	∞	3177.5	3240	3125	3115	3305	3115	125
250	∞	∞	3510	3582.5	3445	3427	3410	3410	150
275	∞	∞	∞	3915	3787.5	3740	3722.5	3722.5	150
300	∞	∞	∞	∞	4120	4082.5	4025	4035	150
325	∞	∞	∞	∞	∞	4415	4377.5	4377.5	150
350	∞	∞	∞	∞	∞	∞	4710	4710	150

Last of all dispatch generator 3 with the other two

$$f_3 = f_2(D - P_3) + F_3(P_3)$$

	$P_3 = 0$	50	75	100	125	150	175	(MW)		
	$F_3(P_3) = \infty$	806	1108.5	1411	1704.5	1998	2358	(\$/h)		
D	f_2									
(MW)	(\$ / h)									
									f_3	P_3^*
.			.							
.				.						
.					.					
300	4035	∞	4216	4223.5	4171	4169.5	<u>4168</u>	4323	4168	150
325	4377.5	∞	4528.5	4518.5	4526	4464	<u>4463</u>	4528	4463	150
.			.							
.				.						
.					.					

Adjust for demand = 310 MW

D	Cost	P_3*	P_2*	P_1*
300	4168	150	100	50
325	4463	150	125	50

Between 300 MW and 325 MW the marginal unit is Generator 2 so the solution to this dispatch is:

$P_1 = 50, P_2 = 110, P_3 = 150$ for a total cost of 4286 $\$/h$

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Economic Dispatch of Generators by considering Transmission Losses

- The losses on the transmission system are a function of the generation dispatch.
- In general, using generators closer to the load results in lower losses.
- This impact on losses should be included when doing the economic dispatch.
- Losses can be included by slightly rewriting the Lagrangian:

$$\mathrm{L}(\mathbf{P}_G, \lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda\left(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi}\right)$$

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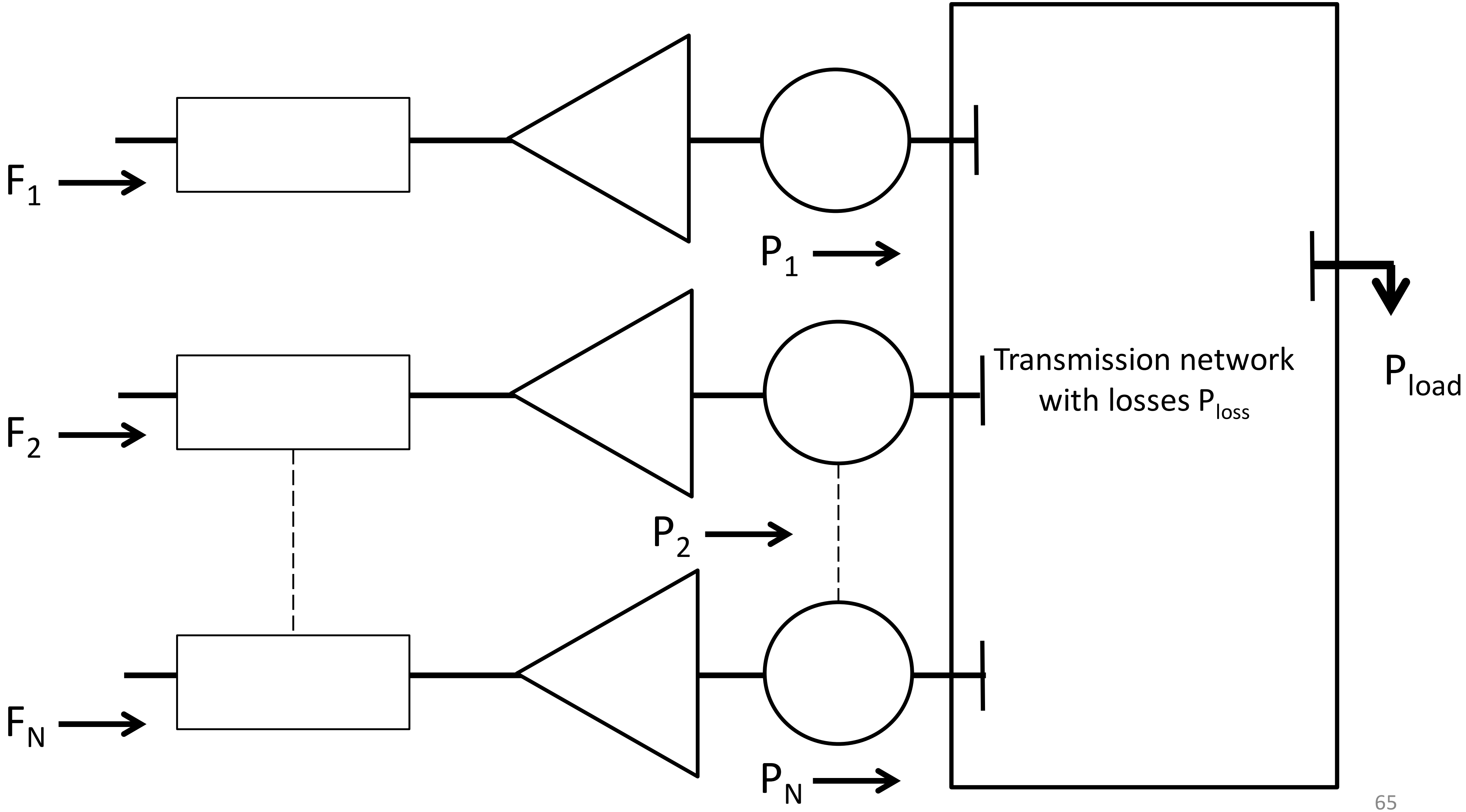
Inclusion of Transmission Losses



Impact of Transmission Losses

The inclusion of losses then impacts the necessary conditions for an optimal economic dispatch:

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda\left(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi} \right).$$

The necessary conditions for a minimum are now:

$$\frac{\partial \mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right) = 0$$

$$P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi} = 0$$

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Impact of Transmission Losses

Solving for λ, we get: $\frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right) = 0$

$$\lambda = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)} \frac{dC_i(P_{Gi})}{dP_{Gi}}$$

Define the penalty factor L_i for the i^{th} generator (don't confuse with Lagrangian L!!!)

$$L_i = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$$

The penalty factor at the slack bus is always unity!

Impact of Transmission Losses

The condition for optimal dispatch with losses is then

$$L_1 IC_1(P_{G1}) = L_2 IC_2(P_{G2}) = L_m IC_m(P_{Gm}) = \lambda$$

$L_i = \dfrac{1}{\left(1 - \dfrac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$. So, if increasing P_{Gi} increases

the losses then $\dfrac{\partial P_L(P_G)}{\partial P_{Gi}} > 0 \Rightarrow L_i > 1.0$

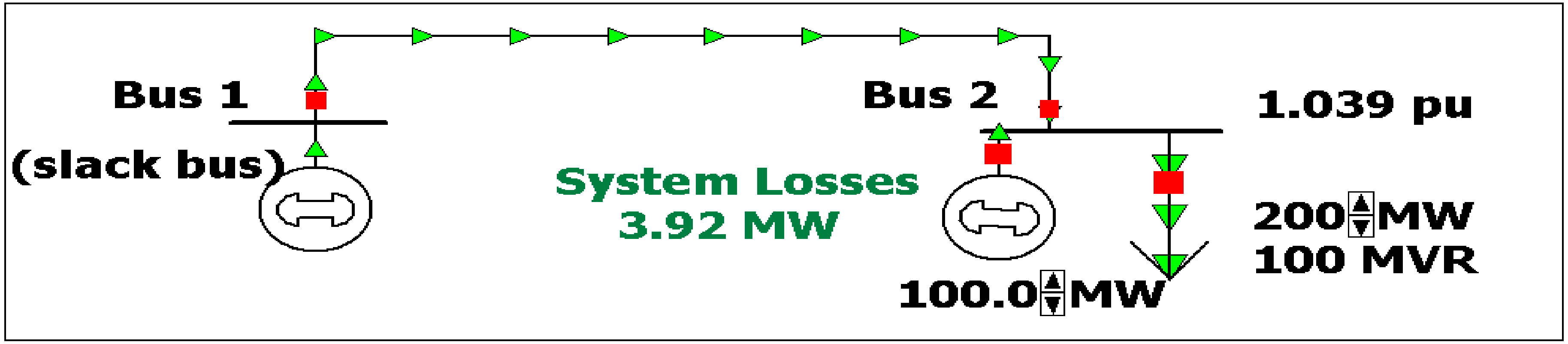
This makes generator i appear to be more expensive (i.e., it is penalized). Likewise $L_i < 1.0$ makes a generator appear less expensive.

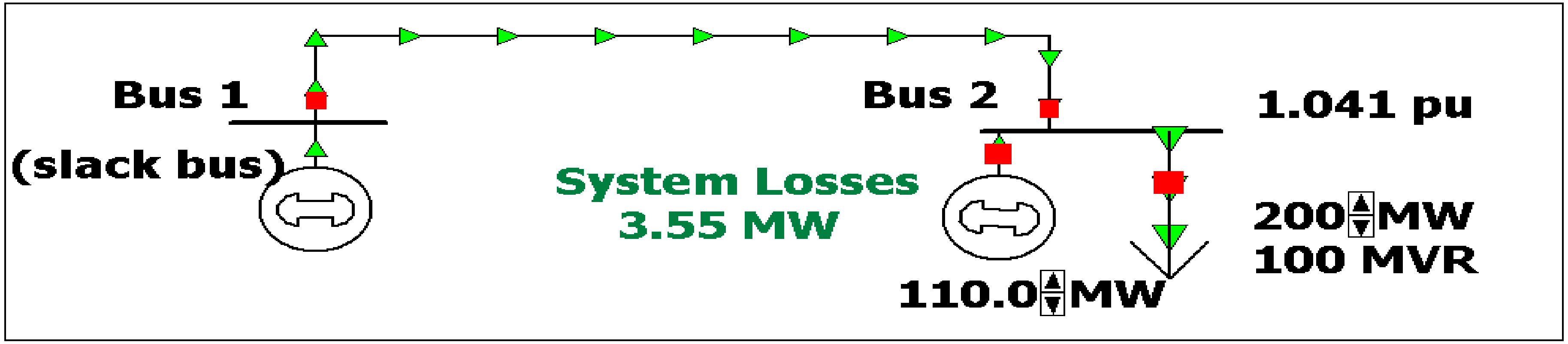
Calculation of Penalty Factors

Unfortunately, the analytic calculation of L_i is somewhat involved. The problem is a small change in the generation at P_{Gi} impacts the flows and hence the losses throughout the entire system. However, using a power flow you can approximate this function by making a small change to P_{Gi} and then seeing how the losses change:

$$\frac{\partial P_L(P_G)}{\partial P_{Gi}} \approx \frac{\Delta P_L(P_G)}{\Delta P_{Gi}} \qquad L_i \approx \frac{1}{1-\dfrac{\Delta P_L(P_G)}{\Delta P_{Gi}}}$$

Example #9: Two Bus Penalty Factor





$$\frac{\partial P_L(P_G)}{\partial P_{G2}} = -0.0387$$

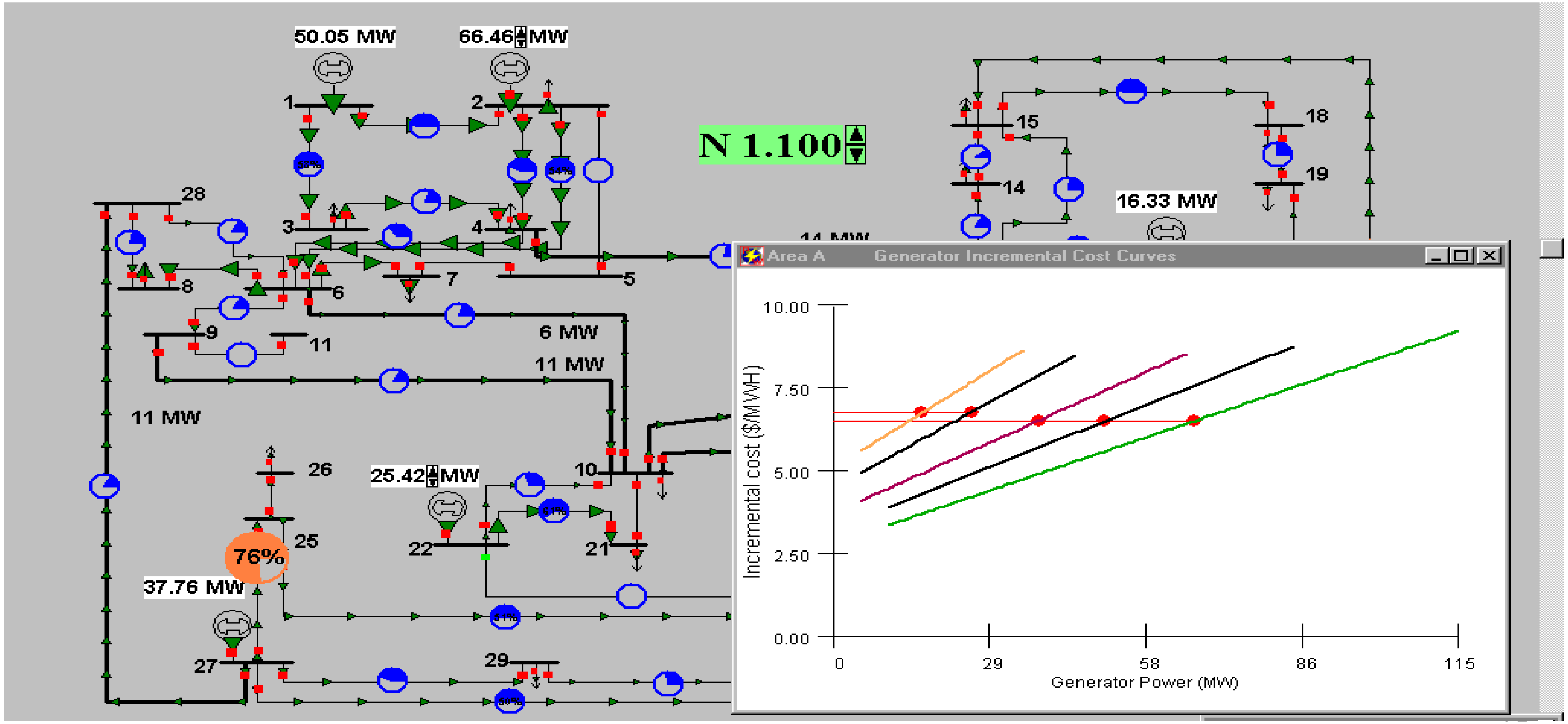
$$\frac{\Delta P_L(P_G)}{\Delta P_{G2}} = \frac{-0.37MW}{10MW} = -0.037$$

$$L_2 = 0.9627$$

$$L_2 \approx 0.9643$$

Thirty Bus ED Example

Now consider losses, because of the penalty factors the generator's incremental costs are no longer identical.



Area Supply Curve

Supply curve for thirty bus system

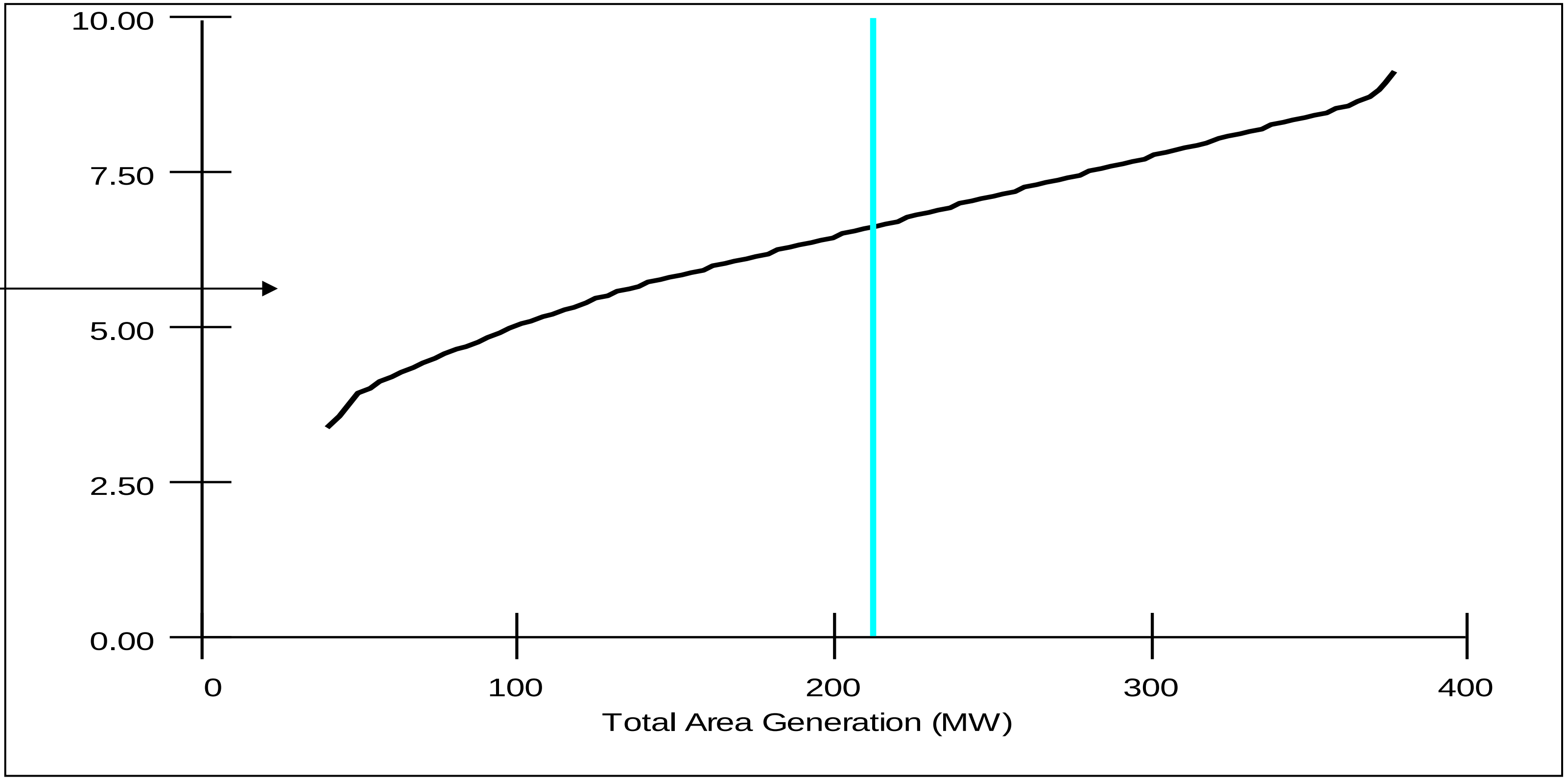
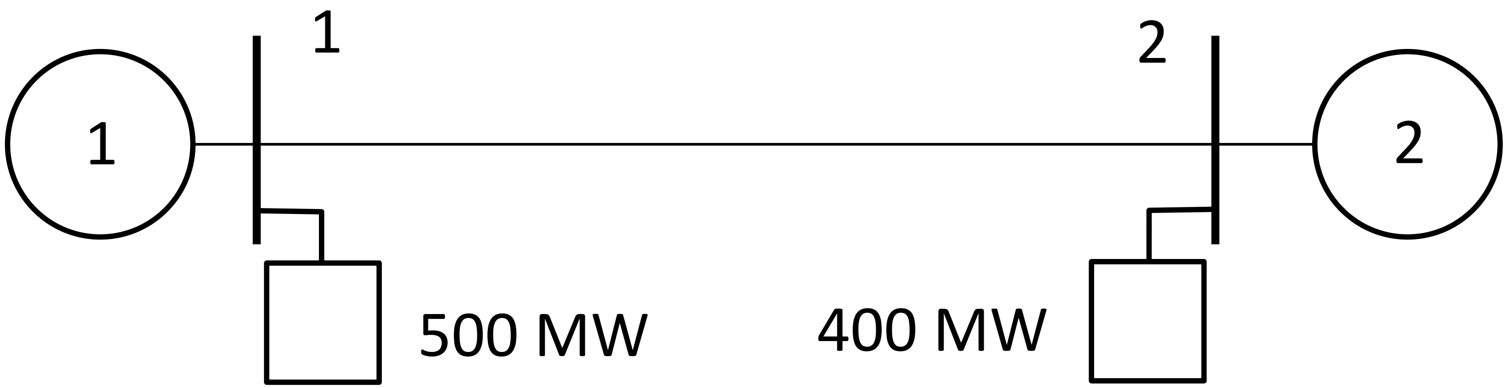


Figure 1.) The area supply curve shows the cost to produce the next MW of electricity, assuming area is economically dispatched.

Summary: Economic Dispatch

- Economic dispatch determines the best way to minimize the current generator operating costs.
- The lambda-iteration method is a good approach for solving the economic dispatch problem:
  - ➢ Generator limits are easily handled
  - ➢ Penalty factors are used to consider the impact of losses
- Economic dispatch is not concerned with determining which units to turn on/off (this is the unit commitment problem).
- Basic form of economic dispatch ignores the transmission system limitations.

Locational Marginal Price (LMP)



Generator 1	MW	Marginal Cost ($/MWh)	Generator 2	Bid MW	Marginal Cost ($/MWh)
Segment A	400	5.00	Segment C	200	6.50
Segment B	800	7.50	Segment D	400	8.00

Generally, LMP determines an energy **price** for each electrical node on the grid as well as the transmission congestion **price** (if any) to serve that node. For the above reason, LMP is often referred to as "**nodal pricing**".

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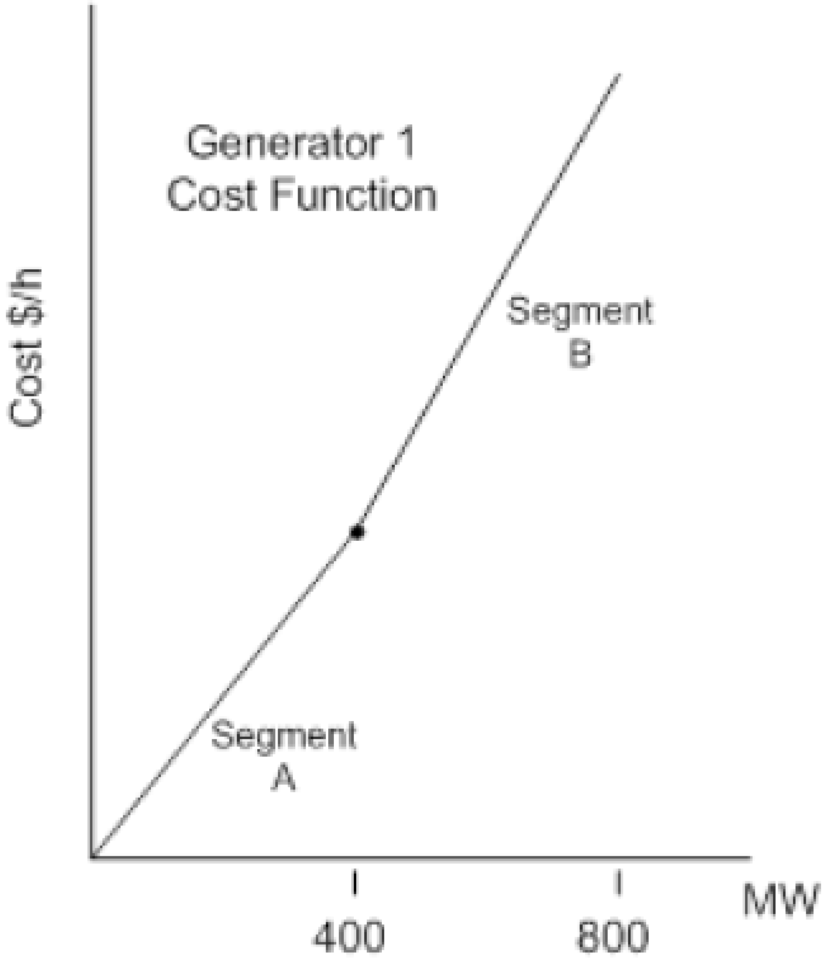
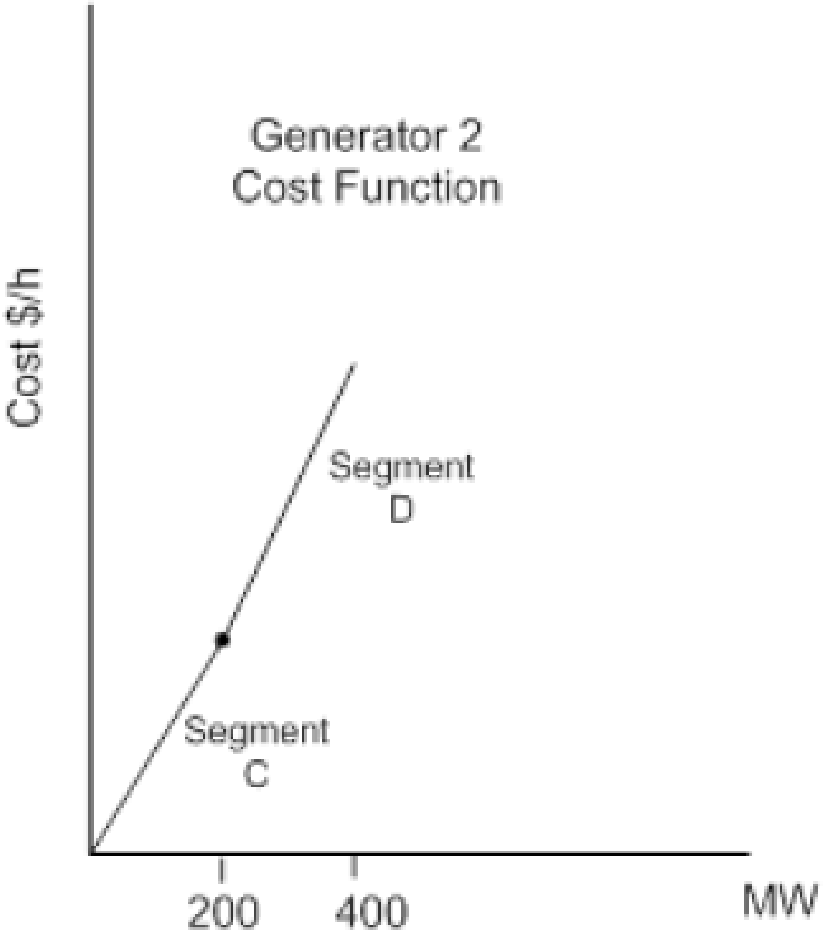
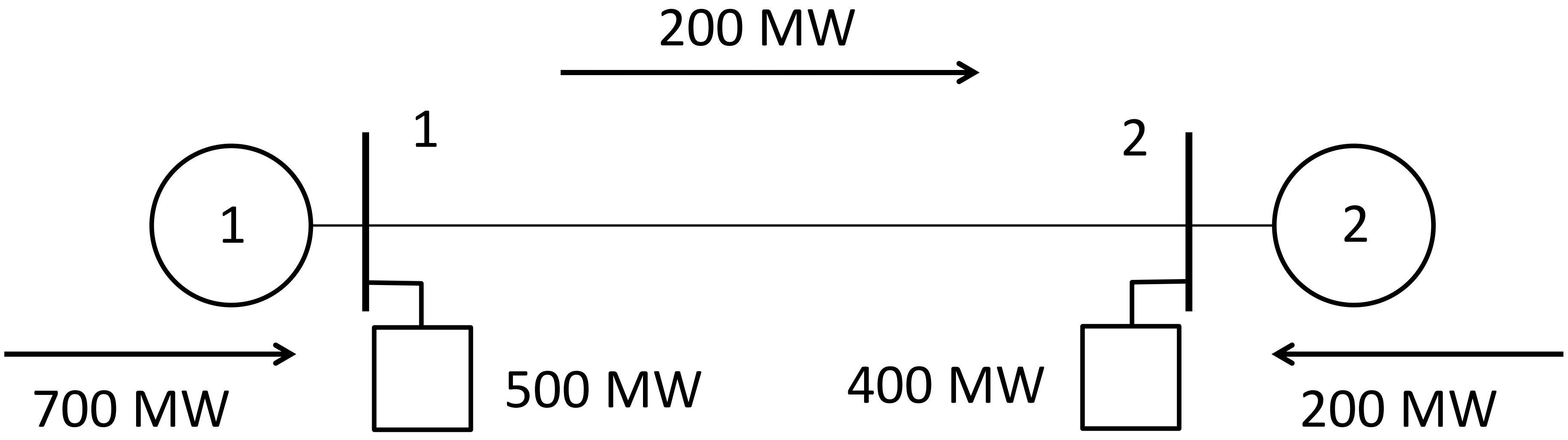
Generator Linear Segment Cost Functions





Base dispatch with no line flow limit

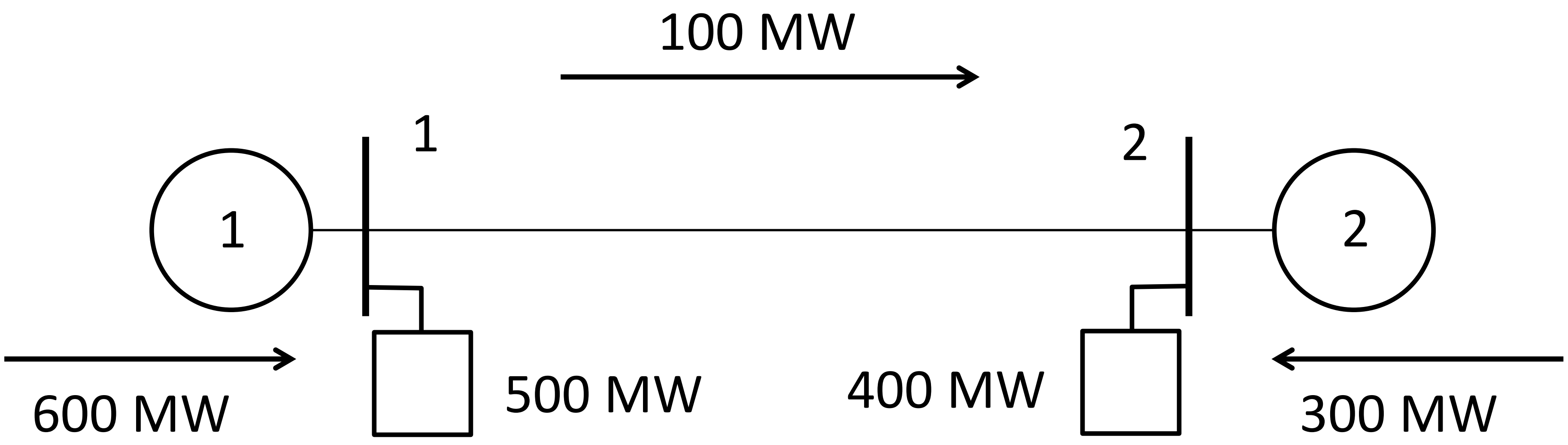


Generation dispatch:

Segment	MW	Price
A	400	5.00
C	200	6.50
B	300	7.50

Note: LMP at both buses is 7.5

Dispatch with 100 MW line flow limit

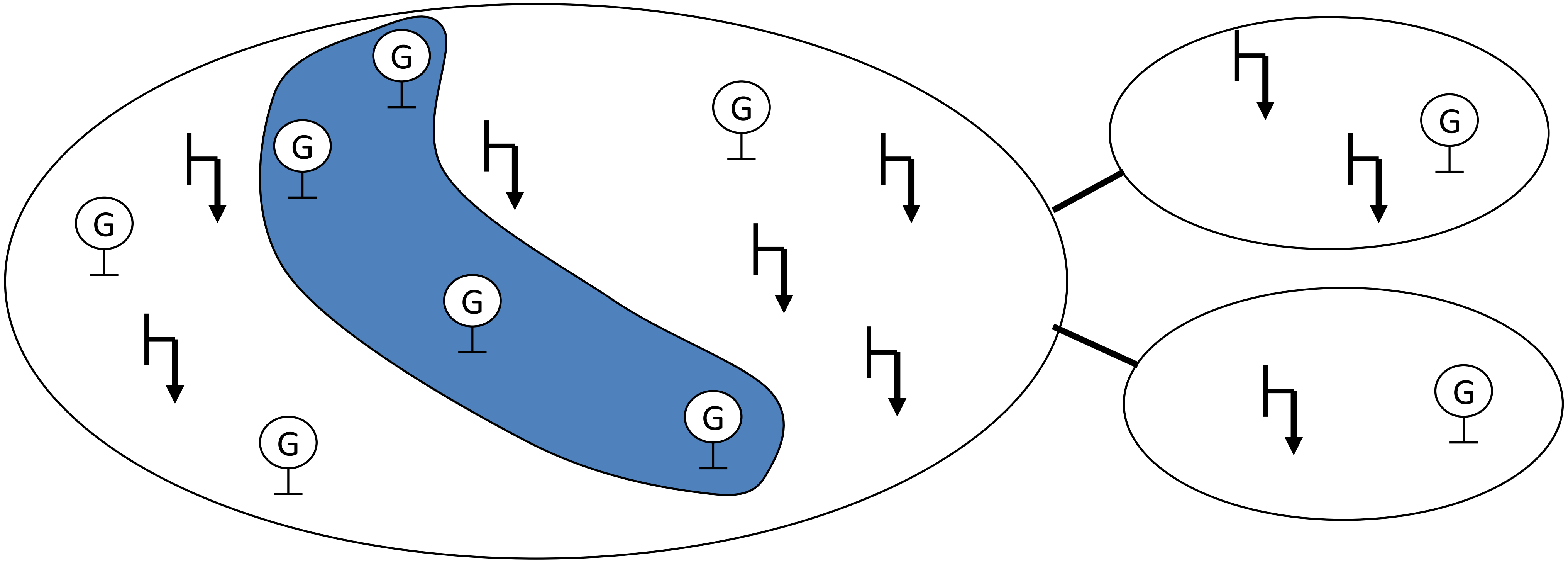


Generation dispatch:

Segment	MW	Price
A	400	5.00
C	200	6.50
B	200	7.50
D	100	8.00

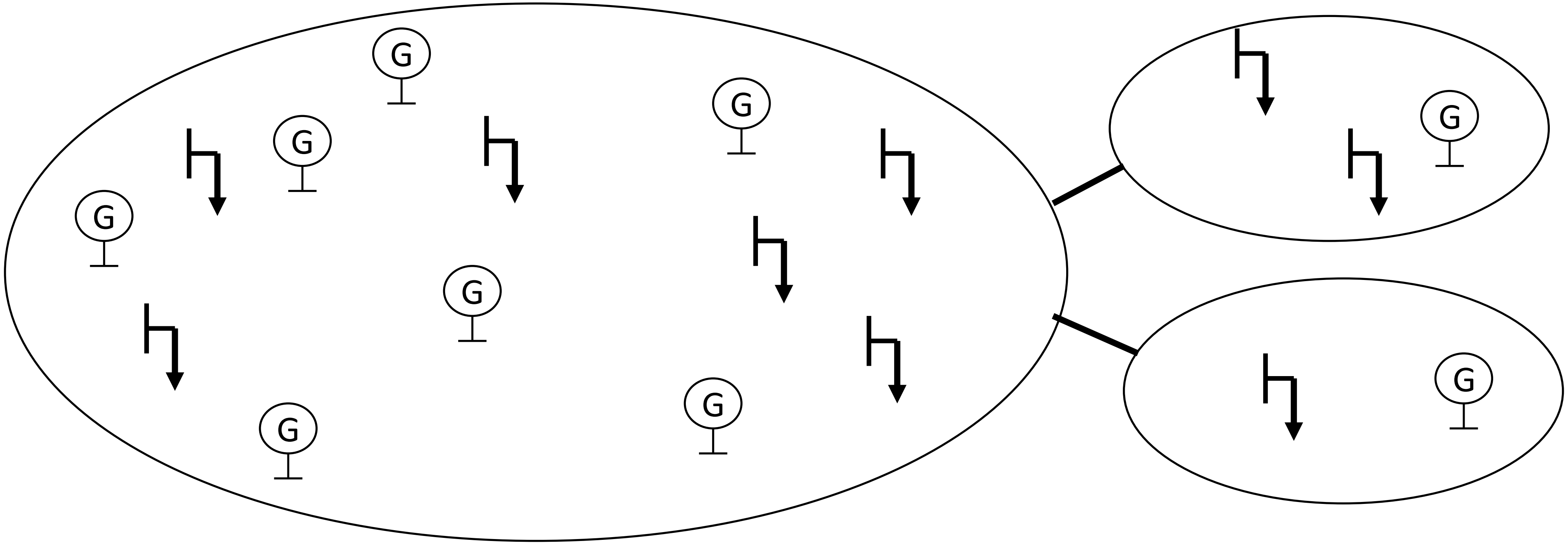
Note: LMP at bus 1 is 7.5; LMP at bus 2 is 8.0

Economic Dispatch & Unit Commitment



The following is the question of Economic dispatch. With a given set of units running, how of the load much should be generated at each to cover the load and losses?

Deciding Which Units to “Commit”



How does one define “economic operation?” Profit maximizing? Cost minimizing? This all depends on the market in which you are.

What is Unit Commitment

- We have a few generators (units) and some forecasted load.
- Besides the cost of running the units, we have additional costs and constraints:
  - Start-up cost
  - Shut-down cost
  - Spinning reserve
  - Ramp-up time... and more

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Assignment 2

Unit1 : Min = 150 MW

Max = 600 MW

$$H_1 = 510.0 + 7.2P_1 + 0.00142P_1^2 \text{ MBtu/h}$$

Unit2 : Min = 100 MW

Max = 400 MW

$$H_2 = 310.0 + 7.85P_2 + 0.00194P_2^2 \text{ MBtu/h}$$

Unit3 : Min = 50 MW

Max = 200 MW

$$H_3 = 78.0 + 7.97P_3 + 0.00482P_3^2 \text{ MBtu/h}$$

with fuel costs:

$$\text{Fuel cost}_1 = 1.1\text{R/MBtu}$$

$$\text{Fuel cost}_2 = 1.0\text{R/MBtu}$$

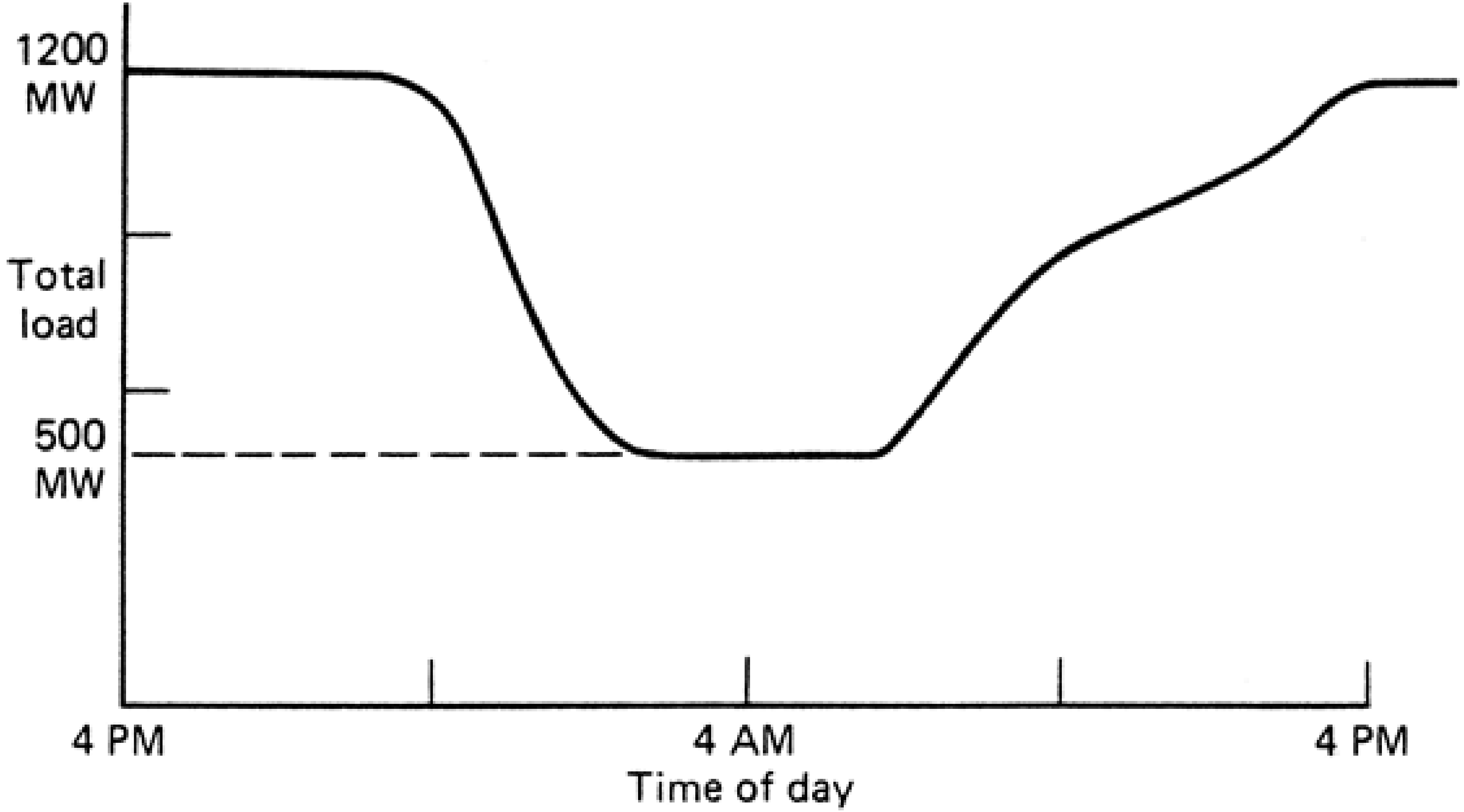
$$\text{Fuel cost}_3 = 1.2\text{R/MBtu}$$

What combination of on line units should be used to supply 550 MW?

Unit Combinations to supply 550 MW

Unit 1	Unit 2	Unit 3	Max Generation	Min Generation	P_1	P_2	P_3	F_1	F_2	F_3	Total Generation Cost $F_1 + F_2 + F_3$
Off	Off	Off	0	0	-	-	-	-	Infeasible	-	-
Off	Off	On	200	50	-	-	-	-	Infeasible	-	-
Off	On	Off	400	100	-	-	-	-	Infeasible	-	-
Off	On	On	600	150	0	400	150	0	3760	1658	5418
On	Off	Off	600	150	550	0	0	5389	0	0	5389
On	Off	On	800	200	500	0	50	4911	0	586	5497
On	On	Off	1000	250	295	255	0	3030	2440	0	5471
On	On	On	1200	300	267	233	50	2787	2244	586	5617

Simple Peak and Valley Pattern



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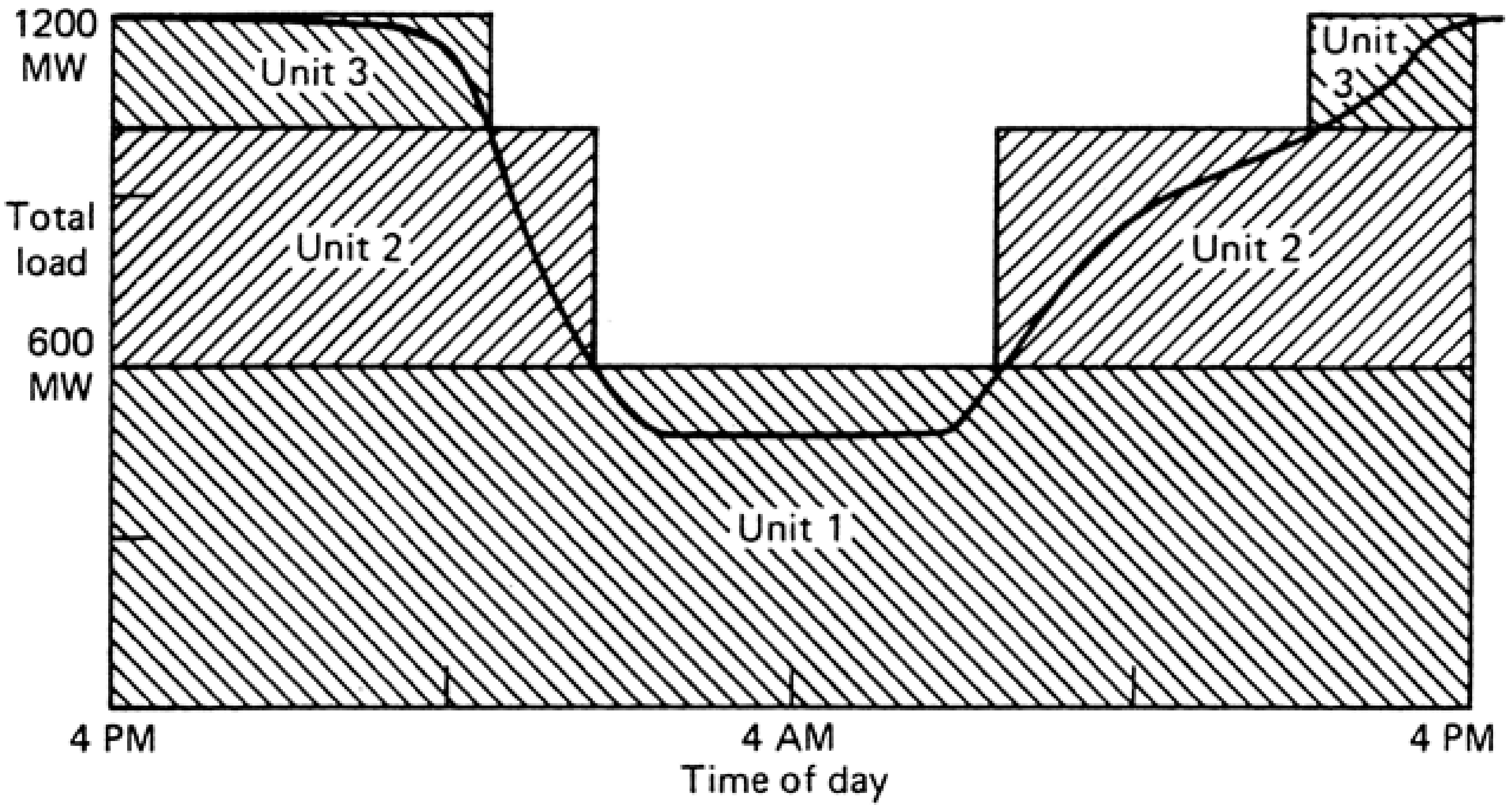
"Shut-down Rule"

	Optimum Combination		
Load	Unit 1	Unit 2	Unit 3
1200	On	On	On
1150	On	On	On
1100	On	On	On
1050	On	On	On
1000	On	On	Off
950	On	On	Off
900	On	On	Off
850	On	On	Off
800	On	On	Off
750	On	On	Off
700	On	On	Off
650	On	On	Off
600	On	Off	Off
550	On	Off	Off
500	On	Off	Off

When load is above 1000 MW, run all three units; between 1000 MW and 600 MW, run units 1 and 2; below 600 MW, run only unit 1.

Shut-down Rule applied to Load Pattern



Unit Commitment Solution Methods

1. Priority-list Schemes
2. Dynamic Programming (DP)
3. Lagrange Relaxation (LR)
4. Integer Programming (IP)

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Priority List Solution

Unit	Full Load Average Production Cost (R /MWh)
1	9.79
2	9.48
3	11.188

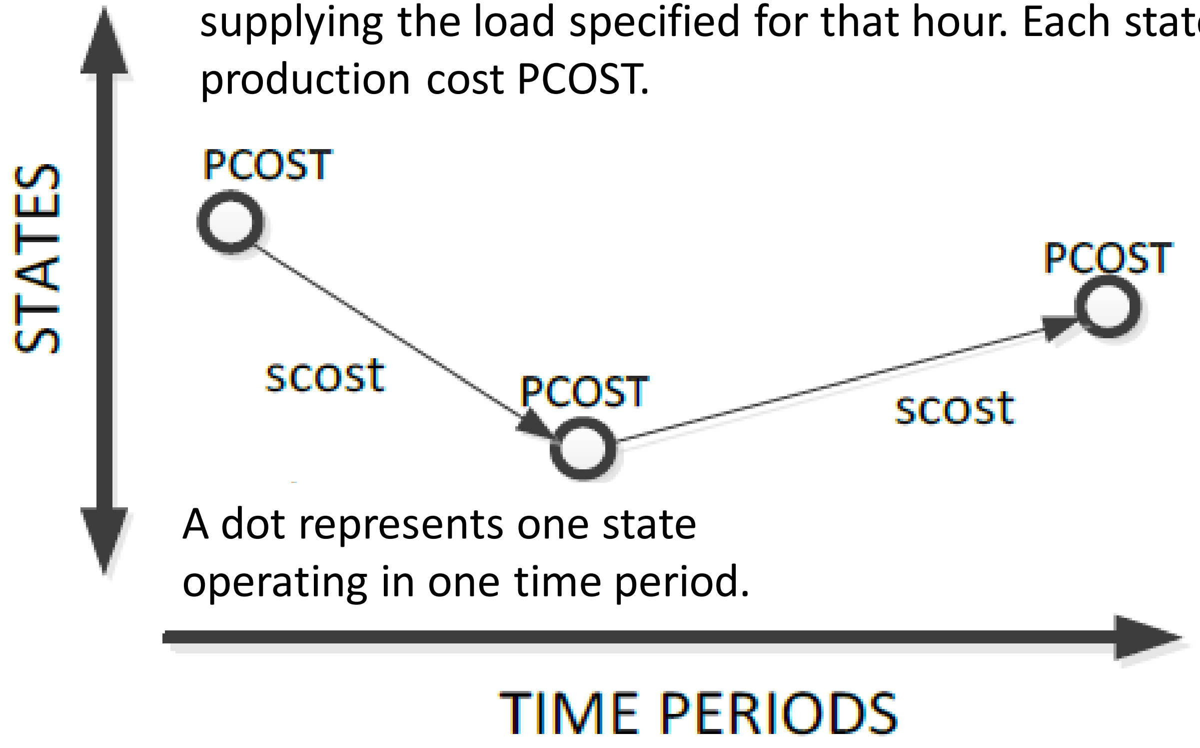
Priority order

Unit	R /MWh	Min MW	Max MW
2	9.48	100	400
1	9.79	150	600
3	11.188	50	200

Combination	Min MW from Combination	Max MW Combination
2+1+3	300	1200
2+1	250	1000
2	100	400

Dynamic Programming Paths and Unit Commitment

Each state represents a combination of generating units supplying the load specified for that hour. Each state has a production cost PCOST.



A dot represents one state operating in one time period.

FCOST is the accumulated cost to get to a state from the start through optimum path leading to that state.

FCOST (end of path) =
PCOST (start of path) + SCOST (along path) + FCOST (start of path)

Dynamic Programming Solution

State Number	Unit 1	Unit 2	Unit 3	Max Capacity
8	1	1	1	1200
7	1	1	0	1000
6	1	0	1	800
5	1	0	0	600
4	0	1	1	600
3	0	1	0	400
2	0	0	1	200
1	0	0	0	0

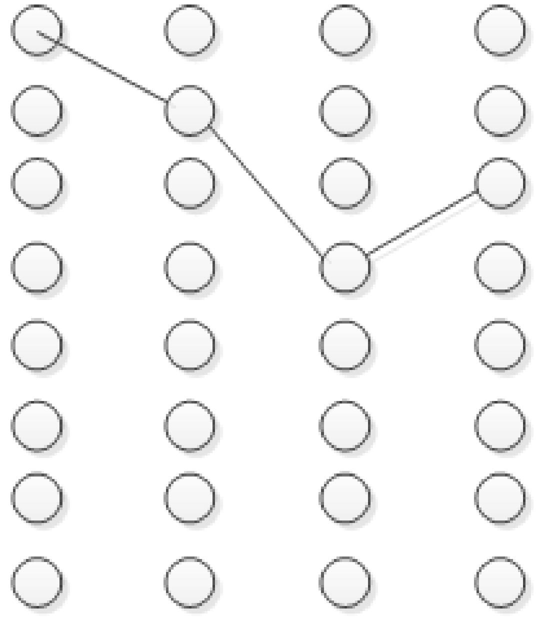
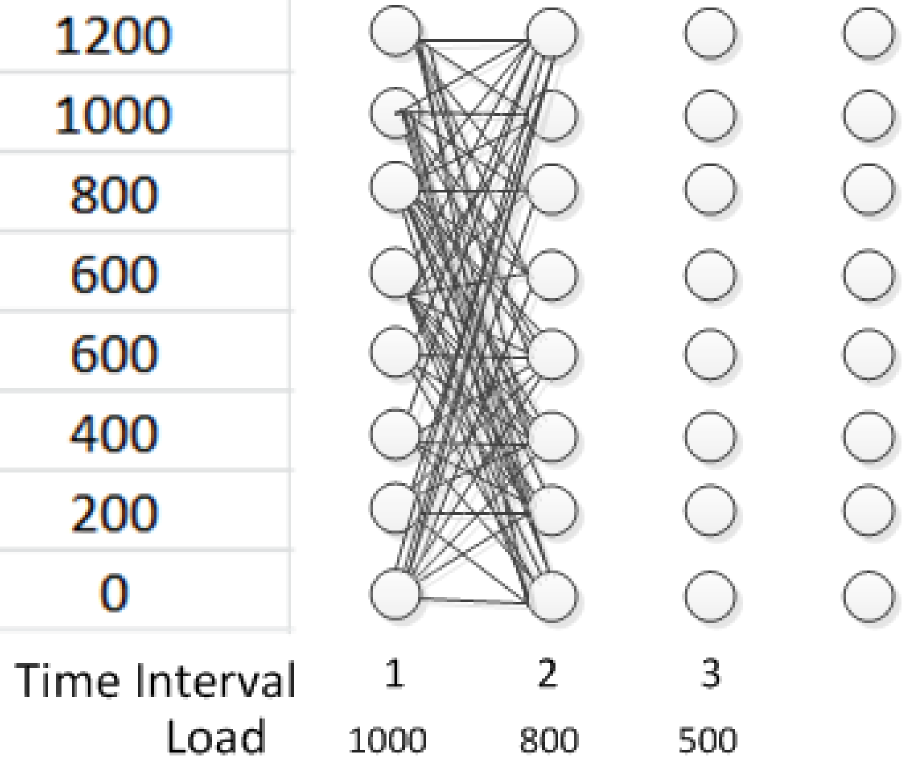


Figure 2.) Schedule shown is: 111 interval 1, 110 interval 2, 100 interval 3, 101 interval 4

Path Multiplication

State Number	Unit 1	Unit 2	Unit 3	Max Capacity
8	1	1	1	1200
7	1	1	0	1000
6	1	0	1	800
5	1	0	0	600
4	0	1	1	600
3	0	1	0	400
2	0	0	1	200
1	0	0	0	0



All possible paths between states in period 1 and period 2

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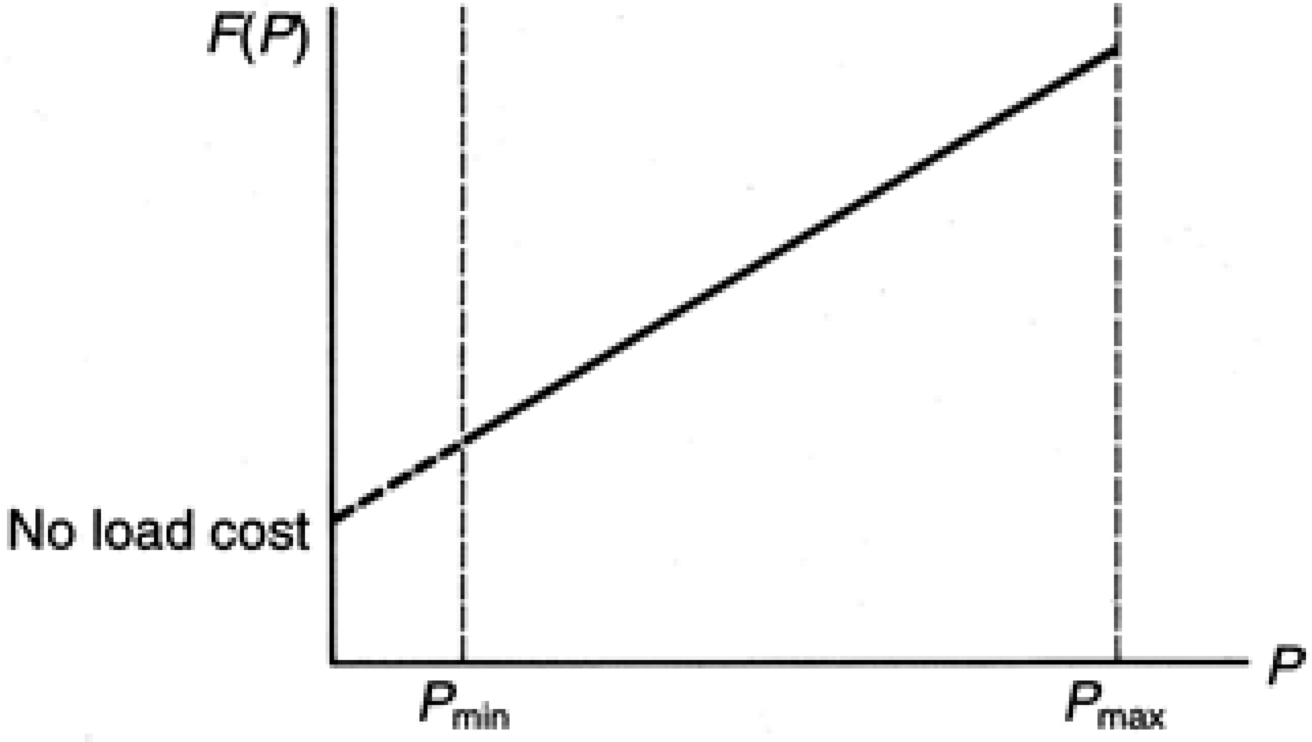
Dynamic Programming Example

Unit	Max (MW)	Min (MW)	Incremental Heat Rate (Btu/kWh)	No-Load Cost (R/h)	Full-Load Ave. cost (R/mWh)	Minimum Times (h)	
						Up	Down
1	80	25	10440	213.00	23.54	4	2
2	250	60	900	585.62	20.34	5	3
3	300	75	8730	684.74	19.74	5	4
4	60	20	11900	252.00	28.00	1	1

Unit	Hours Off-line(-) or On-line(+)	Hot (R)	Cold (R)	Cold Start (h)
1	-5	150	350	4
2	8	170	400	5
3	8	500	1100	5
4	-6	0	0.02	0

Hour	Load (MW)
1	450
2	530
3	600
4	540
5	400
6	280
7	290
8	500

Simplified Generator Cost Function



$$F(P) = \text{No} + \text{Load Cost} + \text{Inc Cost} \times \text{P}$$

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Full Set of Unit Combinations

State	Unit Combination[a]				Maximum Net Capacity for Combination
15	1	1	1	1	690
14	1	1	1	0	630
13	0	1	1	1	610
12	0	1	1	0	550
11	1	0	1	1	440
10	1	1	0	1	390
9	1	0	1	0	380
8	0	0	1	1	360
7	1	1	0	0	330
6	0	1	0	1	310
5	0	0	1	0	300
4	0	1	0	0	250
3	1	0	0	1	140
2	1	0	0	0	80
1	0	0	0	1	60
0	0	0	0	0	0
Unit	1	2	3	4	

a, 1=on 0=off

Case 1: Strict Priority Order

State No.	Unit Status	Capacity (MW)
5	0 0 1 0	300
12	0 1 1 0	550
14	1 1 1 0	630
15	1 1 1 1	690

Case 2: Complete Enumeration

Sample calculations for Case 1

$$F_{\cos t}(J,K) = \min_{\{L\}}\left[P_{\cos t}(J,K) + S_{\cos t}(J-1,L:J,K) + F_{\cos t}(J-1,L)\right]$$

Allowable states are

$$\{\} = \{0010,0110,1110,1111\} = \{5,12,14,15\}$$

At hour $0\{L\} = \{12\}$, initial condition.

$J = 1$: 1st hour

$$\frac{K}{15} F_{\cos t}(1,15) = P_{\cos t}(1,15) + S_{\cos t}(0,12:1,15) = 9861 + 350 + 10211$$

$$14 F_{\cos t}(1,14) = 9493 + 350 = 9843$$

$$12 F_{\cos t}(1,12) = 9208 + 0 = 9208$$

$J = 1$: 1st hour

Feasible states are $\{12,14,15\} = \{K\}$, so $X = 3$.

Suppose two strategies are saved at each stage, so $N = 2, \{L\} = \{12,14\}$,

$$\frac{K}{15} F_{\cos t}(2,15) = \min_{\{12,14\}}\left[P_{\cos t}(2,15) + S_{\cos t}(1,L:2,15) + F_{\cos t}(1,L)\right]$$

$$= 11301 + \min\begin{bmatrix}(350 + 9208) \\ (0 + 9843)\end{bmatrix} = 20859$$

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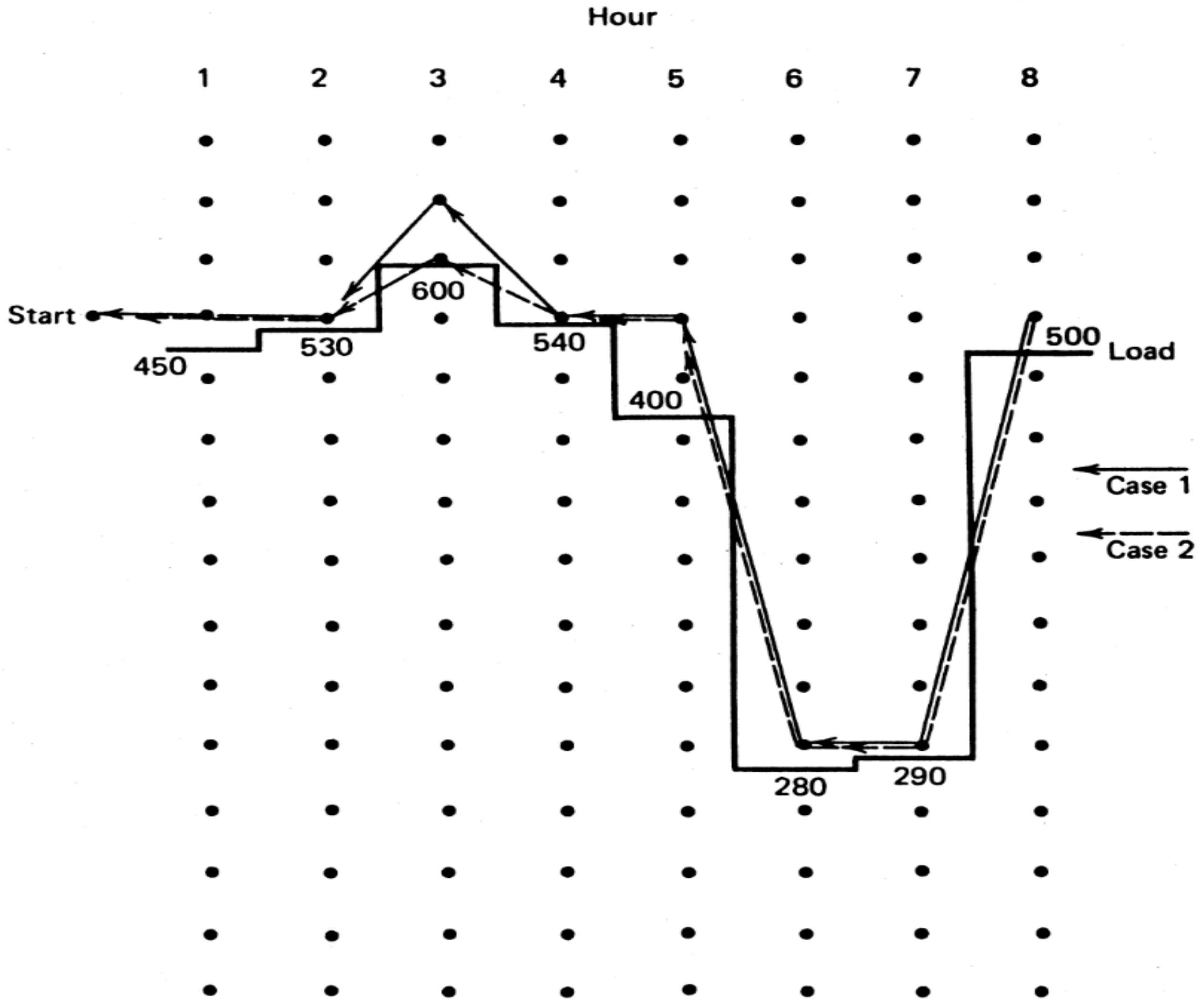
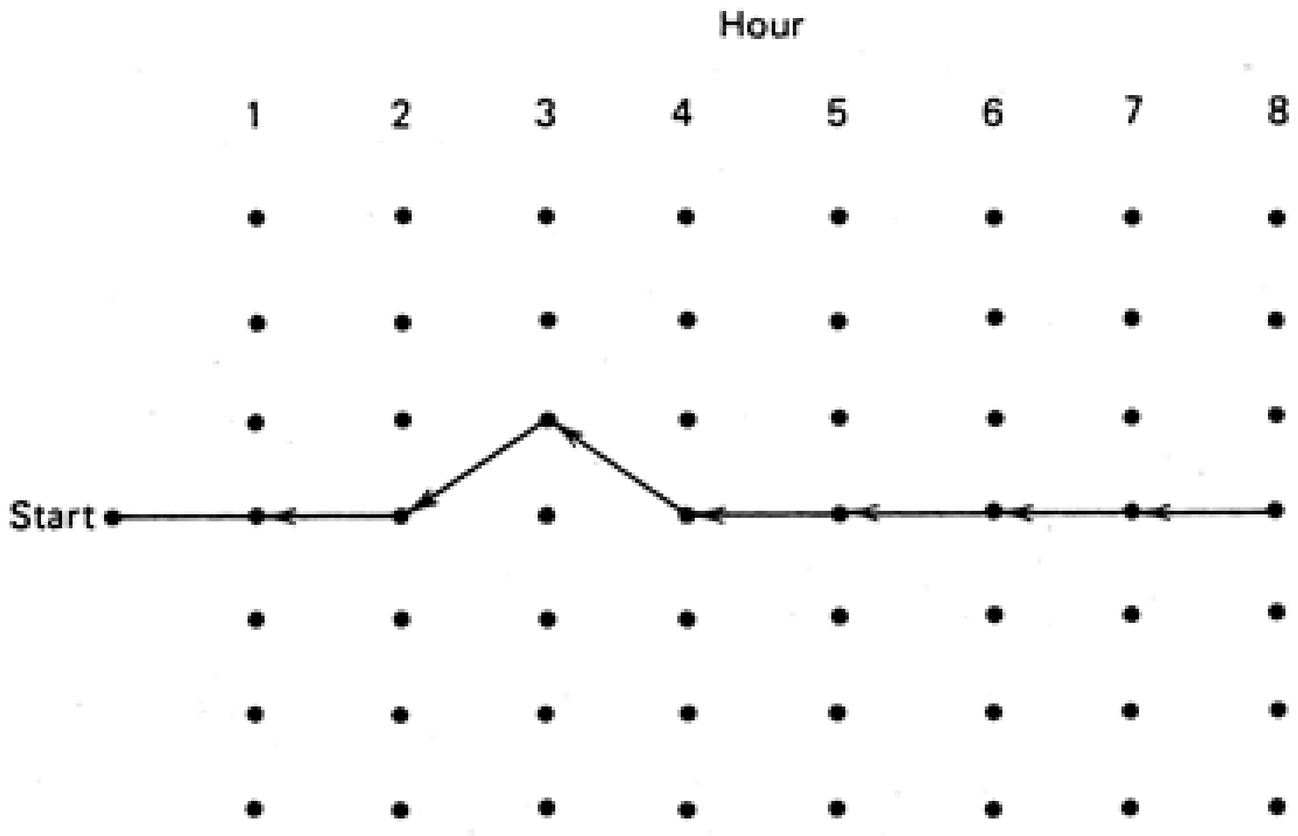
Results

Status number	Unit status	Total capacity
15	1111	690
14	1110	630
13	0111	610
12	0110	550
11	1011	440
10	1101	390
9	1010	380
8	0011	360
7	1100	330
6	0101	310
5	0010	300
4	0100	250
3	1001	140
2	1000	80
1	0001	60



Case 3: Using Minimum Shut-Down Rules

State number	Unit status	Total capacity
15	1111	690
14	1110	630
13	0111	610
12	0110	550
11	1011	440
10	1101	390
9	1010	380
1	1	
1	1	
1	1	



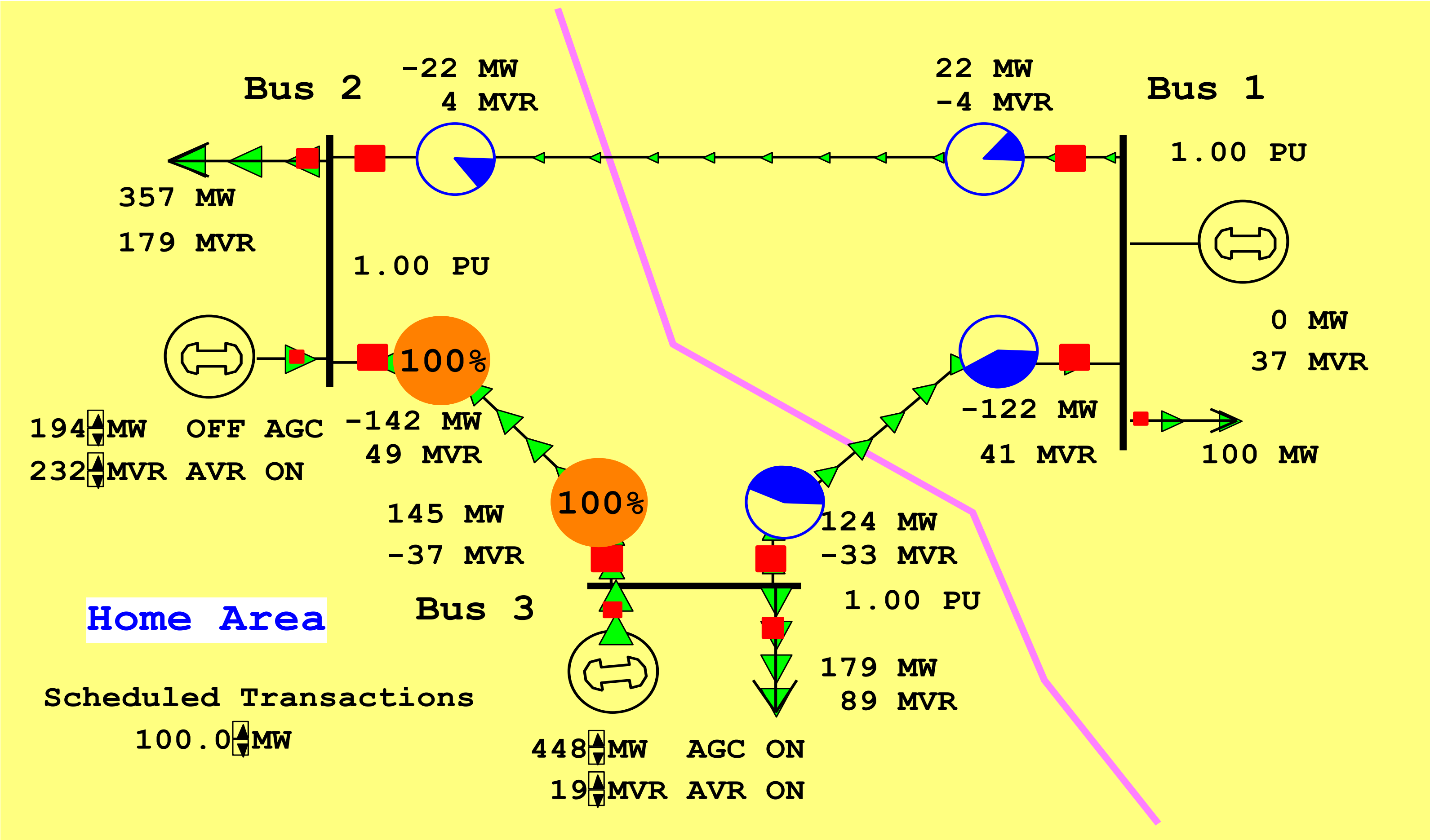
Security Constrained ED or Optimal Power Flow

- Transmission constraints often limit ability to use lower cost power.
- Such limits require deviations from what would otherwise be minimum cost dispatch in order to maintain system "security."

Security Constrained ED or Optimal Power Flow

- The goal of a security constrained ED or optimal power flow (OPF) is to determine the "best" way to instantaneously operate a power system, considering transmission limits.
- Usually "best" = minimizing operating cost, while keeping flows on transmission below limits.
- In three bus case the generation at bus 3 must be limited to avoid overloading the line from bus 3 to bus 2.

Security Constrained Dispatch

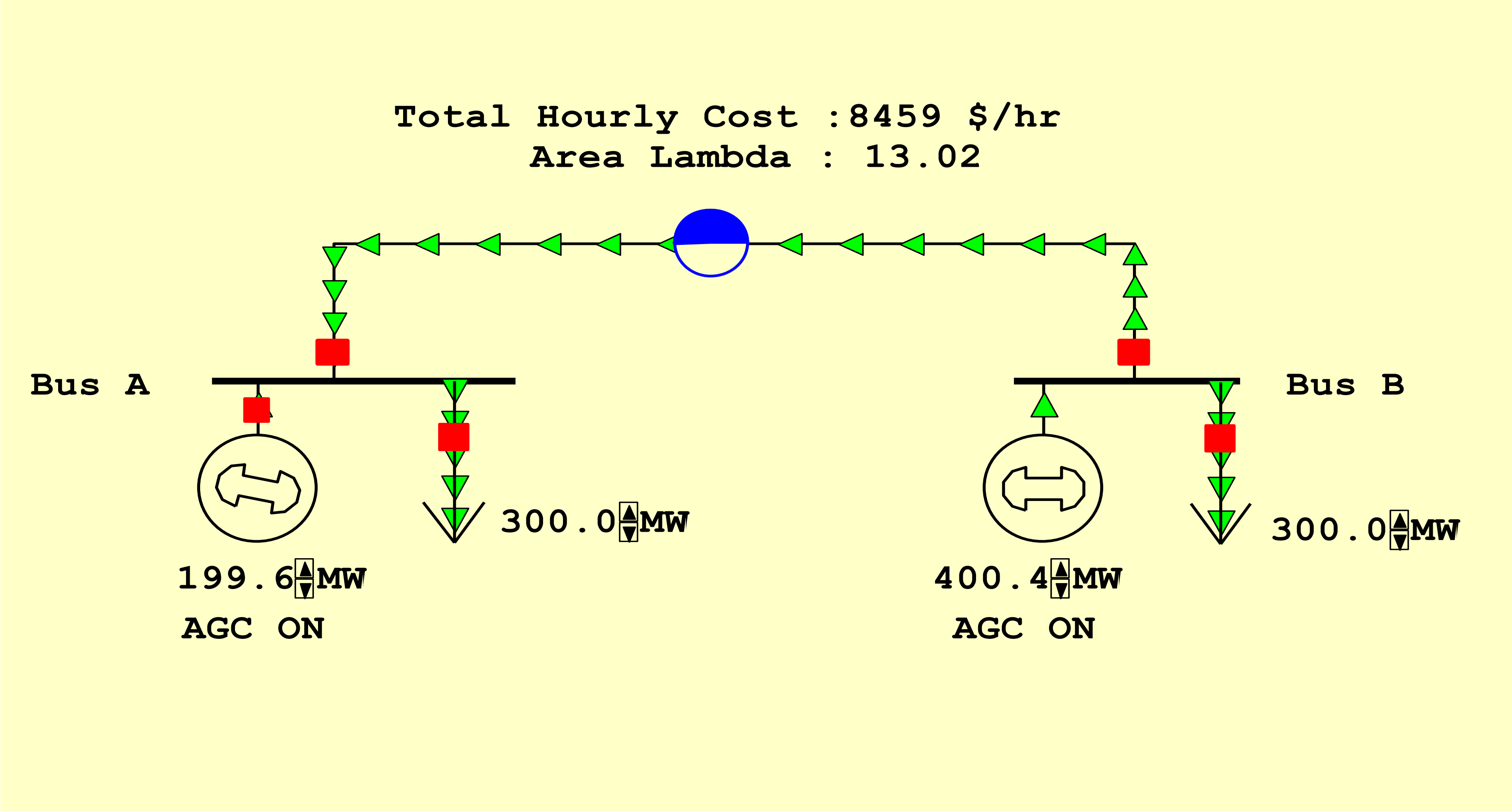


Need to dispatch to keep line from bus 3 to bus 2 from overloading.

Optimal Power Flow

- The goal of an optimal power flow (OPF) is to determine the "best" way to instantaneously operate a power system.
- Usually "best" = minimizing operating cost.
- OPF considers the impact of the transmission system
- OPF is used as basis for real-time pricing in major US electricity markets such as Midcontinent Independent System Operator (MISO) and PJM Energy Market.

Two Bus ED Example



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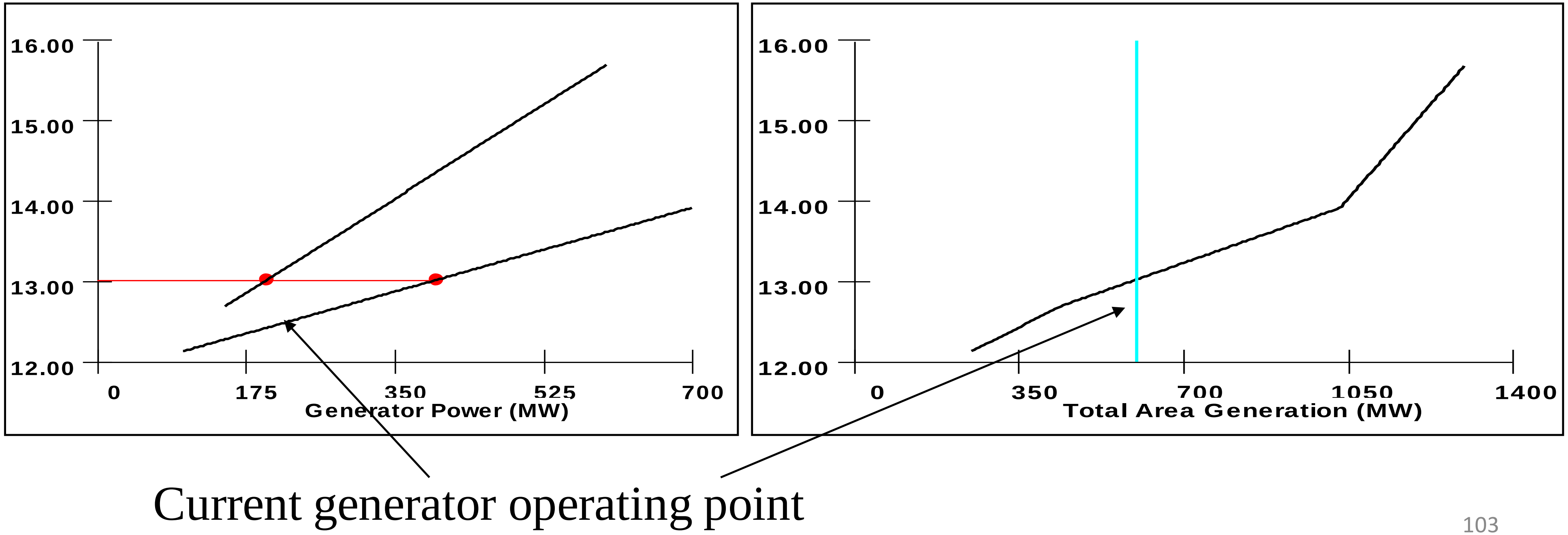
Market Marginal (Incremental) Cost

Below are some graphs associated with this two bus system. The graph on left shows the marginal cost for each of the generators. The graph on the right shows the system supply curve, assuming the system is optimally dispatched.



Real Power Markets

- Different operating regions impose constraints
  - Total demand in region must equal total supply
- Transmission system imposes constraints on the market.
- Marginal costs become localized
- Requires solution by an optimal power flow

Optimal Power Flow (OPF)

- OPF functionally combines the power flow with the economic dispatch.
- Minimizes cost function, such as operating cost, taking into account realistic equality and inequality constraints
- Equality constraints
  - Bus real and reactive power balance
  - Generator voltage set points
  - Area MW interchange

OPF, cont'd

- Inequality constraints:
  - Transmission line/transformer/interface flow limits
  - Generator MW limits
  - Generator reactive power capability curves
  - Bus voltage magnitudes (not yet implemented in Simulator OPF)
- Available Controls:
  - Generator MW outputs
  - Transformer taps and phase angles

OPF Solution Methods

- Non-linear approach using Newton's method
  - Handles marginal losses well, but is relatively slow and has problems determining binding constraints
- Linear Programming
  - Fast and efficient in determining binding constraints, but can have difficulty with marginal losses.
  - Used in Power World Simulator

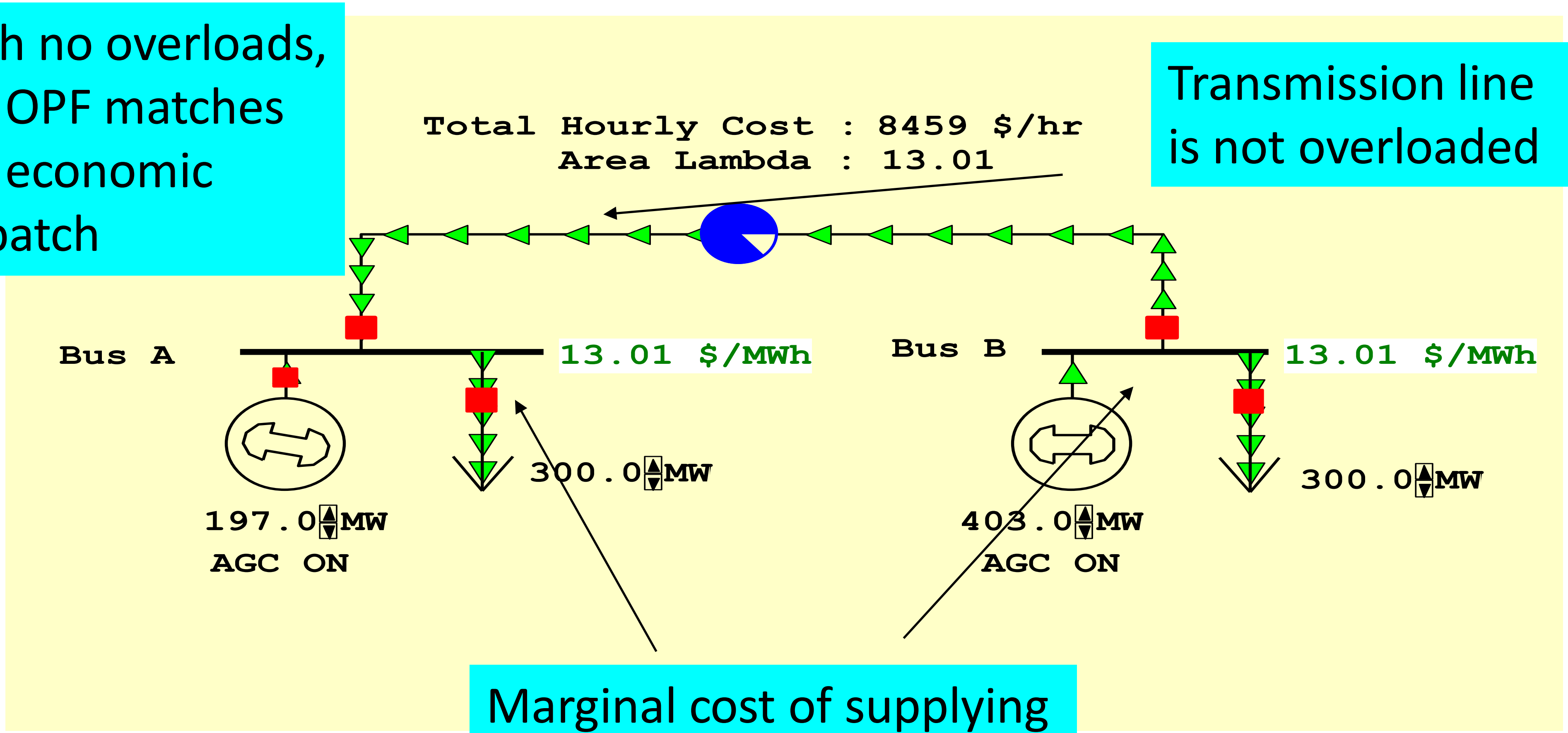
LP OPF Solution Method

- Solution iterates between
  - ➢ Solving a full ac power flow solution
    - ❖ Enforces real/reactive power balance at each bus
    - ❖ Enforces generator reactive limits
    - ❖ System controls are assumed fixed
    - ❖ Takes into account non-linearities
  - ➢ Solving a primal LP
    - ❖ Changes system controls to enforce linearized constraints while minimizing cost

Two Bus with Unconstrained Line

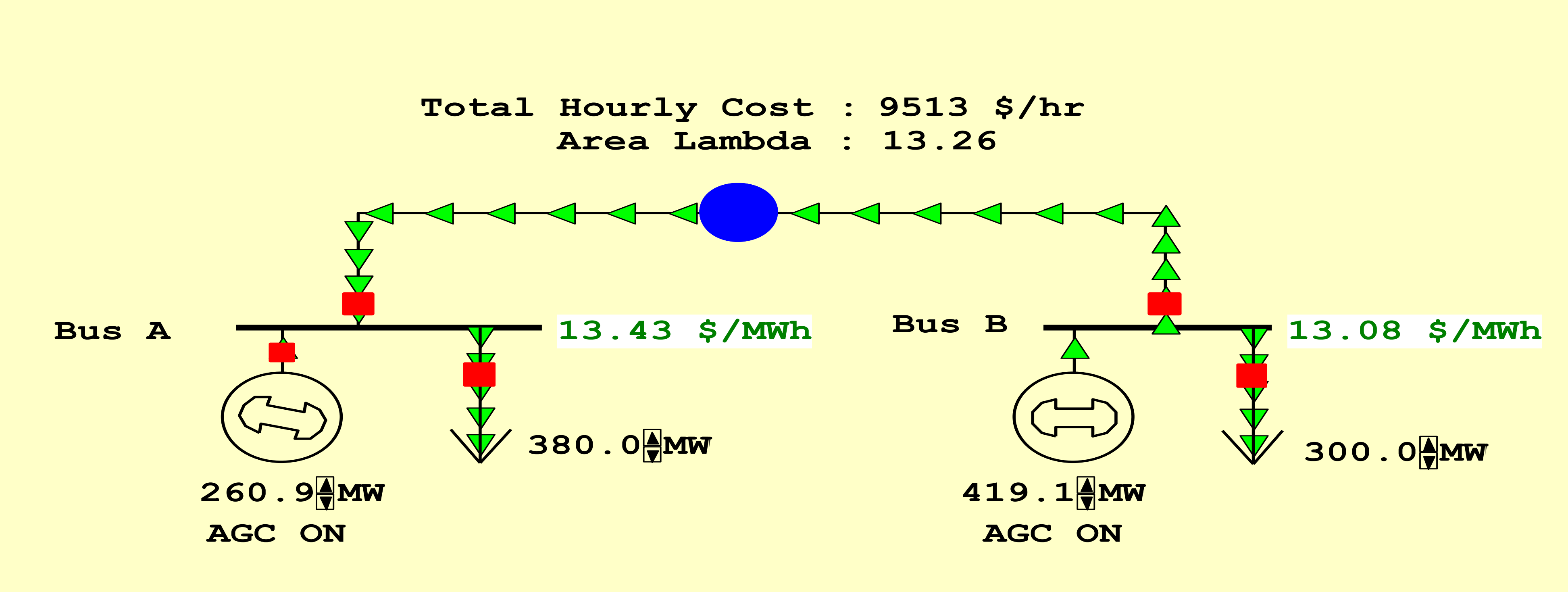
With no overloads, the OPF matches the economic dispatch

Transmission line is not overloaded



Marginal cost of supplying power to each bus (locational marginal costs)

Two Bus with Constrained Line



With the line loaded to its limit, additional load at Bus A must be supplied locally, causing the marginal costs to diverge.

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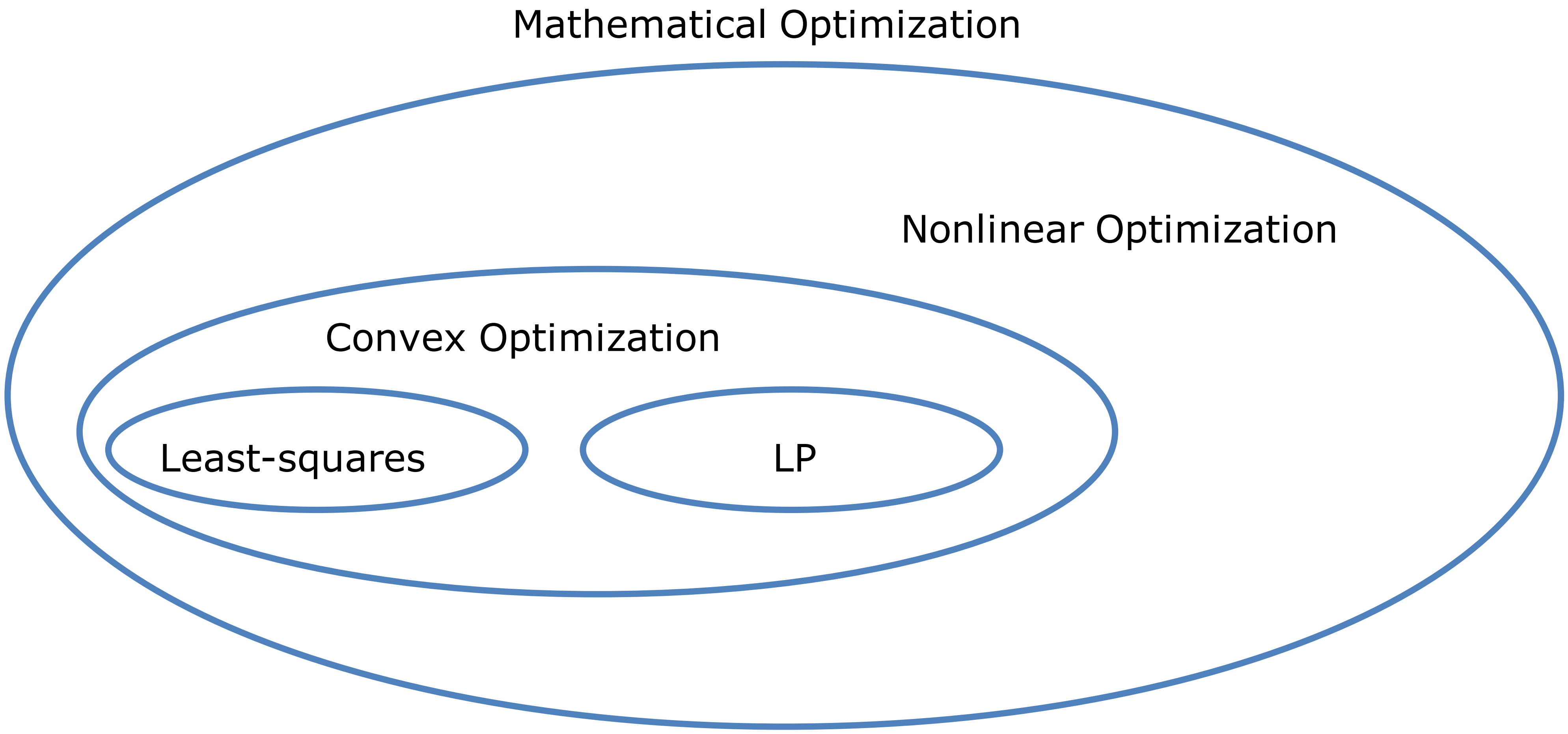
Convex Optimization



Mathematical Optimization

(Mathematical Optimization Problem)

minimize $\mathrm{f_o(x)}$

subject to $\mathrm{f_i(x)} \leq \mathrm{b_i}, i = 1, \ldots\ldots\ldots\ldots m$

- $x = (x_1, \ldots\ldots\ldots\ldots x_n) : optimization\ \mathrm{var}\,iables$
- $f_o = R^n \to R : Objective\ Function$
- $f_4 = R^n \to R : i = 1, \ldots m : Constra\,\mathrm{int}\,Functions$

$Optimal$ Solution x^* has smallest value of $\mathrm{f_o}$ among all vectors that staisfy the constraints

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Convex Optimization

$$Minimize\ \mathrm{f_o(x)}$$

$$\text{subject to } \mathrm{f_i(x)} \le \mathrm{b_i}, i = 1,\ldots., m$$

$$Objective\ and\ Constra\mathrm{int}\ Functions\ are\ convex$$

$$f_i(\alpha x + \beta y) \le \alpha f_i(x) + \beta f_i(y)$$

$$if\ \alpha + \beta = 1, \alpha \ge 0, \beta \ge 0$$

$$includes\ least\ squares\ problems$$

$$and\ linear\ programs\ as\ special\ caseses$$

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Analytical Solution of Least-squares

$A\ least - squares\ problem$

$is\ an\ optimization\ problem$

with no constraints

(ie., m =0) and an objective

which is a sum of squares of

terms of the form $a_i^T x - b_i$:

Here $A \in \mathrm{R}^{kxn} (with\ K \geq n), a_i^T$

$are\ the\ rows$ of A, and the

vector $x \in \mathrm{R}^n$ $is\ the\ optimization$

$\mathrm{var}iable$

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Linear Programming

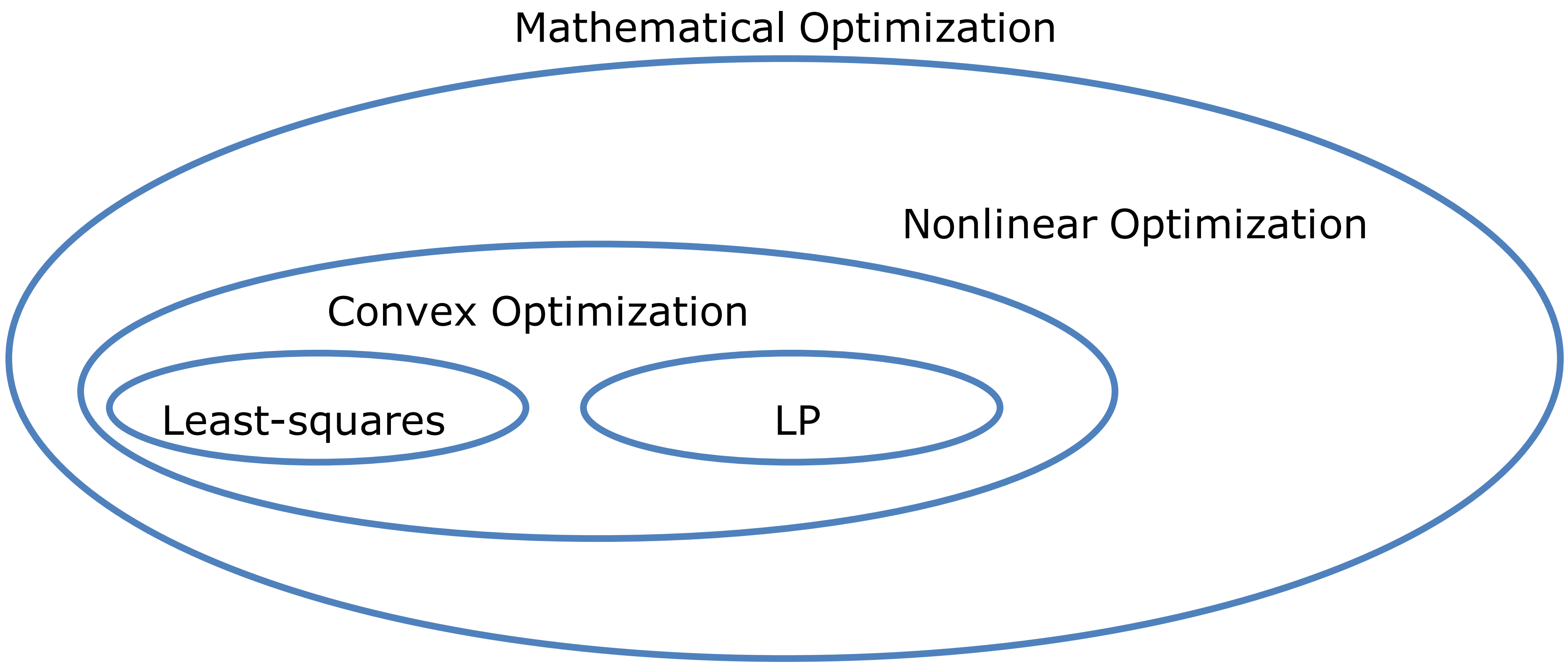
Another Important Class of optimization problems is linear programming in which the objective and all constraints are linear.

minimize $C^T x$

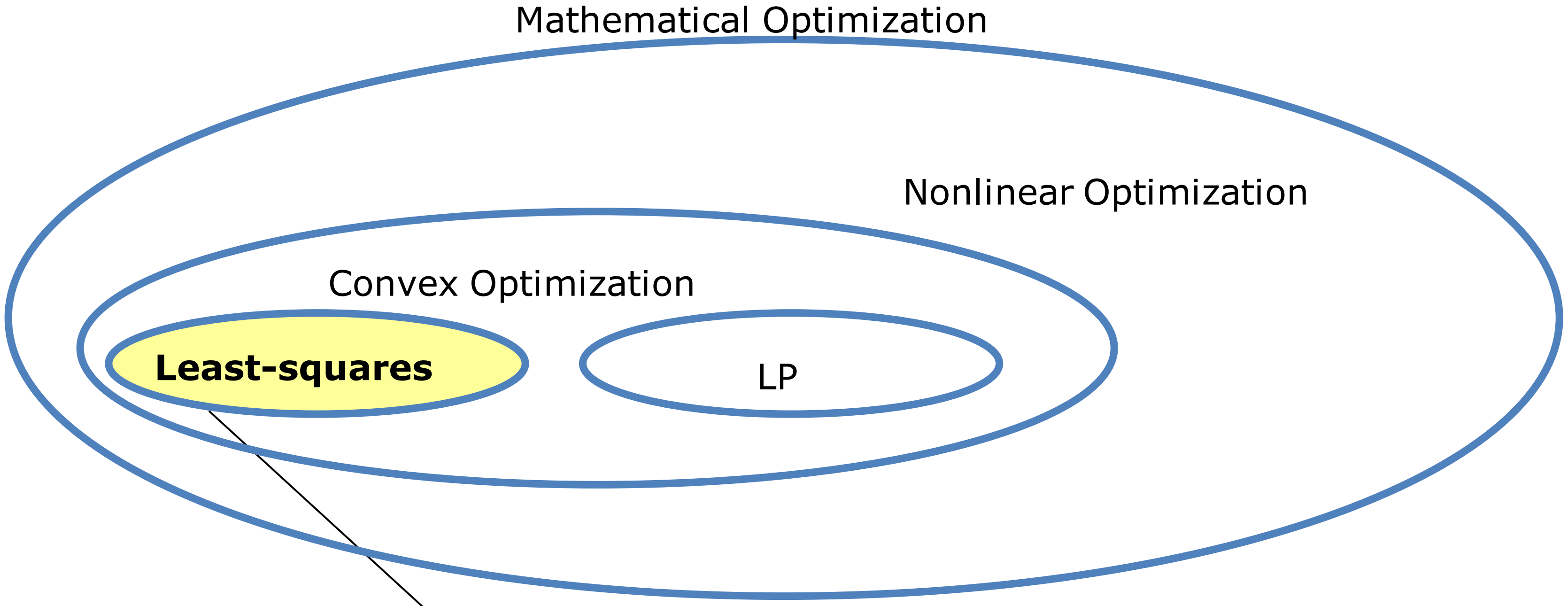
subject to $ai^T x \leq bi$, i = 1,....,m

Here the vectors $c, a_1, \ldots, a_m \in R^n$ *and* scalars $b_1, b_2 \ldots b_m \in R$ are problem parameters that define objective function and constraint.

Solving Optimization Problems



Solving Optimization Problems



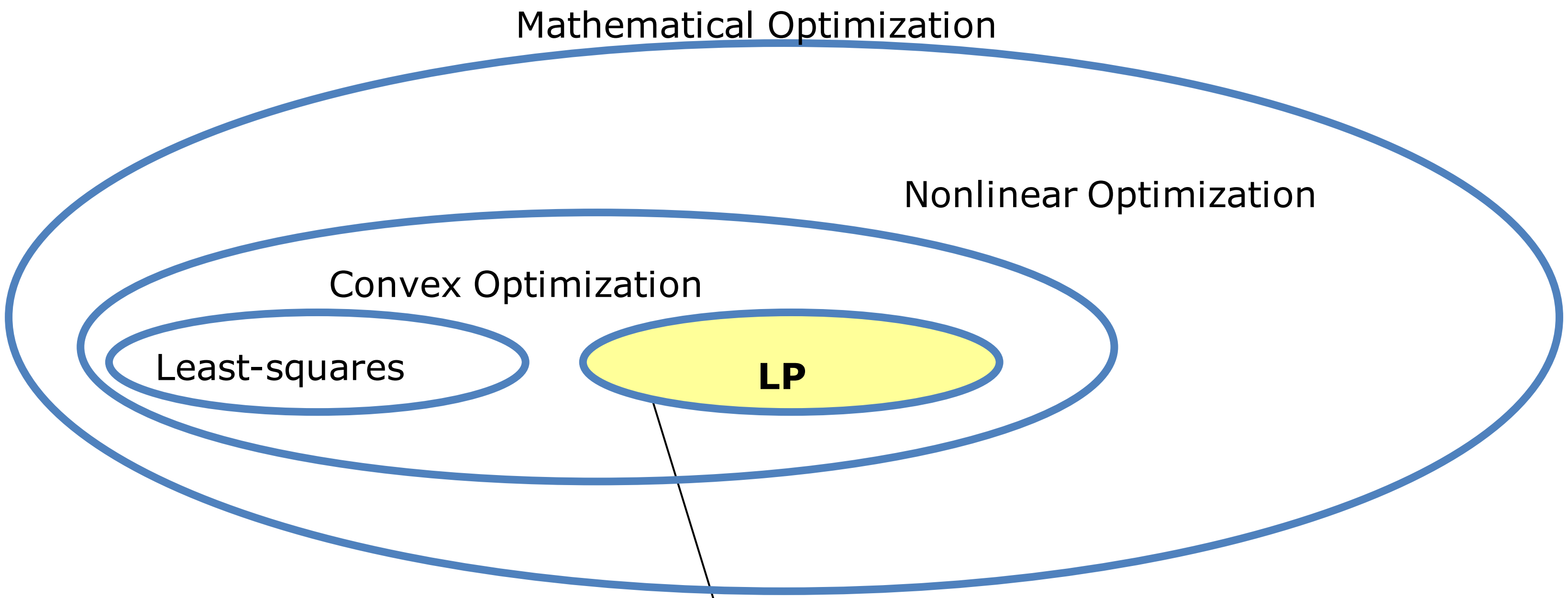
- Analytical solution
- Good algorithms and software
- High accuracy and high reliability
- Time complexity: $C \cdot n^2 k$

$$\text{minimize} \; \|Ax - b\|_2^2$$

A mature technology!

University of Hawai'i Mānoa

Solving Optimization Problems



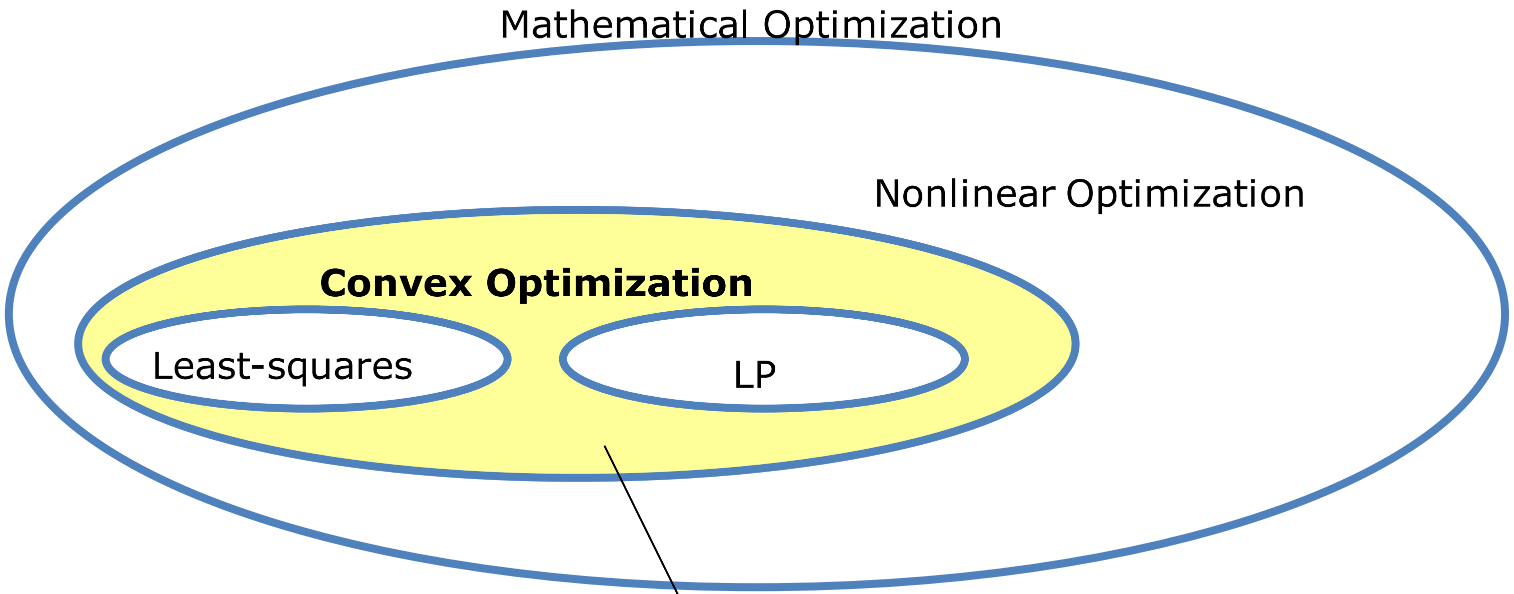
minimize $c^T x$

subject to $a_i^T x \le b_i, i = 1, \ldots, m$

- No analytical solution
- Algorithms and software
- Reliable and efficient
- Time complexity: $C \cdot n^2 m$

Also a mature technology!

Solving Optimization Problems



minimize $f_0(x)$

subject to $f_i(x) \le b_i, i = 1, \ldots, m$

- No analytical solution
- Algorithms and software
- Reliable and efficient
- Time complexity (roughly) $\propto \max\{n^3, n^2 m, F\}$

F is cost of evaluating f_i's and their first and second derivatives

Almost a mature technology!

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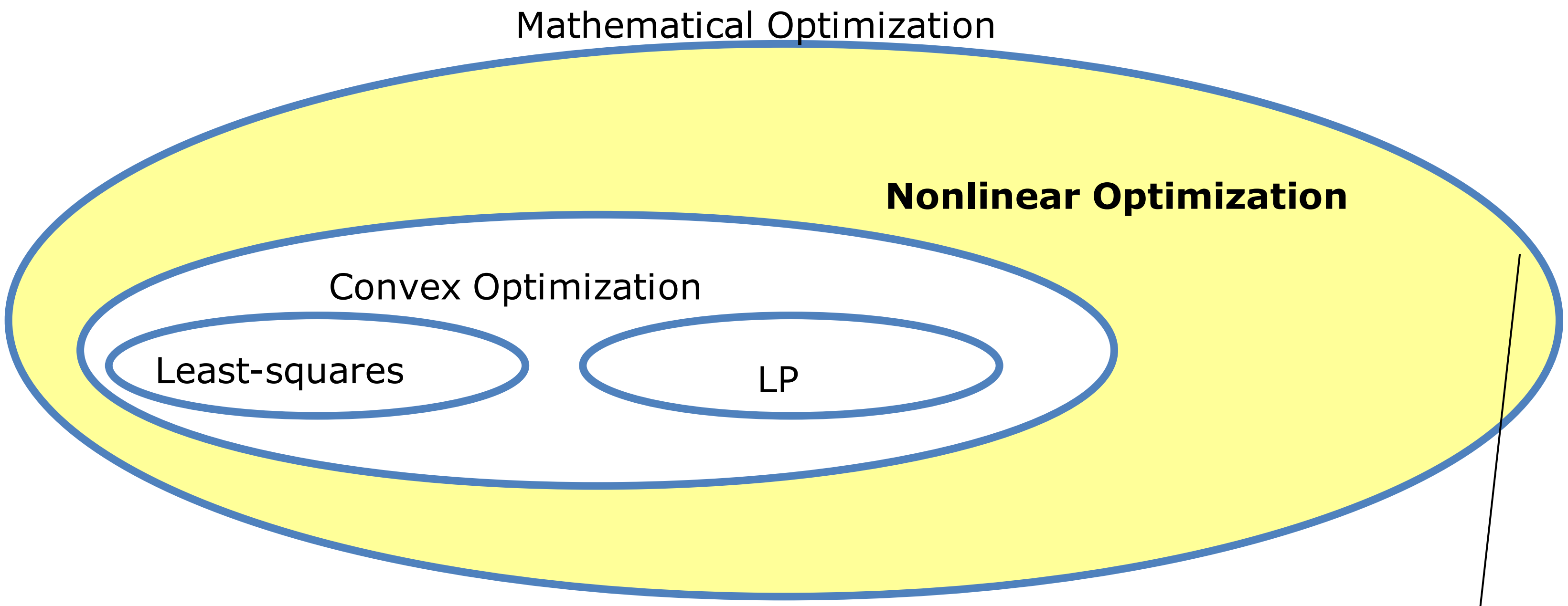
Solving Optimization Problems



- Sadly, no effective methods to solve
- Only approaches with some compromise
- Local optimization: *"more art than technology"*
- Global optimization: greatly compromised efficiency
- Help from convex optimization
  1) Initialization 2) Heuristics 3) Bounds

Far from a technology! (something to avoid)

Example #10: Convex Optimization in Power Flow

- On the figure, generator G1 and G2 connects to Bus1 and Bus2 respectively.
- The two buses are connected via a power line.
- At each bus, there is certain amount of load that is already known.
- We want to determine the power production in G1 and G2; so, that we could achieve the lowest cost for the overall generation cost.

Convex Optimization in Power Flow

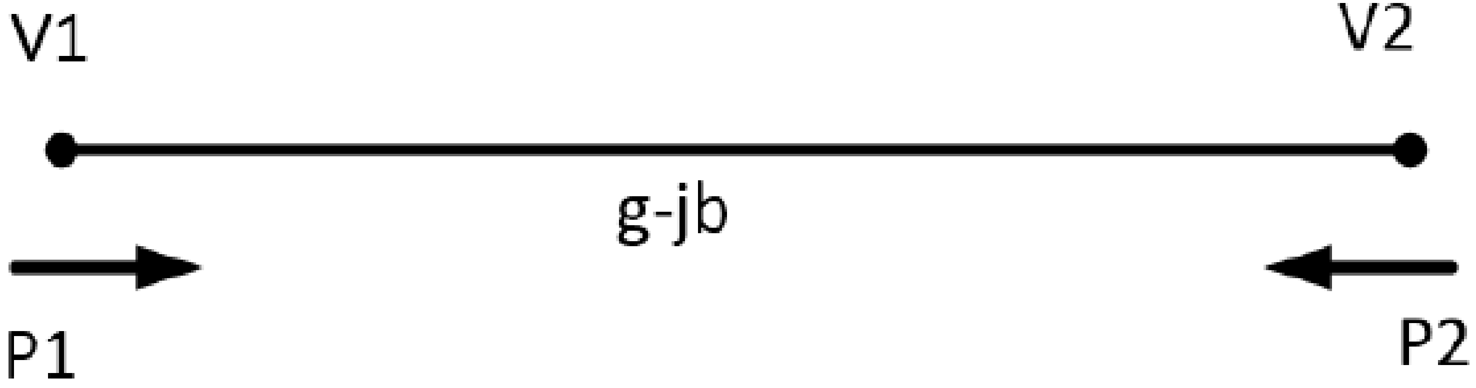
At Bus i, define power flow

into the line: $P_i = P_{Gi} - P_{Di}$, i=1,2;

Our goal is , minimize $f_1(P_1)+f_2(P_2)$, where $f_i(p_i)$ stands for the different cost functions for each generator

We assume that the voltage magnitude at both bus 1 and bus 2 are same and constant, i.e. $|V1|=|V2|=1$. Also, we will ignore reactive power for now.

Convex Optimization in Power Flow



$$P1 = |V1|^2 g + |V1||V2| b\sin(\theta) - |V1||V2| g\cos(\theta)$$

$$P2 = |V2|^2 g - |V1||V2| b\sin(\theta) - |V1||V2| g\cos(\theta)$$

Problem Formulation

Minimize :

$f_1(P1) + f_2(P2)$, with variable θ

Subject to

$$P1 = g + b\sin\theta - g\cos\theta$$

$$P2 = g - b\sin\theta - g\cos\theta$$

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Problem Formulation

By observing the constraints, the linear transformation of the equations leads to an ellipse. As shown in Figure. We note the boundary of this ellipse as S.

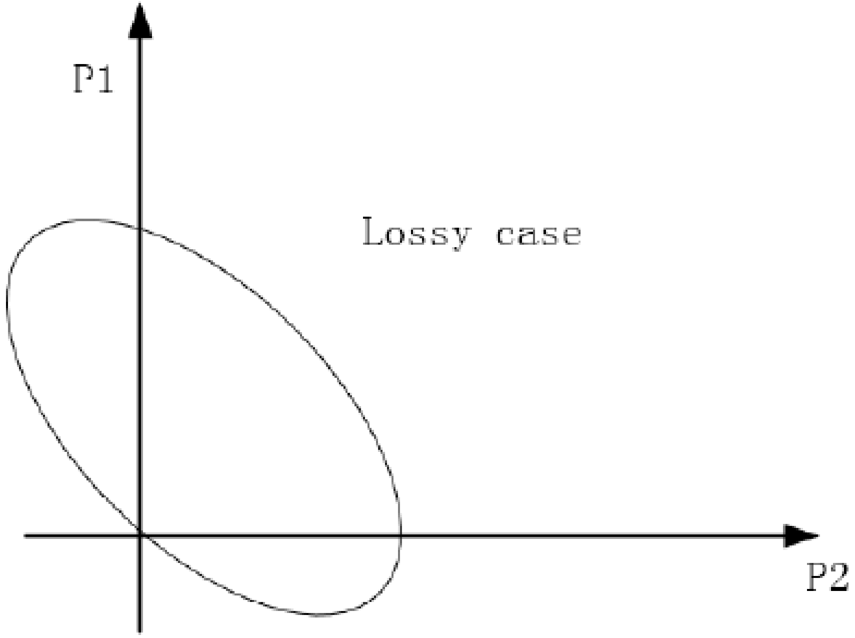

Figure.2

Solution

- The next step is to perform optimization.
- We, however, cannot directly perform convex optimization to it, because the boundary of an ellipse is non-convex.
- Here we have to introduce a concept named convex hull, which means the smallest convex set that contains the original set.
- By obtaining the convex hull of S, we get a set that contains the both the boundary and the area within this ellipse as shown on the next slide.

Solution

Figure.3

$$\text{Minimize}: f_1(P1) + f_2(P2)$$

$$\text{Subject to}: (P1, P2) \in S$$

OPF problem 2:

$$\text{Minimize}: f_1(P1) + f_2(P2)$$

$$\text{Subject to}: (P1, P2) \in S_c$$

Solution

- S_c is convex and can be optimized via convex optimization.
- The idea of solving OPF problem 2 instead of OPF problem 1 is called “convexification”.
- To make this analysis more practical, the upper bound of power production of each generator has to be taken into consideration.
- Therefore, we add one more constrain to achieve it.

Solution

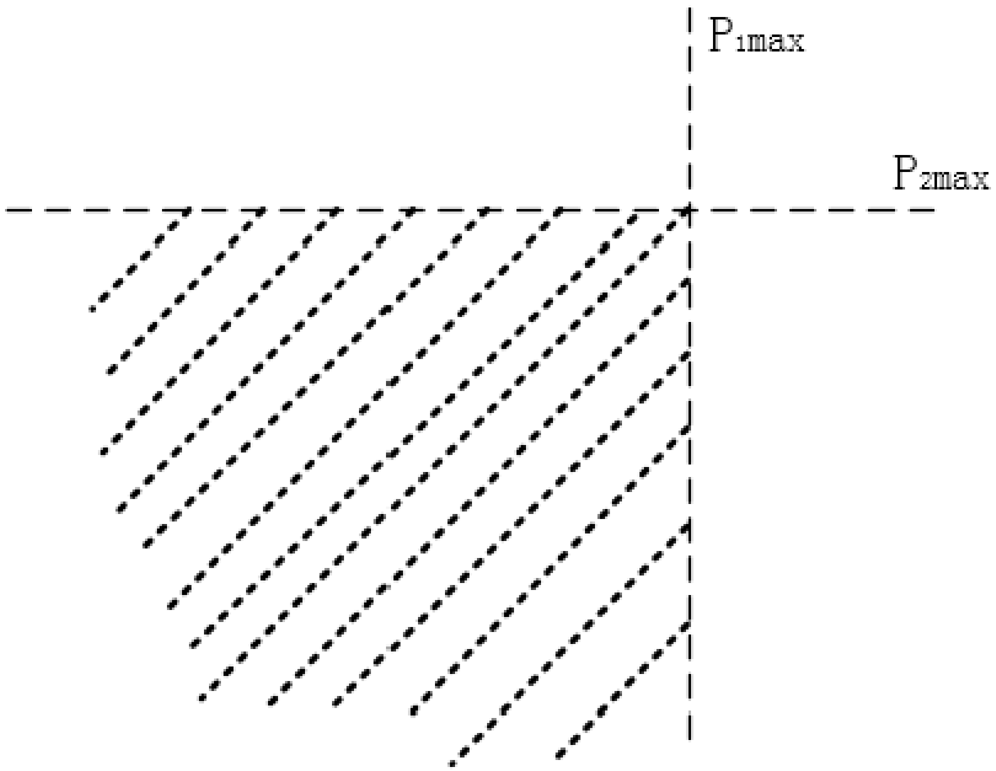

Figure.4

Solution

We want to make sure the power need of the loads are satisfied. Hence,

$$P_{G1}+P_{G2} \geq P_{D1}+P_{D2}$$

$$P1+P2 \geq 0$$

By combining the two feasible sets in Figure 2. and Figure 4.

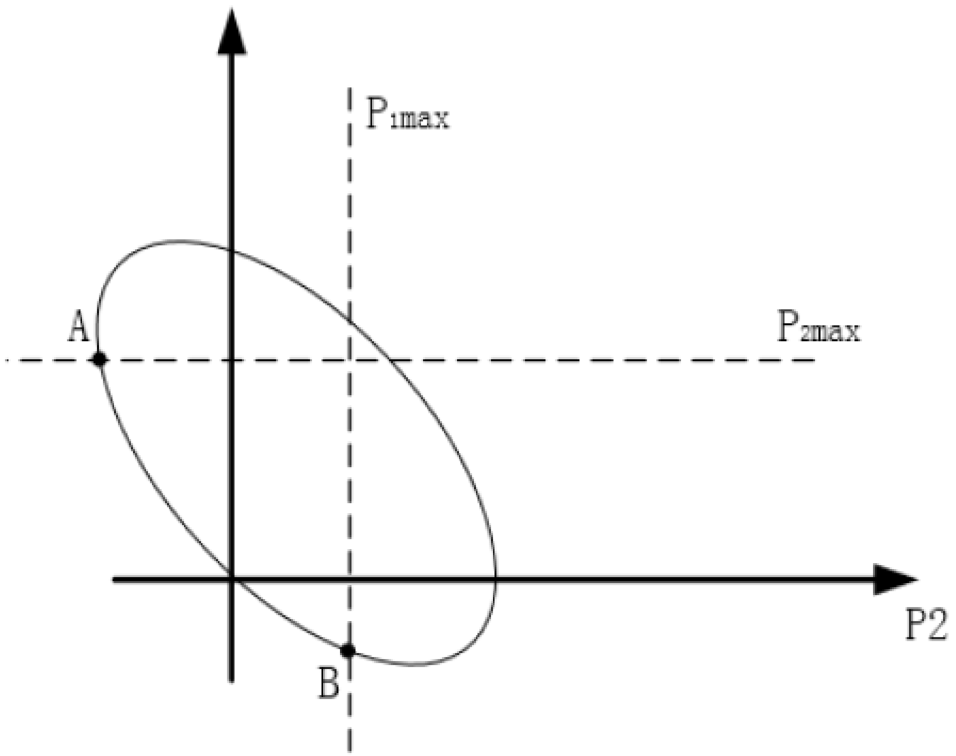

Figure.5

Convex Optimization Problem Formulation for Distribution Networks

Figure.6

Convex Optimization Problem Formulation for General Distribution Networks

Minimize :

$$\sum_{i,j=1}^{n}[f_i(P_i)+f_j(P_j)]$$

Subject to

$$V_i^{\min} \le |V_i| \le V_i^{\max}, i = 1,2,\ldots n$$

$$P_{ij} \le P_{ij}^{\max}, i, j = 1,2,3\ldots n$$

$$P_i \le P_i^{\max}, i = 1,2,\ldots n$$

$$P_i = \mathrm{Re}[V_i(Vi - V_j)*y_{ij}^{*}]$$

$$P_j = \mathrm{Re}[V_j(V_j - V_i)*y_{ij}^{*}]$$

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Convex Optimization Links

1. http://web.stanford.edu/class/ee392o/links.html
2. http://www.ee.columbia.edu/~lavaei/Students_Projects_Power.html

Convex Optimization: Software

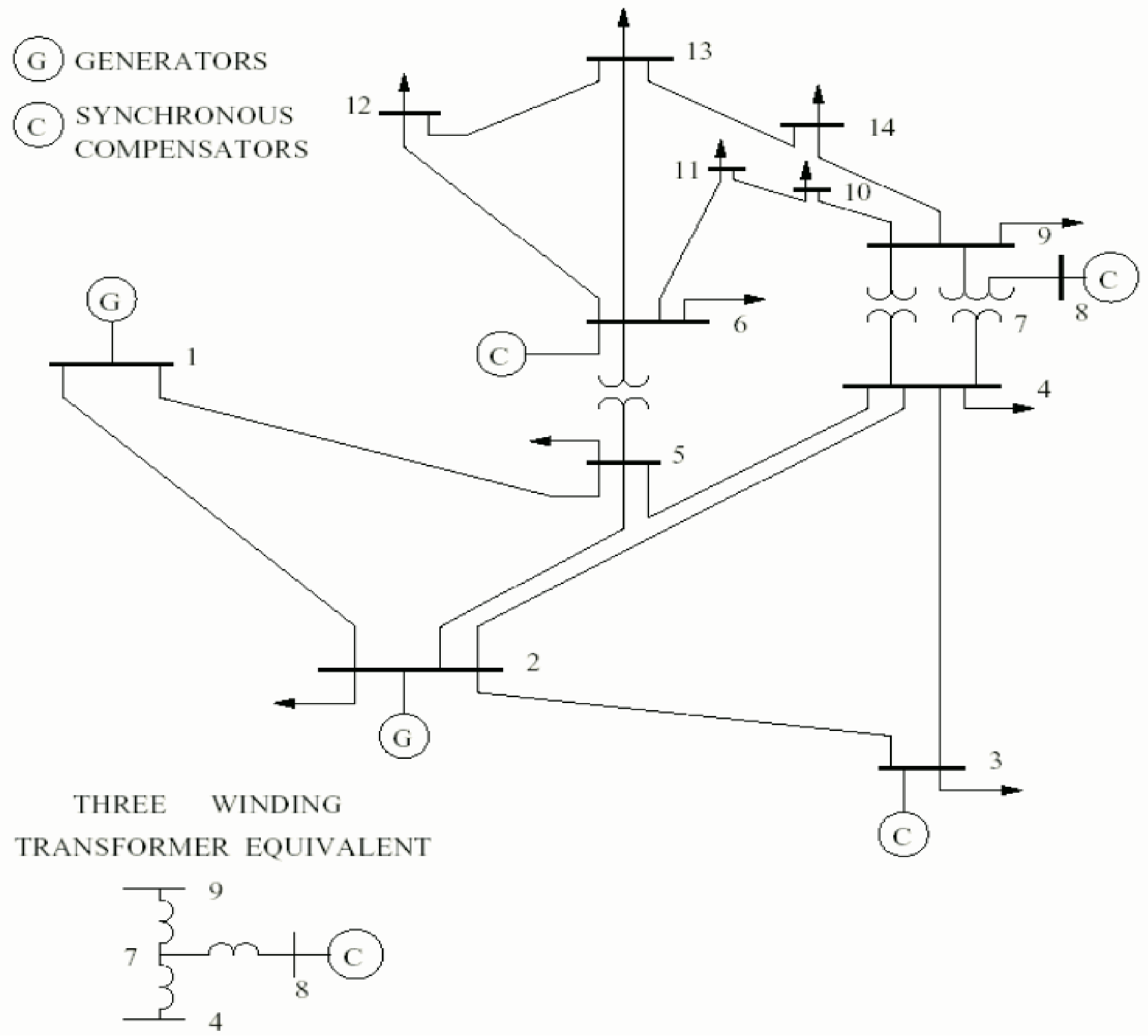
Recommended Software:

http://www.ee.columbia.edu/~lavaei/Software.html

Assignment

1. By using any of the Convex Optimization toolbox/software obtain the OPF solution for IEEE 14 bus system.
2. Obtain the OPF solution for the IEEE 14 bus system with the help of any standard software say PSAT of Waterloo.
3. Compare the solution and comment on the results.
4. What are the benefits and drawbacks of the convex optimization when compared to the conventional optimization methods for solving algorithms such as OPF.

Any other data if required can be assumed.



Questions?

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