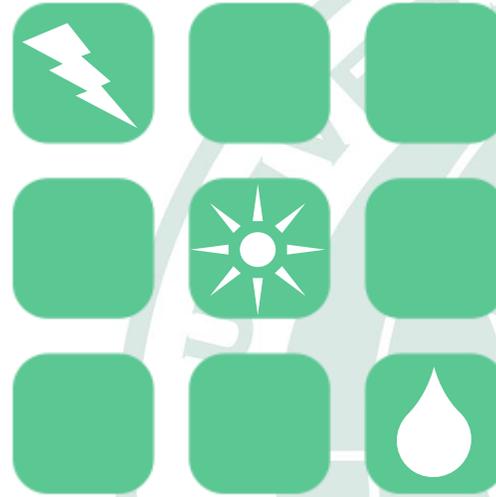


Smart Grid – Modeling and Control



Micro Grid Stability

University of Hawaii's Renewable Energy Design Laboratory (REDLab)
in collaboration with Powersim Inc. and MyWay

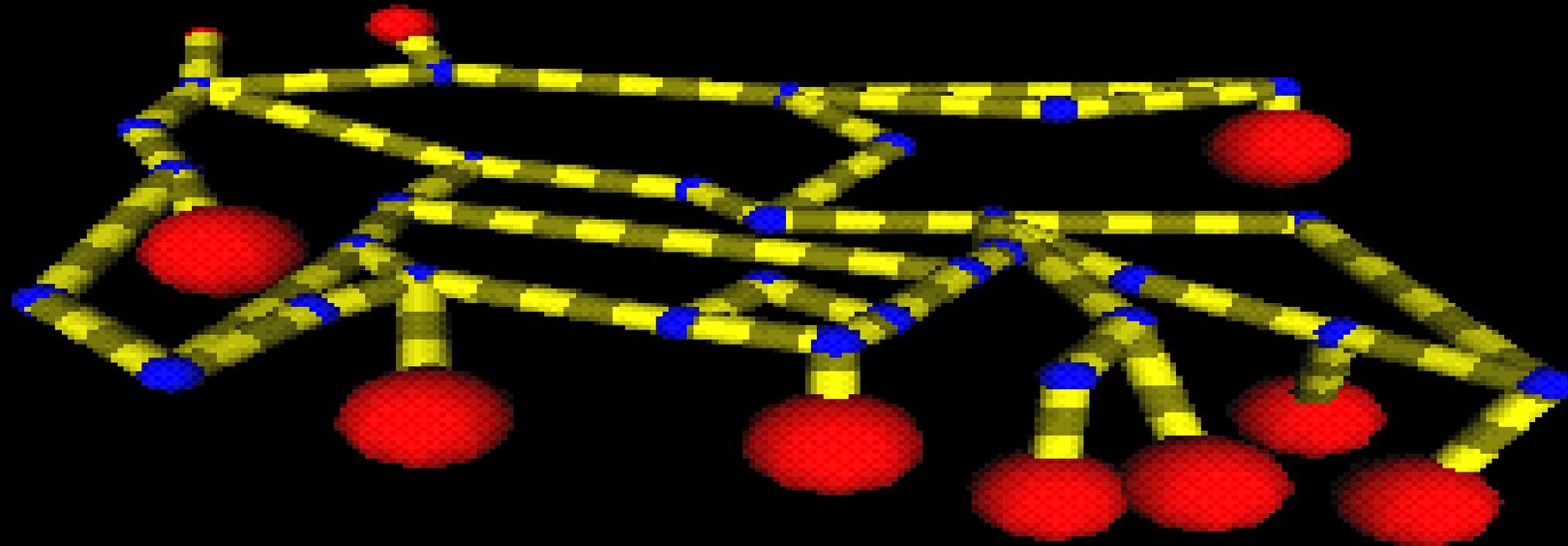


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Smart Grid – Modeling and Control

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ENERGY FLOW IN TRANSMISSION NETWORK

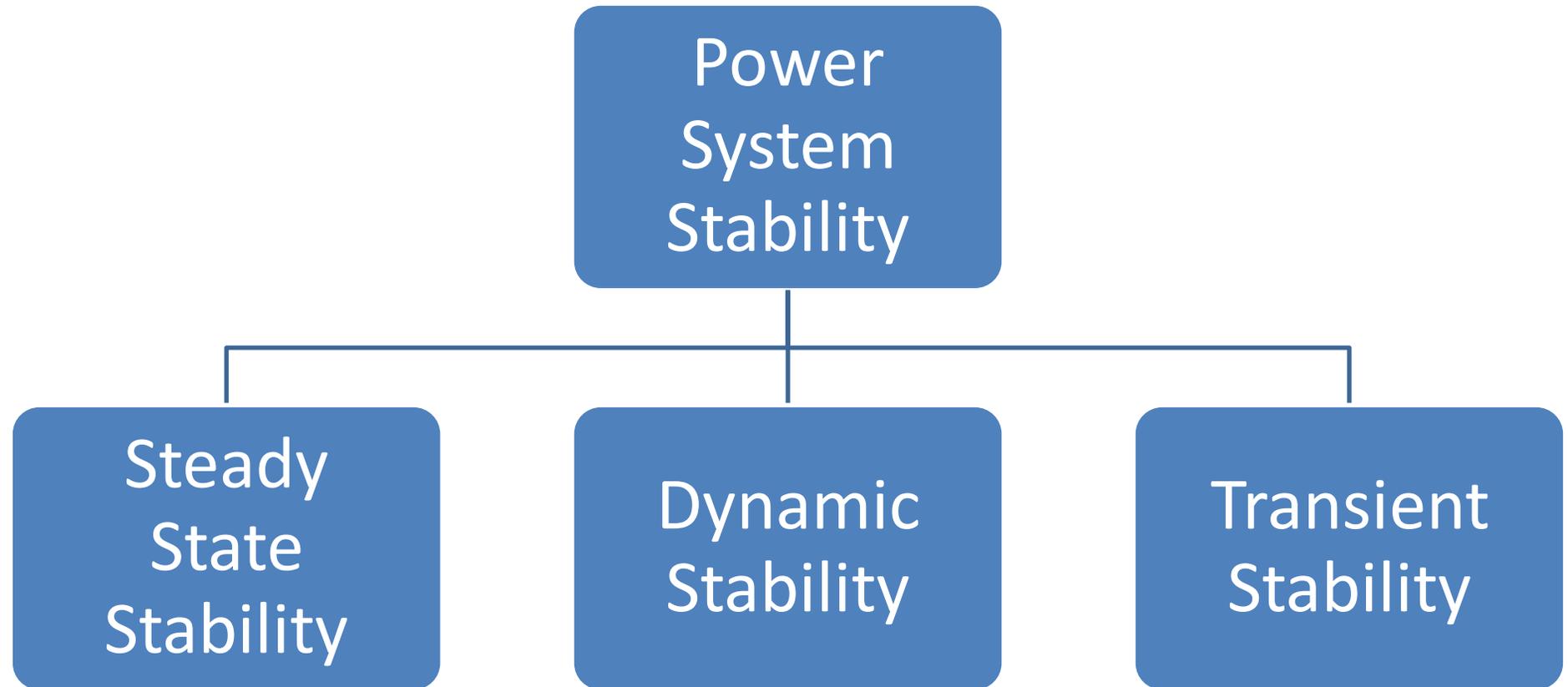


Basic Concepts

It is the property of a **dynamical system** or its ability to remain in state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance.



Power System Stability



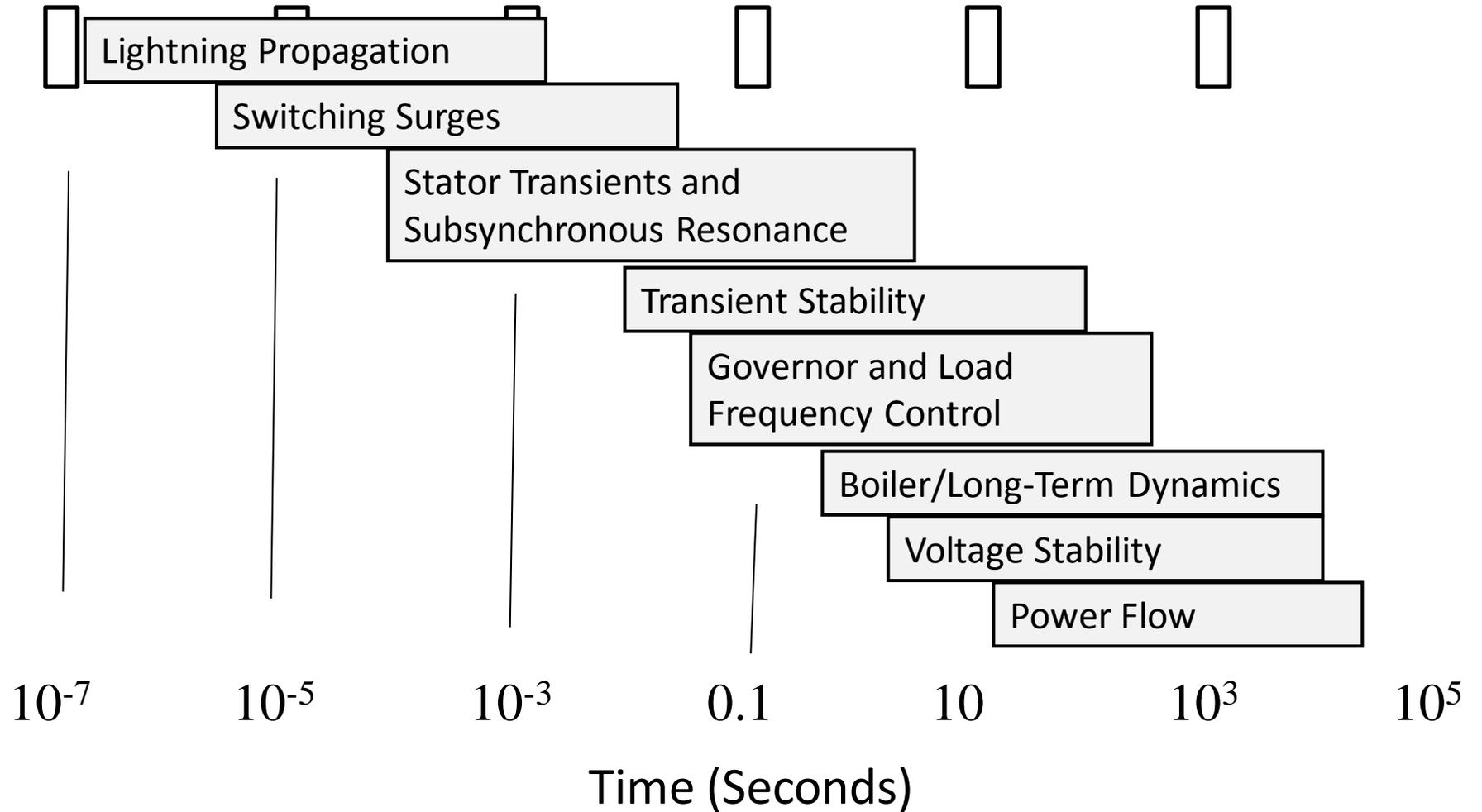
Steady State Stability : Gradual changes in the system operating conditions

Transient Stability: Study of the power system following a major disturbance

Dynamic Stability/Small Signal Stability: Ability of a power system to maintain stability under continuous small disturbances



Power System Time Scales



Source: P.W. Sauer, M.A. Pai, Power System Dynamics and Stability, 1997, Fig 1.2, modified

Power Grid Disturbance Example

Figure 1. Time in Seconds

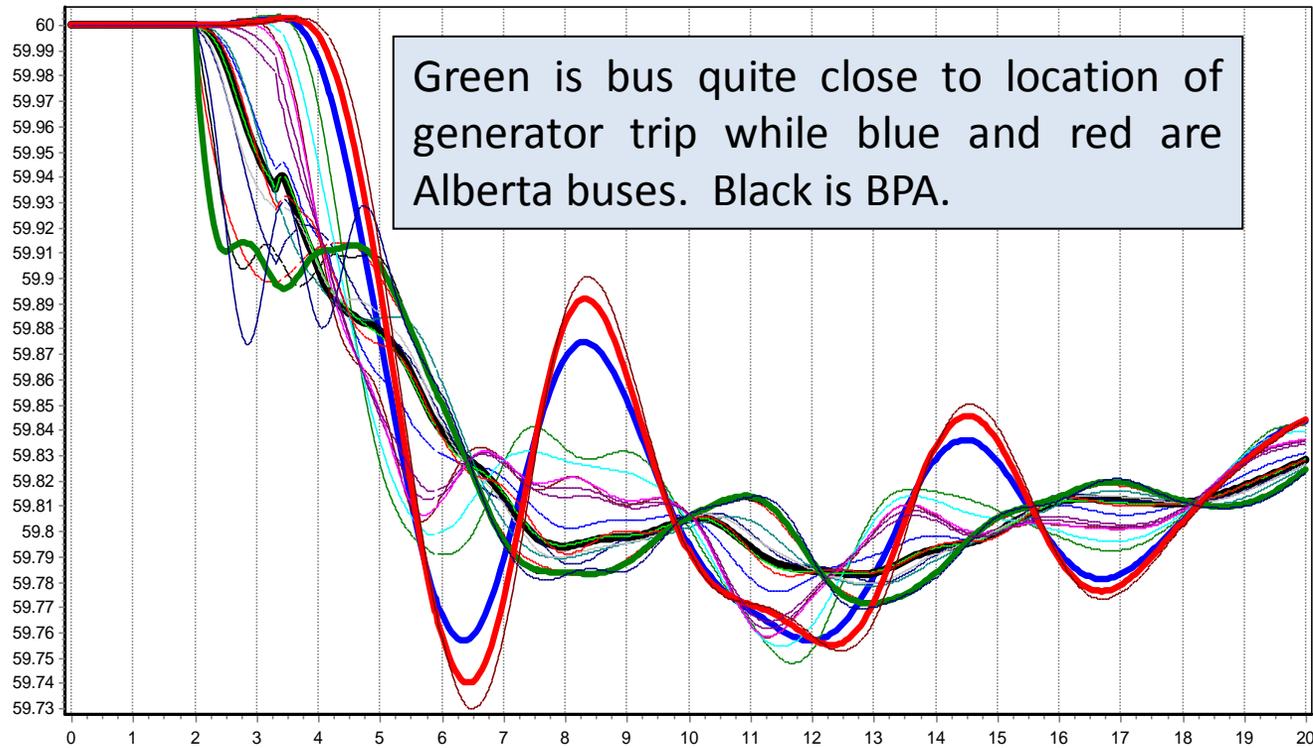
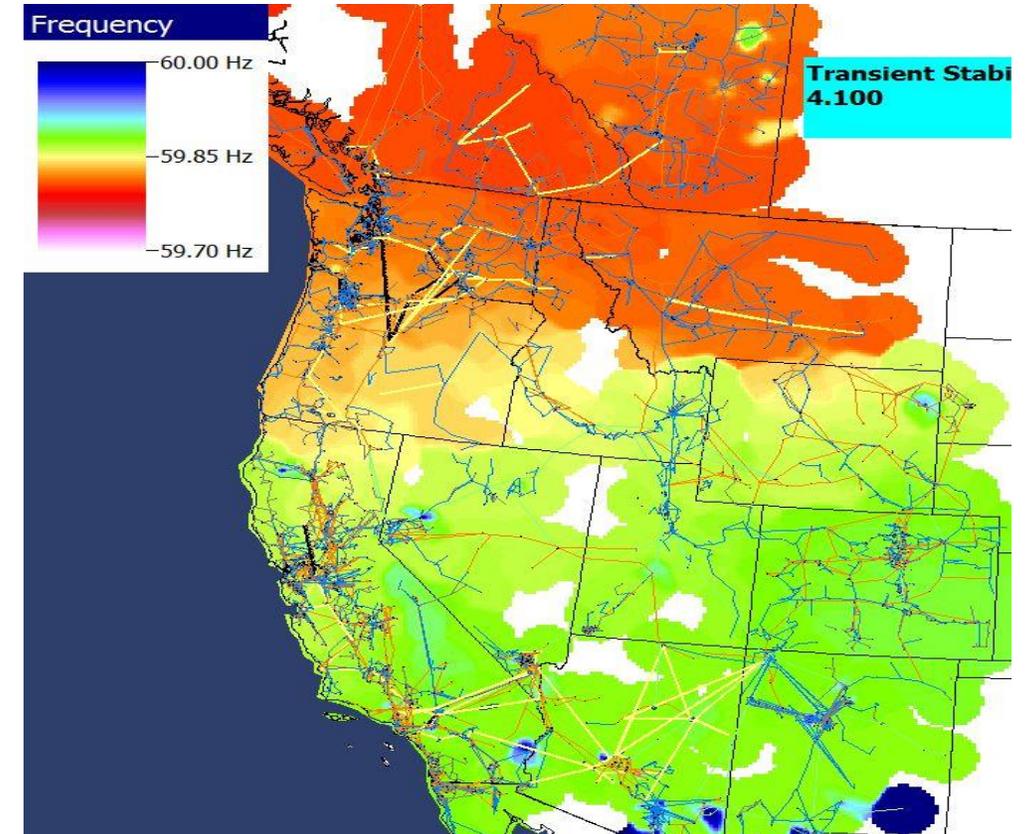


Figure 2. Frequency Contour

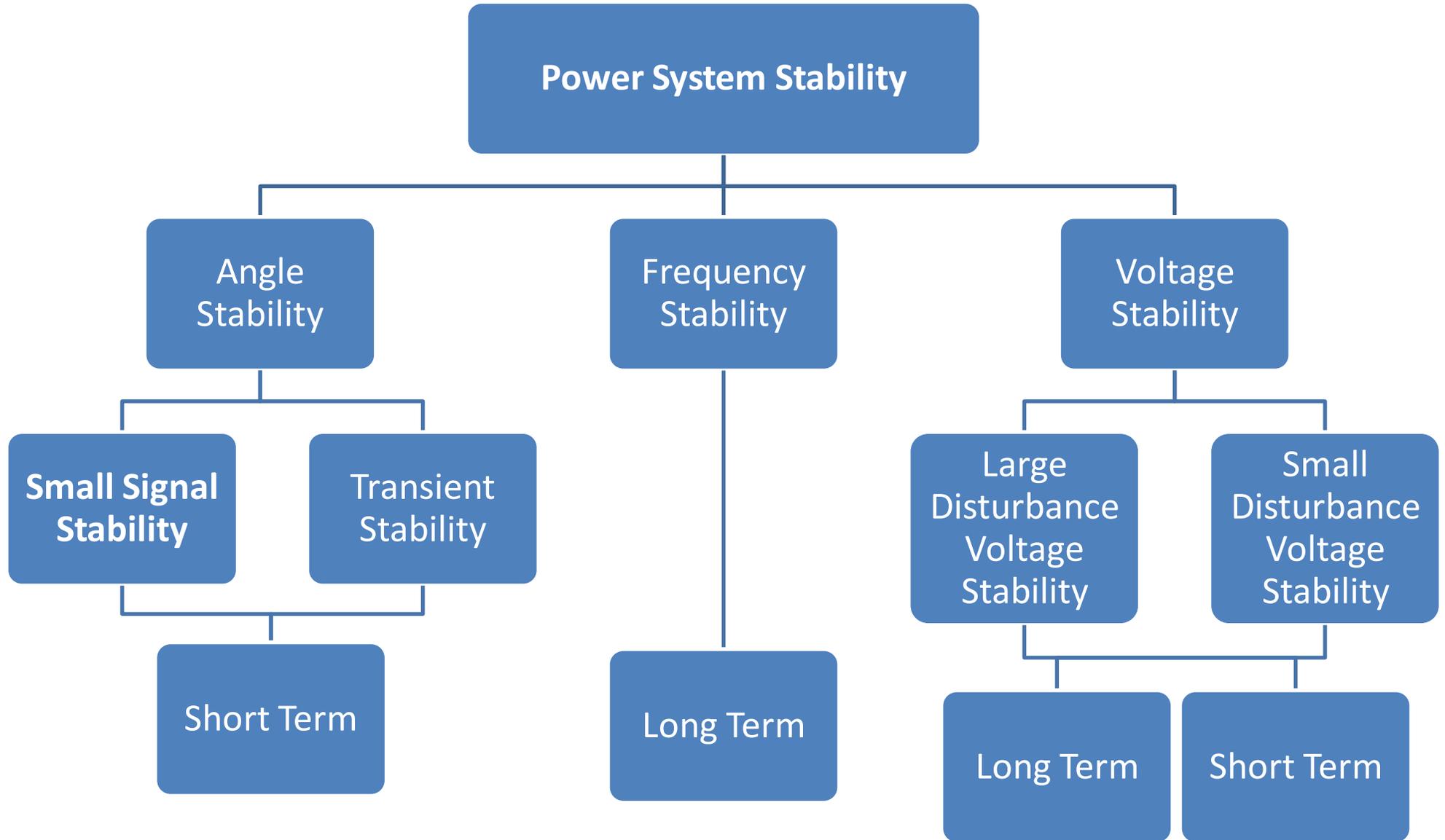


Figures show the frequency change as a result of a sudden loss of a large amount of generation in the Southern Western Electricity Coordination Council (WECC).

Source: WECC



Classification of Power System Stability



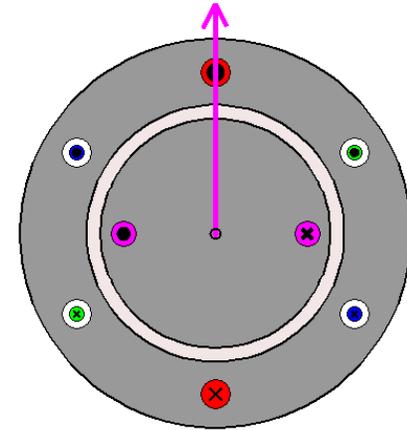
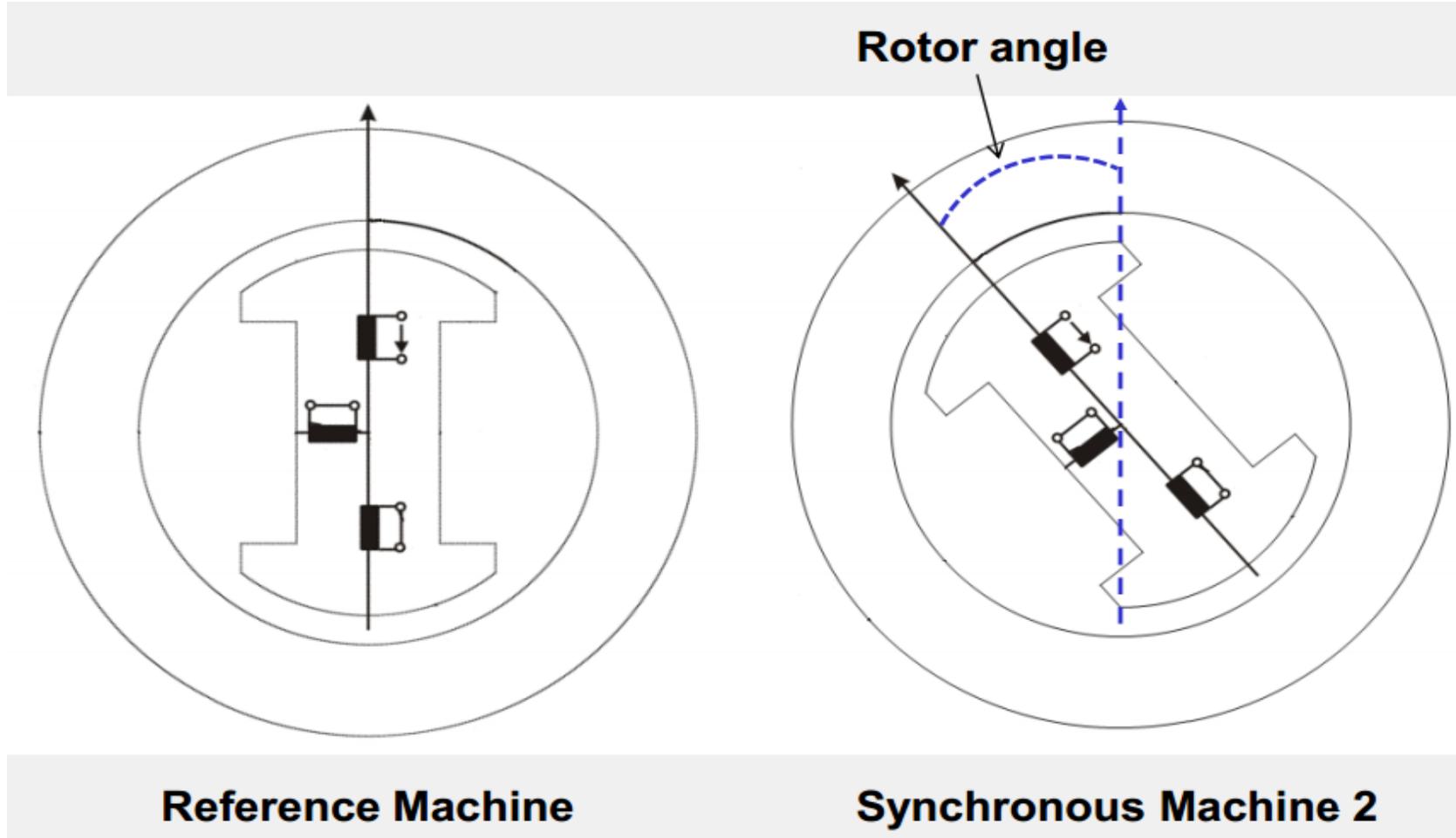


Rotor Angle Stability

- Ability of interconnected **synchronous** machines to remain in synchronism after being subjected to a disturbance
- Depends on the ability to restore **equilibrium between electromagnetic torque** and **mechanical torque** of each synchronous machine
- Instability that may result in the form of increasing angular swings of some generators, leading to **loss of synchronism** with other generators

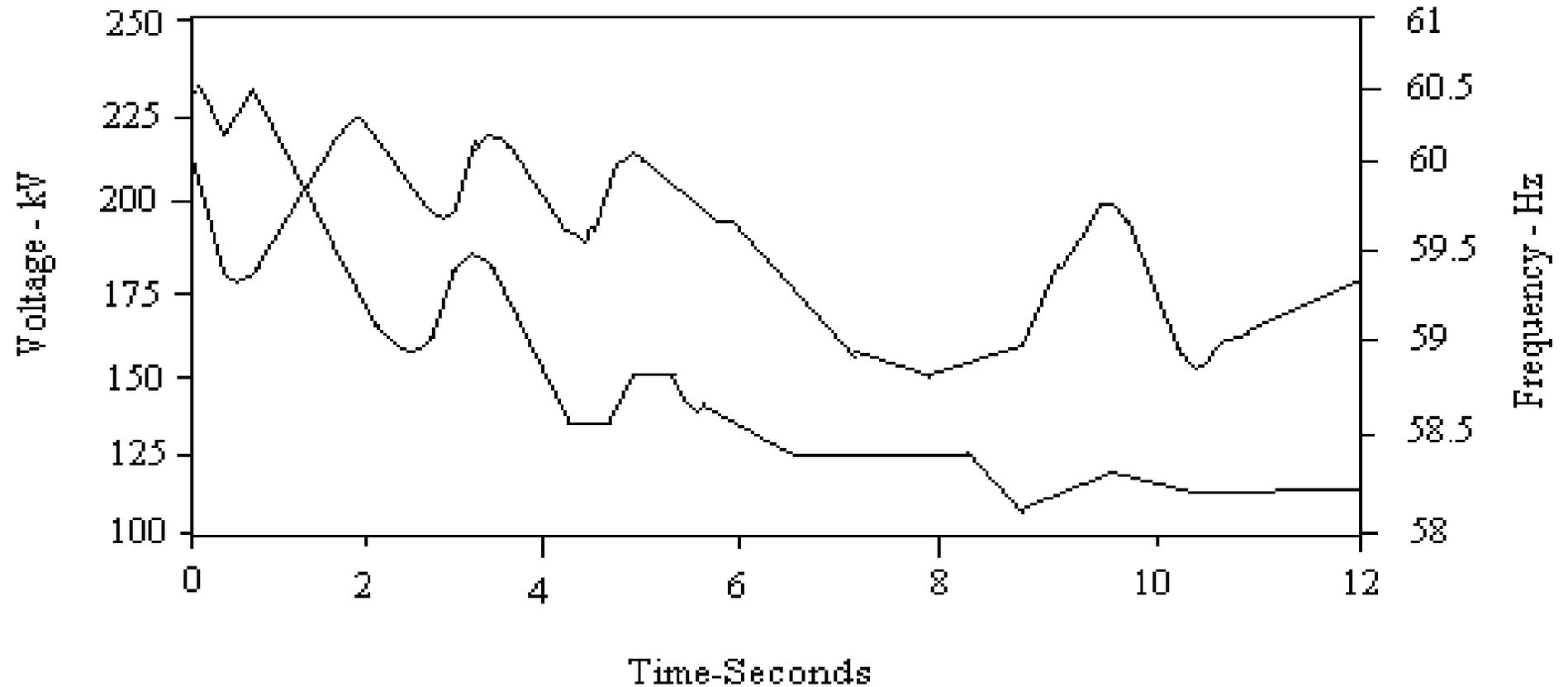


Rotor Angle Stability





Rotor Angle Stability



Voltage and frequency for South Florida blackout in 2008



Transient Stability

- Traditionally used to denote **large-disturbance angle stability**
- Ability of a power system to maintain synchronism when subjected to a severe transient disturbance:
 1. Influenced by the **nonlinear power-angle relationship**
 2. Stability depends on the initial operating condition and severity of the disturbance
- The system is designed and operated to be stable for a selected set of contingencies
 1. Transmission Faults: L-G, L-L-G, Three phase

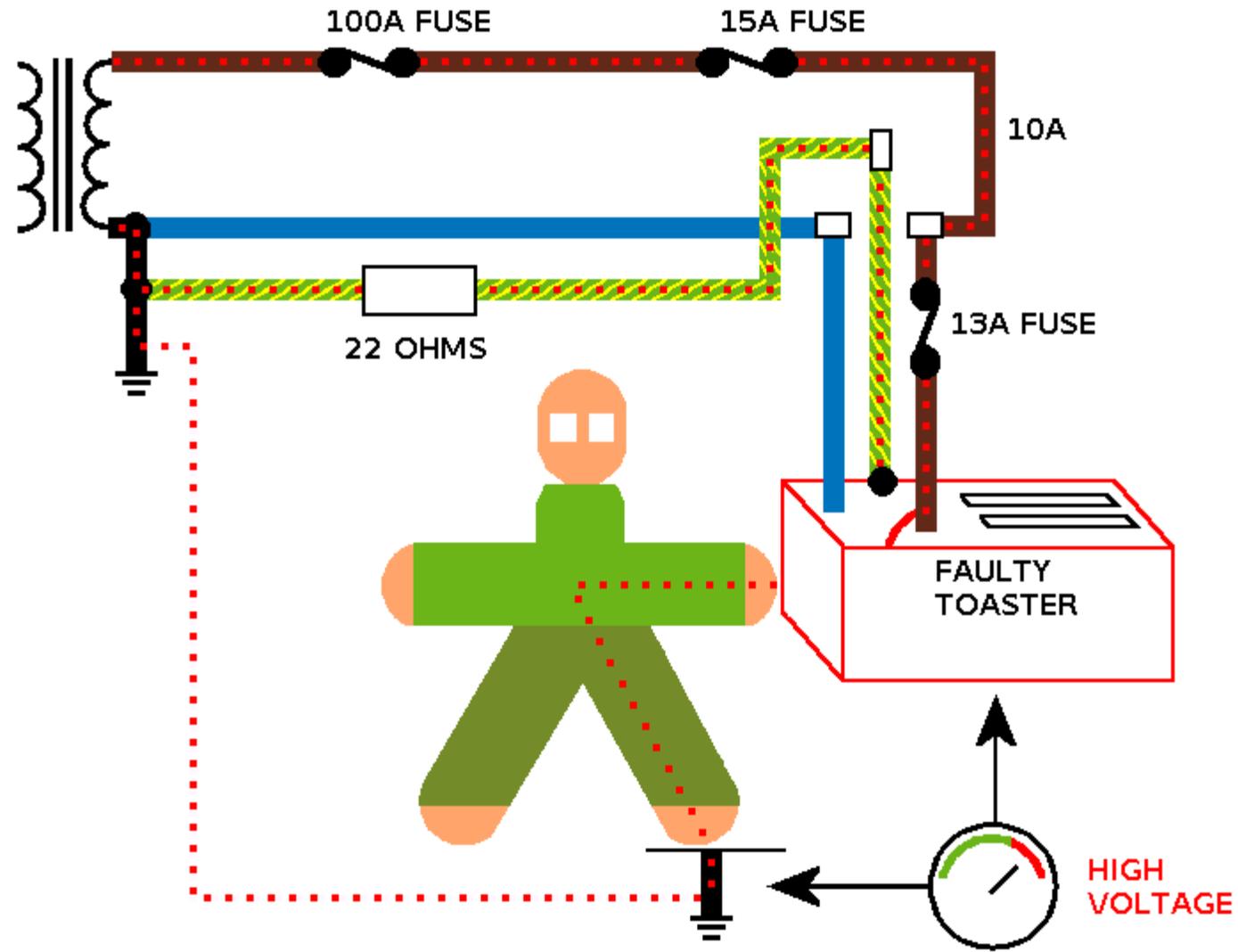


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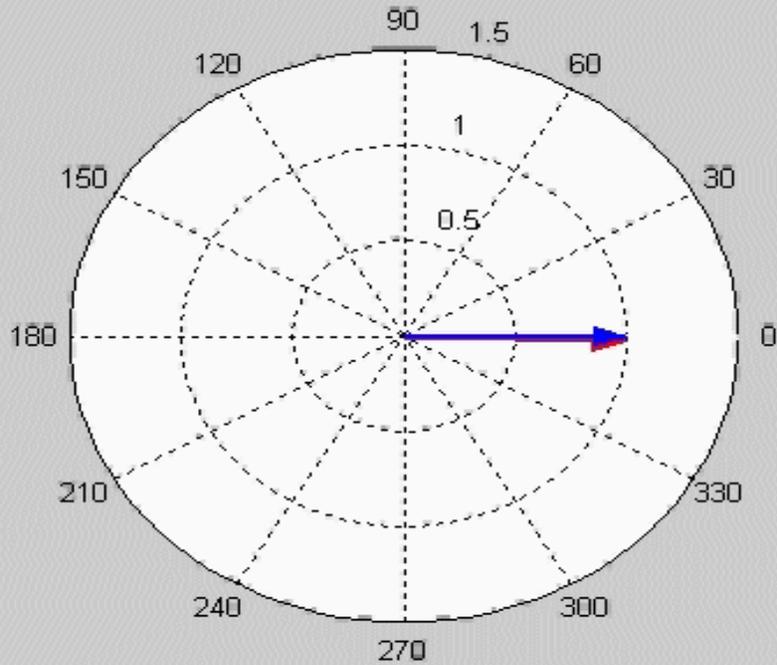
Transient Faults due to tree leaves touching on transmission are being removed



WWW.EEC247.COM

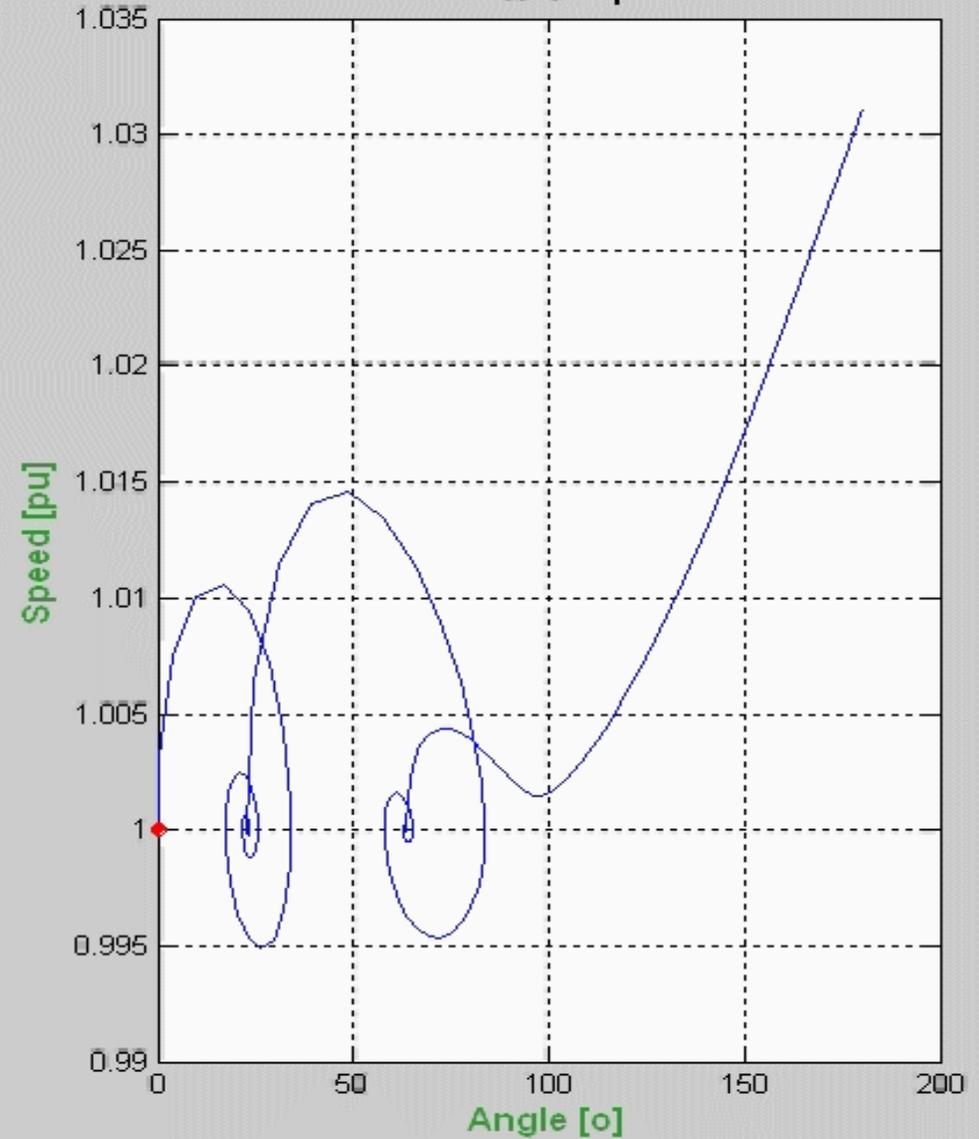
Fault due to faulty appliance

Voltage space vectors in motion



Step load:
 $P = 0$

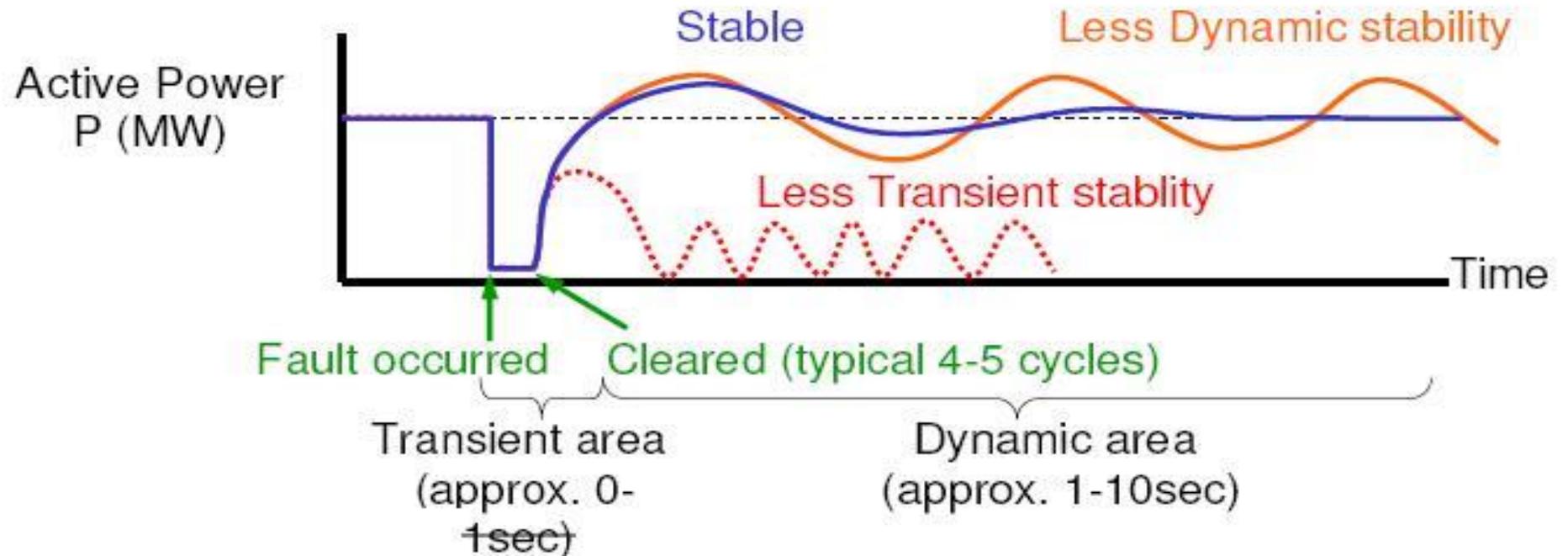
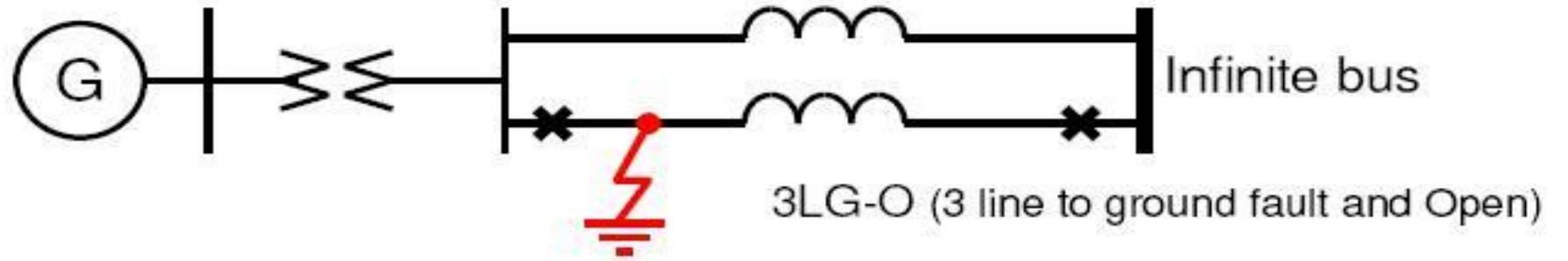
Transient response



Transient Response due to increase in Load



Transient Stability



Clearing the Fault



Small-Signal (Angle) Stability (SSS)

- Ability of a power system to maintain synchronism (disturbance = sufficiently small if **linearization** of system equations is permissible for analysis)
- Instability that may result can be of two forms:
 1. An **aperiodic increase in rotor angle** due to lack of sufficient synchronizing torque
 2. Rotor **oscillations of increasing amplitude** due to lack of sufficient damping torque
- In modern power systems, SSS problems are mostly associated with oscillatory modes
 - Local plant mode oscillations: **0.8 to 2.0 Hz**
 - Inter area oscillations: **0.1 to 0.8 Hz**



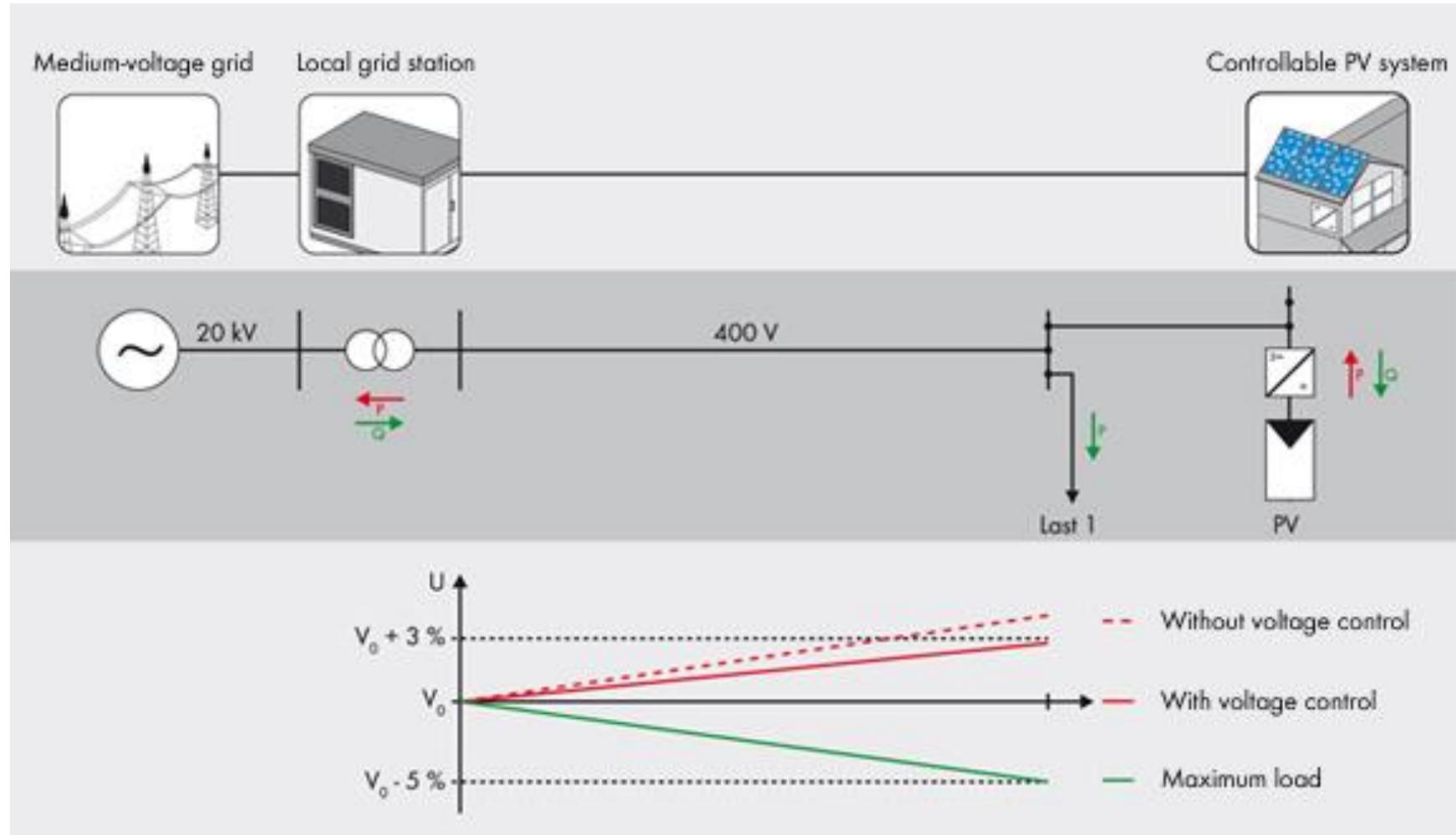
Voltage Stability

- The ability to maintain steady voltages at all buses in the power system after being subjected to a disturbance
- A system experiences voltage instability when a disturbance, increases in load demand, or changes in system condition causes:
 - A progressive and uncontrollable fall or rise in voltage of buses in a small area or a relatively large area
- Main factor causing voltage instability is the inability of power system to maintain a proper balance of reactive power and voltage control actions
 - The driving force is usually the load characteristics



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Voltage Variation in a Micro Grid



Source: Sunny, The SMA Corporate Blog



Short-Term and Long-Term Voltage Stability

- Short-term voltage stability involves dynamics of fast-acting load components, such as induction motors, electronically controlled loads, and HVDC converters
 - Study period can be from several seconds to **several minutes**
 - Dynamic modeling of loads is often essential
 - Faults/short-circuits near loads could be important
(Long-term voltage stability involves slower acting equipment, such as tap-changing transformers, thermostatically controlled loads, and generator field current limiters)



Frequency Stability

- Ability to maintain steady frequency within a nominal range following a disturbance, resulting in a significant imbalance between generation and load
- Instability may occur in the form of sustained frequency swings leading to tripping of generating units and/or loads
- In a small "**island**" system, frequency stability could be of concern for any disturbance causing a significant loss of load or generation



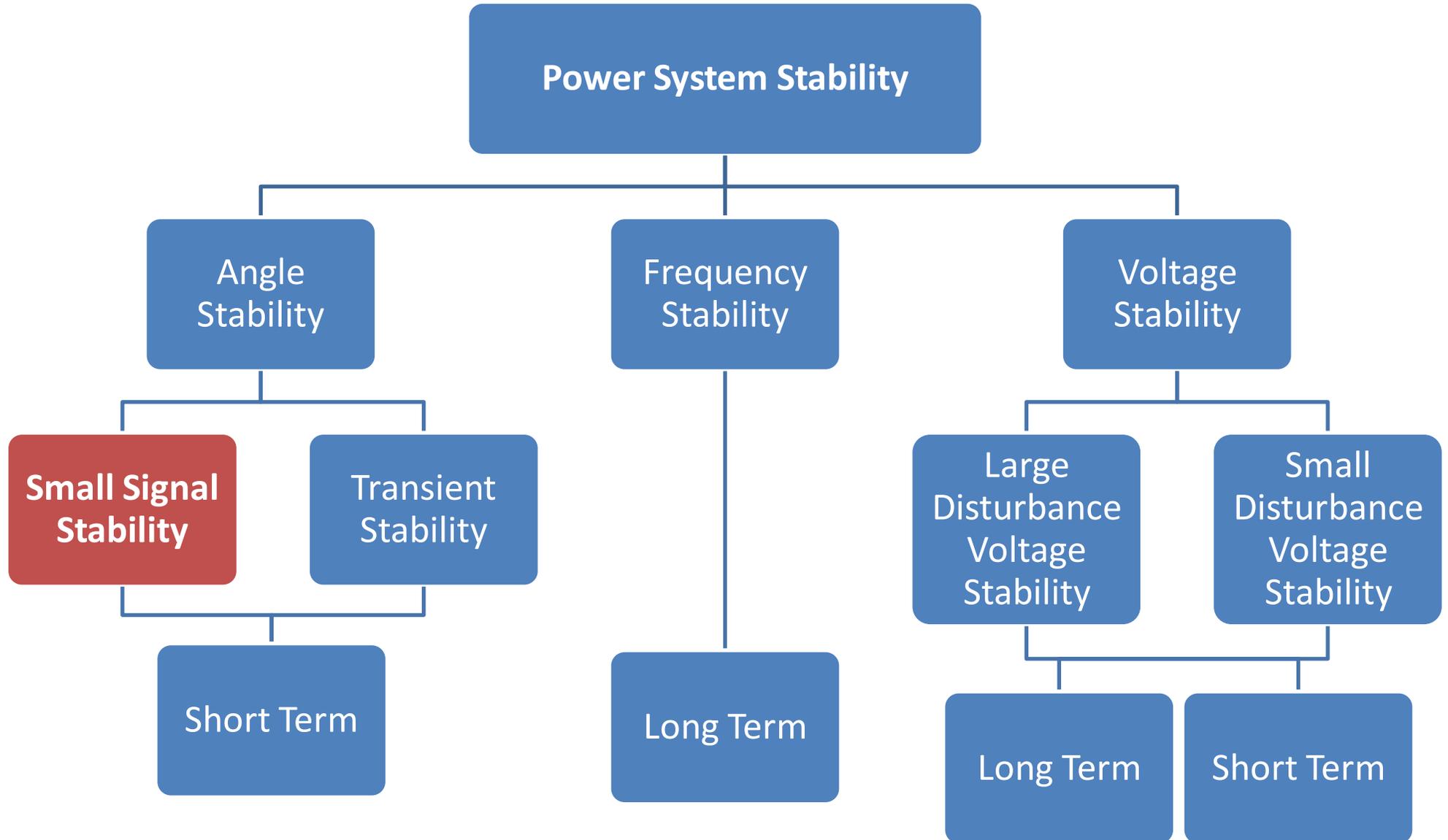
Blackouts in 2003 and 2004

We have had several wake up calls since 2003...

Dates	Locations	Result
August 14, 2003	North East USA and Ontario	63,000 MW load loss affecting 50 million people
September 23, 2003	South Sweden and East Denmark	6,500 MW load loss affecting 4 million people
September 28, 2003	Italy	50,000 MW load unsupplied affecting 60 million people
August 12, 2004	Three Australian States: Queensland, NSW, and Victoria	Load loss of 1,000 MW



Small Signal Stability





Small Signal Stability

- The ability of a power system to maintain synchronism when subjected to small disturbances
- The analyses are carried out on the linearized system equation
 - The linearization process hold since the disturbances considered to be small
- This type of analysis may result in two cases of instability:
 1. Steady increase in generator rotor angle
 2. Rotor oscillation of increasing amplitude



Small Signal Stability

- Stability of the linearized system is described by the eigenvalues of the state matrix
- A real eigenvalue, or a pair of complex eigenvalues, is usually referred to as a **mode**
- For a complex mode $\lambda = \sigma \pm j\omega$, two quantities are of main interest:
 - **Frequency** (in Hz) $f = \frac{\omega}{2\pi}$
 - **Damping ratio** (in %) $\zeta = 100 \times \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$
- The system is unstable if ζ is negative
 - To ensure acceptable performance, a **damping margin** in the range of 3%-5% is normally required

Small Signal Stability

- In the modern power system, the small signal stability is related to **insufficient damping** of system oscillation
- Low frequency oscillation can be related to...
 - The electromechanical mode
 - Local mode (typical frequency range 1-2 Hz)
 - Global mode (typical frequency range 0.1- 1 Hz)
 - The control mode
 - The torsional mode
- Two kinds of analyses are possible:
 - A multi-machine linearized analysis
 - A single machine infinite-bus
 - In multi-machine approach, all the eigenvalues are computed and those machines that contribute to a particular eigenvalue, both local and inter-area oscillations can be studied in such a framework



Small Signal Stability

- Single-machine infinite-bus approach can be used to study only local oscillations
- In a large power system, small signal instability problems may be either local or global in nature

Local Problems

1. Local problems involve a small part of the system
2. They are associated with rotor angle oscillation of a single generator or single plant against the rest of the power system
3. Such oscillations are called ***local plant mode oscillations***
4. These types of problems are widely seen in practical power systems

Small Signal Stability

Linearization

Let x_o be the initial state vector and u_o the input vector corresponding to the equilibrium point about which the small-signal performance is to be investigated.

$$\dot{x} = f(x_o, u_o) = 0$$

Since x_o and u_o satisfy the Ordinary Differential Equation (ODEs)...

A small disturbance in the system can be made by letting $x = x_o + \Delta x$; $u = u_o + \Delta u$; where Δ denotes a small deviation

Small Signal Stability

Linearization

$$\Delta \dot{X} = A\Delta X + B\Delta U$$

$$\Delta Y = C\Delta X + D\Delta U$$

A, B, C and D are matrices with partial derivative terms

- A: State or Plant matrix
- B: Control or Input Matrix
- C: Output Matrix
- D: Feed forward Matrix

The poles of the system is given by the following equation:

$$\det(sI - A) = 0$$

Small Signal Stability

$$\det(sI - A) = 0$$

The values of s which satisfy the above equation are known as eigenvalues of matrix A .

The eigenvalues of a matrix

$$A\phi = \lambda\phi$$

The eigenvalues of matrix A are given by the values of the scalar parameter λ for which there exist non-trivial Solutions (i.e. solutions other than $\phi=0$).



Small Signal Stability

Where A is an n by n matrix (elements of A for physical systems like power system are real) ϕ is an n by 1 vector.

By rearranging the above the following equation can be written as...

$$(A - \lambda I)\phi = 0$$

$$\det(A - \lambda I) = 0$$

for nontrivial solutions.



Small Signal Stability

- Expansion of the determinant gives the characteristics equation
- For an n by n matrix, there will be n solutions of λ , known as the eigenvalues of A
- The eigenvalues may be real or complex
 - If the matrix is real, complex eigenvalues always occur in conjugate pairs



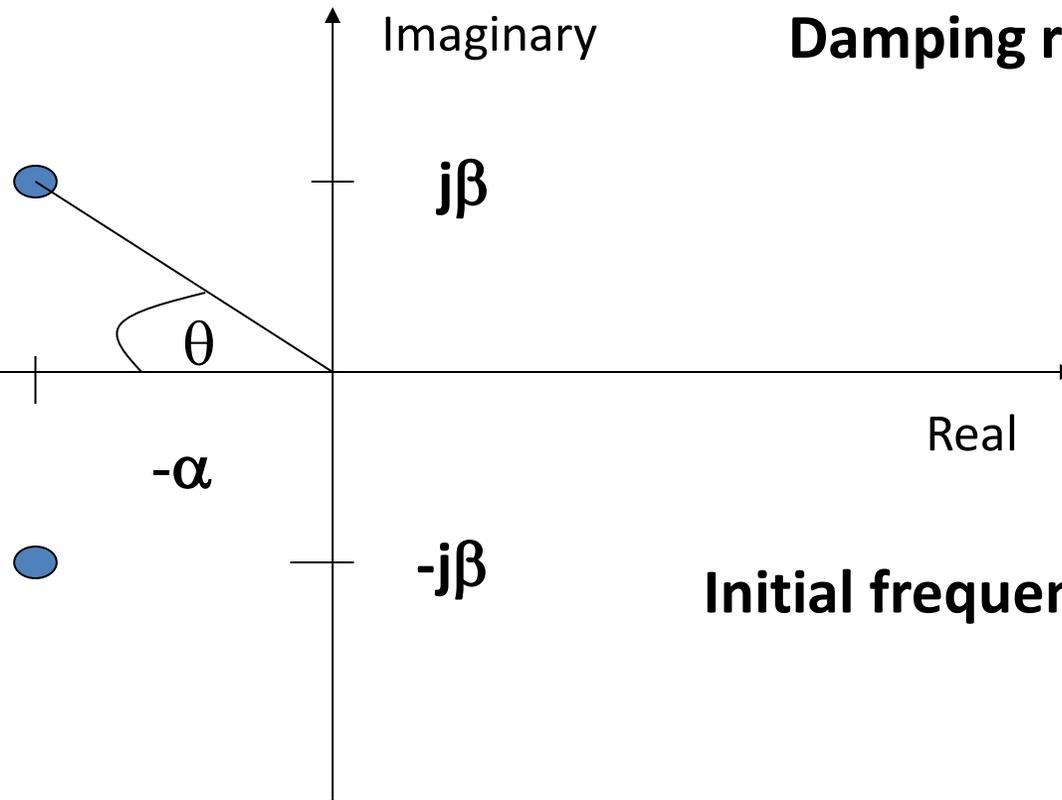
Small Signal Stability

Some properties of eigenvalues in relation of stability

- (i) When all the eigenvalues have negative real parts, the system is **asymptotically stable**
- (ii) When at least one of the eigenvalues has a positive real part, the system is **unstable**
- (iii) When all the eigenvalues have negative real parts except one complex pair having purely imaginary values, the system exhibits **oscillatory motion**

Small Signal Stability

Complex eigenvalues $(-\alpha \pm j\beta)$



Damping ratio = $\cos(\theta) = \zeta$

$$\zeta = \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}}$$

Initial frequency of oscillation

$$f = \frac{\beta}{2\pi}$$

Small Signal Stability

Complex eigenvalues

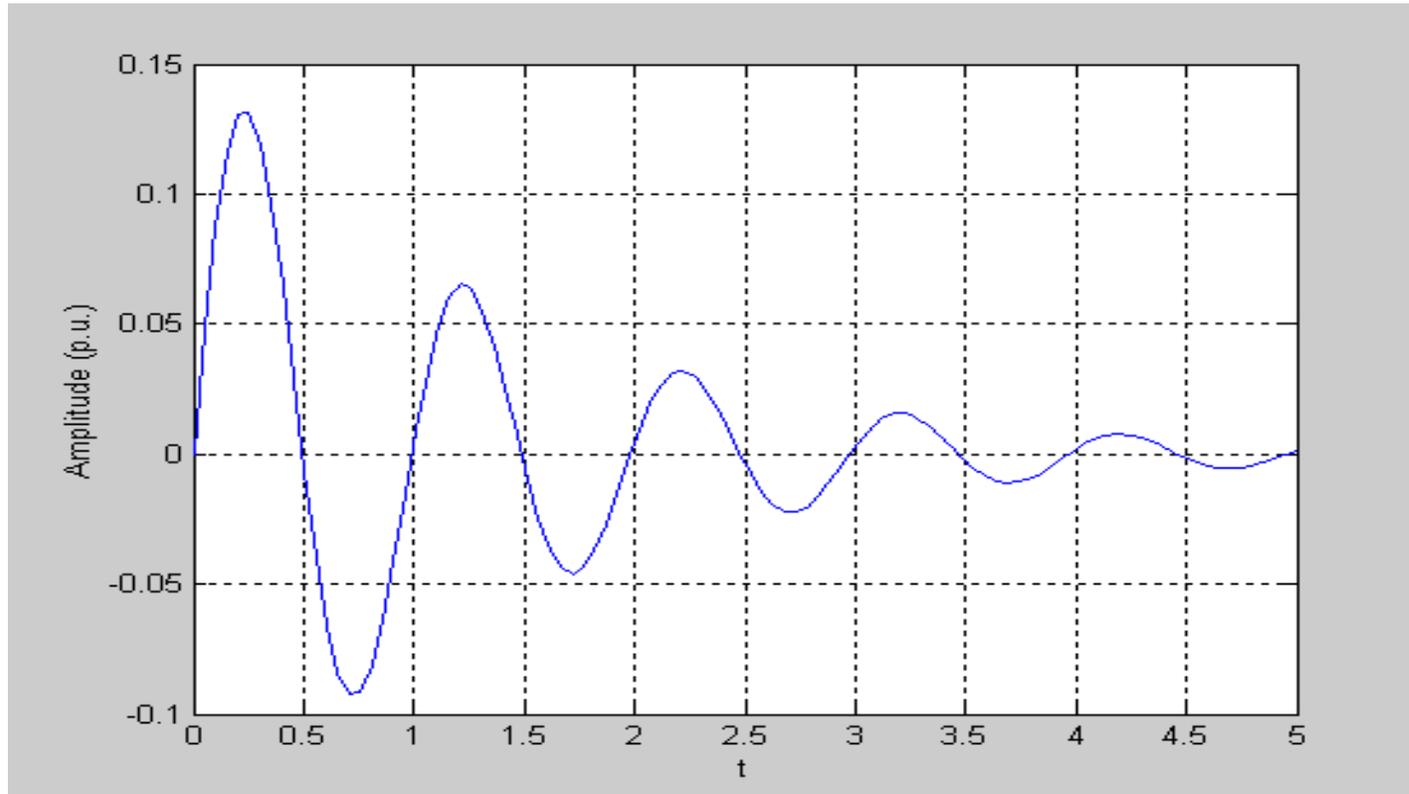
- The damping ratio ζ determines the rate of decay of the amplitude of the oscillation.
- The time constant of the amplitude decay is $1/|\alpha|$
- In other words, the amplitude decays to $1/e$ or 37% of the initial amplitude in $1/|\alpha|$ seconds or in $1/(2\pi\zeta)$ cycles of oscillation

Small Signal Stability

Example of oscillation (damped case):

A simple nonlinear system with two complex eigenvalues

Characteristics equations: $S^2 + 1.43S + 40.716$



Eigenvalues are
 $\lambda_{1,2} = -0.714 \pm j6.35$

Damping ratio
 $\zeta = 0.111$

Frequency of oscillation
 $f = 1.0106$ Hz

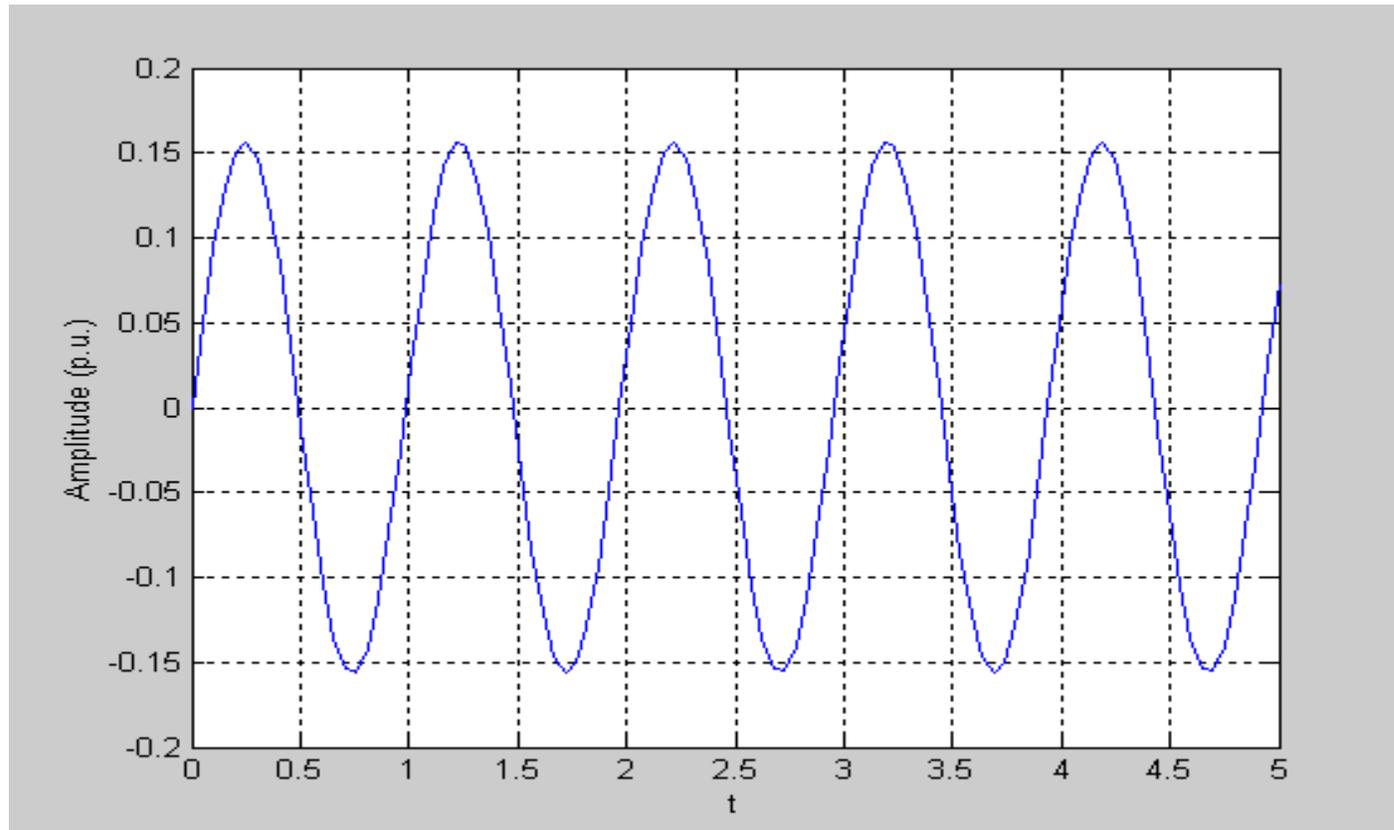


Small Signal Stability

Example of oscillation:

A simple nonlinear system with two complex eigenvalues

Characteristics equations: $S^2 + 40.716$



Eigenvalues are

$$\lambda_{1,2} = \pm j6.38$$

Damping ratio

$$\zeta = 0$$

Frequency of oscillation

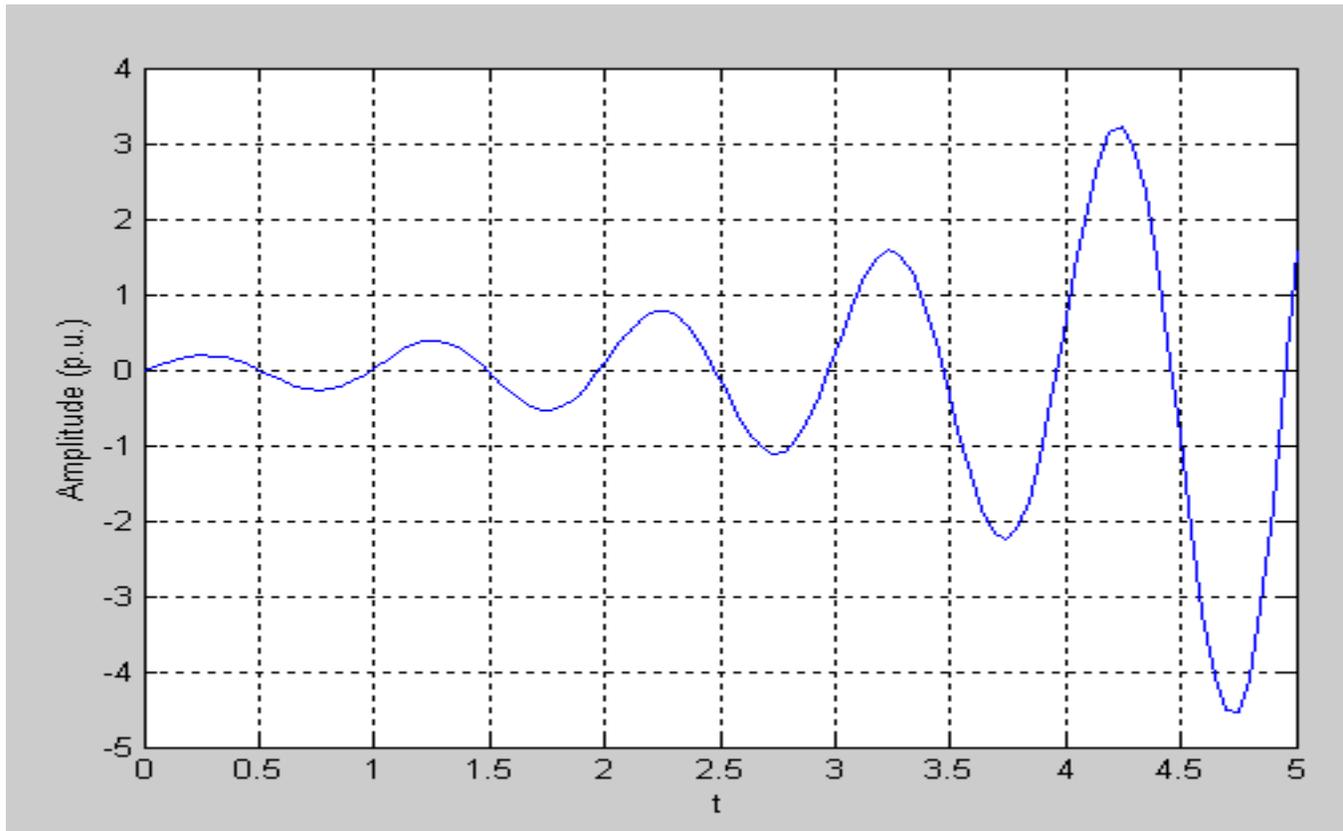
$$f = 1.0156 \text{ Hz}$$

Small Signal Stability

Example of oscillation (undamped case):

A simple nonlinear system with two complex eigenvalues

Characteristics equations: $S^2 - 1.43S + 40.716$



Eigenvalues are
 $\lambda_{1,2} = 0.714 \pm j6.38$

Damping ratio
 $\zeta = -0.111$

Frequency of oscillation
 $f = 1.0106$ Hz



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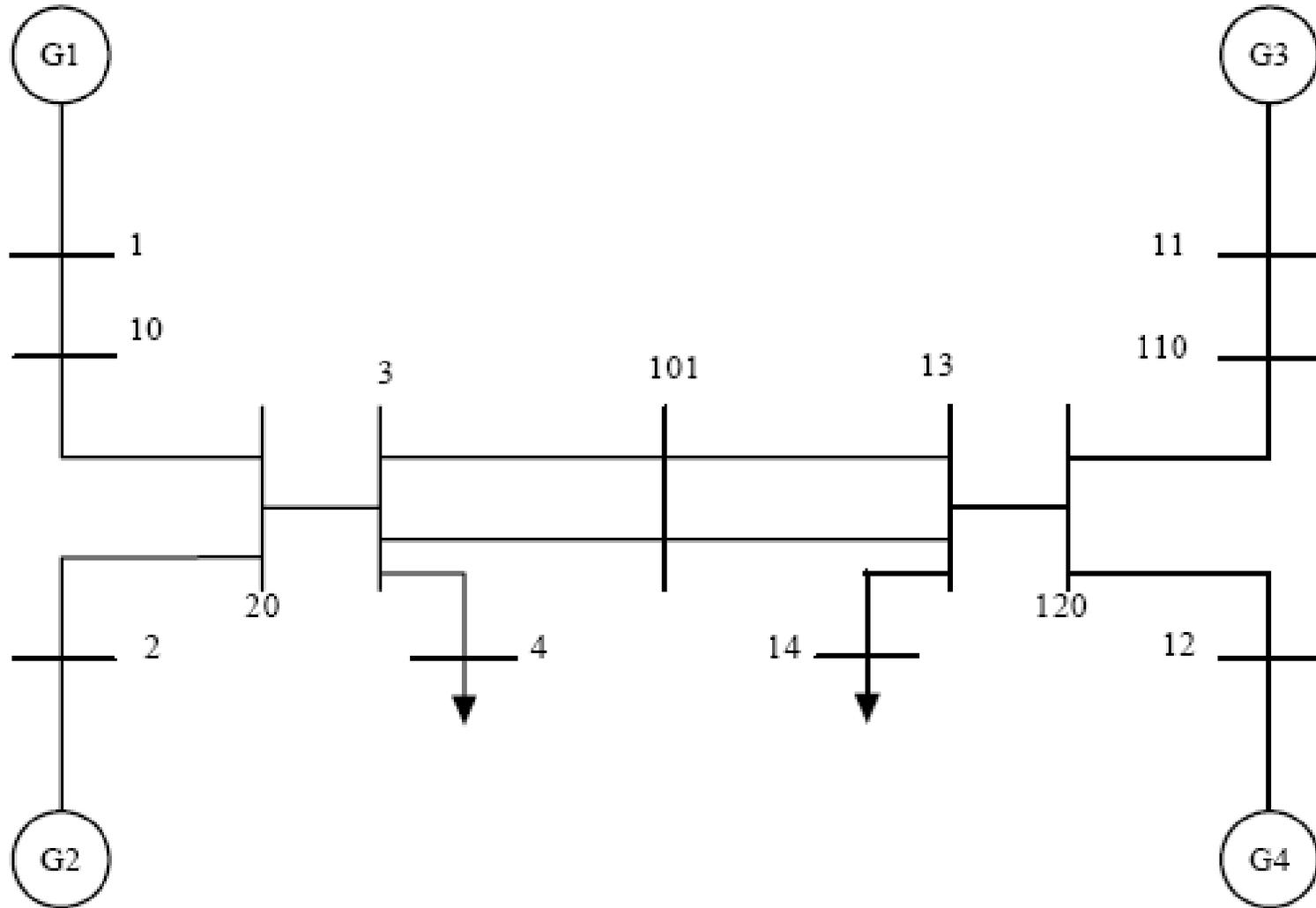
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Small Signal Stability

Case Study & Assignments

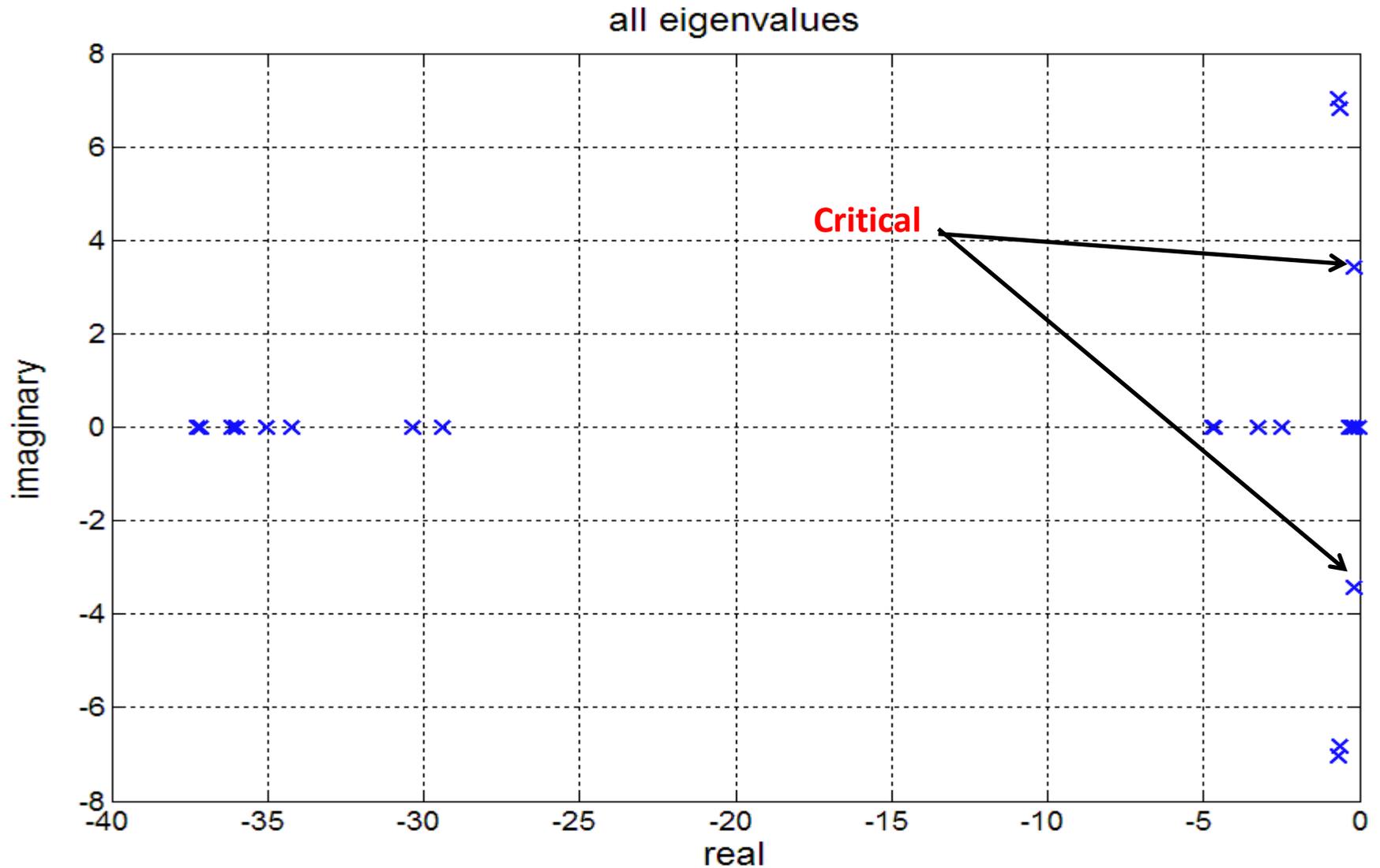


Case study: Two-area system





Case (a): Only generators are activated



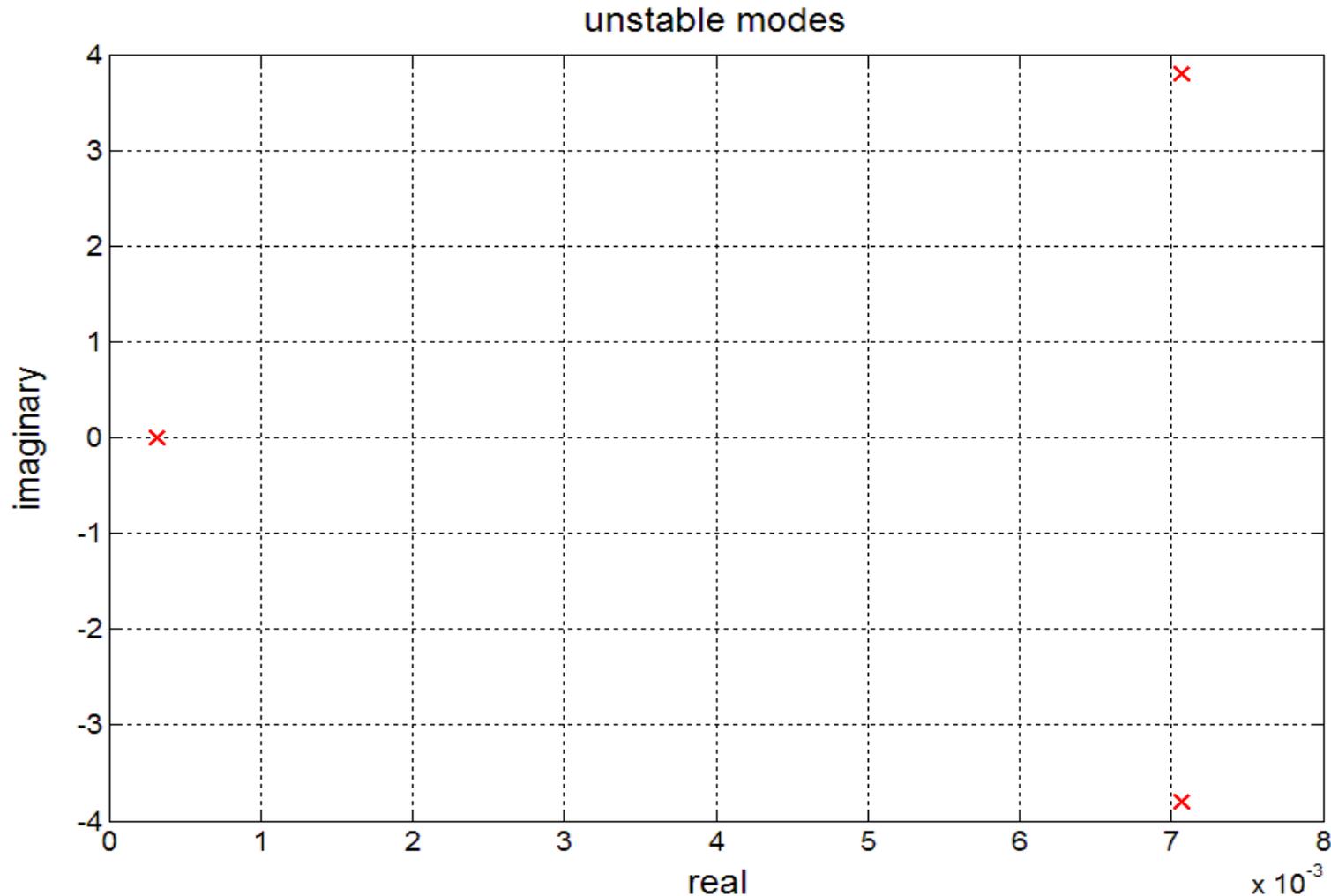


Modes and Oscillation

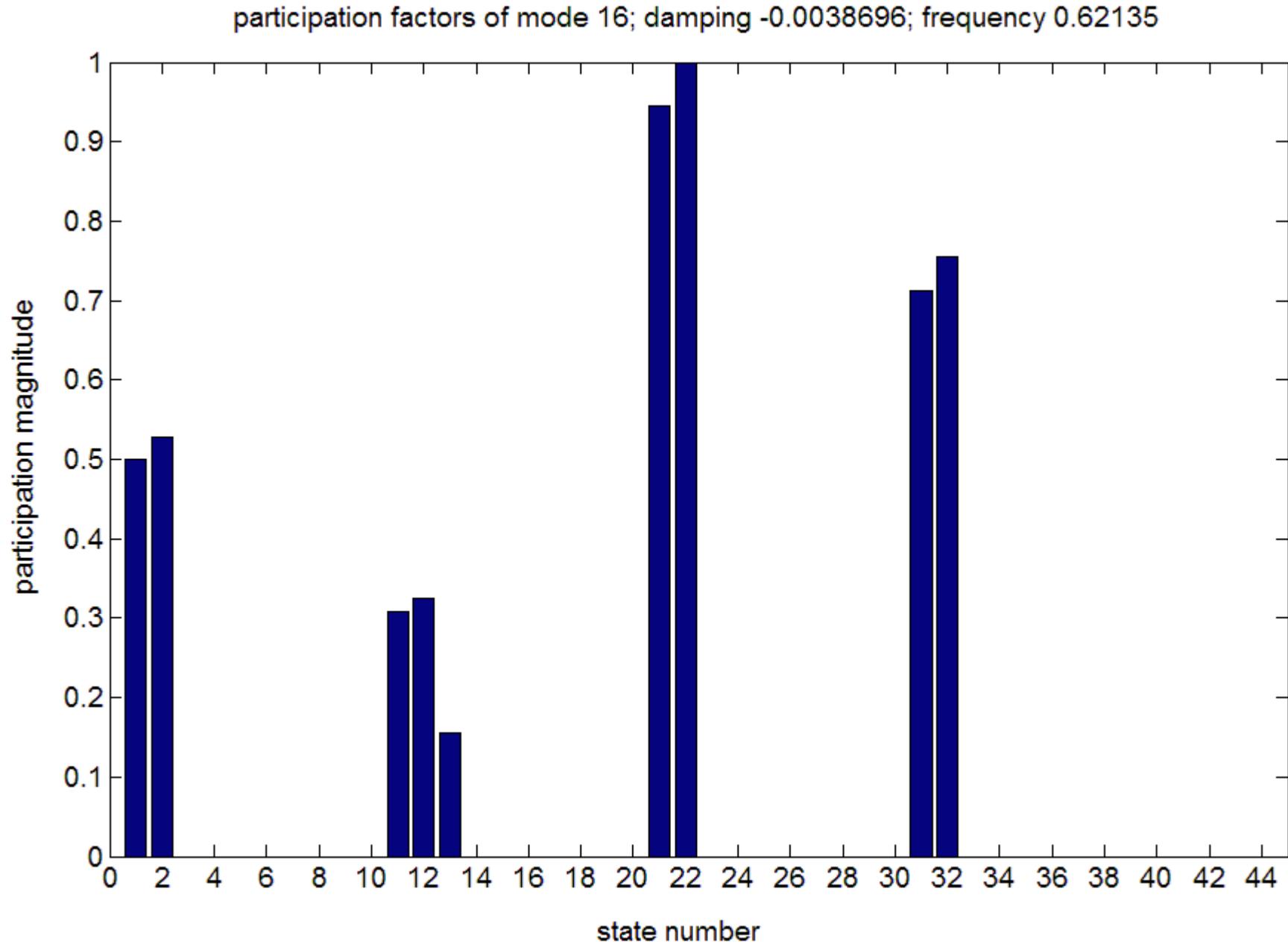
Mode No	Eigenvalue	Damping ratio	Frequency
9,10	$-0.1716 \pm 3.4260i$	0.0500	0.5453
13,14	$-0.6534 \pm 6.8104i$	0.0955	1.0839
15,16	$-0.6590 \pm 7.0346i$	0.0933	1.1196

Modes of Oscillation	Swinging
Inter-area mode $f=0.5453$ Hz, damping=0.0500	G1,G2 of area 1 swinging against G3, G4 of area 2
Area 1 local mode $f=1.0839$ Hz, damping=0.0955	G1 swinging against generator G2
Area 2 local mode $f=1.1196$ Hz, damping=0.0933	G3 swinging against G4

Case (b): Introduce exciters and governors in each generator



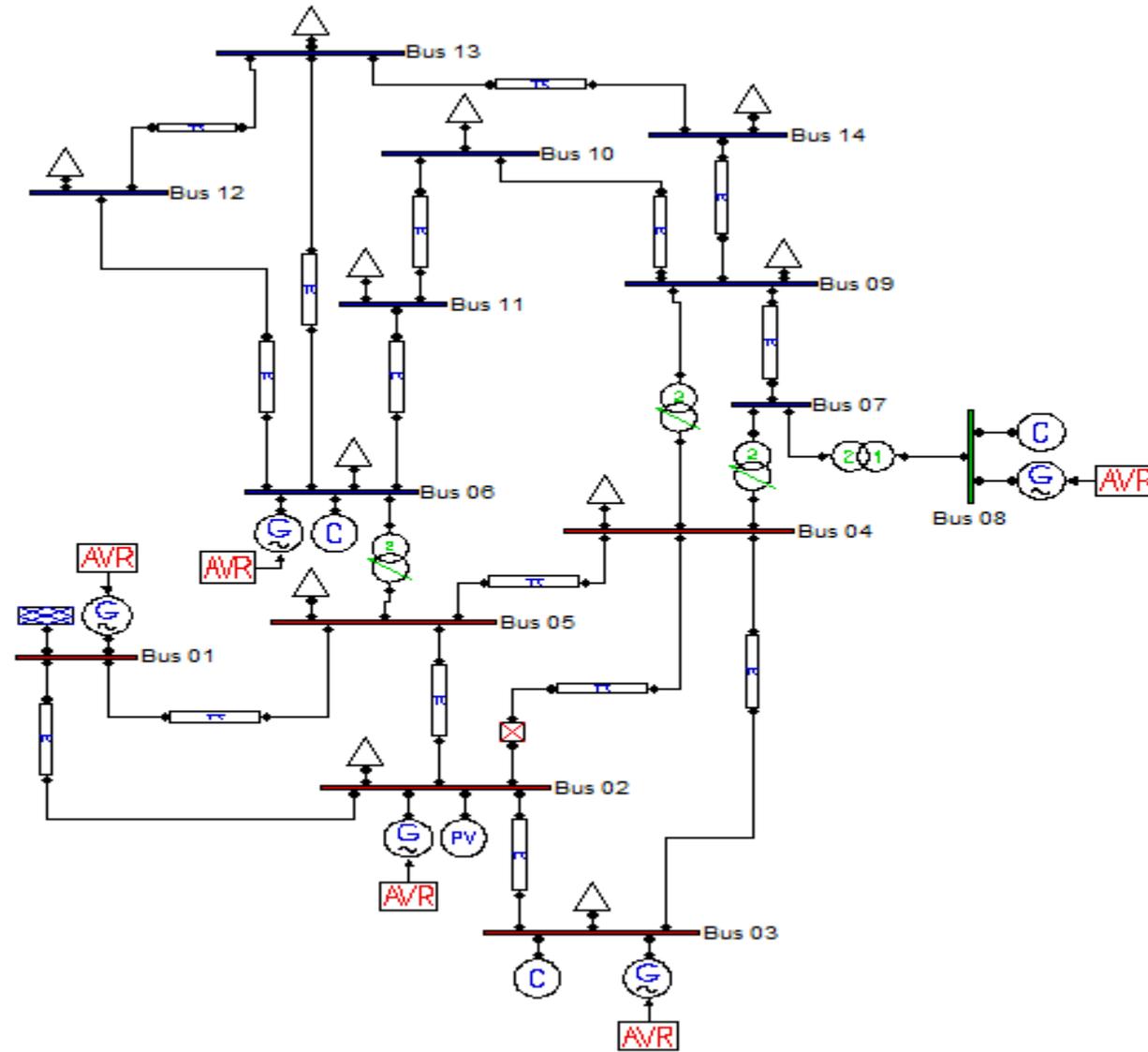
- Overall stability is worse
- From eigenvalue, the unstable modes are 15 and 16, which related to inter-area mode



Using the participation factor analysis, the dominant states are state number 21 and 22. It corresponds to the δ and ω of machine 3.

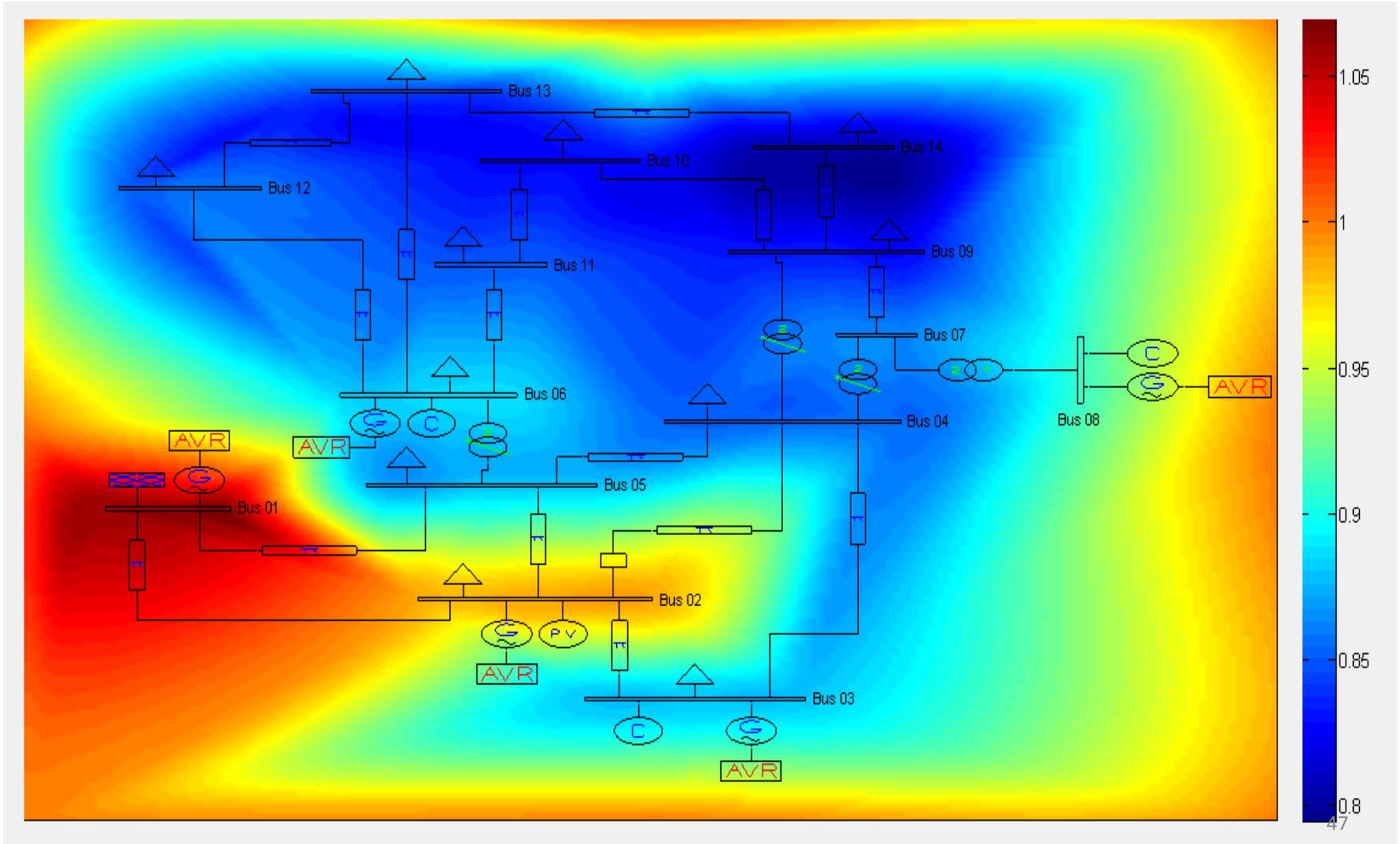


IEEE 14 bus Dynamic Model





Static Voltage Profile (Base Case)



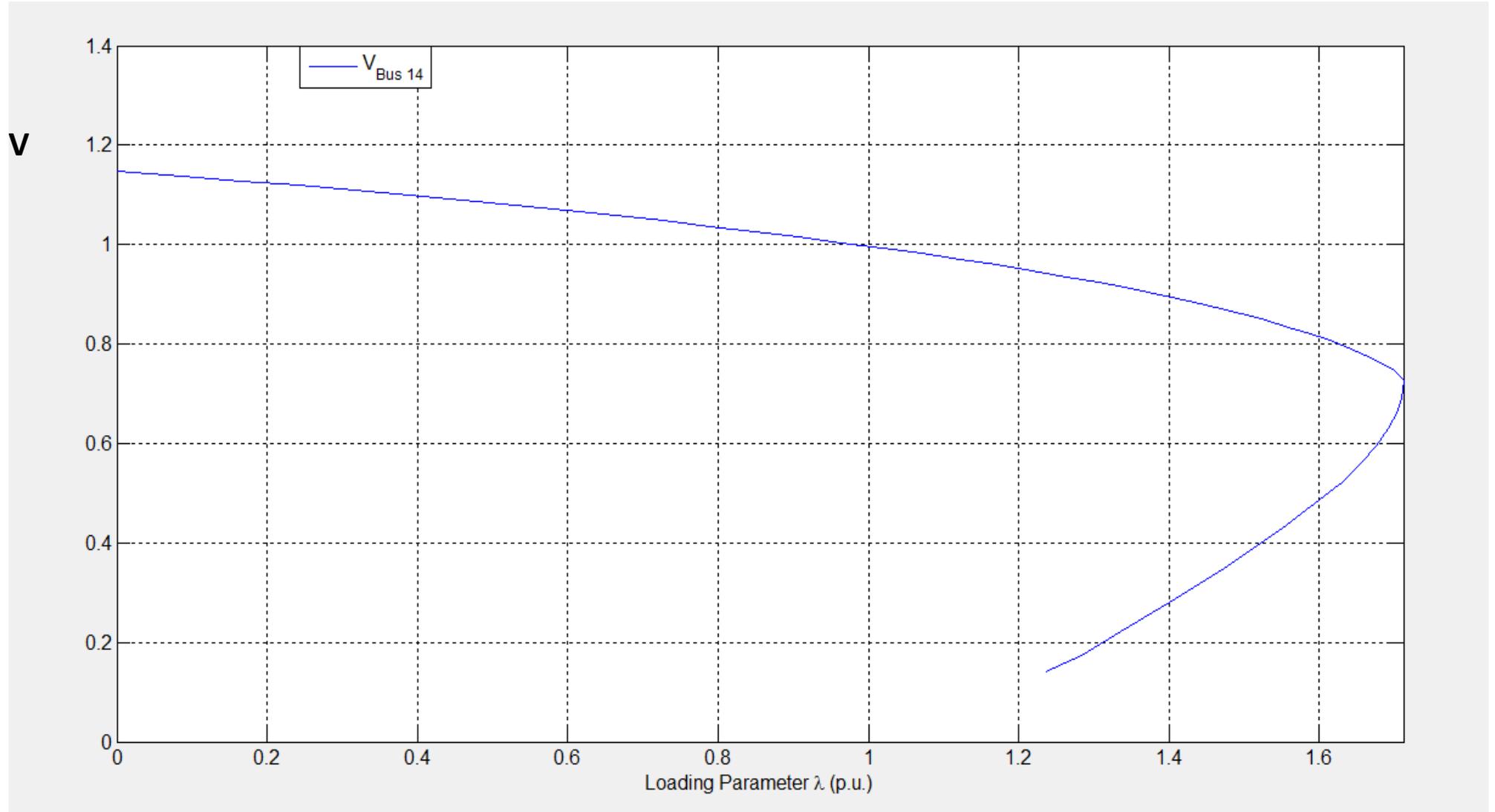


IEEE 14 bus test system

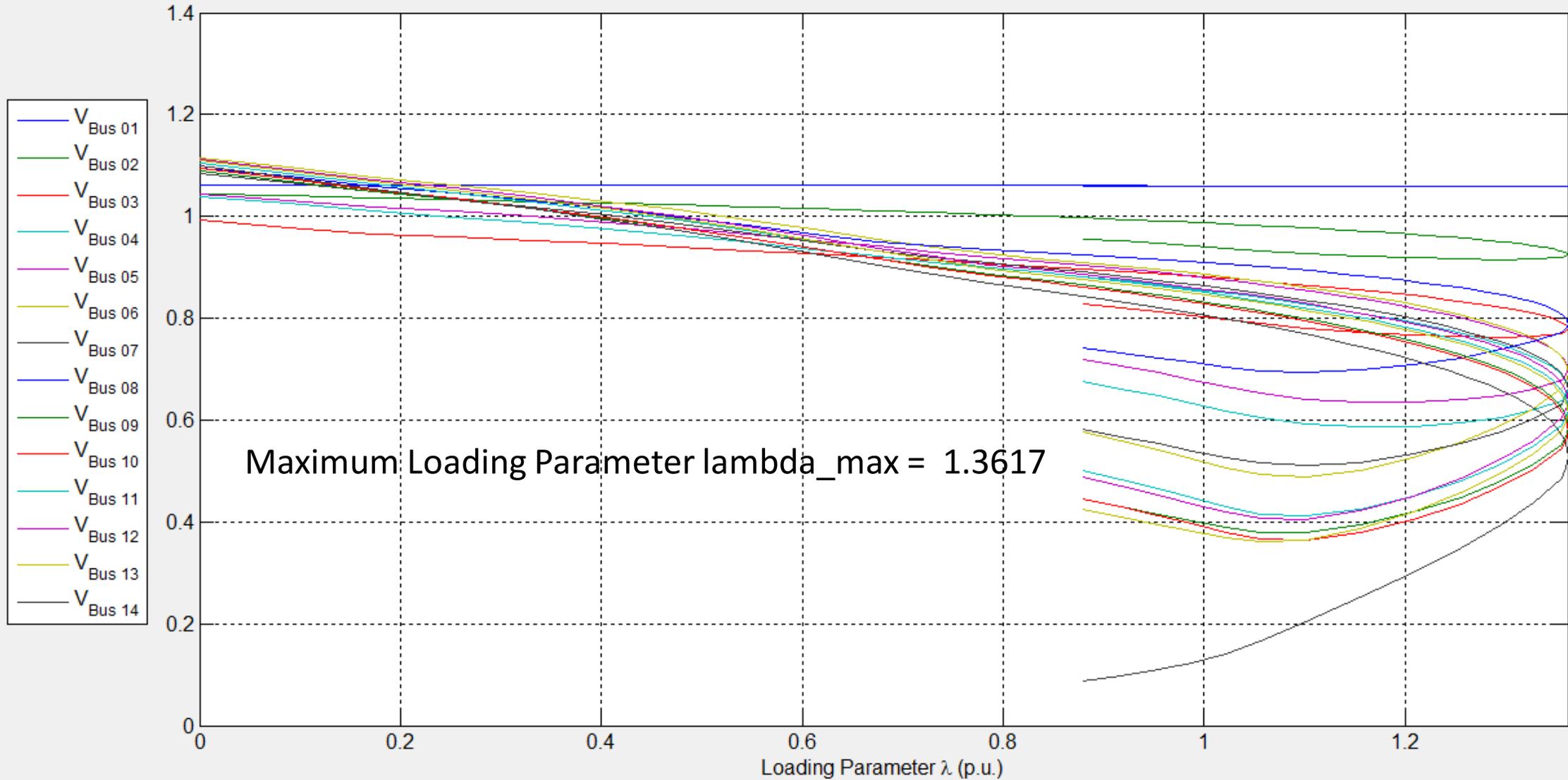
- **Modeling**
- All the generators were modeled in detail with IEEE Type 2 exciters on them
 - Generators at bus 3, 6 and 8 are synchronous compensators
- All the loads were modeled using the constant PQ loads



CPF: PV curve at bus 14



CPF: PV curve at all the buses



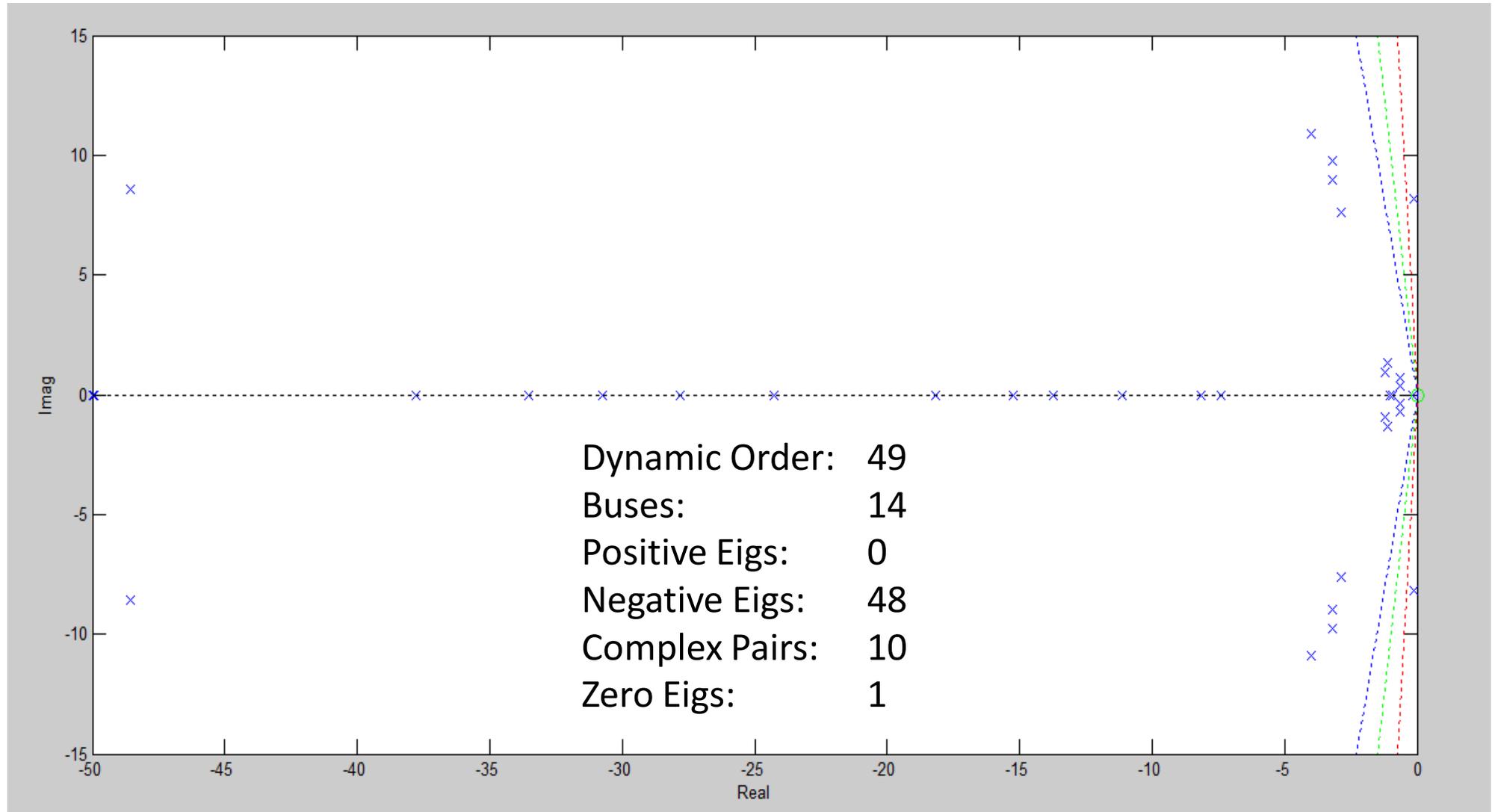
Small Signal Stability Analysis

Total state variables:

- DYNAMIC ORDER 49
- # OF EIGS WITH $\text{Re}(\mu) < 0$ 48
- # OF EIGS WITH $\text{Re}(\mu) > 0$ 0
- # OF REAL EIGS 29
- # OF COMPLEX PAIRS 10
- # OF ZERO EIGS 1



Eigenvalues at Base Case





Assignment 1

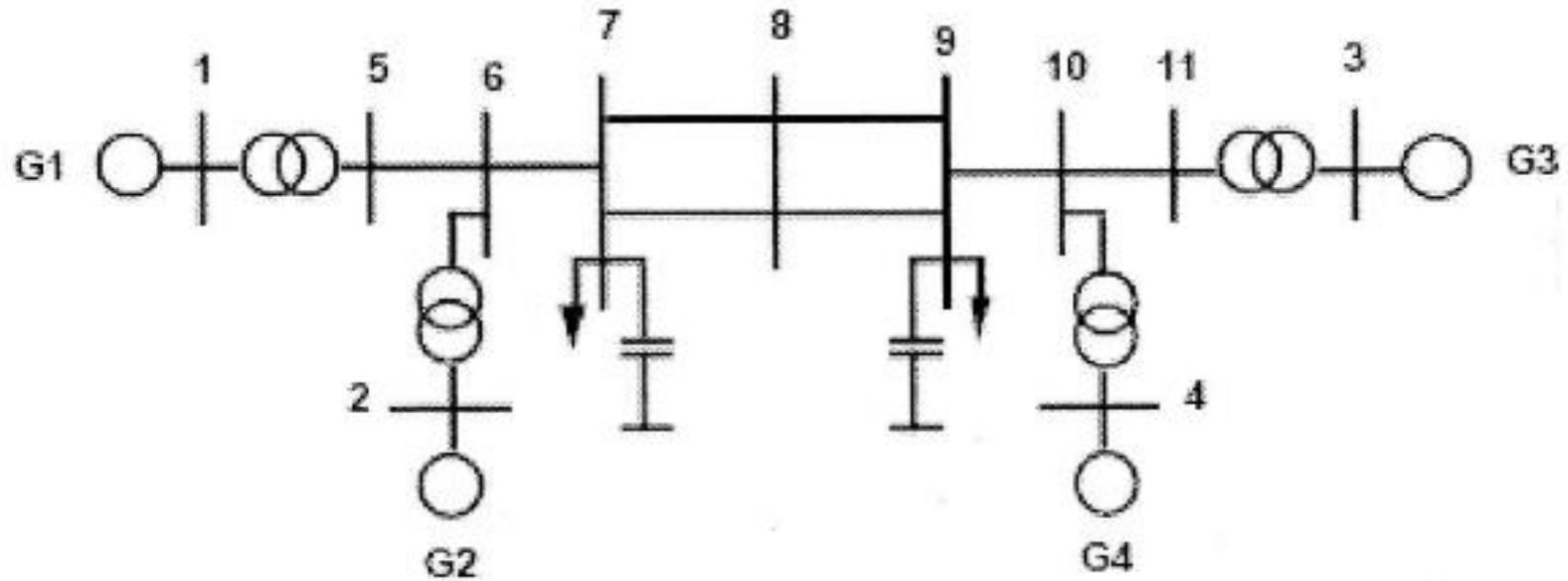


Figure 1 A single line diagram of two-area test system.



Assignment 1

The figure 1 shows a single line diagram of two-area power system, typically used for power system oscillation studies. The system consists of eleven nodes, including four generators, two loads, thirteen branches, and four transformers.

Questions

1. Model all the generators in detail. All the generators have identical parameters. Calculate the eigen values of the system at the base case loading point.
2. Introduce exciters (may be a simple model) in each of those generators and repeat the eigen value calculation.



Assignment 1

1. Now introduce governor models and repeat the calculation again.
2. Using the participation factor analysis, find out what are the dominant state variables in the critical mode and the corresponding machine (problematic one).
3. Introduce a **Power System Stabilizer (PSS)** in the problematic machine and calculate the eigenvalues again.
4. Is there any additional method to regain the stability other than introducing the PSS? Comment on the same.
5. Repeat the questions (a) to (e) by using PSAT. Compare the results.



Remarks

- Oscillation can be triggered by line outages
- PSS is not effective on generators, which deliver reactive power only (synchronous compensators)
- SVC and TCSC are can add damping on the critical mode
- Overall, PSS is very effective in adding damping on the critical mode
- SVC and TCSC not only add damping on critical mode but also increase the loadability of the system



Remedial Measures

- Tuning existing controllers
 - Exciters
 - PSS
- Add new PSS in the problematic machines
- Install FACTS devices:
 - SVC, STATCOM
 - TCSC, SSSC
 - UPFC
- Re-dispatching generators



Sub-synchronous Resonance

Resonance

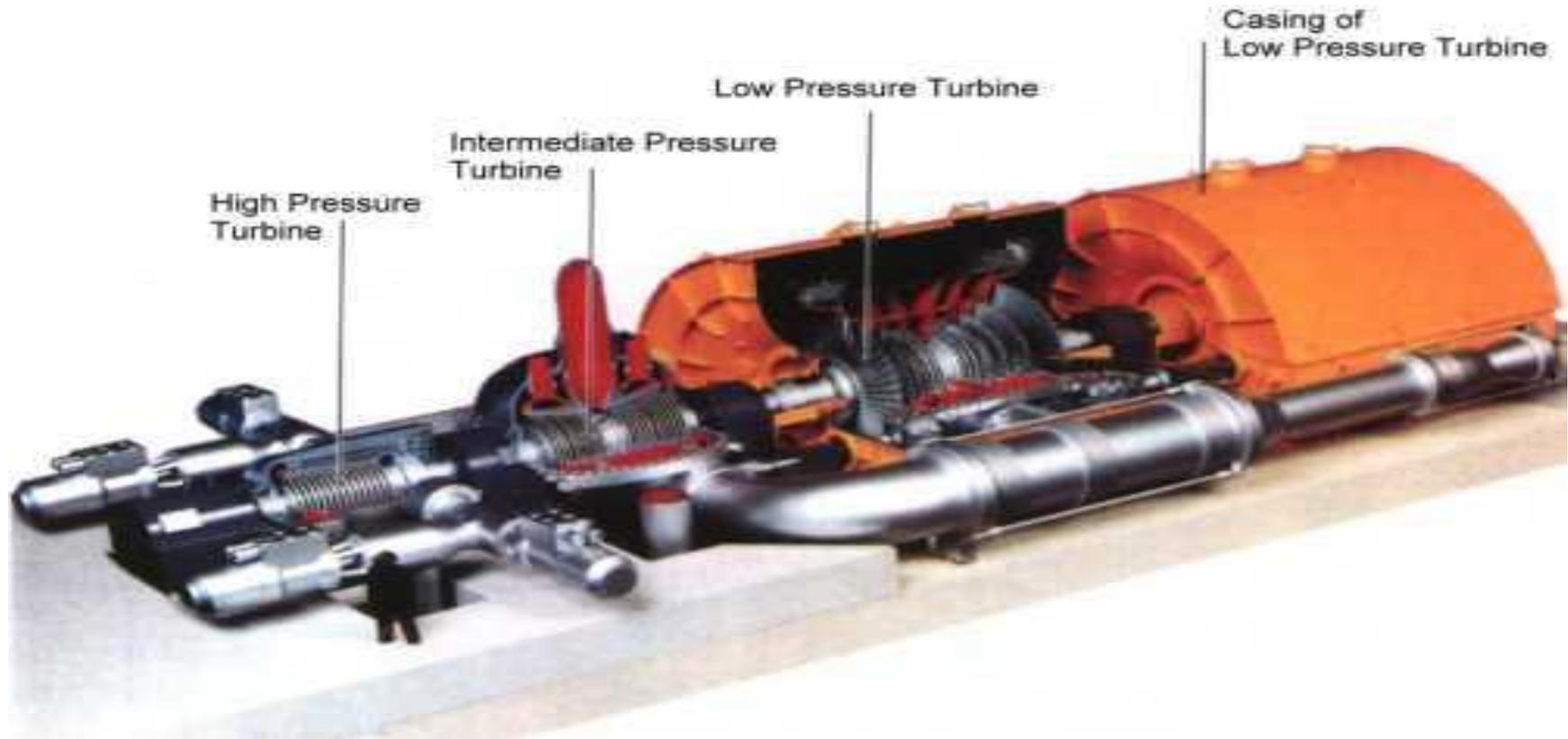
- Assume a system is derived by another system
- If the driving frequency of the system is equal to the natural frequency of the first system, the system will oscillate with an amplitude much larger than the amplitude of the driving force
- This phenomenon is known as **resonance**



Sub-synchronous Resonance

- In the transient stability and small signal stability, the rotor of a turbine generator was assumed as a single mass
- Such a representation accounts for the oscillation of the entire turbine-generator rotor with respect to other generators
- The frequency of this mode of oscillation is usually in the range of **0.1 to 2 Hz**
- In practice, a steam turbine-generator rotor has a very complex mechanical structure consisting of several predominant masses connected by shafts of finite stiffness

Sub-synchronous Resonance



Source: Kraft-Wärme-Kopplung

Sub-synchronous Resonance



Rotor of 8 MW Steam Cogeneration Unit

Source: Peter Brotherhood Ltd.

Sub-synchronous Resonance

- When the generator is perturbed, torsional oscillations result between different sections of the turbine-generator rotor
- Torsional oscillations could be in the range of **sub-synchronous** or **super-synchronous** frequency
- The torsional oscillations in the **sub-synchronous** range could, under certain conditions, interact with the electrical system in an adverse manner
 - Torsional interaction with power system controls
 - Sub-synchronous resonance with series capacitor-compensated transmission lines
 - Torsional fatigue due to network switching.



Sub-synchronous Resonance

Definition of SSR

“Sub-synchronous resonance is an electric power system condition where the electric network exchanges energy with a turbine generator at one or more of natural frequencies of the combined system below the synchronous frequency of the system.”



Sub-synchronous Resonance

- Application of **series capacitors** in long power transmission lines helps in improving power transfer
- It is economical compared to addition of new lines, or series FACTS controllers
- Series capacitors have been extensively used in Western USA and Sweden
- The phenomenon of **SSR occurs mainly in series capacitor compensated transmission systems**
- The first SSR problem was experienced in 1970 resulting in the failure of a turbine-generator shaft at Mohave power plant in southern California



Sub-synchronous Resonance

- It was not until a second shaft failure, which occurred in 1971, that the real cause of the failure was recognized as sub-synchronous resonance
- First, let us examine the characteristics of a series capacitor-compensated system and the possibility of sub-synchronous oscillations
- In an uncompensated transmission system, faults and other disturbances result in DC offset components in the generator stator windings



Sub-synchronous Resonance

- In series capacitor-compensated transmission systems, the situation can be very different
- Instead of the DC component of fault current, the offset transmission current is also the alternating current of frequency equal to the natural frequency, ω_n of the circuit inductance and capacitance

$$\omega_n = \frac{1}{\sqrt{LC}} = \omega_o \sqrt{\frac{X_c}{X_L}} \quad f_n = f_o \sqrt{\frac{X_c}{X_L}}$$



Sub-synchronous Resonance

- Generator stator current components of frequency f_n induce rotor currents (and hence torque) of slip frequency $(50-f_n)$ Hz

Natural and slip frequencies as a function of compensation

Percentage Compensation (X_c/X_L)*100 %	Natural Frequency f_n (Hz)	Slip Frequency $50-f_n$ (Hz)
10	15.81	34.19
25	25	25
30	27.39	22.07
35	29.58	20.42
40	31.62	18.38

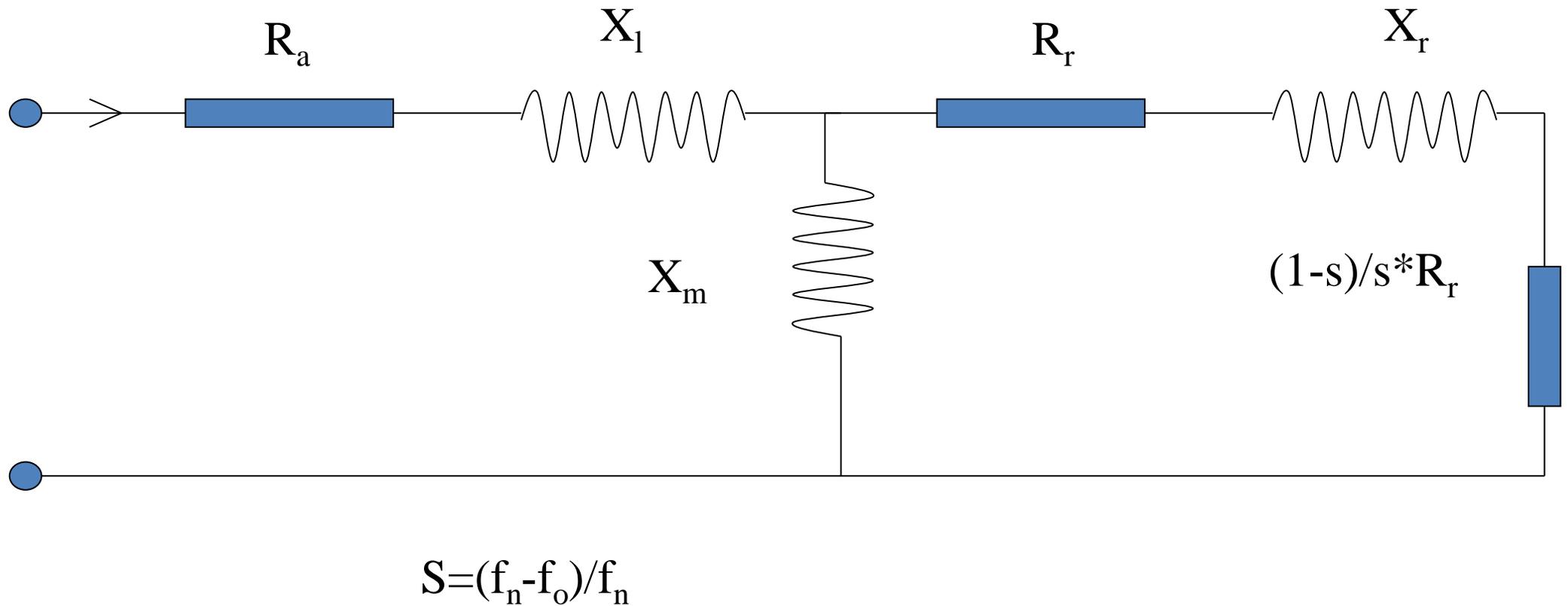


Sub-synchronous Resonance

- The frequency-dependent characteristic of the effective impedance of a network is complex and a more detailed system representation is required
- The sum of frequencies of the induced rotor current and the sub-synchronous natural frequency is equal to synchronous frequency.
- A series capacitor-compensated transmission network can cause sustained or negative damped sub-synchronous oscillation due to...
 - **Self-excitation due to induction generator effect**
 - **Interactions with torsional oscillations**

Sub-synchronous Resonance

Self-excitation due to Induction Generator Effect





Sub-synchronous Resonance

- Depending on f_n , the effective resistance can be negative
- At the high degree of compensation, this negative resistance may exceed the network resistance
- This will result in an RLC circuit with negative resistance (negative damping)
- As a result of this self excited oscillation could cause electrical oscillation of intolerable levels



Sub-synchronous Resonance

- The tendency toward this electrical synchronous instability is decreased by increasing the network resistance and by decreasing the resistance of generator rotor circuits.
- This form of self-excitation is purely an electrical phenomenon and is not dependent on the shaft torsional characteristics



Sub-synchronous Resonance

Torsional Interaction Resulting in SSR

- Torsional oscillation can be excited, if the complement of the natural frequency of the network is close to one of the torsional frequencies of the turbine-generator shaft system
- Under such conditions, a small voltage induced by rotor oscillation can result in larger sub-synchronous currents
- This current will produce an oscillatory component of rotor torque whose phase is such that it enhances the rotor oscillation
- When this torque is large than that resulting from mechanical damping, the coupled electromechanical system will experience growing oscillations



Sub-synchronous Resonance

Analytical Methods of SSR

- Torsional interaction results in energy exchange between the generator shaft and the inductance/capacitances of the network
- Therefore, the analysis of SSR problems requires representation of both the electromechanical dynamics of the generating units and the electromagnetic dynamics of the transmission network

Sub-synchronous Resonance

Analytical Methods of SSR

Several methods have been used for the study of SSR:

1. Eigenvalue (model) Analysis
2. Frequency Scanning
3. Frequency Response Analysis of the Full System
4. Approximate Frequency-domain Analysis
5. Time Domain Analysis



Sub-synchronous Resonance

Modeling Requirement

- Since the interactions of network and the machines, especially the rotating part, are considered, the network and rotor of the synchronous machines need to be modeled
- Network: Dynamic modeling
 - Including the equations of series capacitors
- Rotor: Dynamic modeling
 - Including the differential equations relating to different segments of the rotor



Countermeasures of the SSR Problems

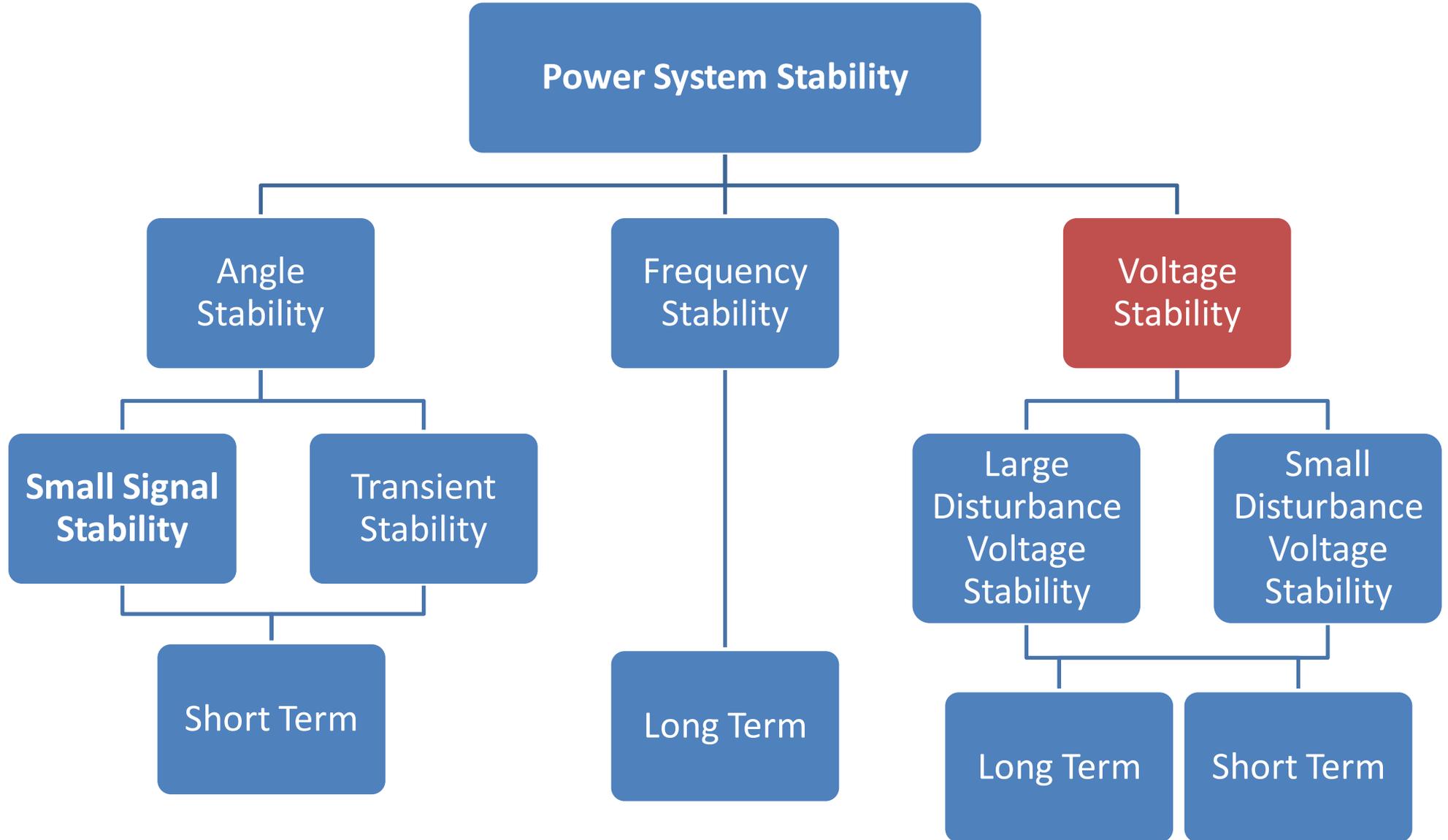
1. On detection of SSR, trip the generator unit, even though it may not be desirable
2. Use of **Static blocking filters** near generator or **Bypass filter** (damping circuit) in parallel with series capacitors
3. Use **Dynamic filter** (active device) in series with generator. It produces voltage in phase opposition with SSR voltage generated in armature (sensed from rotor motion)
4. Use of **Dynamic stabilizer** (Thyristor modulated shunt reactor) at generator terminals (signal from shaft speed)

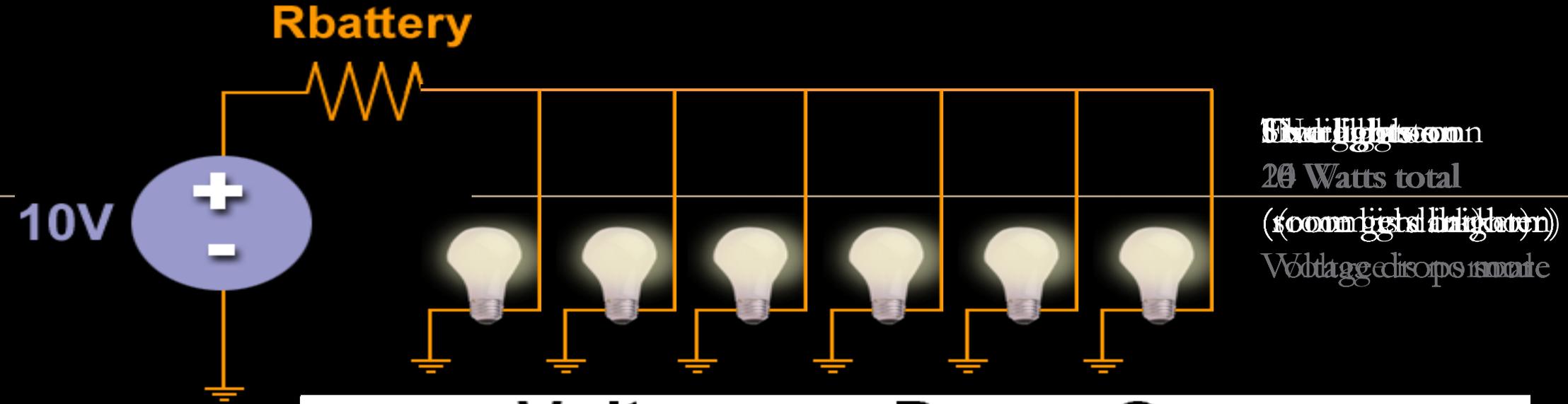
Countermeasures of the SSR Problems

1. The **excitation system damper**, which modulates generator excitation (PSS)
2. Bypass part of series capacitor banks on local detection of SSR
3. The **NGH** (Narain G. Hingorani) scheme consists of linear resistors in series with thyristors connected across series capacitors. SSR component in capacitor voltage is detected and reduced by the discharge circuit
4. Use of supplementary controls of HVDC, TCSC and SVC to damp out SSR oscillations

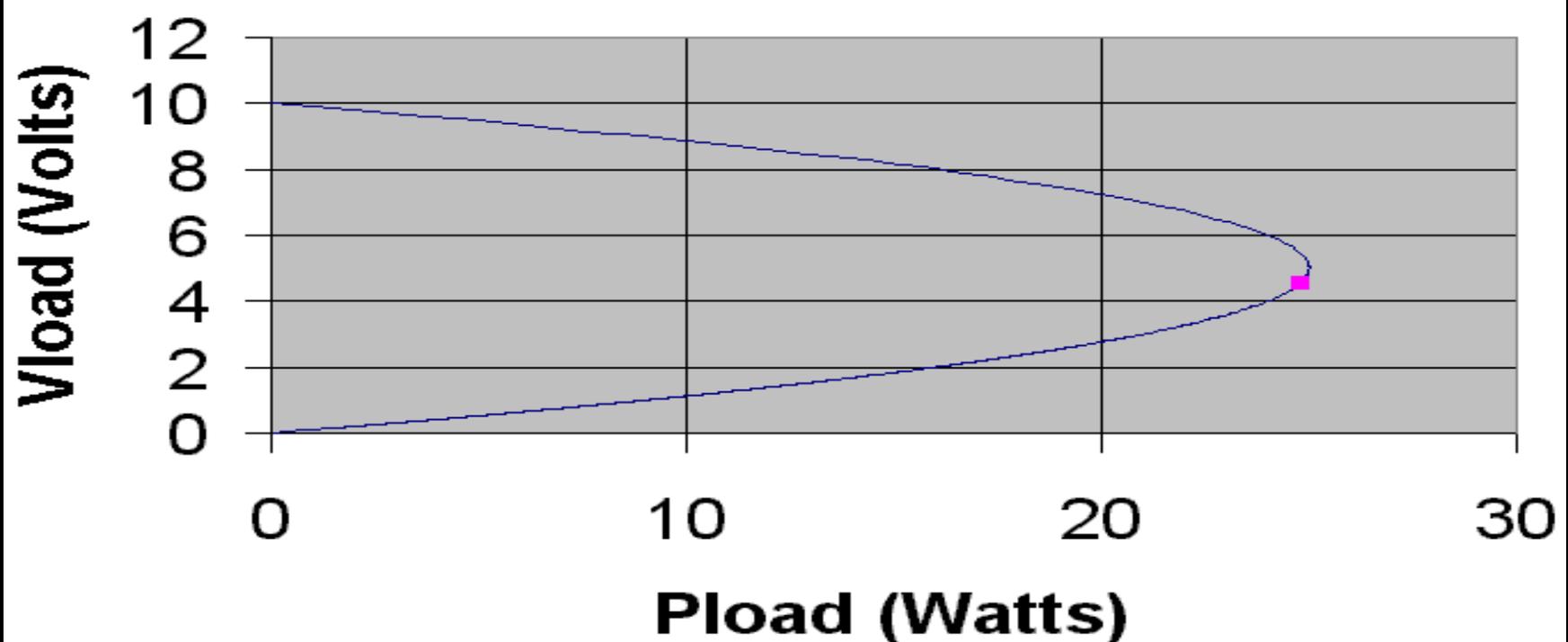


Voltage Stability



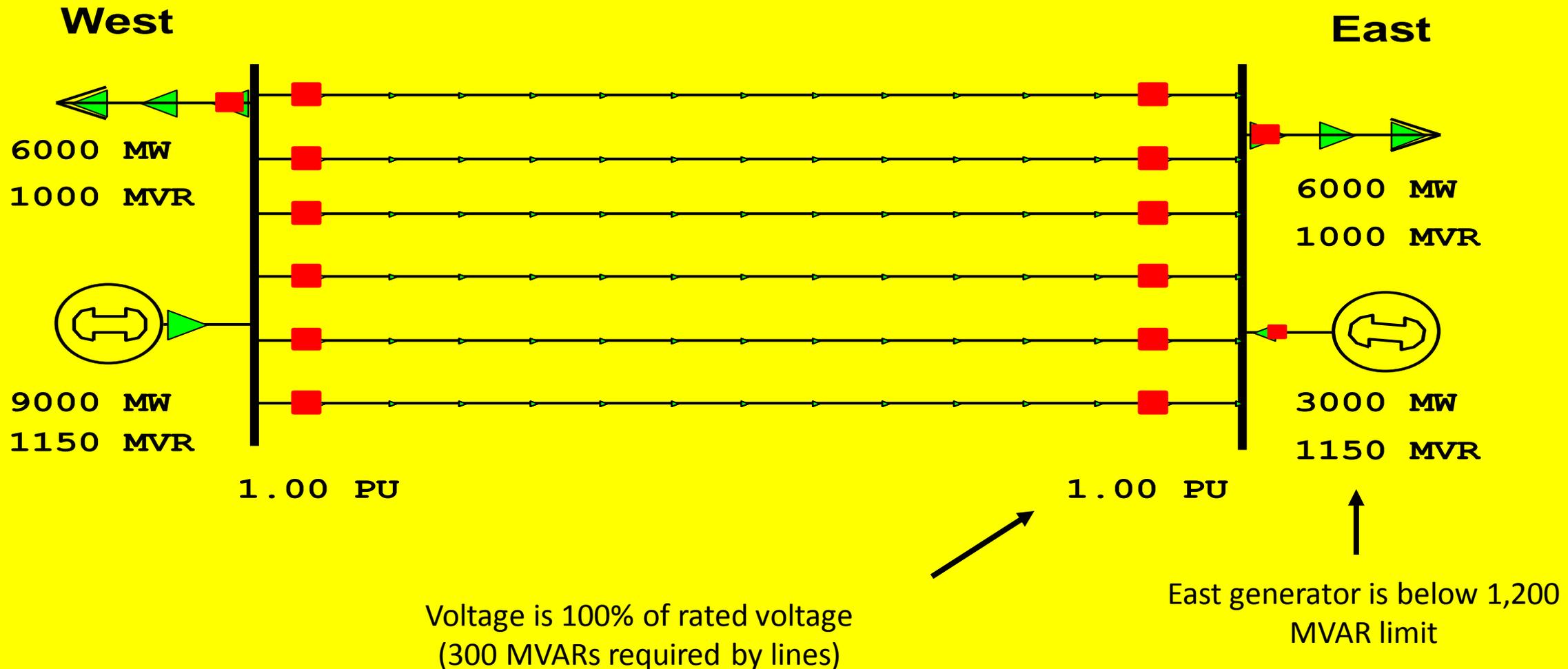


Voltage vs. Power Curve



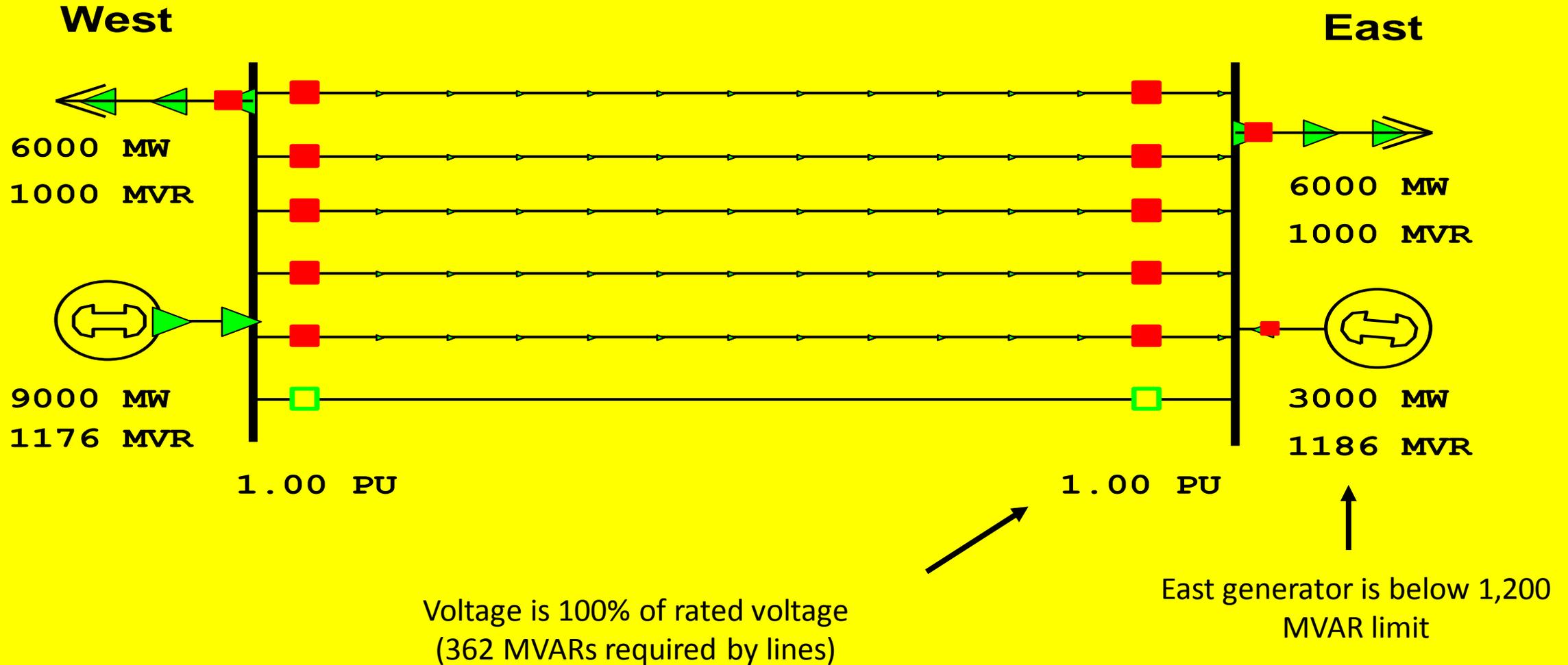
Case 1: All Lines In-Service

3,000 MW transfer – 500 MW per line



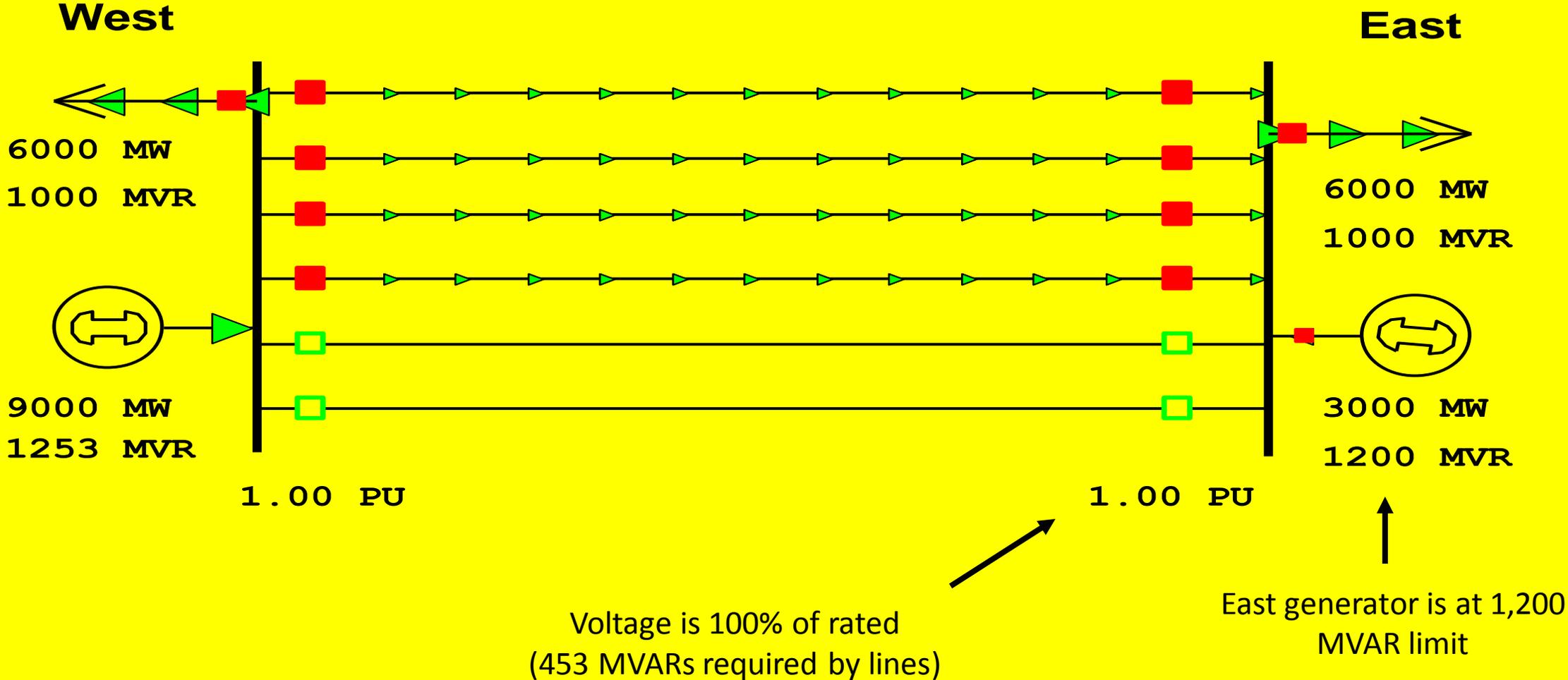
Case 2: One Line Out

3,000 MW transfer – 600 MW per line



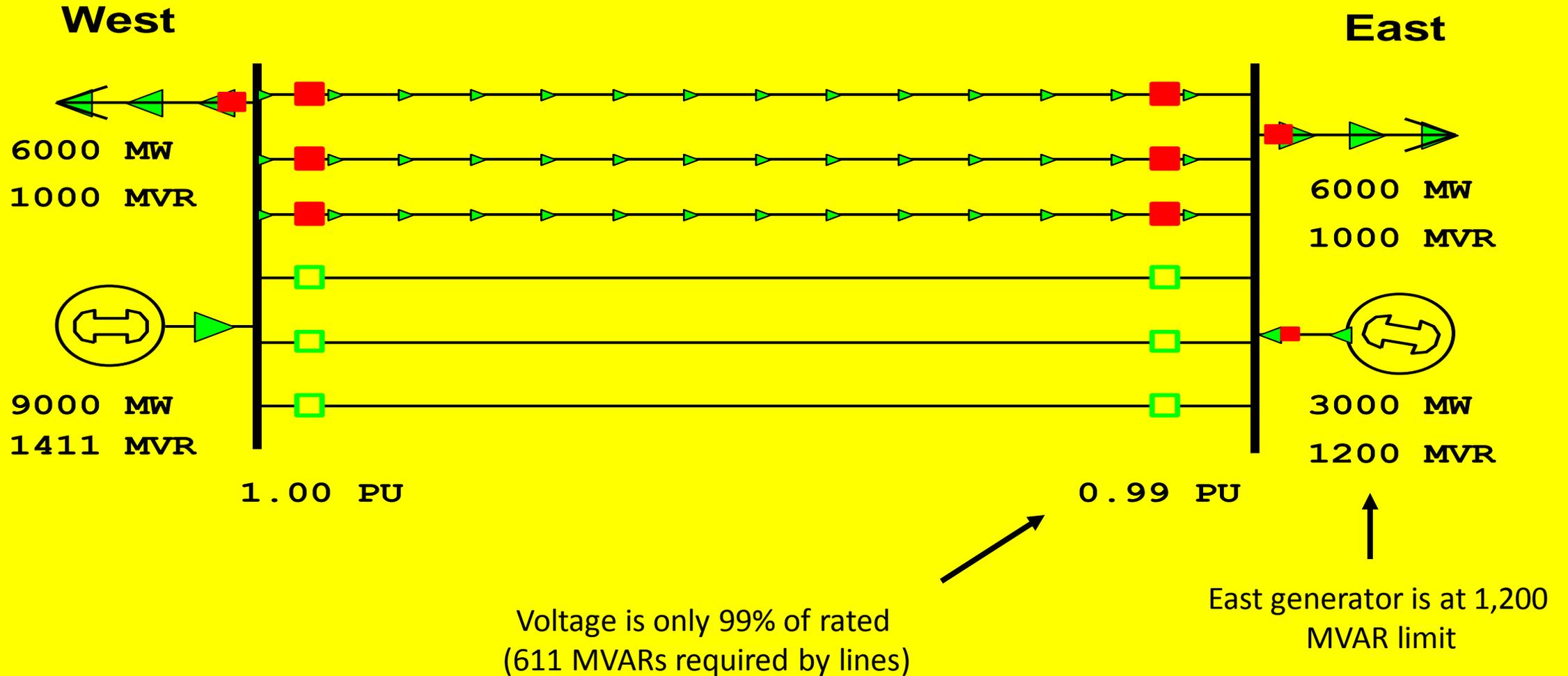
Case 3: Two Lines Out

3,000 MW transfer – 750 MW per line



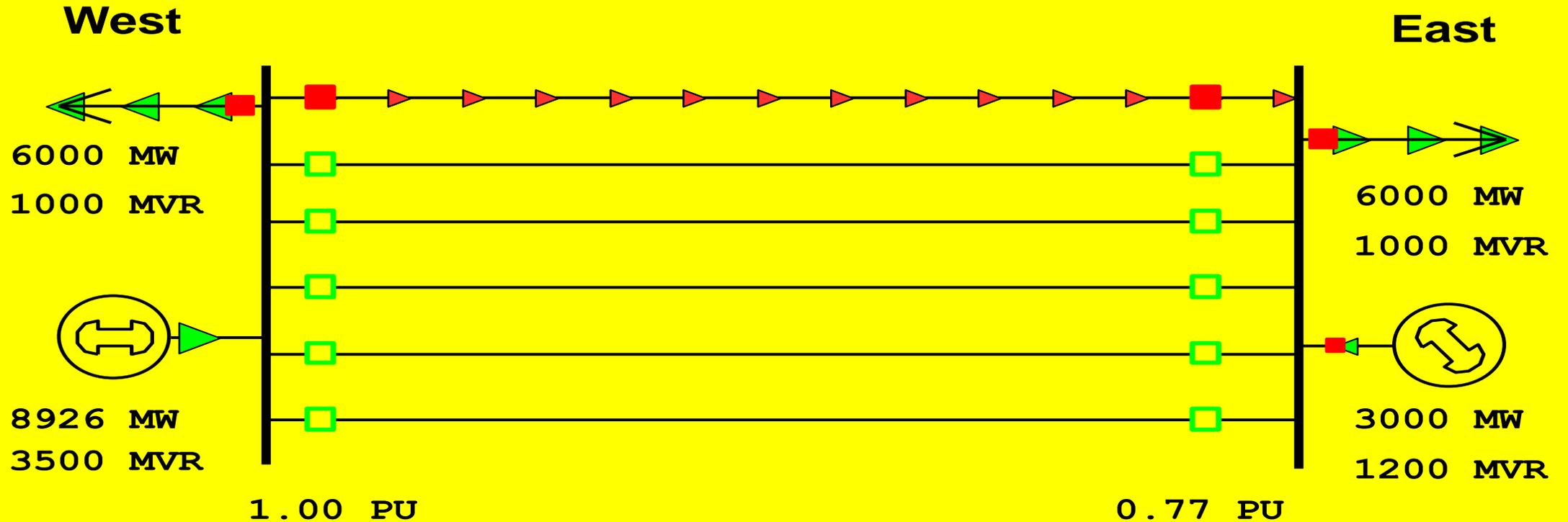
Case 4: Three Lines Out

3,000 MW transfer – 1,000 MW per line



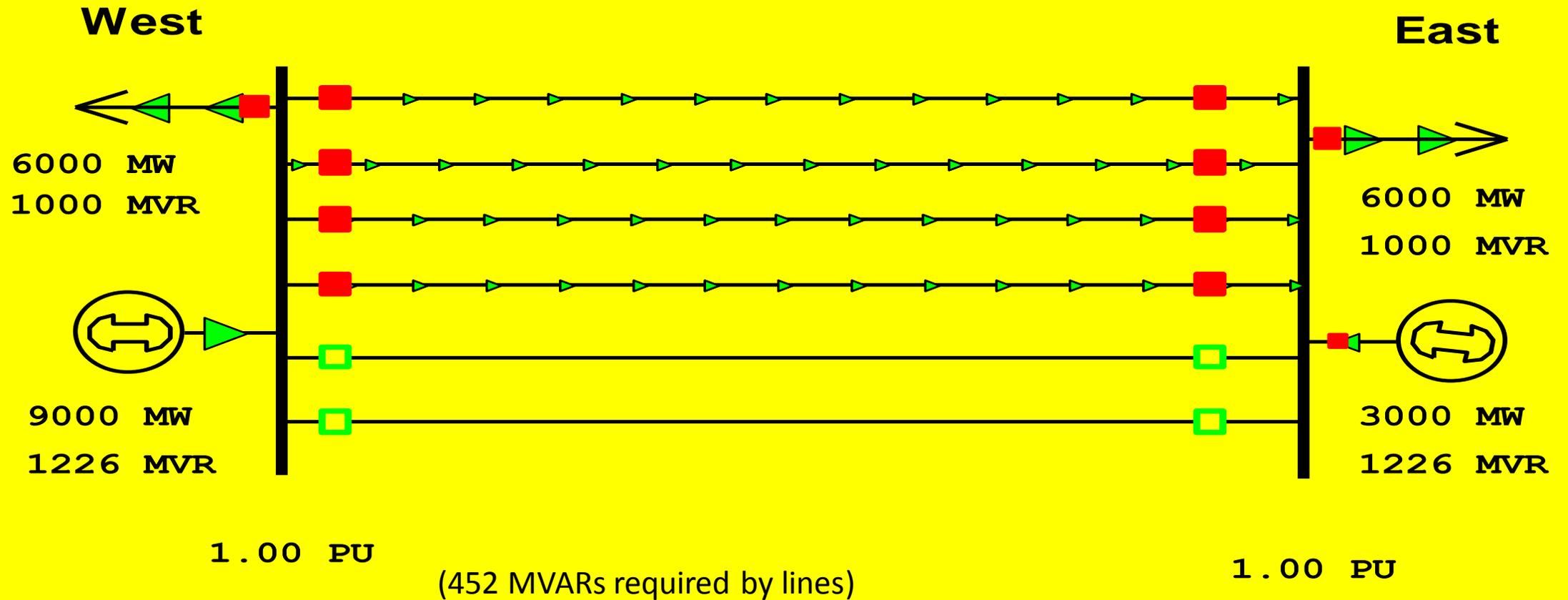
Case 6: Five Lines Out

System Collapse



This simulation could not solve the case of 3,000 MW transfer with five lines out. Numbers shown are from the model's last attempt to solve. The West generator's unlimited supply of VARs is still not sufficient to maintain the voltage at the East bus.

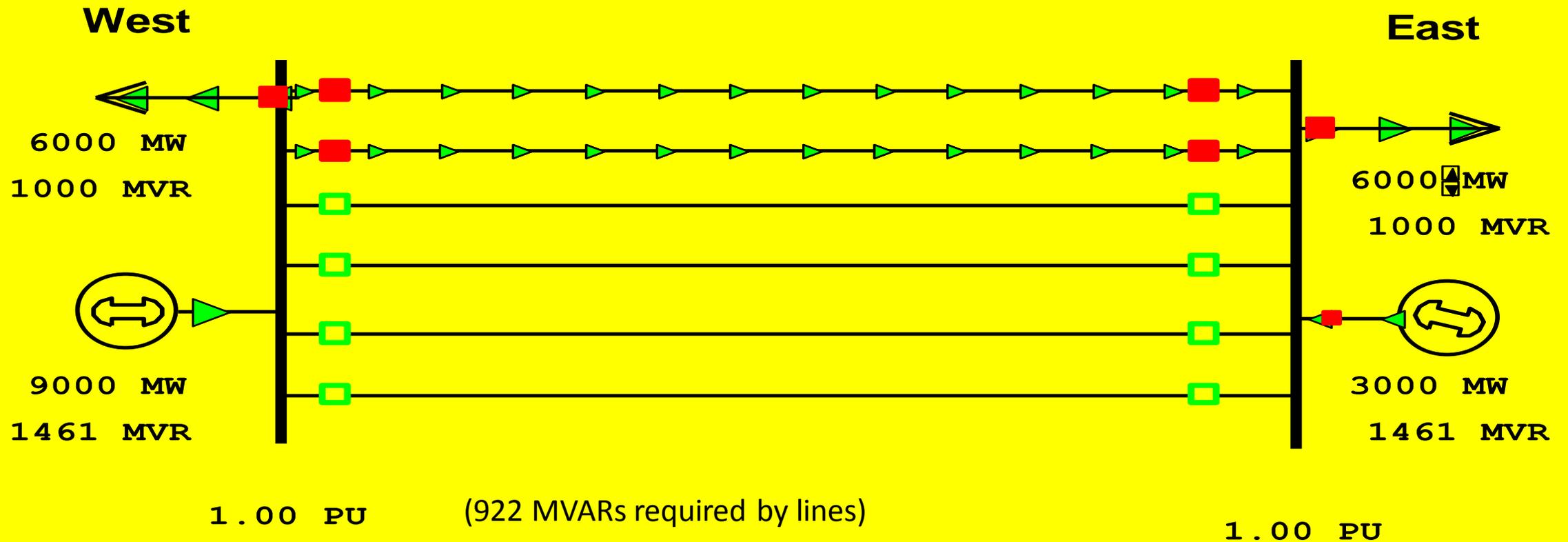
Case 7: Two lines out: Full voltage control



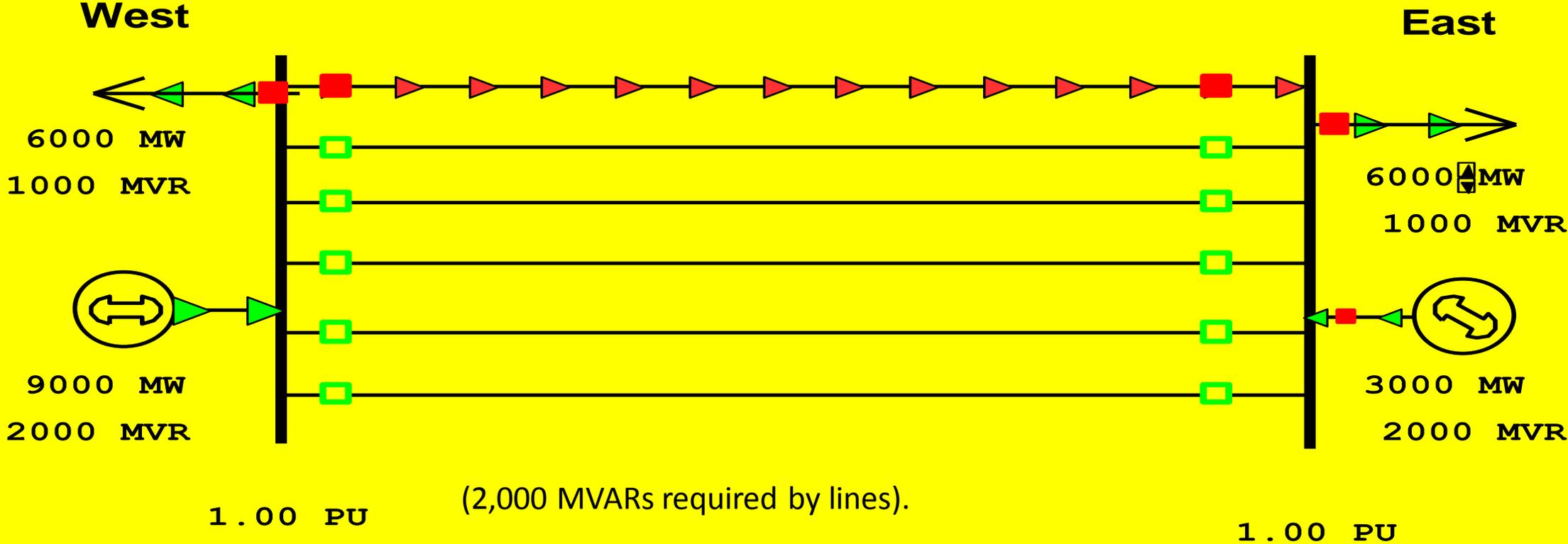
Case 8: Three lines out: Full voltage control



Case 9: Four lines out: Full voltage control

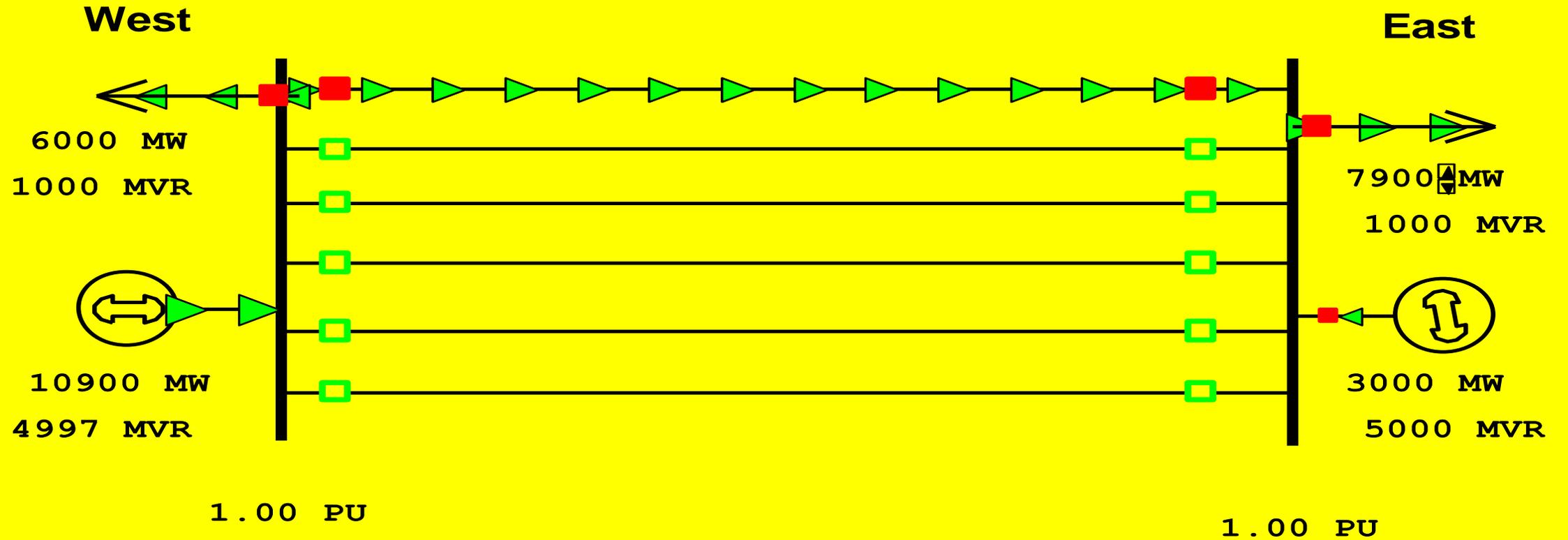


Case 10: Five lines out: Full voltage control



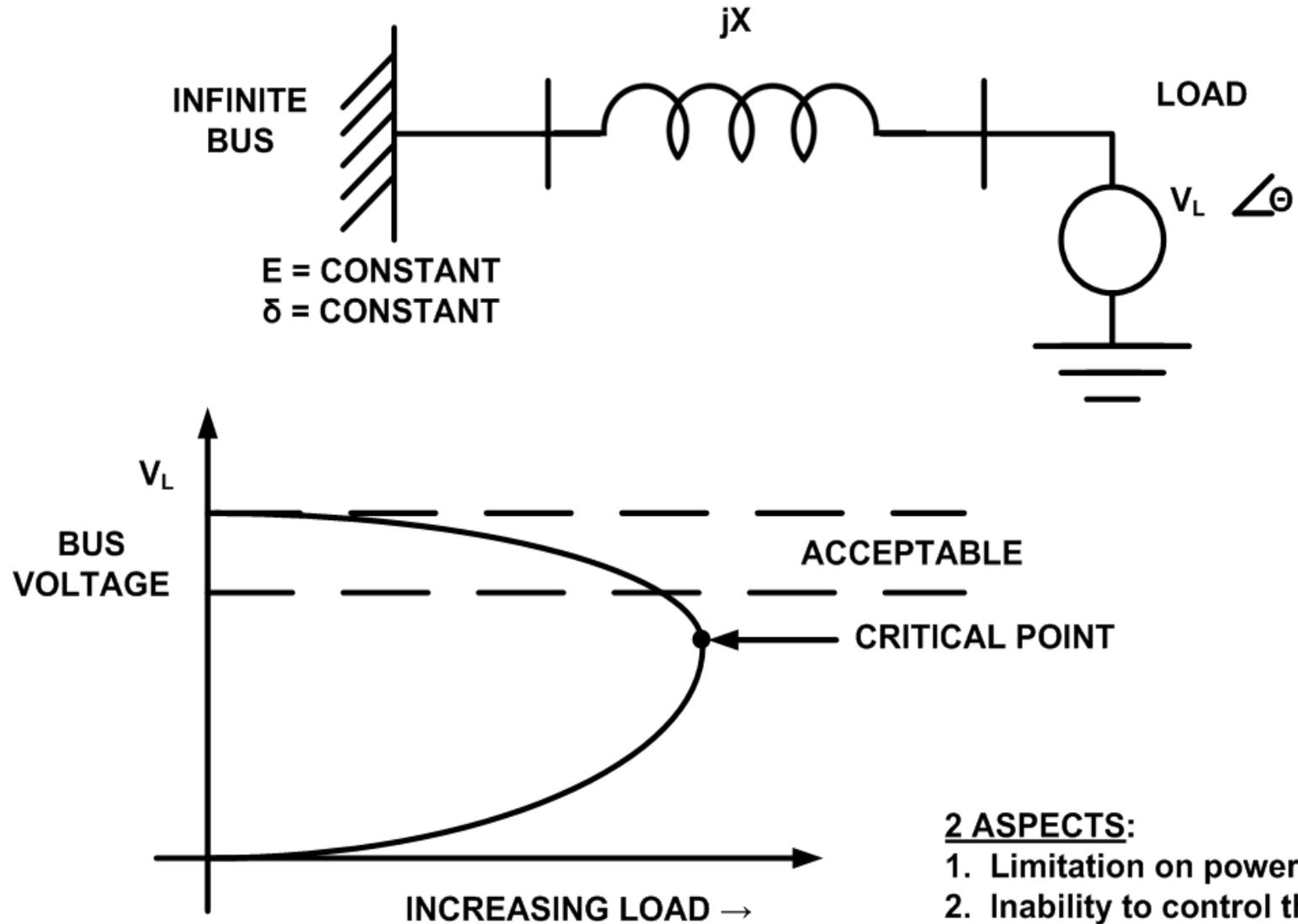
Case 11: How much could this have handled?

4,900 MW



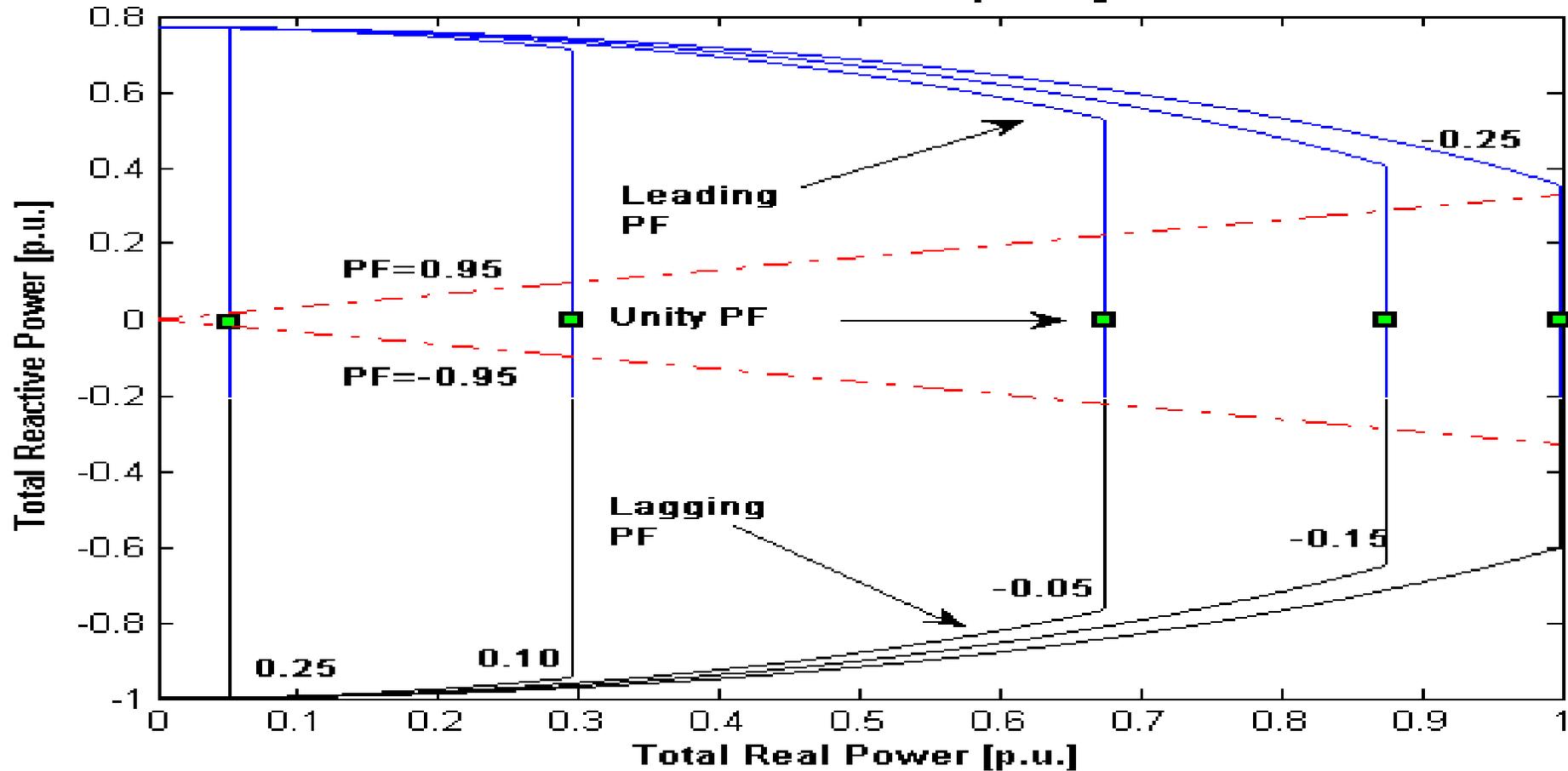


VOLTAGE INSTABILITY



Capability curve of a 1.5 MW machine

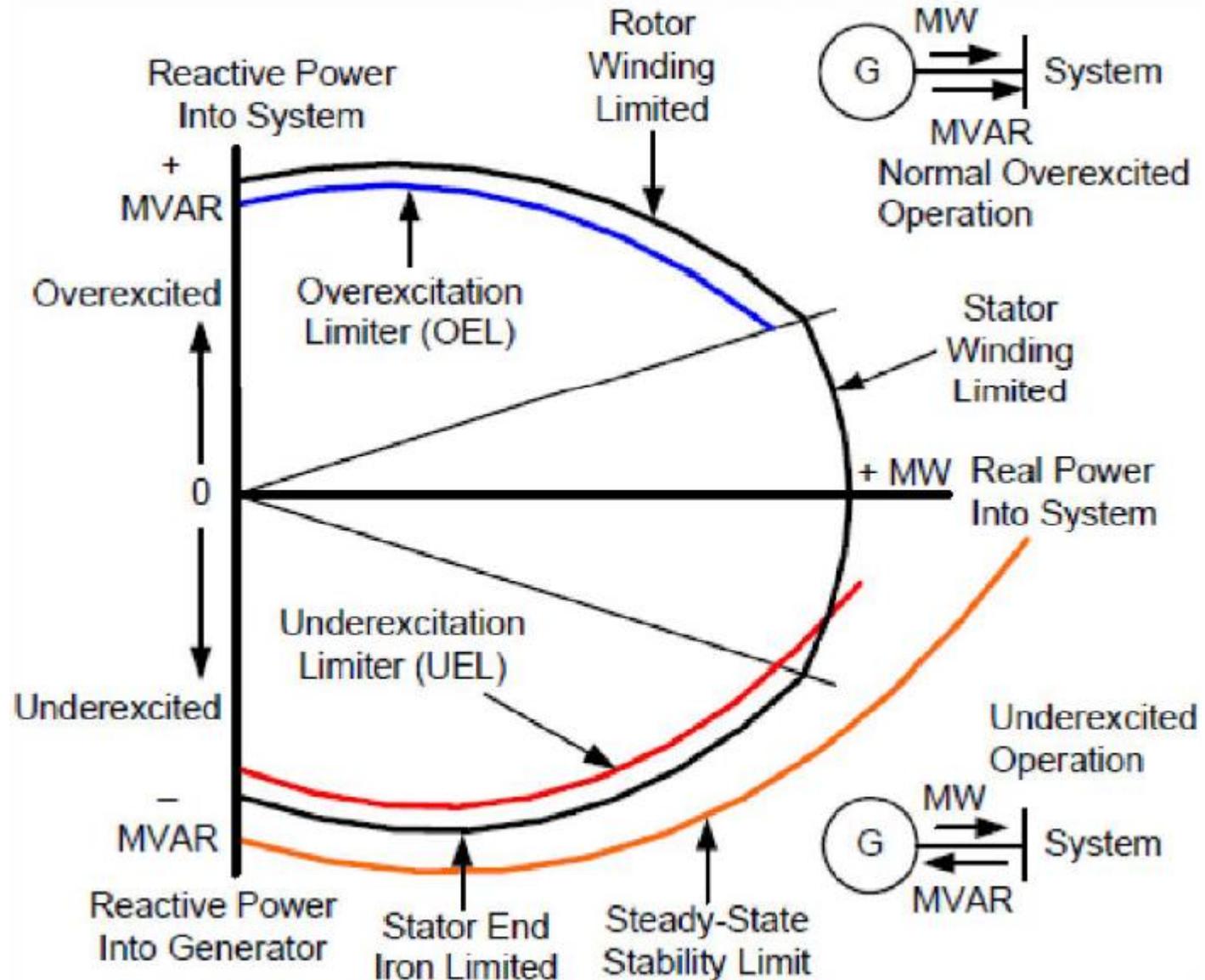
DFIG Wind Park Capability



Rated electrical power	1.5 MW
Rated generator power	1.3 MW
Rated stator voltage	575 V
Rotor to stator turns ratio	3
Machine inertia	30 kgm ²
Rotor inertia	610000 kgm ²
Inductance: mutual, stator, rotor	4.7351, 0.1107, 0.1193 p.u.
Resistance: stator, rotor	0.0059, 0.0066 p.u.
Number of poles	3
Grid frequency	60 Hz
Gearbox ratio	1:72
Nominal rotor speed	16.67 rpm
Rotor radius	42 m
Maximum slip range	+/- 30%



Capability curve – End Limits





Classification of Voltage Stability

1. Large disturbance voltage instability:
 - Large disturbance such as faults, loss of generation or contingencies
 - Requires long-term transient simulation
2. Small disturbance voltage instability:
 - Small disturbance such as incremental changes in loads and etc
 - Static analysis used to determine margin and effect of various factors



Classification of Voltage Stability

Other Classification

1. Long term stability
 - Few minutes to hours
2. Mid term stability
 - 10 seconds to few minutes
3. Short term or transient voltage stability
 - 0-10 seconds

Classification of Voltage Stability

	Small Signal: - Small disturbance	Large Signal: - System fault - Loss of generation
Long –Term	-PV Curves (load flows) - QV Curves - Long-term dynamic models, including tap-changers, var-control, excitation limiters, etc.	-PV Curves (load flows) of faulted state -Long-term dynamic models, including tap-changers, var-control, excitation limiters, etc.
Short –Term	- Typically not a problem and not studied	- Dynamic models (short-term), special importance on dynamic load modeling, stall effects, etc.



Voltage Collapse Incidents Around the World

1. Winnipeg, Canada Nelson River HVDC links, April 1986, 1 second
2. SE Brazil, Paraguay, November 1986, 2 seconds
3. South Florida, May 17, 1985, 4 seconds
4. Western Tennessee, August 22, 1987, 10 seconds
5. Sweden, December 27, 1983, 55 seconds
6. Northern California, May 21, 1983, 2 minutes
7. Florida, September 2, November 26, December 28 & 30, 1982, 1-3 minutes
8. Jacksonville, Florida, September 22, 1977, few minutes
9. Belgium, August 4, 1982, 4.5 minutes



Voltage Collapse Incidents Around the World

1. England, May 20, 1986, 5 minutes
2. Western France, January 12, 1987, 6-7 minutes
3. Tokyo, July 23, 1987, 20 minutes
4. France, December 19, 1978, 26 minutes
5. Japan, August 22, 1970, 30 minutes
6. New York, September 22, 1970, several hours
7. Illinois and Indian, July 20, 1987, hours
8. Northeastern USA, June 11, 1984, hours



Voltage Stability and Reactive Power Transmission

Since real power is the main variable to rotor angle stability analysis, reactive power is key to voltage stability analysis.

- Reactive power is easier to generate compared to real power but more difficult to transmit
- Deficit of reactive power either locally or globally leads to poor voltage profile and with the increase in loading, it may lead to voltage collapse
- The transmission of reactive power becomes difficult in heavily loaded lines. This can be illustrated through an example given below

Voltage Stability and Reactive Power Transmission

Case	V_s	V_r	Angle δ	Q_s	Q_r
a. Lightly Loaded	1.10	1.00	10	0.845	0.555
b. Moderately Loaded	1.05	0.90	20	1.894	0.056
c. Heavily Loaded	1.00	0.80	50	3.238	-1.048

This table clearly indicates that at higher loading conditions transmission lines are unable to transfer reactive power



Voltage Stability and Reactive Power Transmission

It has also been observed that as the distance of the reactive power sources from the reactive power demand become larger...

1. The voltage gradient on line supplying reactive power becomes greater
2. More reactive power compensation is required
3. It becomes more difficult to control the voltage level

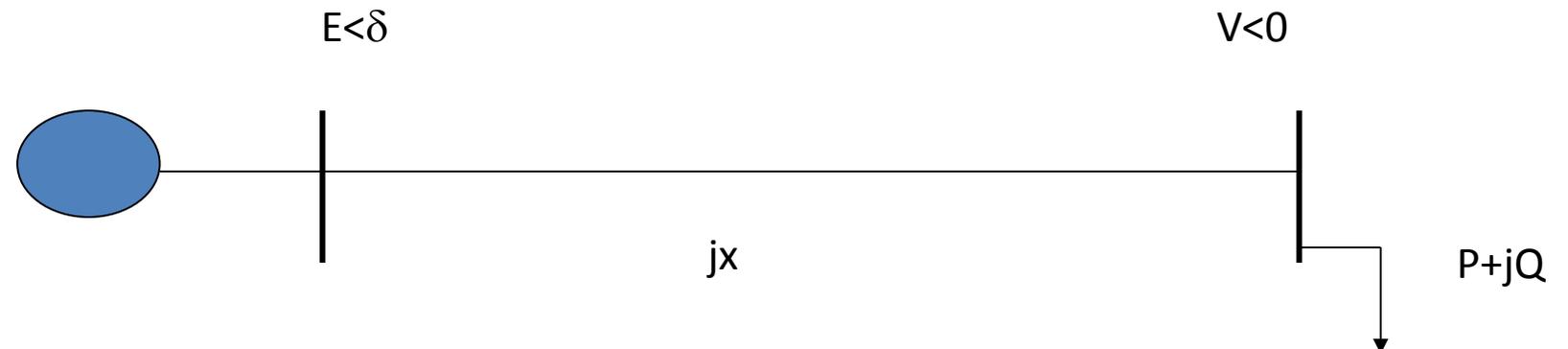


Static Voltage Stability Analysis using P-V and V-Q curves

- Most of the voltage stability analysis is static in nature
- This is due to the fact that variations are over a long period of time (several minutes) followed by rapid change when it reaches very close to the instability
- Some of the concerns of operating engineers:
 - Knowing the steady state loadability limit
 - Effect of control actions
 - Effect of system parameter changes on the system stability or its loadability

Static Voltage Stability Analysis using P-V and V-Q curves

- Two curves, which are frequently used, are the P-V curve and V-Q curve (or Q-V curve)
- These are especially useful for conceptual analysis of voltage stability of radial systems or longitudinal transmission lines
- Once again consider a simple radial system having a load of $P+jQ$;



Static Voltage Stability Analysis using P-V and V-Q curves

- The load voltage equation can be written as:

$$V^2 = \left(\frac{PX}{E} \right)^2 + \left(\frac{QX + V^2}{E} \right)^2$$

- Solutions to the above polynomial equation are:

$$V = \left\{ \frac{E^2 - 2QX \pm \left((E^2 - 2QX)^2 - 4(P^2 + Q^2)X^2 \right)^{\frac{1}{2}}}{2} \right\}^{\frac{1}{2}}$$

- For a given values of loading conditions (i.e. P,Q, and E), there are four values for V
 - Only two of them are real

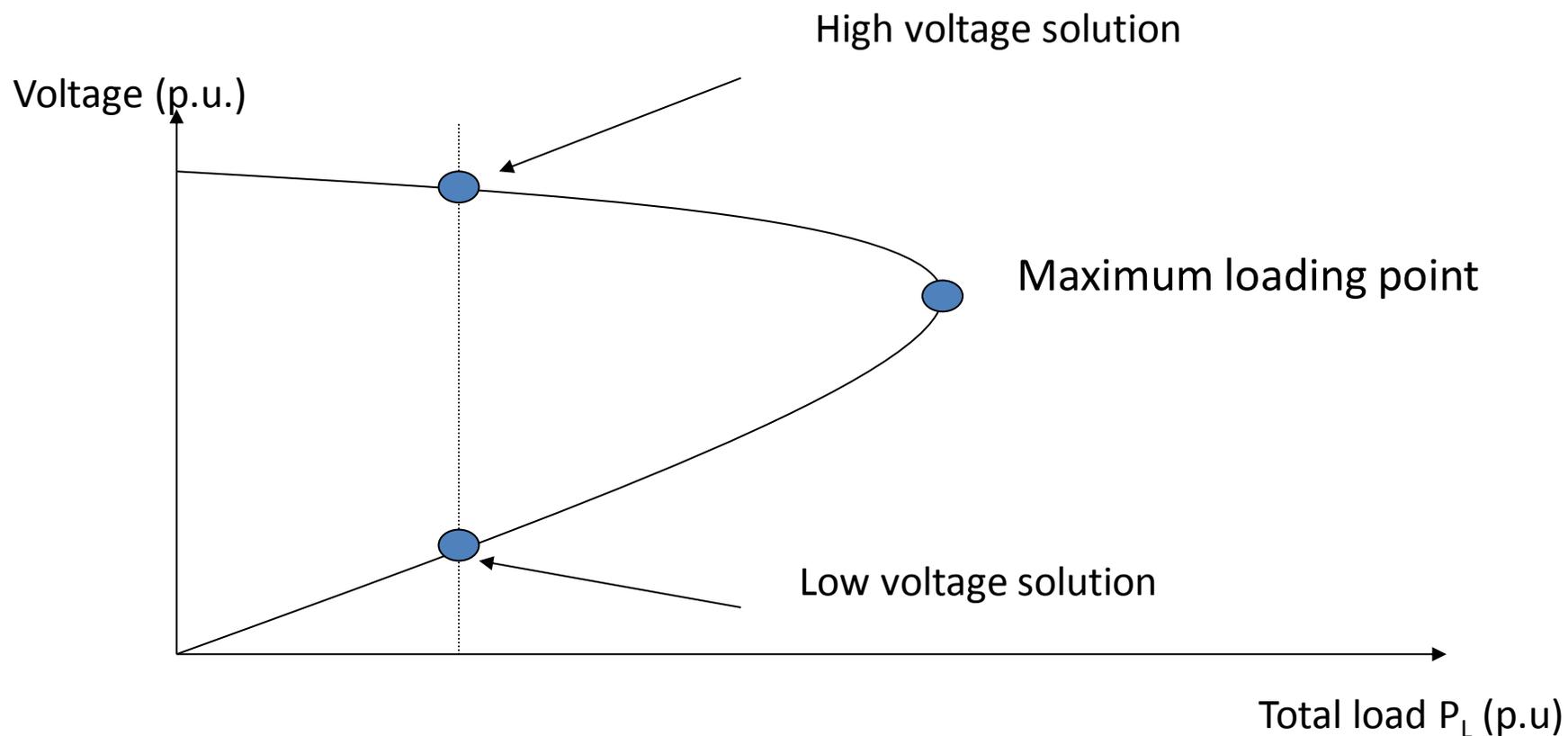


Static Voltage Stability Analysis using P-V and V-Q curves

- When $(E^2 - 2QX)^2 = 4(P^2 + Q^2)X^2$ has only one solution
- For a fixed load power factor and different values of P, the curve obtained is known as P-V curve
- When both solutions coalesce with each other, the system loading corresponds to maximum real power loading (loading margin of the system) and the voltage is known as “**Critical Voltage**”

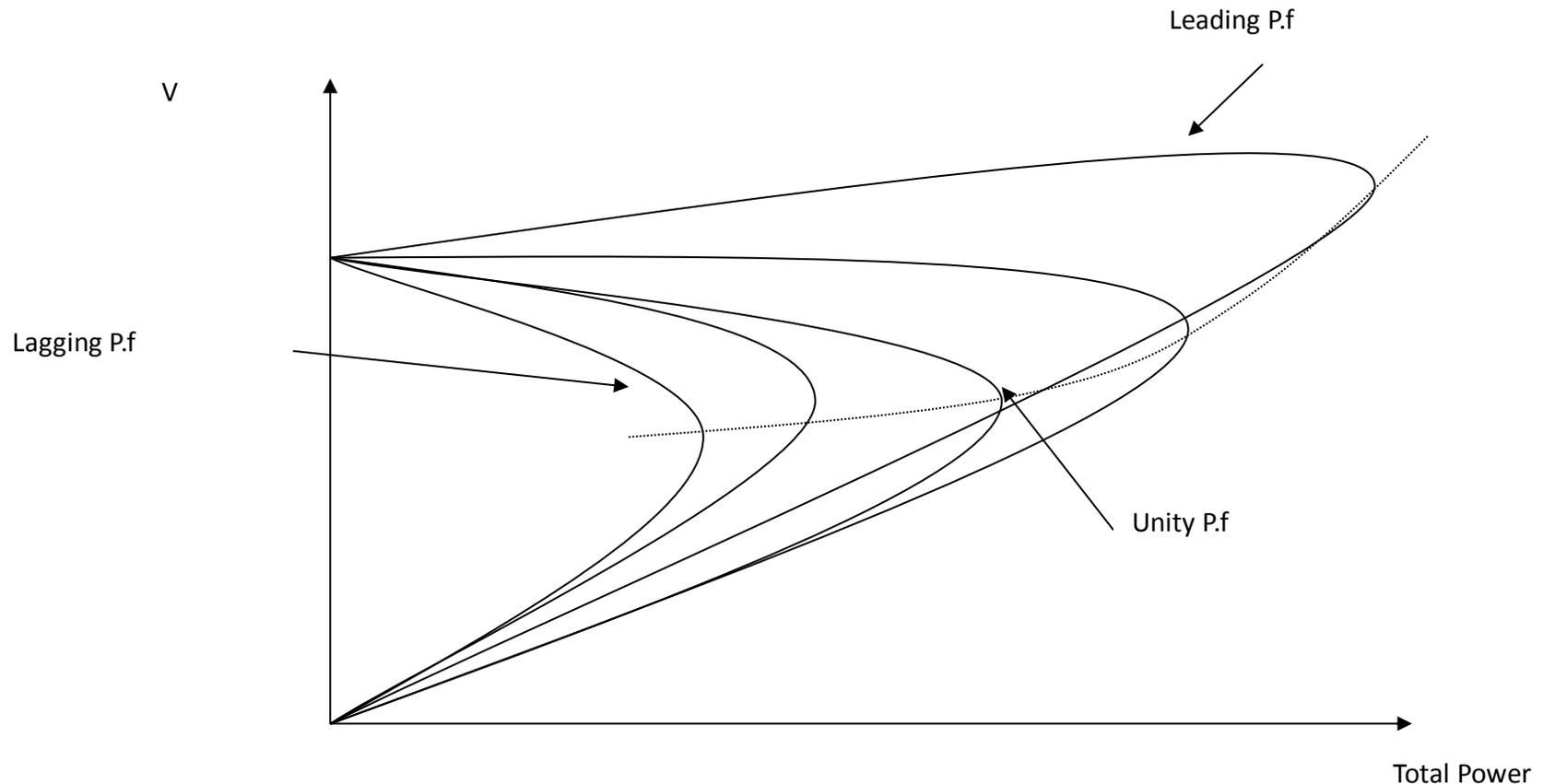
Static Voltage Stability Analysis using P-V and V-Q curves

PV curve or Nose curve



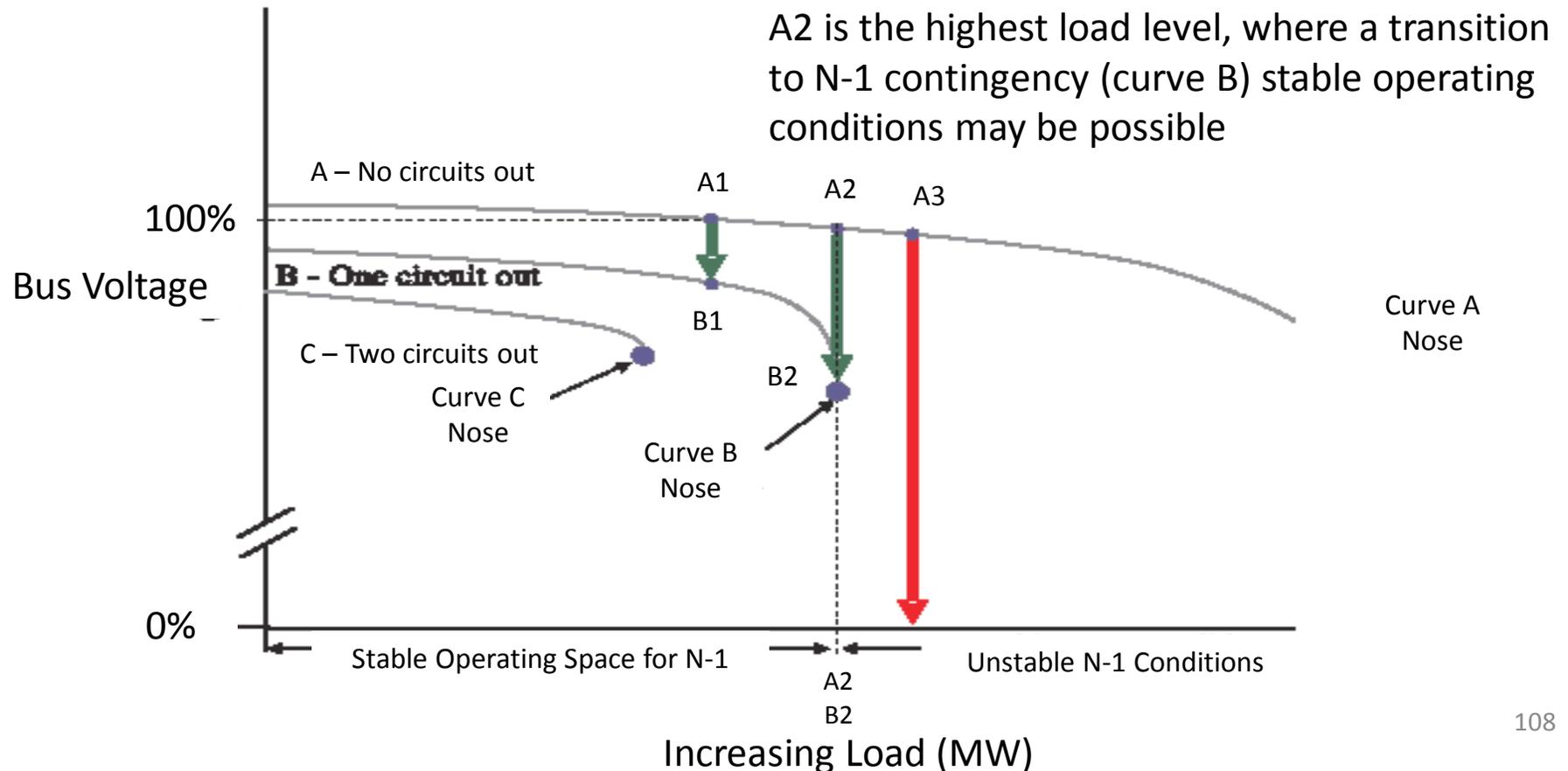
Static Voltage Stability Analysis using P-V and V-Q curves

The maximum loading point shifts with different power factor.



Static Voltage Stability Analysis using P-V and V-Q curves

The maximum loading point shifts with contingency in the system.



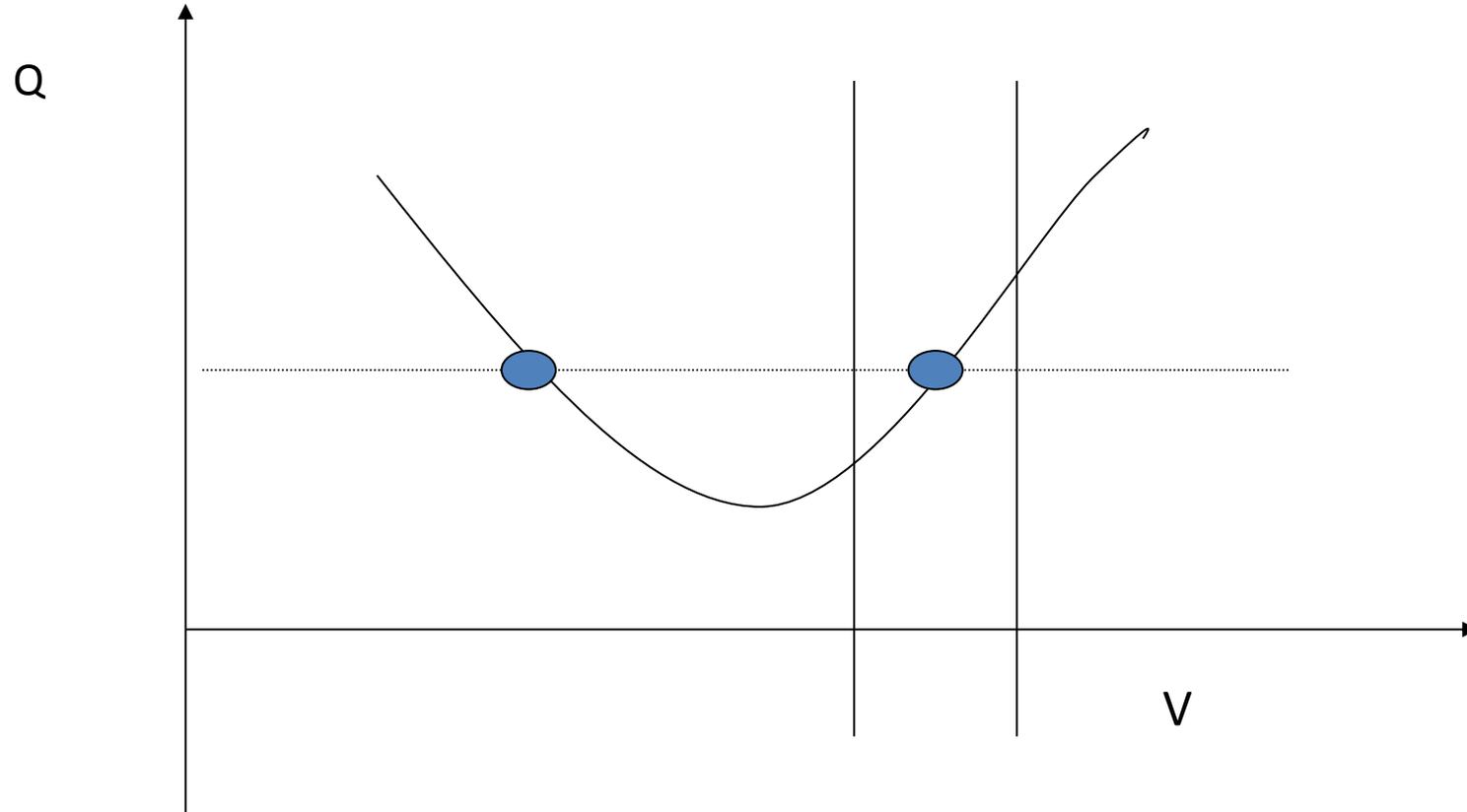


Static Voltage Stability Analysis using P-V and V-Q curves

- Similarly for a given P , variation of voltage with Q can be plotted, and V-Q can be obtained
- In an interconnected system, P-V and Q-V curves can be used to study the voltage variation at a bus for change in the loading at that bus
- These curves can be generated by running several load flow solutions at different loading conditions

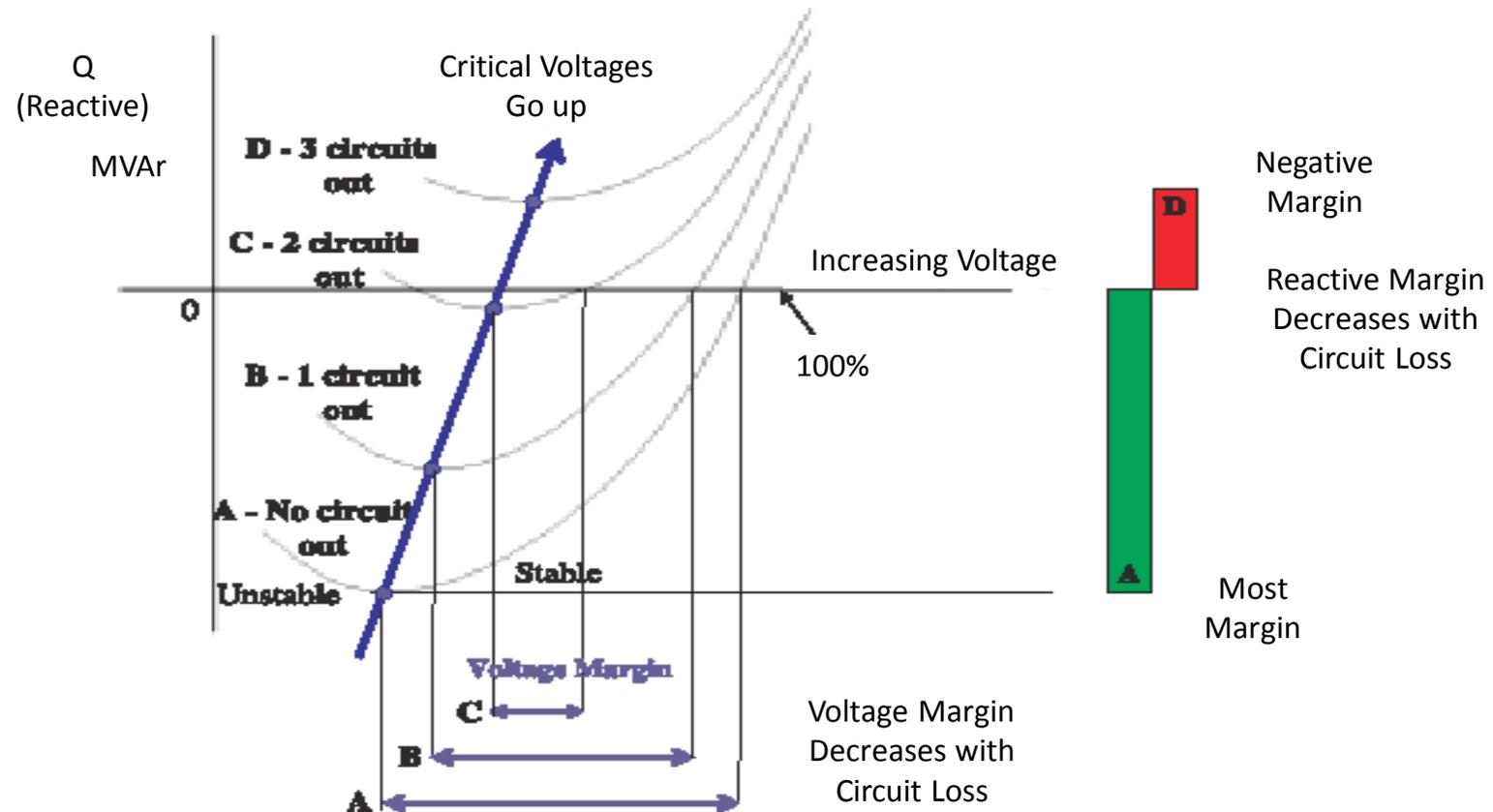
Static Voltage Stability Analysis using P-V and V-Q curves

A typical Q-V curve



Static Voltage Stability Analysis using P-V and V-Q curves

Q-V curves and Reactive Margins





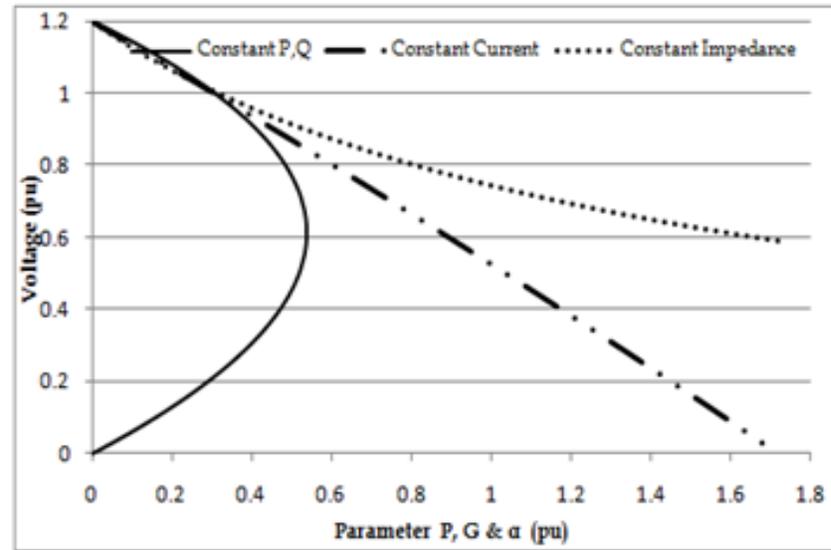
Static Voltage Stability Analysis using P-V and V-Q curves

- Conventional load flow techniques generally provide the **high voltage solutions**
- The low voltage solutions and hence complete P-V and V-Q curves can be obtained by **multiple load flow solution** method or **continuation power flow** method (CPF)
- The P-V and V-Q curves can also be used to study impacts of load variations, load type and controls on system maximum loadability, and voltage collapse

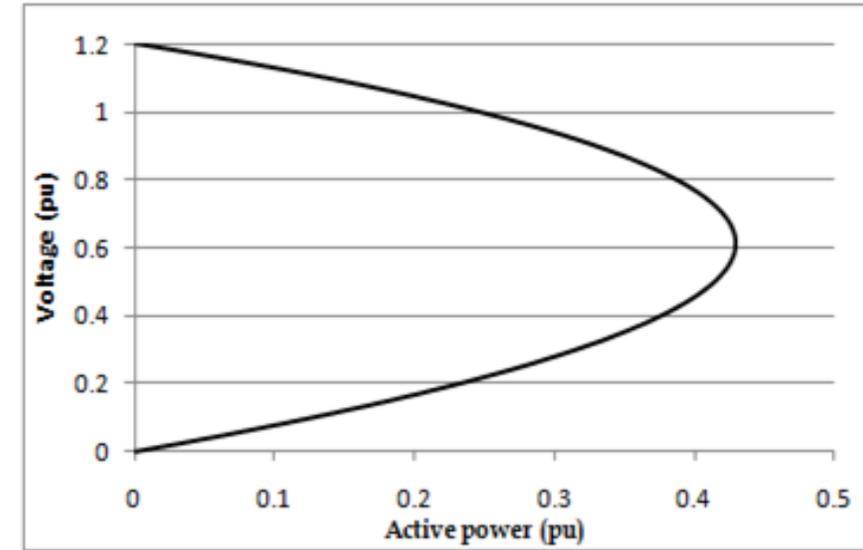


Impact of Load Type

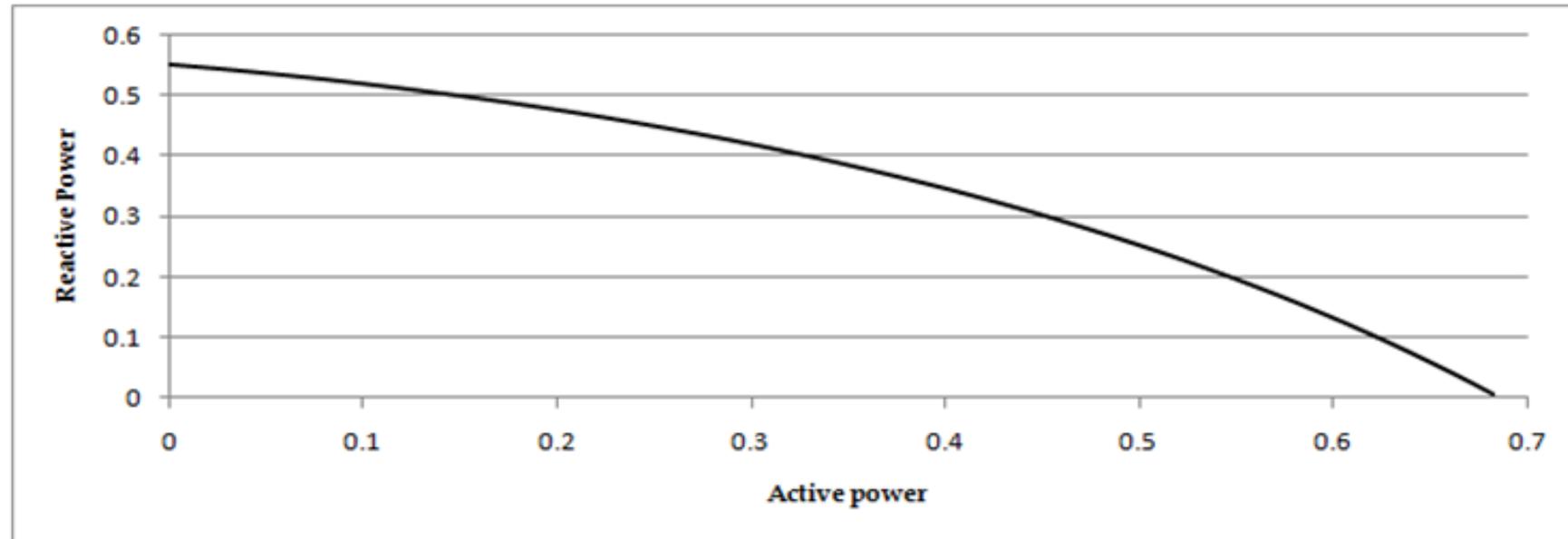
- Load characteristics have a profound effect on voltage instability and collapse
- Usually, three Static load models are used in the stability analysis:
 - **Constant PQ,**
 - **Constant current**
 - **Constant impedance**
- Their relative severity to the voltage instability is in the same order. In other words, the constant PQ load is the most tedious load in the system followed by constant current then constant impedance.



a. Voltage/Load parameters



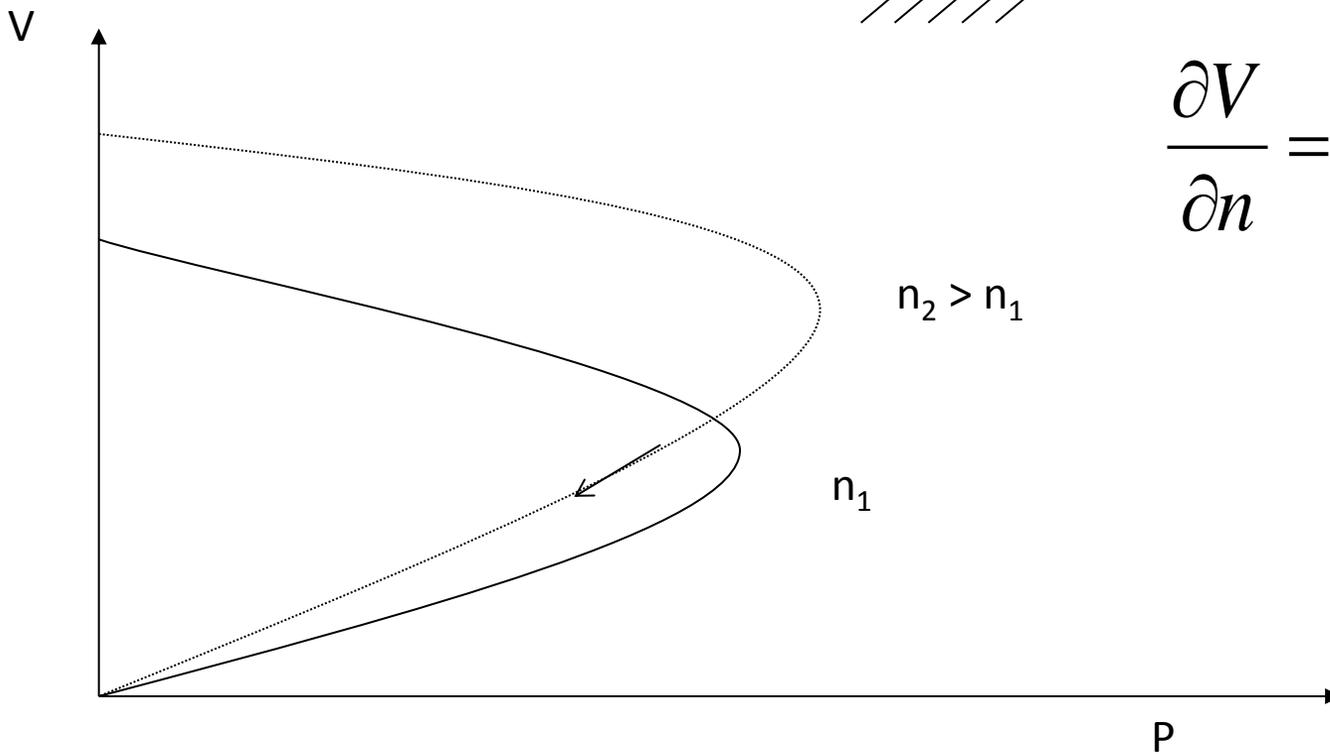
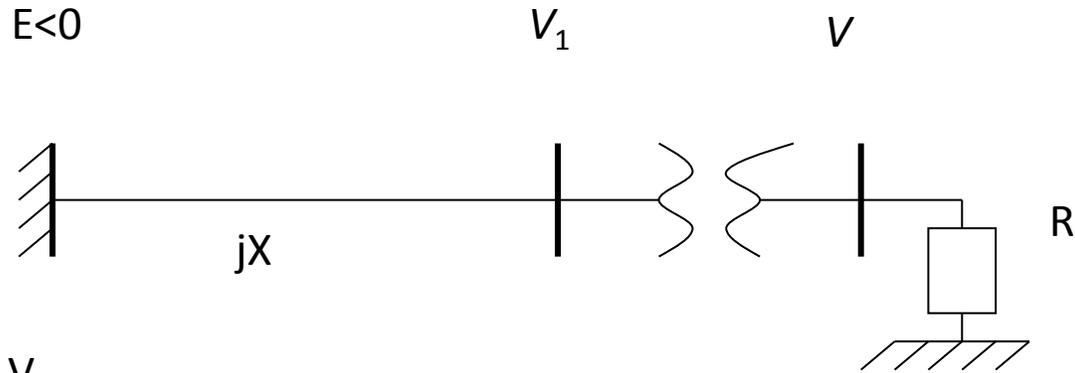
b. Voltage/Active power



c. Voltage stability limit in the complex power plane



Reverse action of transformer OLTC



$$V = \frac{R}{\sqrt{R^2 + (n^2 X)^2}} nE$$

$$\frac{\partial V}{\partial n} = \frac{R(R + n^2 X)(R - n^2 X)}{\{R^2 + (n^2 X)^2\}^{\frac{3}{2}}}$$

Voltage Stability Analysis

- **The analysis of voltage stability involves...**
 - The examination of **proximity to voltage instability**
 - **Mechanism of voltage instability**
- **Proximity to voltage instability:**

How close the system is to voltage instability?
- This involves measuring the proximity of instability in terms of measurable quantities such as real and reactive power
- The most appropriate measures for any given situation depends on the specific system and the intended use of margin
(e.g. planning vs. operating margin)



Voltage Stability Analysis

- Consideration must be given to possible contingencies (e.g. line outages, loss of generating units, other reactive power source, or etc.)
- Mechanism of voltage instability:
 1. How and why does instability occur?
 2. What are the factors contributing to instability?
 3. What are the voltage weak-areas?
 4. Which measures are the most effective in improving voltage stability?



Voltage Stability Analysis

- Time-domain simulations, which include appropriate modeling, capture the events and their chronological events leading to instability
- The problems with this kind of simulation are computationally time consuming, do not provide sensitivity information and the degree of stability, readily
- System dynamics influencing voltage instability are usually slow
- Many aspects of the problem can be effectively analyzed by using static analysis methods



Voltage Stability Analysis

- The **static analysis** techniques allow examination of a wide range of the system conditions of the power system and provide much insight of the nature of the problem and various contributing factors
- **Dynamic analysis**, on the other hand, is useful for detailed study of voltage collapse situations



Modeling Requirement

- The following components, which have significant impact on voltage stability, have to be taken into account in the analysis:
 1. Loads (Voltage and frequency dependence of the load, induction machines, expansion of sub-transmission network, including OLTC, compensators and regulators)
 2. Generator excitation control (excitation limits)
 3. FACTS controllers
 4. Automatic generation control
 5. Protection and control



Dynamic Analysis

- The overall system equations include a set of first-order differential equation and a set of algebraic equations
- Since we include the representation of transformer tap-changer and phase-shift angle controls, the elements of the network node admittance matrix changes as a function of bus voltage and time
- The set of ODE and algebraic equations can be solved in the time-domain by using any of the numerical integration methods
- The study period typically lasts for several minutes



Static Analysis

VQ Sensitivity Analysis

- For static voltage stability analysis of an interconnected system, power flow models have been considered
- It has been proved that at **near system maximum loading** point the load flow diverges and the power flow **Jacobian (Newton Raphsons Load Flow, NRLF)** becomes singular
- The NRLF equations in polar coordinates can be written as
$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$



Static Analysis

- Singularity of the reduced Jacobian, J_R defining Q-V has also been used to determine static voltage instability

$$\Delta Q = J_R \Delta V$$

$$J_R = J_4 - J_3 J_1^{-1} J_2$$

- At system maximum loadability or the point of static voltage instability, **the determinant of full Jacobian J**, or the **reduced Jacobian J_R becomes zero** or the NRLF diverges
- Divergence of load flow can be due to some other reasons as well



Static Analysis

Singular value decomposition Technique

The Jacobian J is decomposed into three matrices as given below:

$$[J]_{n \times n} = [U][D][V]^T$$

Where, U and V are left and right eigenvectors, respectively. D is a diagonal matrix, containing singular values of J



Static Analysis

- When J become singular, the minimum singular value of the matrix J is assumed to be zero.
- Ratios of the maximum to minimum singular values of the Jacobian is know as the **condition number** of J .
- The condition number is assumed to be a larger number (infinite) when J becomes singular.



Static Analysis

Q-V modal Analysis

- Voltage stability characteristics of the system can be identified by computing the eigenvalues and eigenvectors of the reduced Jacobian matrix J_R

$$J_R = \xi \Lambda \eta$$

ξ : *Right* Eigen Vector J_R

η : Left Eigen Vector of J_R

Λ : *Diagonal* Eigen Value Matrix of J_R



Static Analysis

Q-V modal Analysis

$$J_R^{-1} = \xi \Lambda^{-1} \eta$$

But $\Delta V = J_R^{-1} \Delta Q$

Substituting

$$\Delta V = \xi \Lambda^{-1} \eta \Delta Q$$

$$\xi^{-1} = \eta$$

$$\eta \Delta V = \Lambda^{-1} \eta \Delta Q$$

$v = \eta \Delta V$, *the* vector of modal voltage variations

$q = \eta \Delta Q$, *the* vector of modal reactive power variations



Static Analysis

- The previous equation can be written in the following form...

$$v_i = \frac{1}{\lambda_i} q_i$$

- If $\lambda_i > 0$, the i^{th} modal voltage and the i^{th} modal reactive power variations are along the same direction, indicating that the system voltage is stable
- If $\lambda_i < 0$, the i^{th} modal voltage and the i^{th} modal reactive variation are along in the opposite directions, indicating the system voltage is unstable

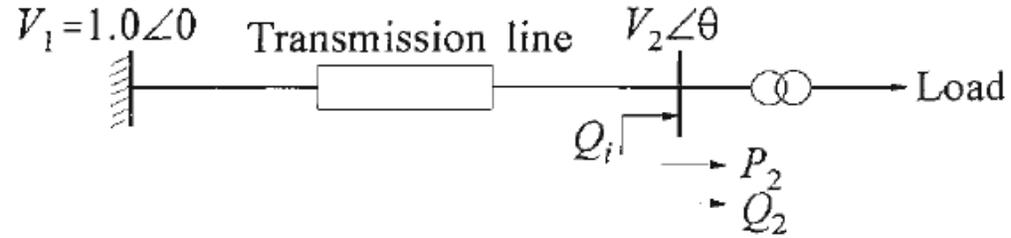


Static Analysis

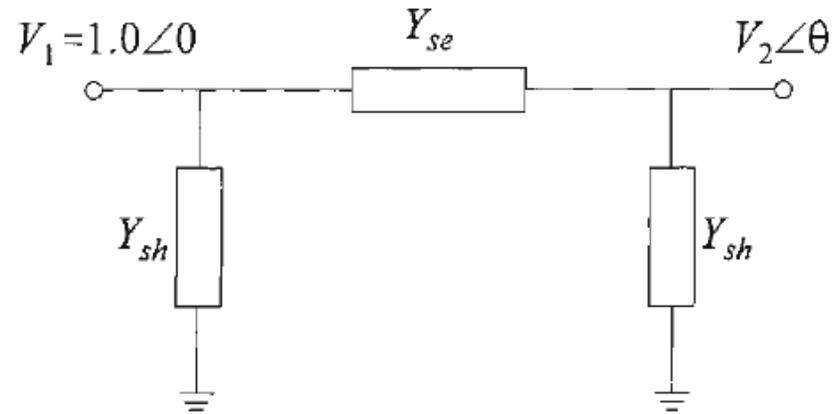
- The magnitude of each modal voltage variation is equal to the inverse of λ_i times the magnitude of the modal reactive power variation
- Therefore, the magnitude λ_i determines the degree of stability of the i^{th} modal voltage
- The smaller the magnitude of positive λ_i , the closer the i^{th} modal voltage is to being unstable
- When $\lambda_i = 0$, the i^{th} modal voltage collapses because any change in that modal reactive power causes infinite change in the modal voltage



Assignment-2



(a) Schematic diagram



$$Y_{se} = 2.142 - j24.973 \text{ pu}$$
$$Y_{sh} = 0 + j2.076 \text{ pu}$$

(b) Equivalent π circuit representation of line

A 322 km, 500 kV line supplying a radial load

The figure shows the system representation applicable to a 322km (200mi), 500 kV transmission line supplying a radial load from a strong system. The line parameters are expressed in per unit on 100MVA and 500kV base

$$P = f(\theta, V)$$

$$Q = g(\theta, V)$$

Formulate the Jacobian Matrix

When $P_2 = 1500$ MW, calculate the eigen values of the reduced Jacobian matrix and V-Q sensitivities with the following reactive power injections for each of the corresponding two voltages on the Q-V curve



$$i) Q_i = 500 \text{MVA}r$$

$$ii) Q_i = 400 \text{MVA}r$$

iii) Values of Q_i close to the bottom of the $Q-V$ Curve

Assume that for the purpose of analysis, the load and reactive power source have constant P,Q characteristics

Determine the voltage stability by computing the eigenvalues of the reduced Q-V Jacobain matrix for the following cases...

$$i) P = 1500 \text{MW}, Q_i = 450 \text{MVA}r$$

$$ii) P = 1900 \text{MW}, Q_i = 950 \text{MVA}r$$

Assume that the reactive power Q_i is supplied by a shunt capacitor



Static Analysis

Continuation Power Flow (CPF)

- CPF methods are typically employed to trace the upper and lower part of P-V curve, including the maximum loading point of the system
- The technique is computationally demanding for larger systems; however, it provides additional information such as sensitivity with respect to parameter variation, which is useful in analyzing the system further



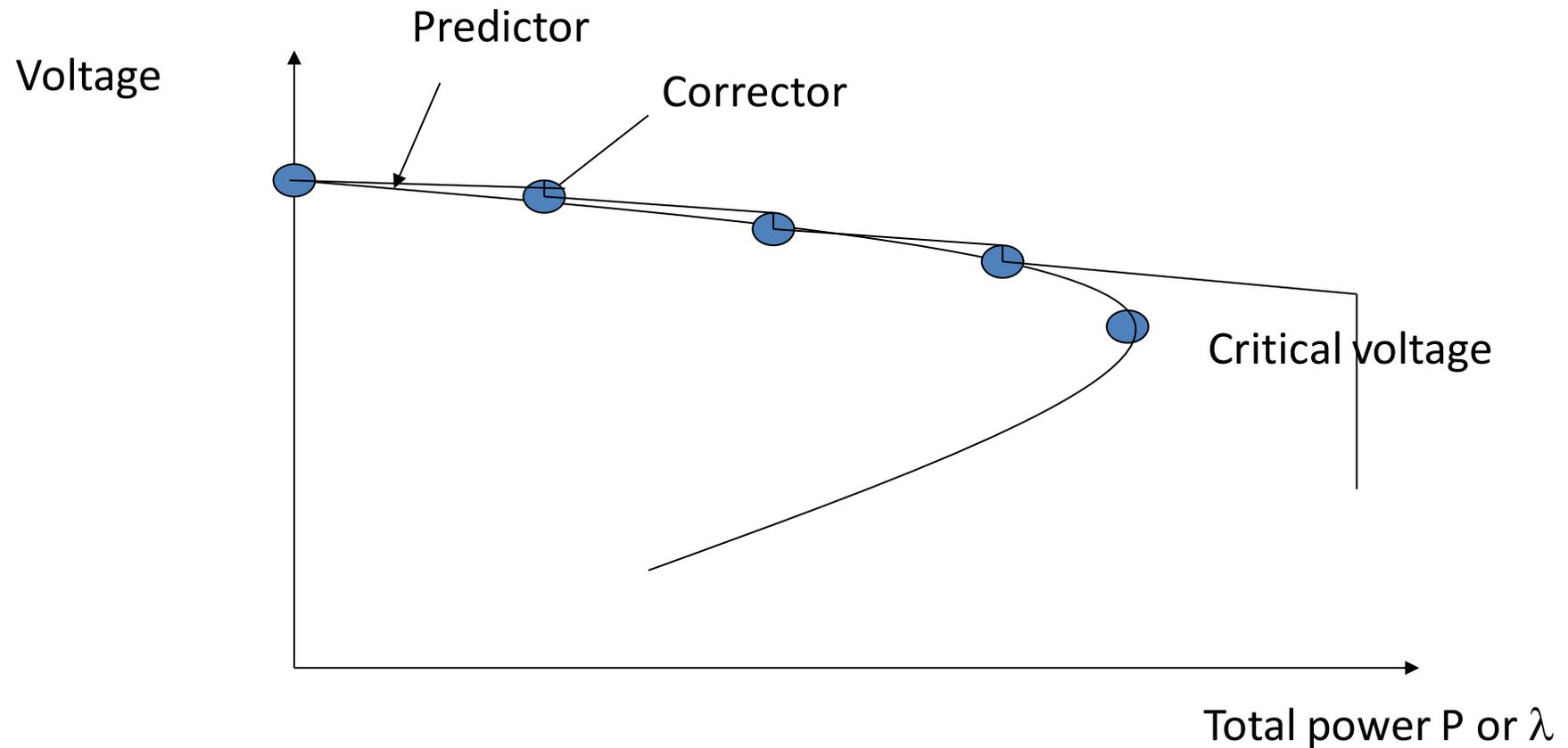
Static Analysis

Continuation Power Flow (CPF)

- The CPF technique uses an iterative process involving **predictor** and **corrector** steps
- In some cases, an additional parameterization step is used to avoid certain convergence issues
- An additional technique like “step cutting” can also be used to solve the divergence problem

Static Analysis

Continuation Power Flow (CPF)





Static Analysis

Continuation Power Flow (CPF)

- From a known initial point A, a tangent predictor step is used to estimate the solution point B for a given load direction defined by the parameter λ
- A corrector step is then used to determine the exact solution C, using a power flow with an additional equation to find out the proper value of λ
- This process is repeated until the desired bifurcation or P-V curve is obtained



Static Analysis

Mathematical Formulation of CPF

- In this technique, the basic equations are similar to the ones used in standard power flow analysis, except that λ is added as a parameter
- The re-formulated power flow equations, with provision for increasing generation as the load is increased, may be expressed as:

$$F(\theta, V) = \lambda K \quad (1)$$



Static Analysis

Mathematical Formulation of CPF

Where,

λ = the load parameter

θ = the vector of bus voltage angles

V = the vector of bus voltage magnitudes

K = the vector representing percentage load change
at each bus

The above set of nonlinear equations is solved by specifying a value for λ , such that λ lies between 0 and its critical value



Static Analysis

Mathematical Formulation of CPF

- Where $\lambda = 0$ represents the base load condition, and $\lambda = \lambda_{\text{critical}}$ represents the critical, or maximum loading condition, for the given load and generation direction
- Equation (1) can be rearranged and written in the following form:

$$F(\theta, V, \lambda) = 0 \quad (2)$$



Static Analysis

Mathematical Formulation of CPF

Predictor Step:

- A linear approximation is used to estimate the next solution for a change in one of the state variables (i.e. θ , V , or λ)
- Taking the derivatives of both sides of Equation (2), with the state variables corresponding to the initial solution, the result will be in the following set of linear equations: $F_{\theta}d\theta + F_v dV + F_{\lambda}d\lambda = 0$



Static Analysis

Mathematical Formulation of CPF

Predictor Step:

- The equation can be written in the following form:

$$\begin{bmatrix} F_{\theta} & F_V & F_{\lambda} \end{bmatrix} \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} = 0$$

- Since the insertion of λ in the power flow equation added an unknown variable, one more equation is needed to solve the above equations



Static Analysis

Mathematical Formulation of CPF

Predictor Step:

- This is carried out by setting one of the components of the tangent vector to +1 or -1 (i.e. $t_k = \pm 1$)
- This component is referred to as the continuation parameter. Now the above equation can be re-written as:

$$\text{where } e_k = (0, 0, \dots, 1, 0, 0) \quad \begin{bmatrix} F_\theta & F_V & F_\lambda \\ & e_k & \end{bmatrix} \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 1 \end{bmatrix}$$



Static Analysis

Mathematical Formulation of CPF

Predictor Step:

- Once the tangent vector has been found, the prediction of states can be done as:

$$\begin{bmatrix} \theta \\ V \\ \lambda \end{bmatrix} = \begin{bmatrix} \theta_0 \\ V_0 \\ \lambda_0 \end{bmatrix} + \sigma \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix}$$

- Where σ is the step size, chosen to make sure the power flow solution exists with the specified continuation parameter

Static Analysis

Mathematical Formulation of CPF

Predictor Step:

- If for a given step size a solution cannot be found in the corrector step, the step size is reduced and the corrector step is repeated until a successful solution is obtained
- This process is commonly known as the “step cutting” method



Static Analysis

Mathematical Formulation of CPF

Corrector Step:

- In the corrector step, the original set of equations $F(\theta, V, \lambda) = 0$ is augmented by one equation that specifies the state variable selected as the continuation parameter
$$\begin{bmatrix} F(\theta, V, \lambda) \\ x_k - \eta \end{bmatrix} = [0]$$
- x_k is the state variable selected as the continuation parameter and η is equal to the predicted value of x_k



Static Analysis

Mathematical Formulation of CPF

Corrector Step:

- This set of equations can be solved using NR (Newton Raphson) power flow method
- The introduction of additional equation specifying x_k makes the Jacobian non-singular at the critical point
- The continuation power flow can be continued up to and beyond the critical point (i.e. **lower portion of the P-V curve**)



Proximity Indicators to Static Voltage Stability Limit

- Proximity indicators or performance indices are used to predict or detect voltage collapse problems. They are very useful for power system operation staffs
- These indices could be used as online or offline tools to help operators determine how close the system is to collapsing or the instability point
- The objective of these indices is to define a scalar magnitude that can be monitored as the system parameter changes (e.g. loading level)



Proximity Indicators to Static Voltage Stability Limit

- These indices should have a “**predictable shape**” and smooth behavior so that a possible prediction can be made well in advance to avoid possible collapses by taking appropriate remedial measures
- Another aspect of these indices is that they should be computationally fast, especially in the **online system monitoring**

Proximity Indicators to Static Voltage Stability Limit

Sensitivity Factors

- Sensitivity factors are used in several utilities throughout the world to detect voltage stability problems and to devise corrective measures
- Some of the sensitivity information can be obtained from load flow Jacobian at no cost
- These indices were first used to predict voltage control problems in generator QV curves



Proximity Indicators to Static Voltage Stability Limit

Sensitivity Factors

$$VSF_i = \max_i \left\{ \frac{dV_i}{dQ_i} \right\}$$

- As generator i approaches the bottom of its QV curve, the value of the VSF of the generator becomes large
- Based on this concept, more general system wide indices have been proposed

Proximity Indicators to Static Voltage Stability Limit

Sensitivity Factors

$$SF = \left\| \frac{dz}{d\lambda} \right\|$$

- When SF becomes larger the system turns “insecure” and eventually collapses, due to all entries $dz_i/d\lambda \rightarrow \pm\infty$
- The system approaches a maximum value of the parameter λ ($\Delta\lambda \rightarrow 0$)
- As λ typically represents the load changes, the collapse point associated with the maximum value of λ is usually referred to as maximum loadability point



Proximity Indicators to Static Voltage Stability Limit

Sensitivity Factors

If only the system voltages V are monitored then VSF can be defined as

$$VSF = \left\| \frac{dV}{d\lambda} \right\|$$

Proximity Indicators to Static Voltage Stability Limit

Singular Value and Eigenvalues

- Singular value or eigenvalue of the power flow Jacobian can be monitored with the loading parameter
- At the collapse or instability point, the singular value index or eigenvalue index becomes zero
- These indices have been used for AC as well as AC-DC networks

Proximity Indicators to Static Voltage Stability Limit

Real and Reactive Power Margin

- Real or reactive power margin can be a more practical indicator to the operators
- Reactive power margin can be expressed as the first or second norm of distance between the present reactive power loading vector, Q_L^0 , and nearest extreme loading point, Q_L^*
- The worst case loading Q_L^* can be determined by minimizing $||Q_L^* - Q_L^0||$ subjected to satisfying the power balance and in addition the singularity condition of the Jacobian

Proximity Indicators to Static Voltage Stability Limit

Other Indices

- Voltage Collapse Proximity Indicators (VCPI)
- System Determinant
- Voltage Controllability Index
- Center Manifold Based Index
- P and Q angles
- Energy Functions
- Reactive Power Margin
- V/Vo Index



Voltage Stability Indicators

Type	Name	Index Calculation	Unstable condition	Stable condition
Jacobian -based VSI	Test function	$t_{cc} = e_c^T J J_{cc}^{-1} e_c $	Quadratic shape	Linear shape
	Second order index	$i = \frac{1}{i_0} \frac{\sigma_{\max}}{d\sigma_{\max}/d\lambda_{total}}$	$i = 0$	$0 < i \leq 1$
	Tangent vector	$TVI_i = \left \frac{dV_i}{d\lambda} \right ^{-1}$	$TVI_i \rightarrow 0$	$TVI_i \neq 0$
	V/V0	V/V0	$V/V0 \rightarrow 0$	$V/V0 \rightarrow 1$



Voltage Stability Indicators

System variable-based VSI	Bus	L index	$L_j = \left \frac{S_{j+}^*}{Y_{jj+} V_j^2} \right $	$L=1$	$L<1$
		VCI	$VCI_i = \left[1 + \frac{I_i \Delta V_i}{V_i \Delta I_i} \right]^\alpha$	$VCI=0$	$0 < VCI \leq 1$
	Line	SI	$SI(r) = 2V_s^2 V_r^2 - V_r^4$ $-2V_r^2(PR + QX)$ $- Z ^2(P^2 + Q^2)$	$SI<0$	$SI \geq 0$
		Lmn	$L_{mn} = \frac{4XQ_r}{[V_s \sin(\theta - \delta)]^2}$	$L_{mn} > 1$	$L_{mn} \leq 1$
	Line	LQP	$LQP = 4 \left(\frac{X}{V_i^2} \right) \left(\frac{X}{V_i^2} P_i^2 + Q_i \right)$	$LQP > 1$	$LQP \leq 1$
		FVSI	$FVSI_{sr} = \frac{4Z^2 Q_r}{V_s^2 X}$	$FVSI > 1$	$FVSI \leq 1$
		VCPI	$VCPI(1) = \frac{P_r}{P_{r(max)}}$	$VCPI > 1$	$VCPI \leq 1$



Prevention of Voltage Collapse

System Design measures

Application of reactive power-compensating devices:

- Adequate stability margin should be ensured by proper selection of compensation schemes
- Some well known reactive power compensation devices are shunt, series capacitors, SVC, and STATCOM



Prevention of Voltage Collapse

Control of network voltage and generator reactive output

- Load compensation of a generator AVR regulates voltage on the high-tension side of or partway through the step-up transformer
- In many situations, this has a beneficial effect on voltage stability by moving the point of constant voltage closer to the loads
- Alternatively, the secondary outer loop of generator excitation may be used to regulate the network side voltage



Prevention of Voltage Collapse

Coordination of Protection/Controls

- Adequate coordination should be ensured based on dynamic simulation studies
- Tripping of equipment to prevent an overload condition should be the last resort
- Wherever possible, adequate control measures (automatic or manual) should be provided for relieving the overload condition before isolating the equipment from the system



Prevention of Voltage Collapse

Control of transformer tap changers

- Tap changers can be controlled, either locally or centrally, to reduce the risk of voltage collapse
- When tap changing is detrimental, a simple method is to block the tap changer when the source side voltage sags then unblock it when the voltage recovers
- There is a potential application of improved Under Load Tap Changes (ULTC) control strategies



Prevention of Voltage Collapse

Under-voltage load shedding

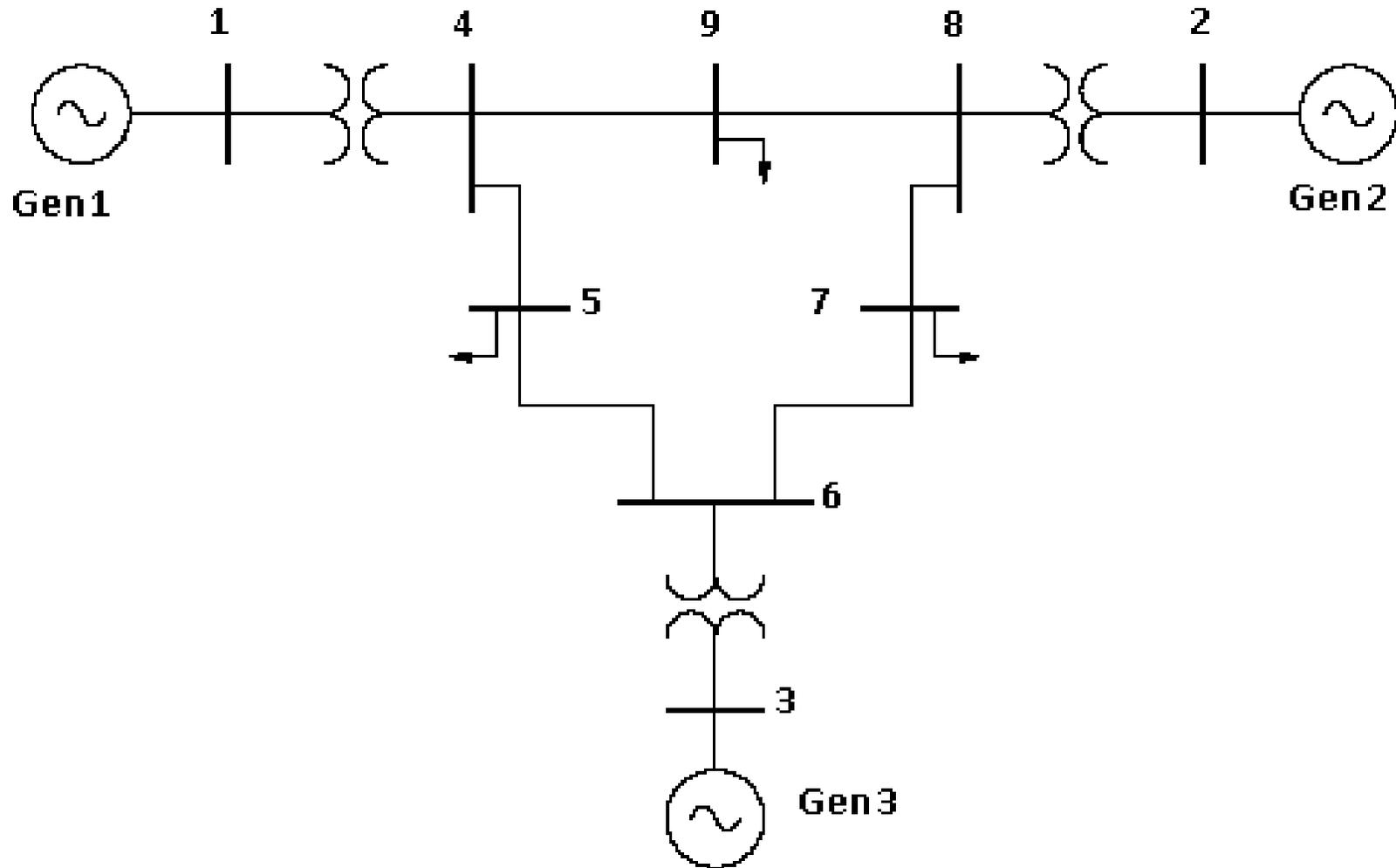
To manage with unplanned or extreme situations, it may be necessary to use under-voltage load-shedding schemes

System Operating Measures

1. Stability margin
2. Spinning reserve
3. Operator's action



Assignment 3

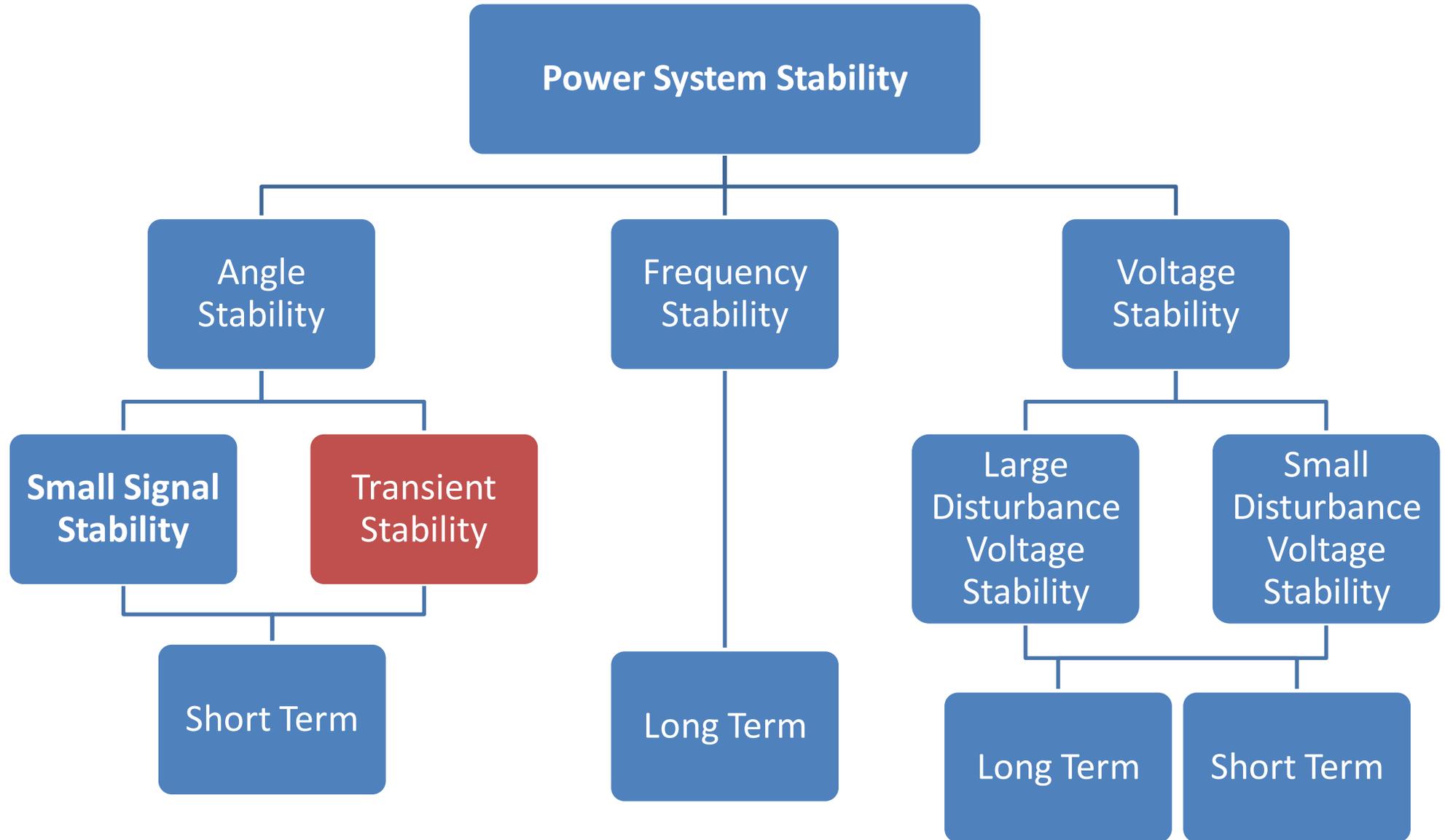




1. Increase the loading level of the system step by step, run a load flow and see whether you are getting convergence or not then report the maximum loading level. For loading “direction,” assume all the loads increase by the same ratio, and only the generator at bus one is allowed to dispatch the required additional real power
2. Plot the voltage against the total load at all the load buses; find out which load bus has the highest dV/dP_{total} at the point close to divergence
3. Run the continuation power flow for the same generation and loading direction in the direct method then obtain the static voltage stability margin of the system



Transient Stability





Transient Stability Definition

- Transient stability is the ability of the power system to maintain synchronism when subjected to severe transient disturbance, such as fault on transmission facilities, loss of generation, or loss of a large load
- The system response to such disturbances involves large excursions of generator rotor angles, power flows, bus voltages, and other system variables

Examples:

1. Sudden Fault or short circuit followed by opening of circuit breakers
2. Sudden application or rejection of large or sudden tripping of generators
3. Switching Operation



Explanation

- Generators run at constant synchronous speed with rotor acceleration of zero
- A disturbance in the electrical network causes real power output of the generators to change, creating an imbalance with the mechanical power input
- Mechanical power input from the turbine cannot change instantaneously
- This causes rotor to accelerate and decelerate



Assumptions in Transient Stability Studies

- Each synchronous machine is represented by a constant voltage source behind the direct axis transient reactance
- The governor's action are neglected and the input powers are assumed to remain constant.
- Using the prefault bus voltages, all loads are converted to equivalent admittances to the ground and are assumed to remain constant
- Damping or asynchronous powers are ignored
- Mechanical rotor angle of each machine coincides with the angle of the voltage behind the machine reactance
- Machines belonging to the same station swing together and are represented by one equivalent machine



Transient Stability Analysis

For transient stability analysis we need to consider three systems...

1. **Prefault** – Before the fault occurs, the system is assumed to be at an equilibrium point
2. **Faulted** – The fault changes the system equations, moving the system away from its equilibrium point
3. **Postfault** – After the fault is cleared, the system hopefully returns to a new operating point



Transient Stability Solution Methods

Numerical Integration

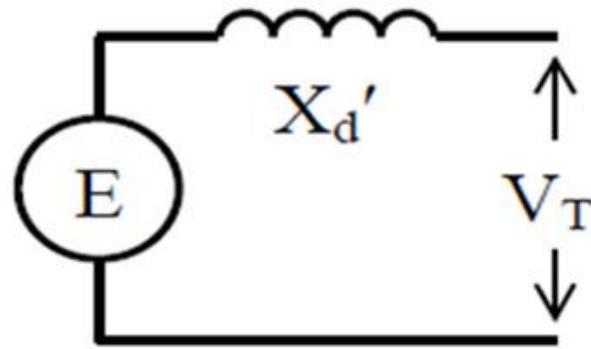
This is by far the most common technique, particularly for large systems. During the fault and after the fault, the power system's differential equations are solved using numerical methods

Direct or Energy Methods

- For two bus system, use the equal area criteria
- Mostly used to provide intuitive insight on the transient stability problem



Synchronous Machine Model

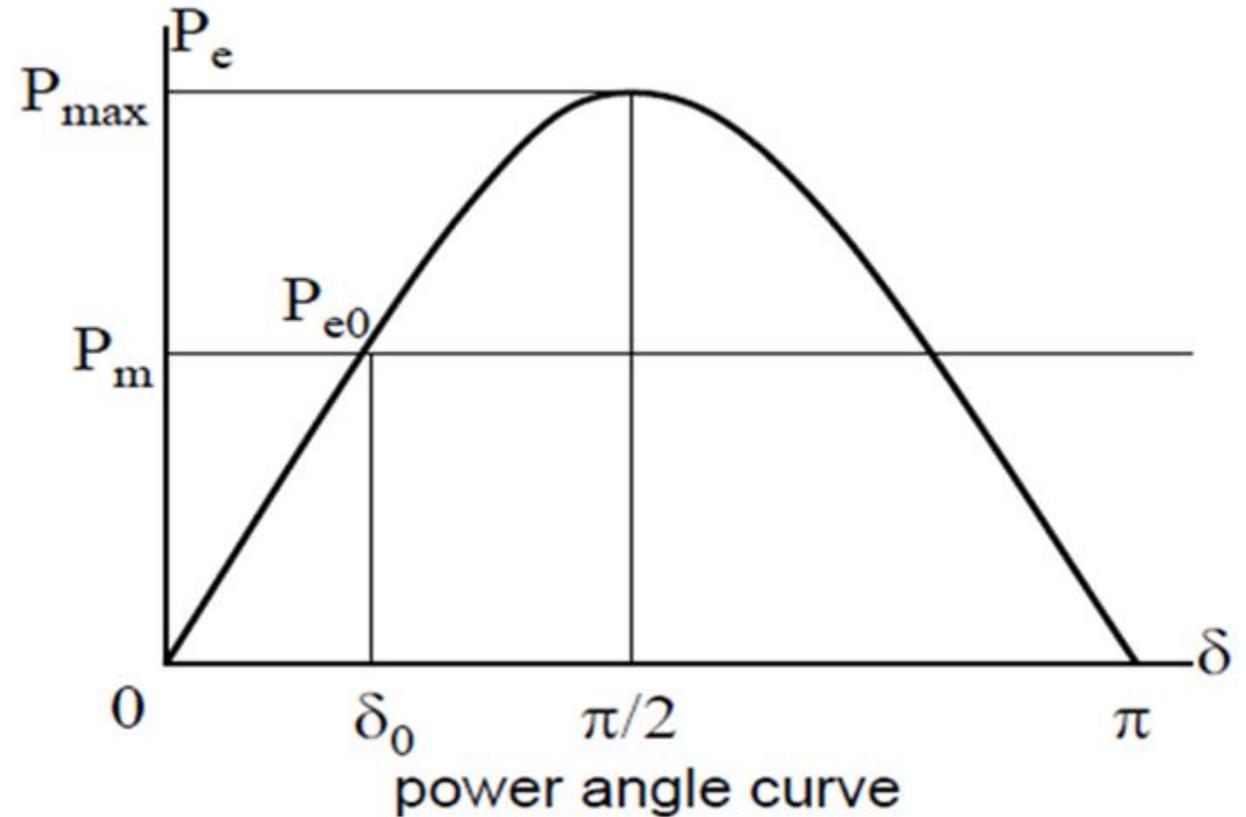


Round Rotor Machine Model

$$E' = |E'| \angle \delta$$

$$V_G = |V_G| \angle 0^\circ$$

$$B = 1/X'_d$$



$$P_e = |E'| |V_G| |B| \cos(\delta - 90^\circ) = \frac{|E'| |V_G|}{X'_d} \sin \delta = P_{max} \sin \delta$$



The Swing Equation

$$\frac{H}{\pi f_0} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

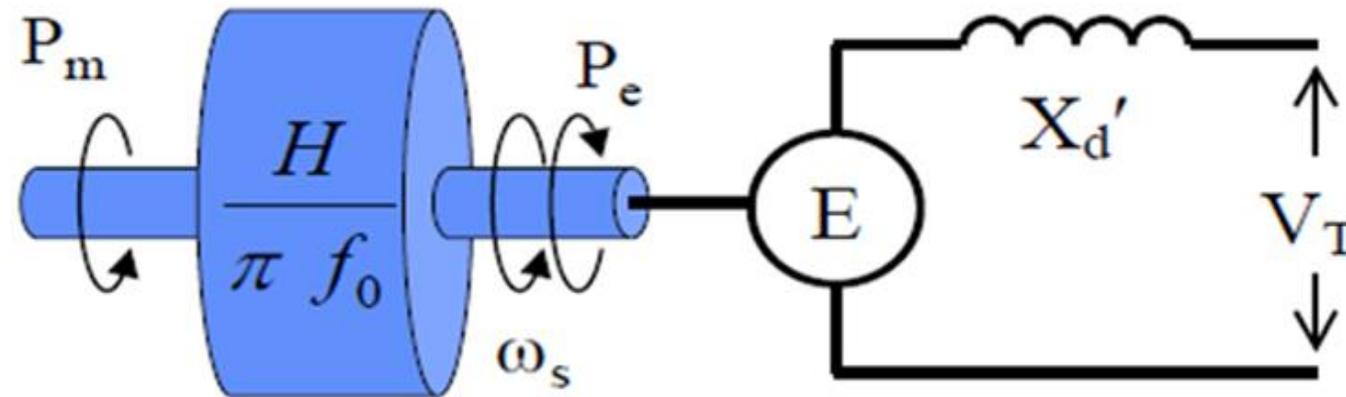
Dynamic Generator Model

$$P_e = P_{\max} \sin \delta$$

Synchronous Machine Model

$$\frac{H}{\pi f_0} \frac{d^2 \delta}{dt^2} = P_m - P_{\max} \sin \delta$$

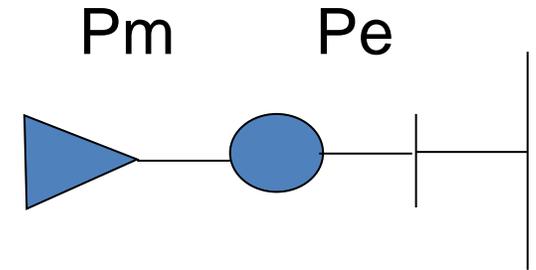
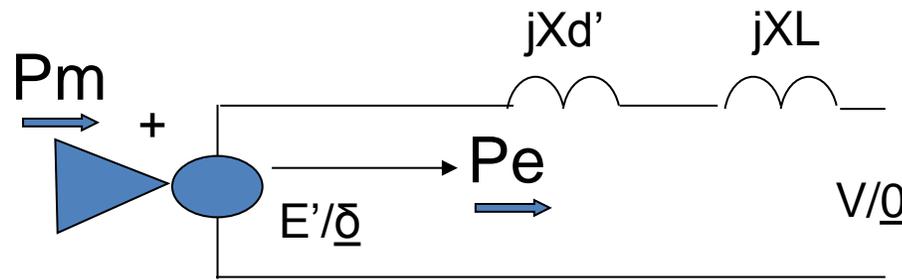
Forming the Swing Equation





First Swing Stability

A generator connected to an infinite bus through a line, initially $P_m = P_e$



Fixed (Infinite Bus)

Stability is governed by the Swing Equation

$$d^2\delta/dt^2 = (\pi f/H) (P_m - P_e)$$

$$d\delta/dt = \omega - \omega_{syn}$$

Swing Equation

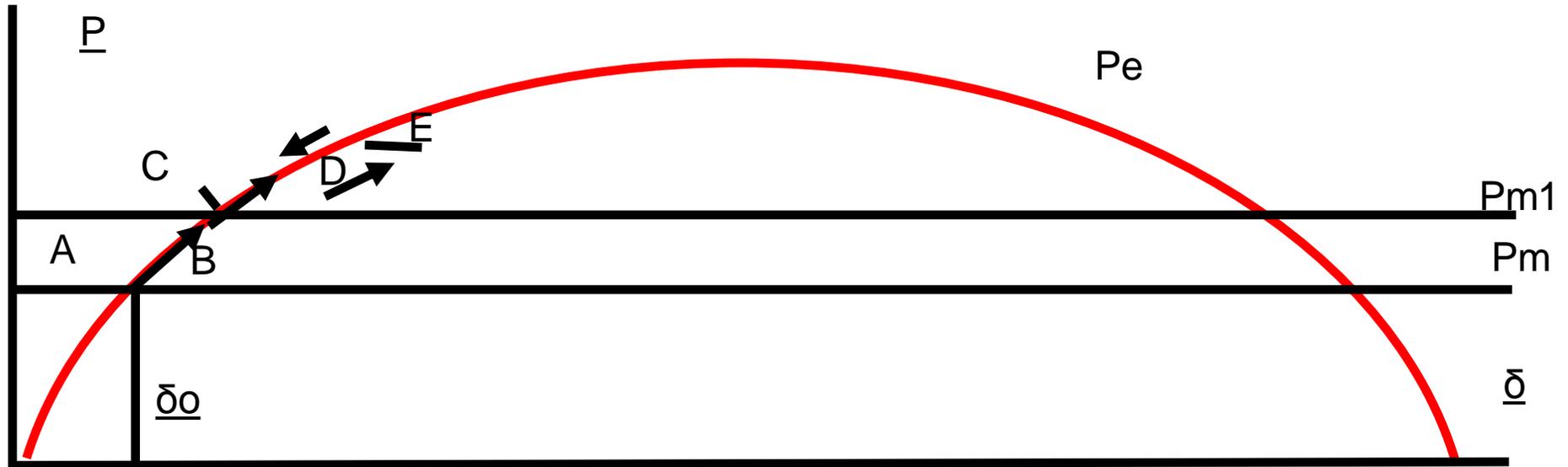
$$P_e = E' V \sin(\delta) / (X + X_L)$$

Power Angle
Equation



Equal Area Criterion

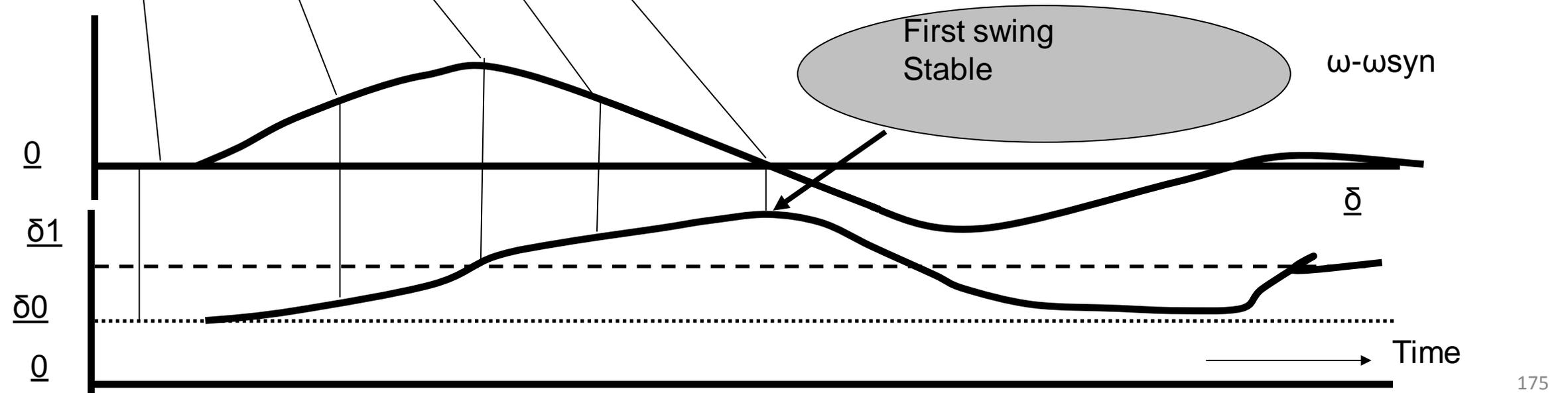
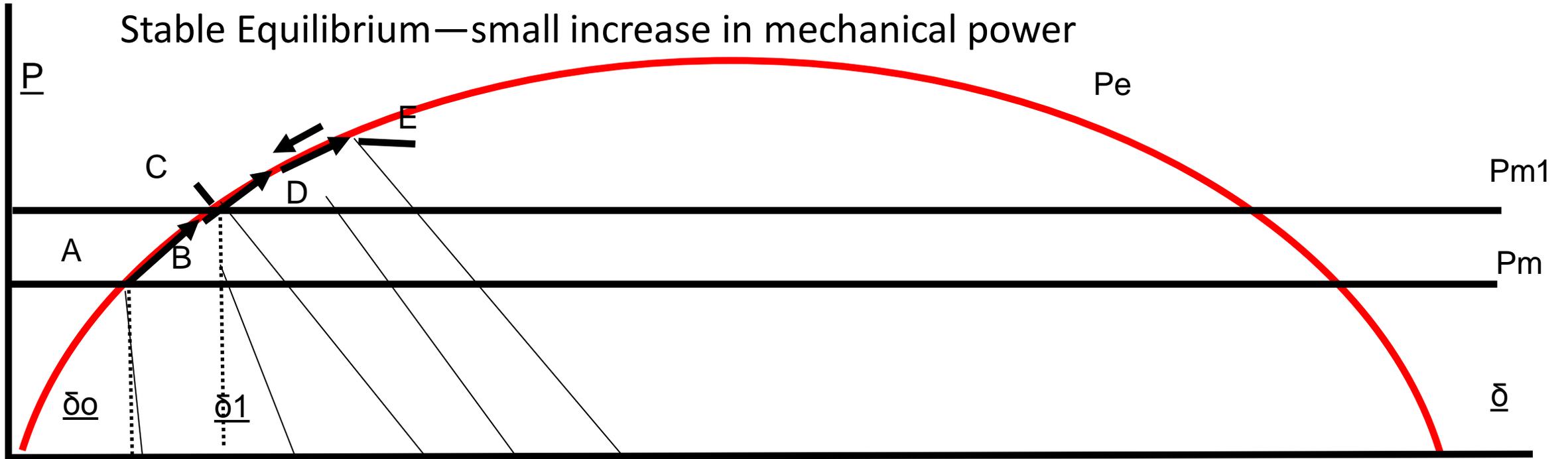
Stable Equilibrium—small increase in mechanical power



At D, ω is decreasing but > 0
 δ increases further say to point E
 By now suppose ω is back to zero and decreasing
 Thus ω becomes < 0 as the generator continues to slow
 Since $\omega < 0$ δ decreases towards B First swing stable!

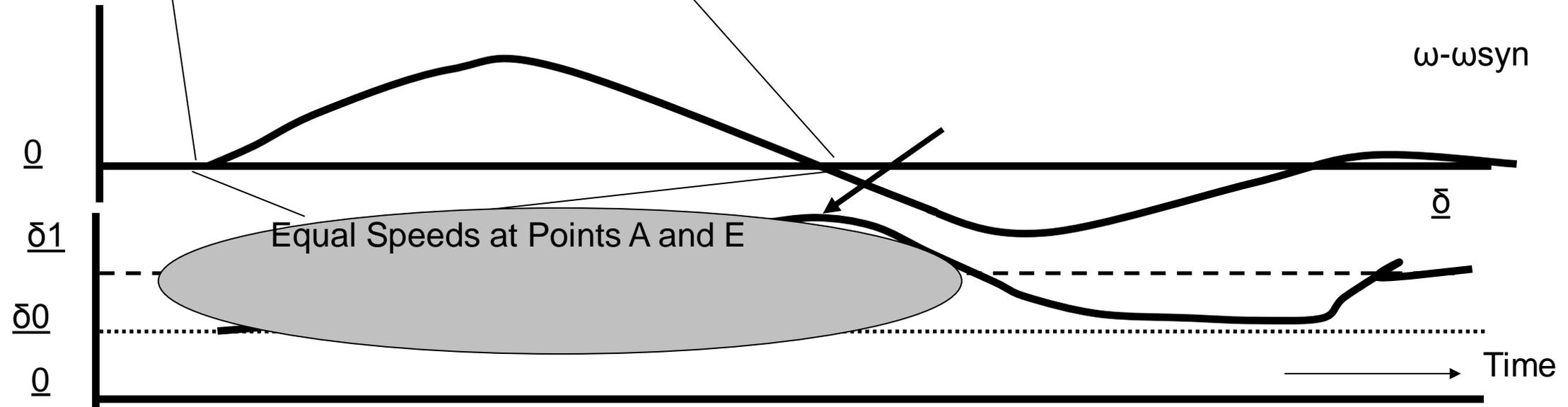
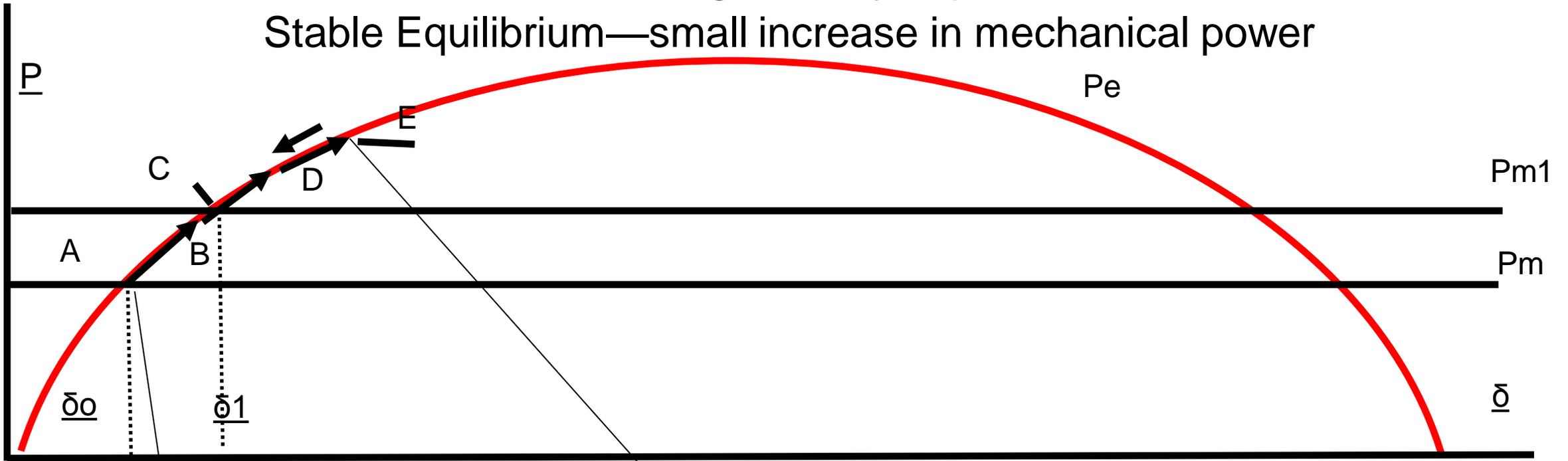
First Swing Stability-Equal Area Criterion

Stable Equilibrium—small increase in mechanical power

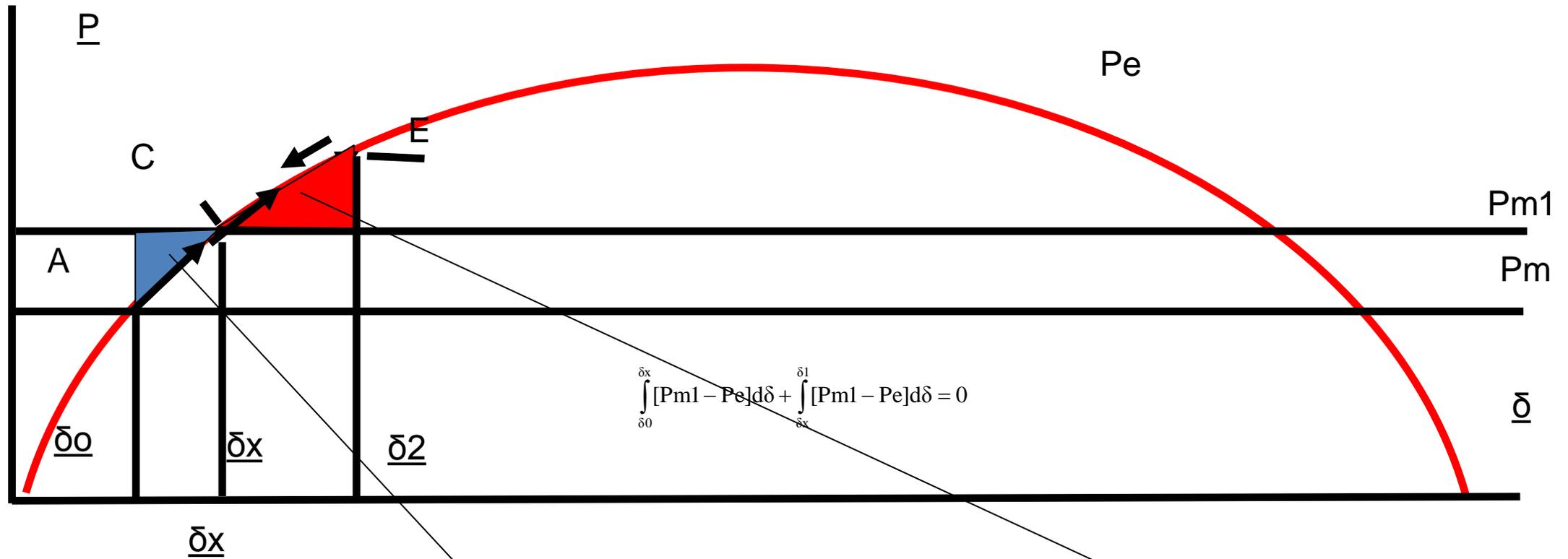


First Swing Stability-Equal Area Criterion

Stable Equilibrium—small increase in mechanical power



First Swing Stability-Equal Area Criterion Basic Principle





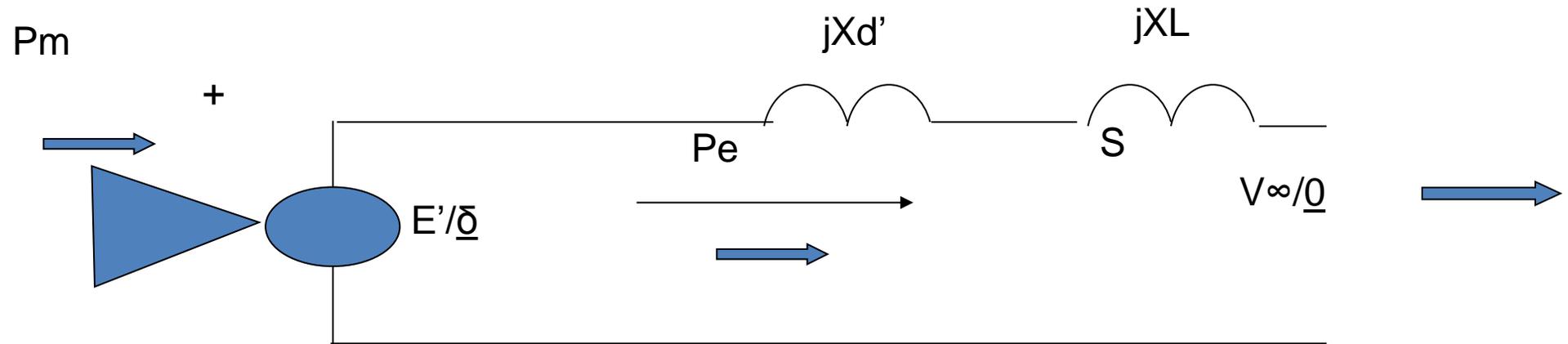
Problems

1. Sudden Change in Mechanical Power Input
2. Sudden Change in Load
3. Application to Three Phase Fault

First Swing Stability-Equal Area Criterion

Example 1

Stability under small change in mechanical power

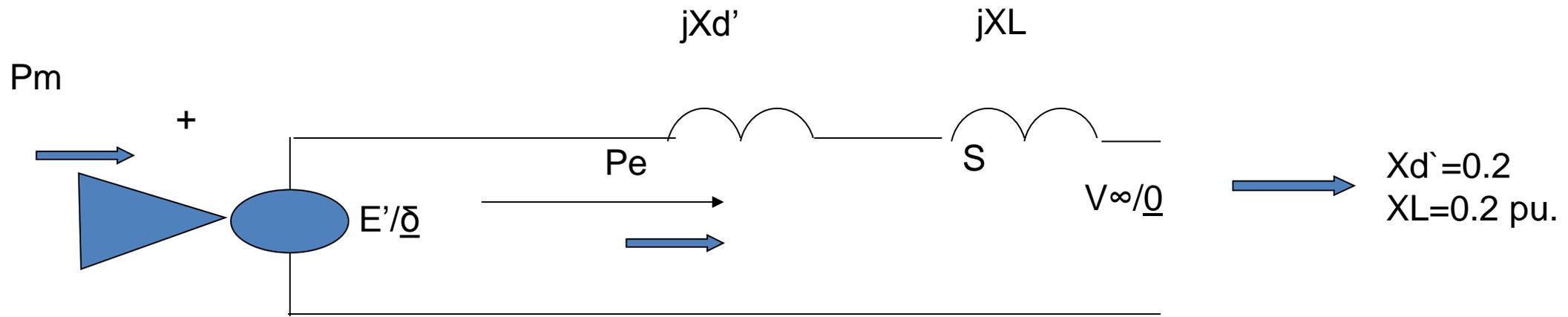


A 10 MVA, 0.8 pf lagging, 4160 V, 60Hz, three-phase generator supplies 50% rated power at .8 pf lagging to a 4160 V infinite bus. Determine if the generator is first-swing stable if the prime mover power is increased by 10%

Equal Area Criterion

Example 1

Stability under small change in mechanical power



$X_{d'} = 0.2$
 $X_L = 0.2$ pu.

1. Initial

$$S := 0.5 \cdot e^{j \cdot \arccos(0.8)} \quad S = 0.4 + 0.3i \text{ pu} \quad V_\infty := 1$$

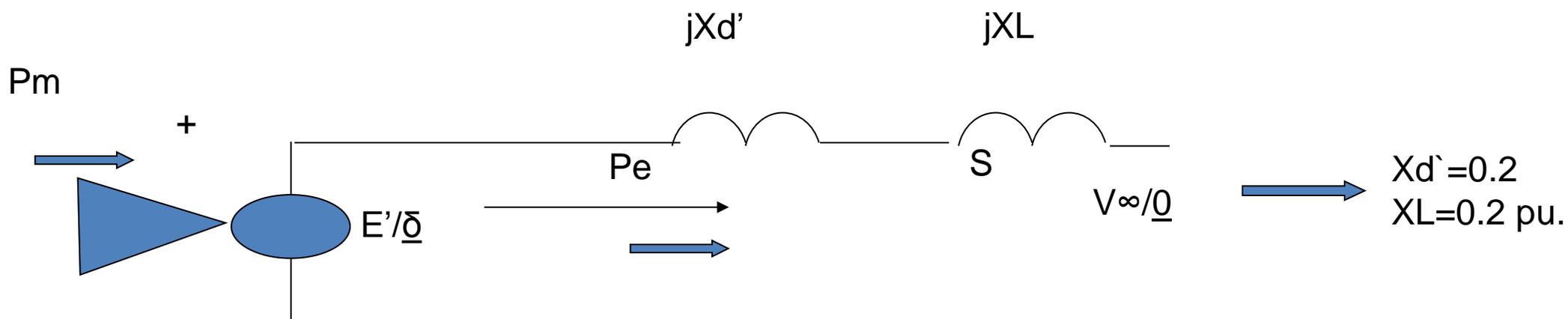
$$I := \overline{\left(\frac{S}{V_\infty} \right)} \quad I = 0.4 - 0.3i$$

$$E' := V_\infty + j \cdot (X_{d'} + X_L) \cdot I \quad |E'| = 1.131 \text{ pu} \quad \arg(E') = 8.13 \text{ deg}$$

First Swing Stability-Equal Area Criterion

Example 1

Stability under small change in mechanical power



$$P_{e0} := \operatorname{Re}(S) \quad P_{e0} = 0.4 \quad \text{pu} \quad \text{initial power}$$

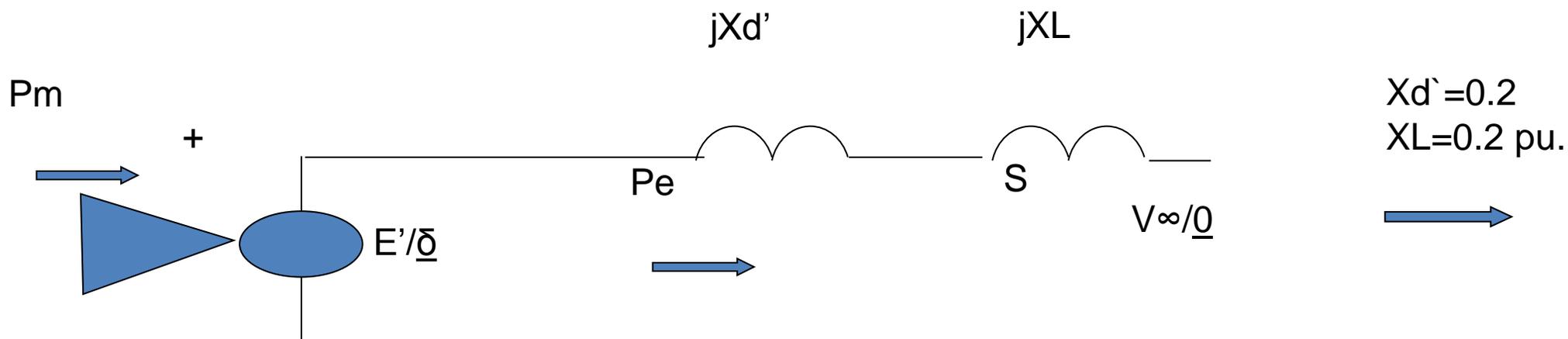
$$P_m := P_{e0} \quad P_m = 0.4 \quad \text{pu}$$

$$\delta_0 := 8.13 \text{deg}$$

First Swing Stability-Equal Area Criterion

Example 1

Stability under small change in mechanical power



2. Power angle equation before and after change in P_m

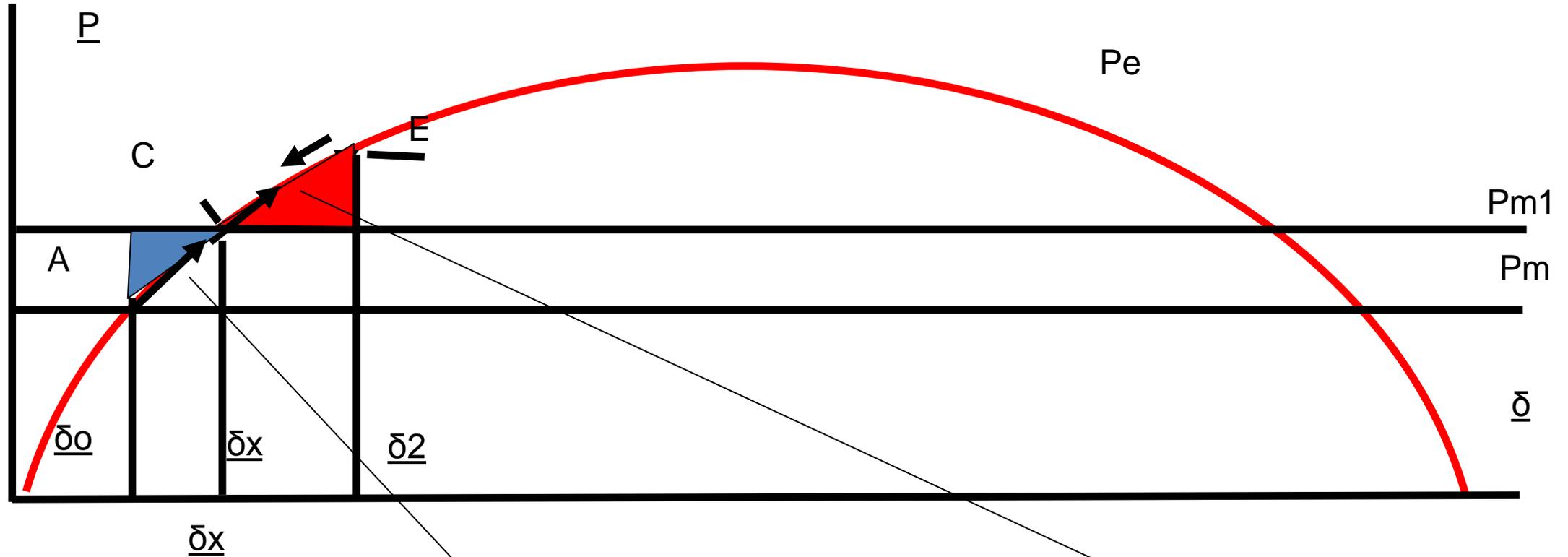
$$P_{\max} = 2.828 \quad P_e(\delta) := P_{\max} \cdot \sin(\delta)$$

$$P_e = P_{\max} \sin(\delta)$$

$$P_{\max} = E'V_\infty / (X_{d'} + X_L)$$

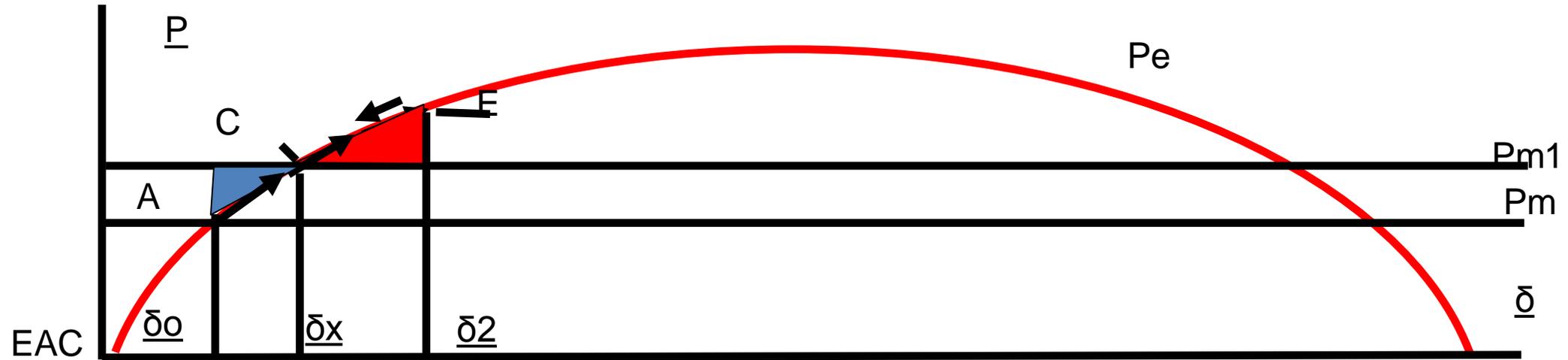
First Swing Stability-Equal Area Criterion

Small change in mechanical power



$$\int_{\delta_0}^{\delta_1} [P_{m1} - P_e(\delta)] d\delta + \int_{\delta_0}^{\delta_x} [P_m - P_e(\delta)] d\delta = 0$$

Equal Area Criterion-Small Change in Mechanical Power



Remember

Guess

$$\delta_2 := 10 \text{deg}$$

δ

Given

$$\left[\int_{\delta_0}^{\delta_2} (P_{m1} - P_e(\delta)) d\delta \right] = 0$$

$$\text{Find}(\delta_2) = 9.77 \text{ deg}$$

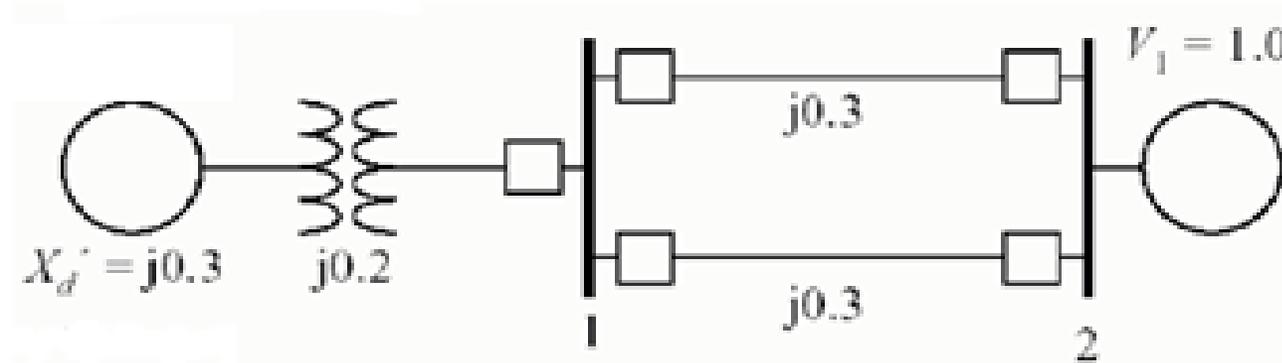
$$\delta_0 := 8.13 \text{ deg}$$

$$\delta_x := \text{asin}\left(\frac{P_{m1}}{P_{\text{max}}}\right)$$

$$\delta_x = 8.949 \text{ deg}$$



Example 2: Sudden Change in Mechanical Power Input



60Hz, $H = 9.94 \text{ MJ/MVA}$

Generator delivering real power of 0.6 pu, 0.8 power factor lagging to infinite bus. Determine the maximum power input that can be applied with out loss of synchronism.

Refer : Hadi. Saadat "Power System Analysis"



Example 2: Sudden Change in Mechanical Power Input

$$X = 0.3 + 0.2 + 0.3/2 = 0.65 \text{ pu}$$

$$I = S^* / V^*$$

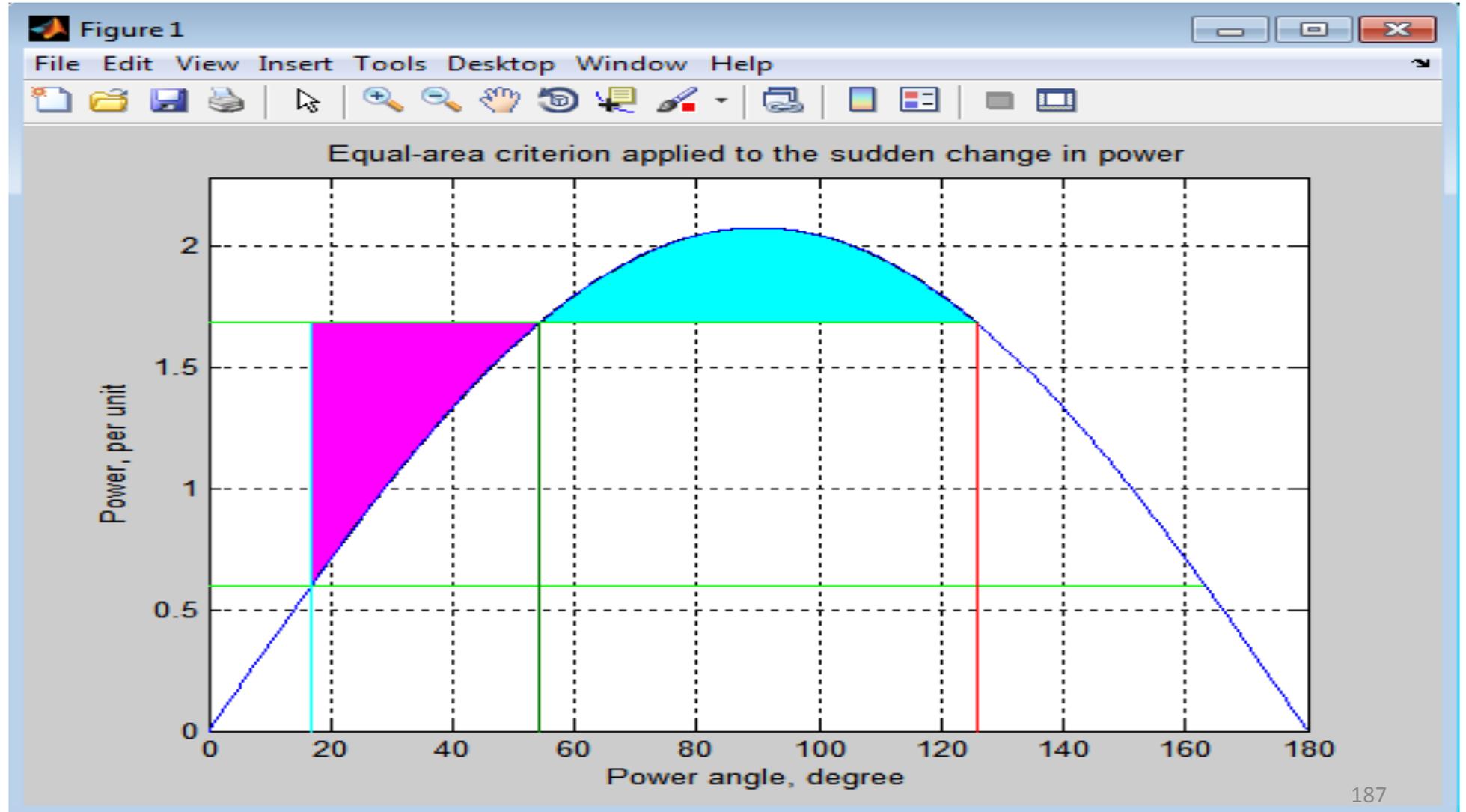
$$E' = V + jXI = 1.35 \text{ pu}$$

$$P_0 = 0.6; E = 1.35; V = 1.0, X = 0.65$$

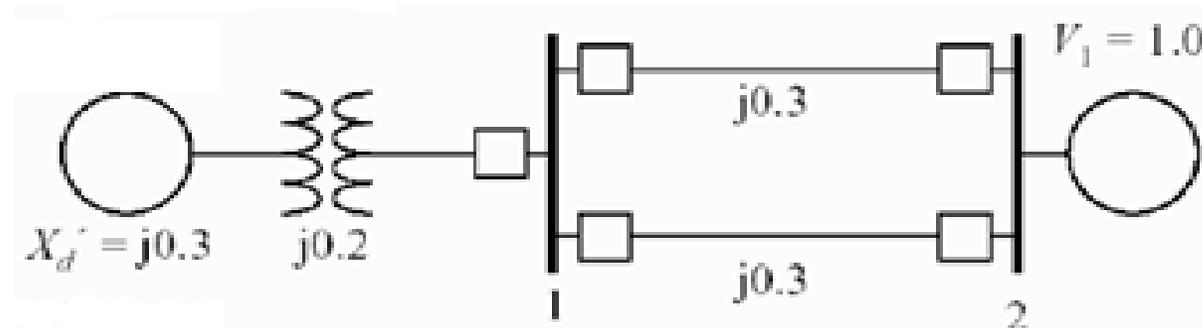
$$\text{eacpower}(P_0, E, V, X)$$

Refer : Hadi. Saadat "Power System Analysis"

Example 2: Sudden Change in Mechanical Power Input



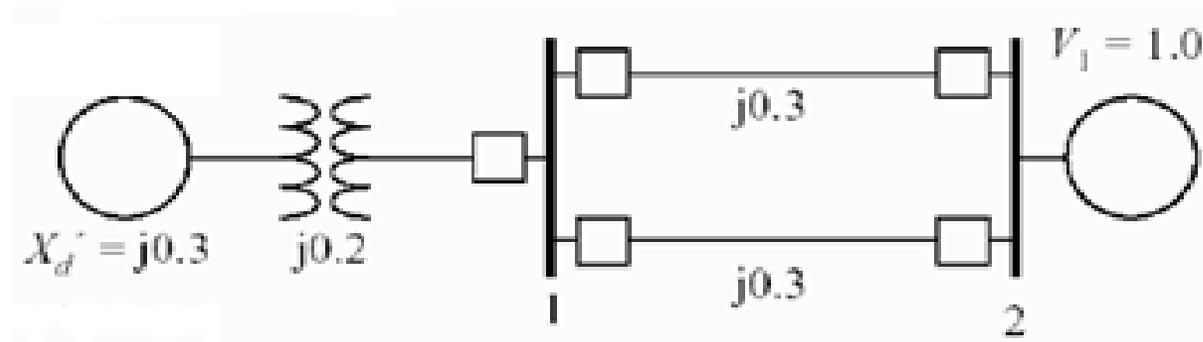
3 Phase Fault – Case I Fault at the Sending End Bus



When the fault is at the sending end bus, no power is transmitted to the infinite bus. $P_e = 0$; and the power angle curve corresponds to horizontal axis

Refer : Hadi. Saadat "Power System Analysis"

Example 4: 3 Phase Fault – Case II Fault in the transmission line



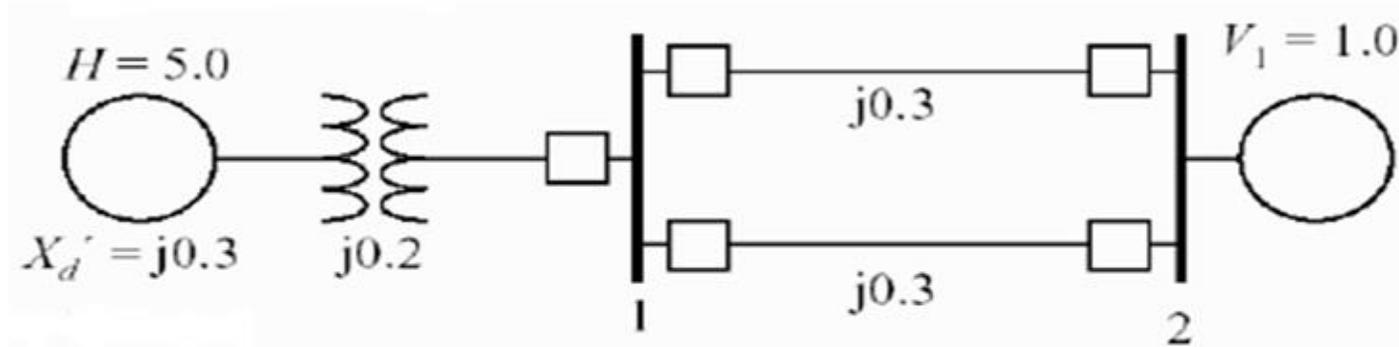
When the fault is in the transmission line,
there will be three power angle curves

Prefault , Fault, and Post-fault

Refer : Hadi. Saadat “Power System Analysis”

Temporary 3 Phase fault at the Sending End Bus

Three-phase fault occurs at the middle of one line and then is cleared by isolating the faulted circuit simultaneously at both ends. Determine the critical-fault clearing time by time-domain simulation



Initial conditions are given $\delta_0 = 26.388^\circ = 0.46055$, $\Delta\omega_0 = 0$ and $P_m = 0.8$

By calculation

during fault

$$P_e = 0.65 \sin \delta$$

post fault

$$P_e = 1.4625 \sin \delta$$

Temporary 3 Phase fault at the Sending End Bus

$P_m = 0.8; E = 1.17; V = 1.0;$

$X_1 = 0.65; X_2 = \infty; X_3 = 0.65$

Eacfault (P_m, E, V, X_1, X_2, X_3)

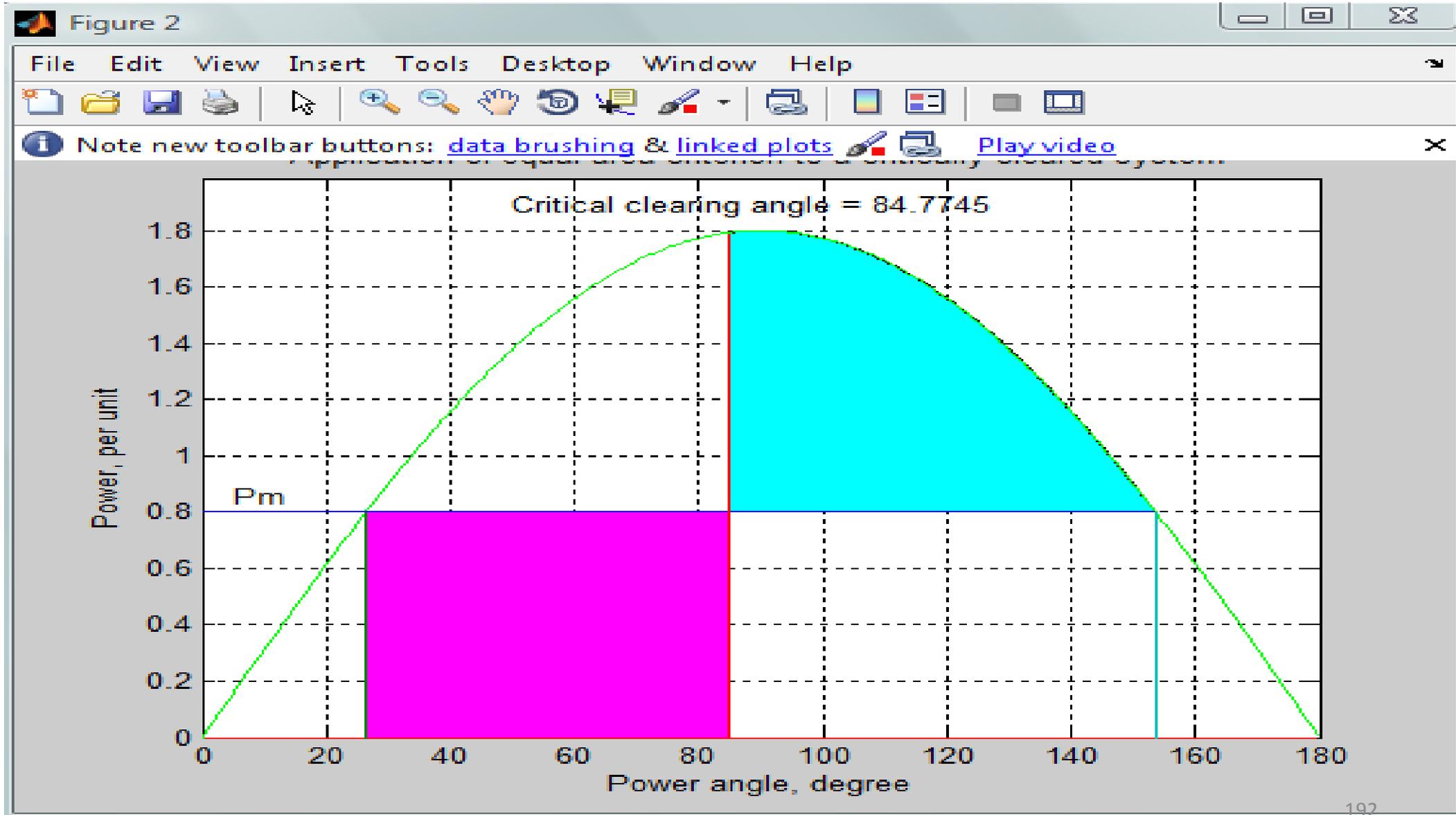
Only two Power Angle Curve

Since Prefault and Post Fault curve is the same

Refer : Hadi. Saadat “Power System Analysis”



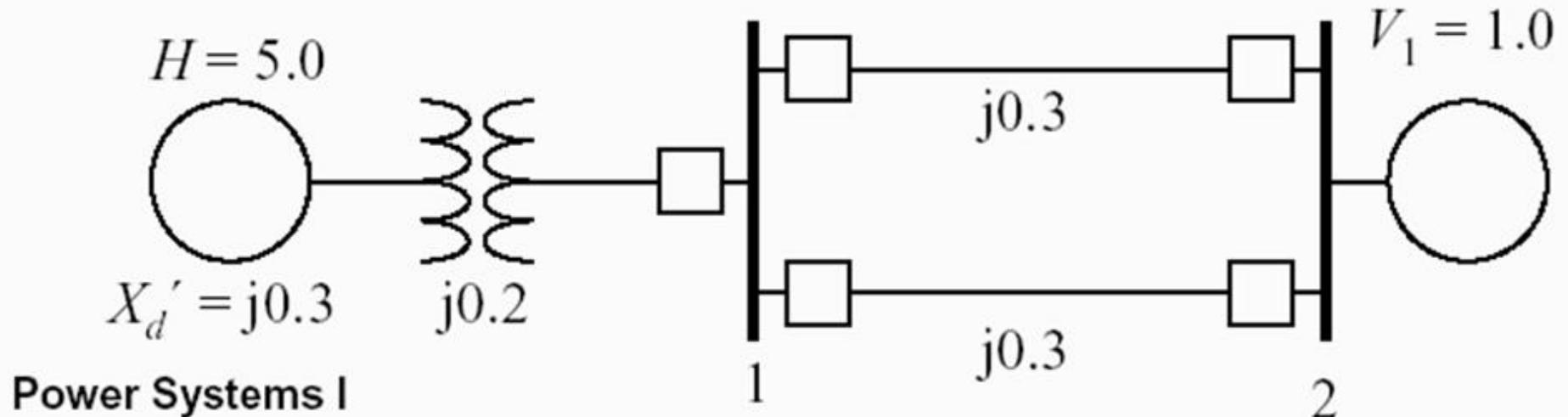
Critical Clearing





Example

- Consider the following system
 - ◆ a three-phase fault at the middle of one line is cleared by isolating the faulted circuit simultaneously at both ends.
 - ◆ The fault is cleared in 0.3 seconds, perform several steps of the numerical solution of the swing equation using the modified Euler method with a step size of $\Delta t = 0.01$ seconds.
 - ◆ graph the swing equation for clearing times of 0.3 s, 0.4 s, and 0.5 s.





Example

$$H = 5 \text{ (Machine Parameters)}$$

$$P_m = 0.8$$

$$E = V + jX_1 I = 1.0 + (j0.65) \frac{0.8 - j0.074}{1.0} = 1.17 \angle 26^\circ$$

$$P_m = P_{\max} \sin \delta = \frac{(1.17)(1.0)}{0.65} \sin \delta = 1.8 \sin \delta \text{ (Pre - fault equation)}$$

$$\delta_0 = 26.4^\circ = 0.4606 \text{ rad (Initial Conditions)}$$

$$P_m = 0.8, \Delta\omega = 0 \text{ rad / s}$$

$$P_{\max}^{[fault]} = \frac{(1.17)(1.0)}{1.8} \sin \delta = 0.65 \sin \delta \text{ (Fault Parameters)}$$

$$P_a = P_m - P_{\max}^{[fault]} = 0.8 - 0.65 \sin \delta$$



Example

$$\left. \frac{d\delta}{dt} \right|_{\Delta\omega_0} = 0 \text{ rad} / s$$

$$\left. \frac{d\Delta\omega}{dt} \right|_{\delta_0} = \frac{\pi \cdot 60}{5} (0.8 - 0.65 \sin(0.4606 \text{ rad})) = 19.27 \text{ rad} / s^2$$

$$\delta_1^p = 0.4606 + (0)(0.01) = 0.4606 \text{ rad}$$

$$\Delta\omega_1^p = 0 + (19.27)(0.01) = 0.1927 \text{ rad} / s$$

$$\left. \frac{d\delta}{dt} \right|_{\Delta\omega_1^p} = 0.1927 \text{ rad} / s$$

$$\left. \frac{d\Delta\omega}{dt} \right|_{\delta_1^p} = \frac{\pi \cdot 60}{5} (0.8 - 0.65 \sin(0.4606 \text{ rad})) = 19.27 \text{ rad} / s^2$$



Example

$$\delta_1^c = 0.4606 + 0.5(0 + 0.1927)(0.01) = 0.4615 \text{ rad}$$

$$\Delta\omega_1^c = 0 + 0.5(19.27 + 19.27)(0.01) = 0.1927 \text{ rad / s}$$

End of the first step. Next step :

$$\left. \frac{d\delta}{dt} \right|_{\Delta\omega_1} = 0.1927 \text{ rad / s}$$

$$\left. \frac{d\Delta\omega}{dt} \right|_{\delta_1^p} = \frac{\pi \cdot 60}{5} (0.8 - 0.65 \sin(0.4615 \text{ rad})) = 19.25 \text{ rad / s}^2$$



Three Phase Fault Middle of one of the Lines

$P_m = 0.8; E = 1.17; V = 1.0;$
 $X_1 = 0.65; X_2 = 1.8; X_3 = 0.8;$
`eacfault(Pm,E,V,X1,X2,X3)`

Refer : Hadi. Saadat “Power System Analysis”

Generator Specifications

- $H = 5 \text{ MJ/MVA}$; $X_d' = 0.3 \text{ pu}$; Direct Axis Transient Reactance
- $P_e = 0.8 \text{ pu}$; Real power delivered
- $Q = 0.074$; Reactive power delivered

Analysis

- Prefault
- current flowing into the infinite bus
- $I = S^*/V^* = 0.8 - j0.0074/1.0 = 0.8 - j0.074 \text{ pu}$
- Transfer reactance between internal voltage and infinite bus before fault
- $X_1 = 0.3 + 0.2 + 0.3 // 0.3 = 0.65 \text{ pu}$
- $E' = V + jX_1 I = 1 + (j0.65)(0.8 - j0.074) = 1.17 / 26.387^\circ$
- $P_{\max} = E' V / X_s = 1.17 * 1.0 / 0.65 = 1.8$
- The generator is delivering real power
- $P_e = 0.8 \text{ pu}$; $0.8 = 1.8 \sin \delta$; $\delta = 26.388^\circ = 0.46055 \text{ rad}$.

Refer : Hadi. Saadat "Power System Analysis"



Fault

$X_2 = 1.8\text{pu}$ (By applying Star Delta Transformation)

$P_a = 0.8 - 0.65 \sin \delta$

Post Fault Condition

$X_3 = 0.8\text{pu}$

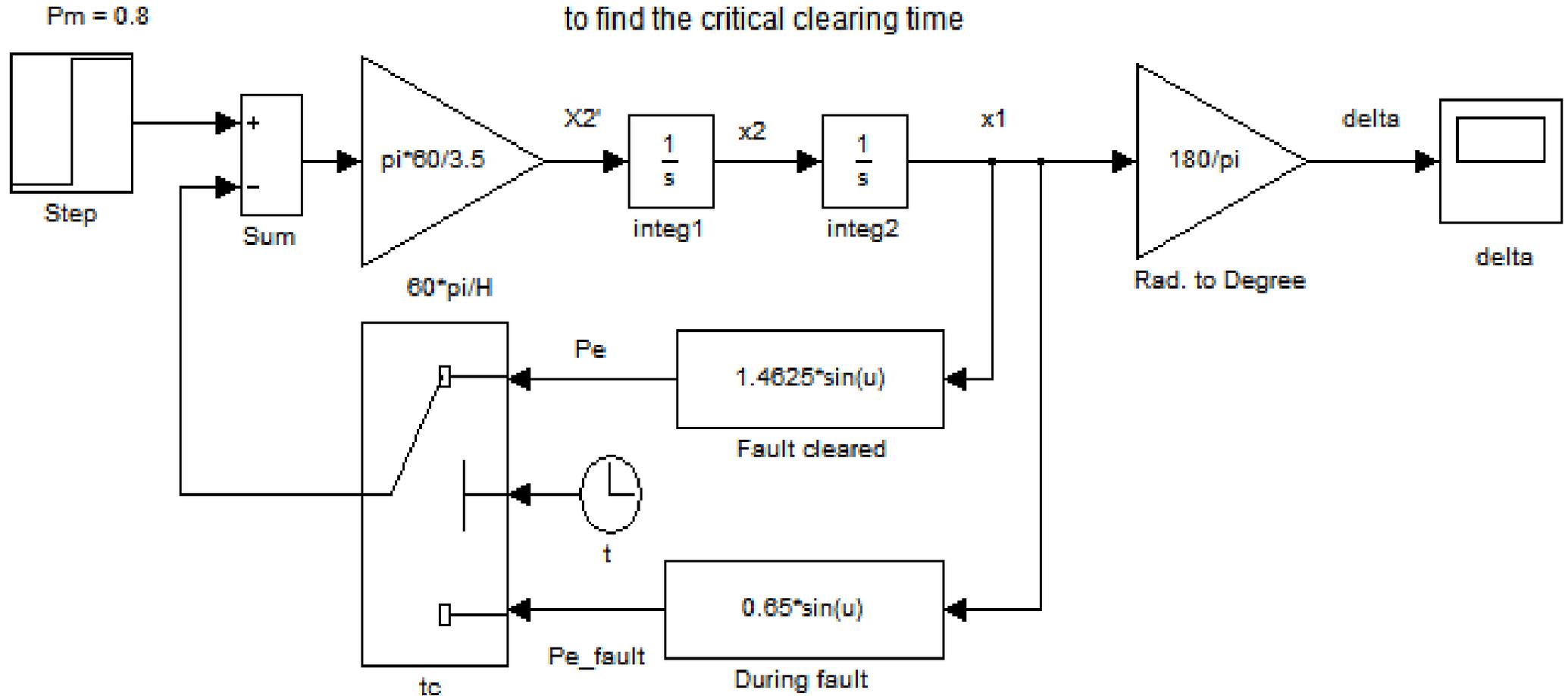
$P_a = 0.8 - 1.4625 \sin \delta$

Refer : Hadi. Saadat "Power System Analysis"



Matlab Model

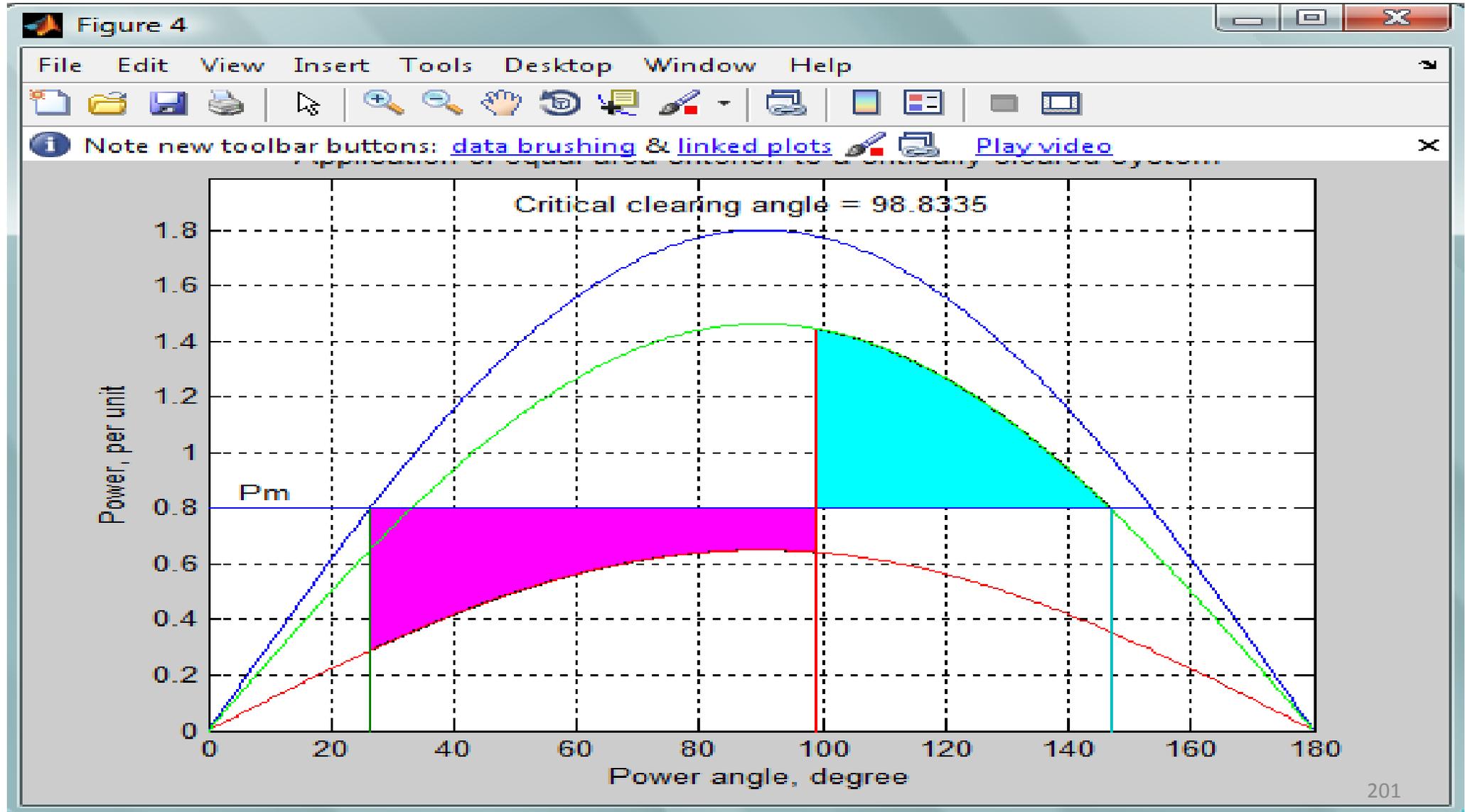
Change the clearing time of fault
to find the critical clearing time



To change the clearing time of fault open the switch
dialog box and change the Threshold setting.

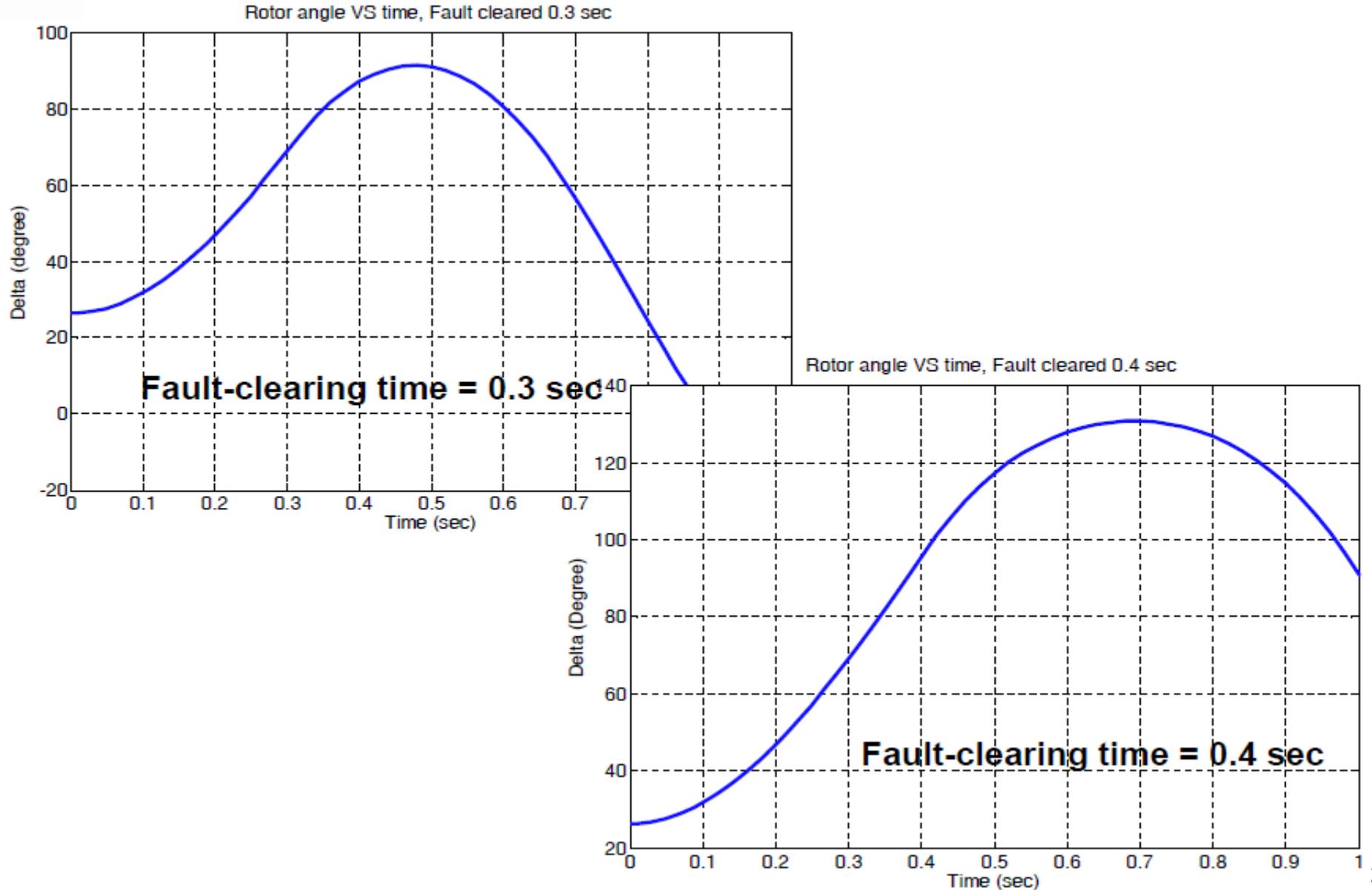


Critical Clearing



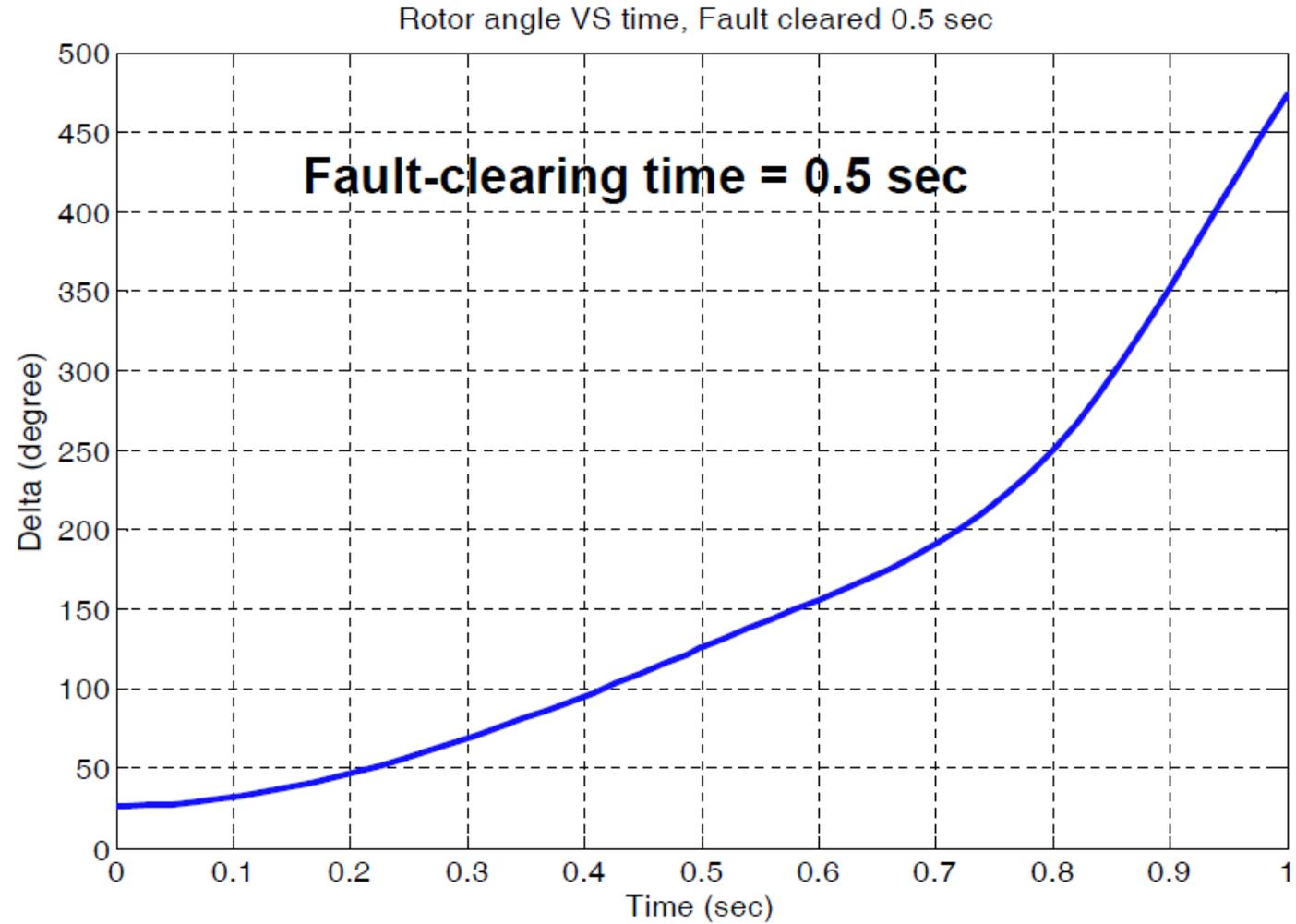


Fault Clearing Time set as 0.3 s



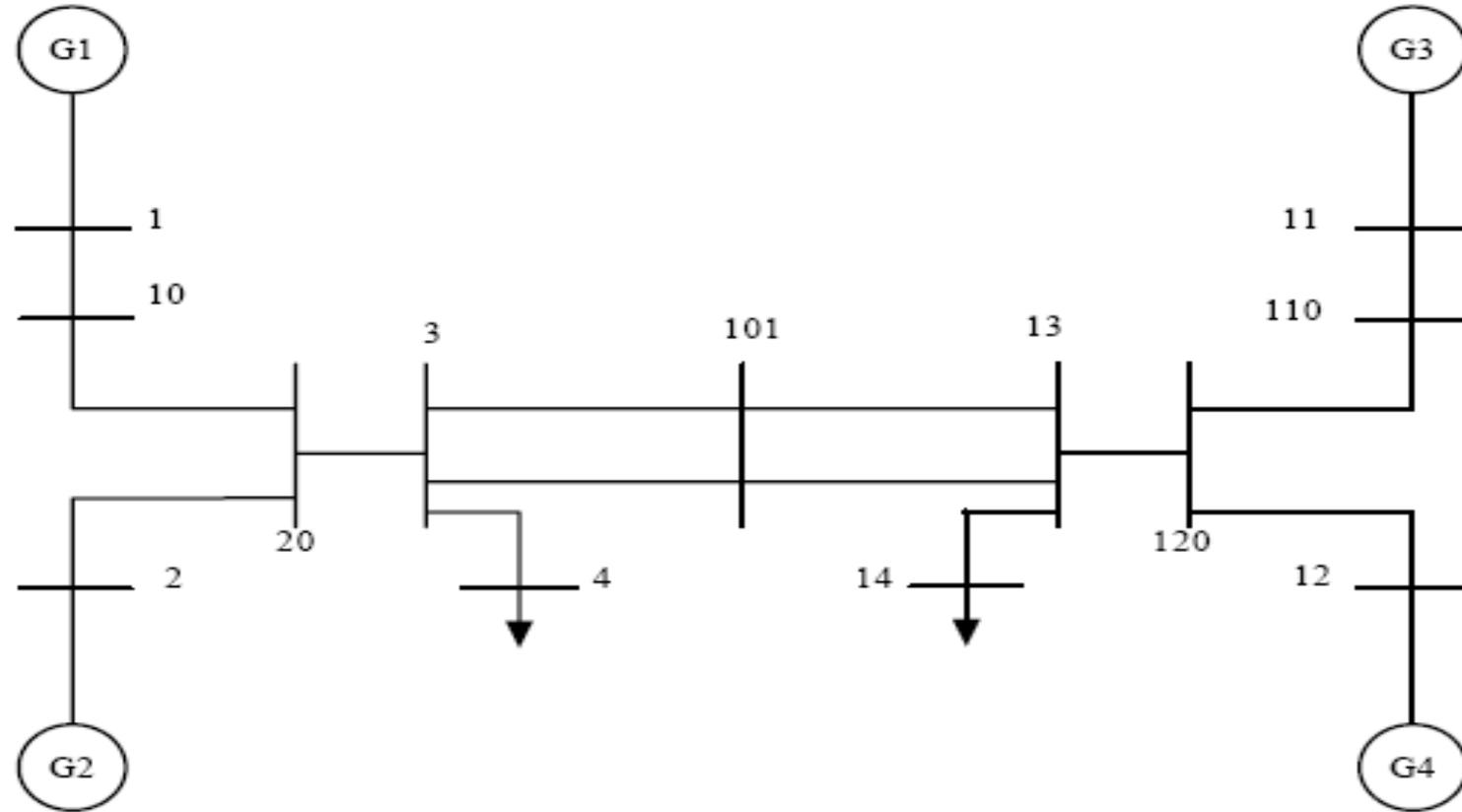


Fault Clearing Time set as 0.5 s





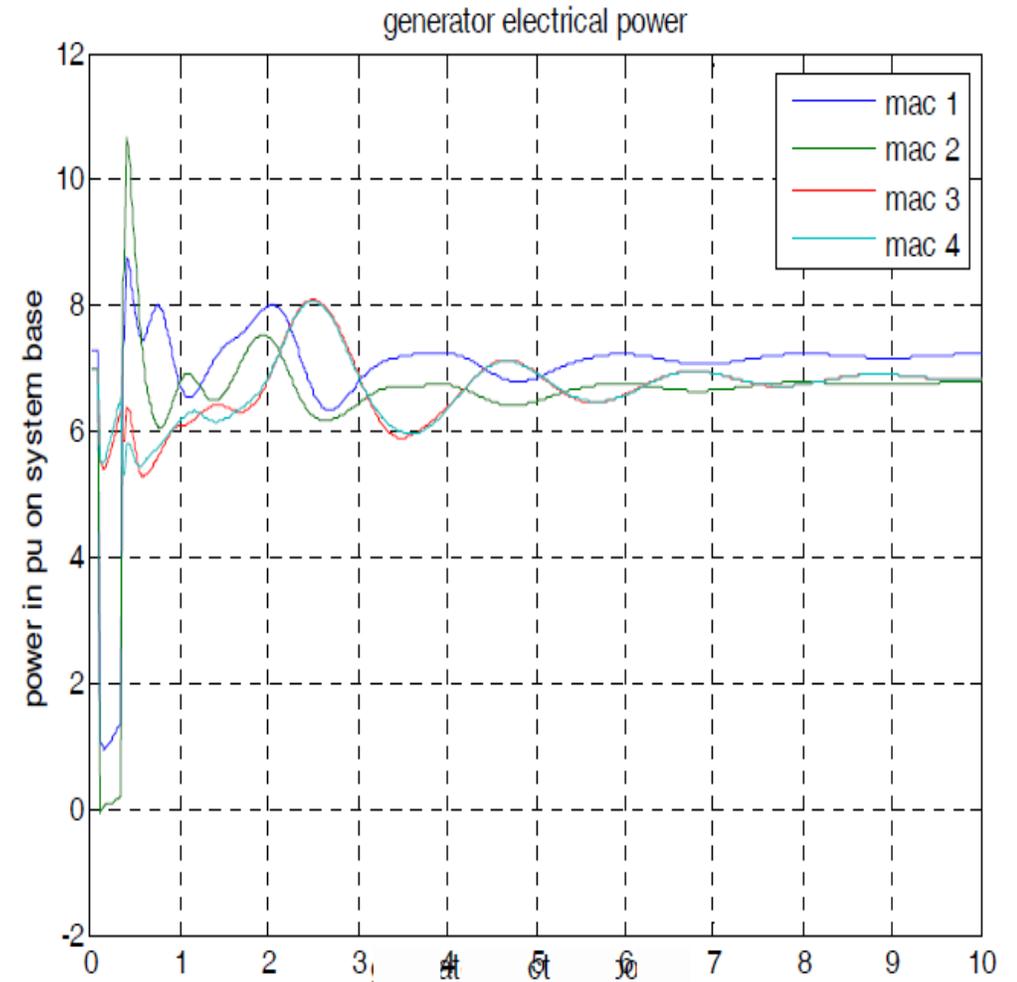
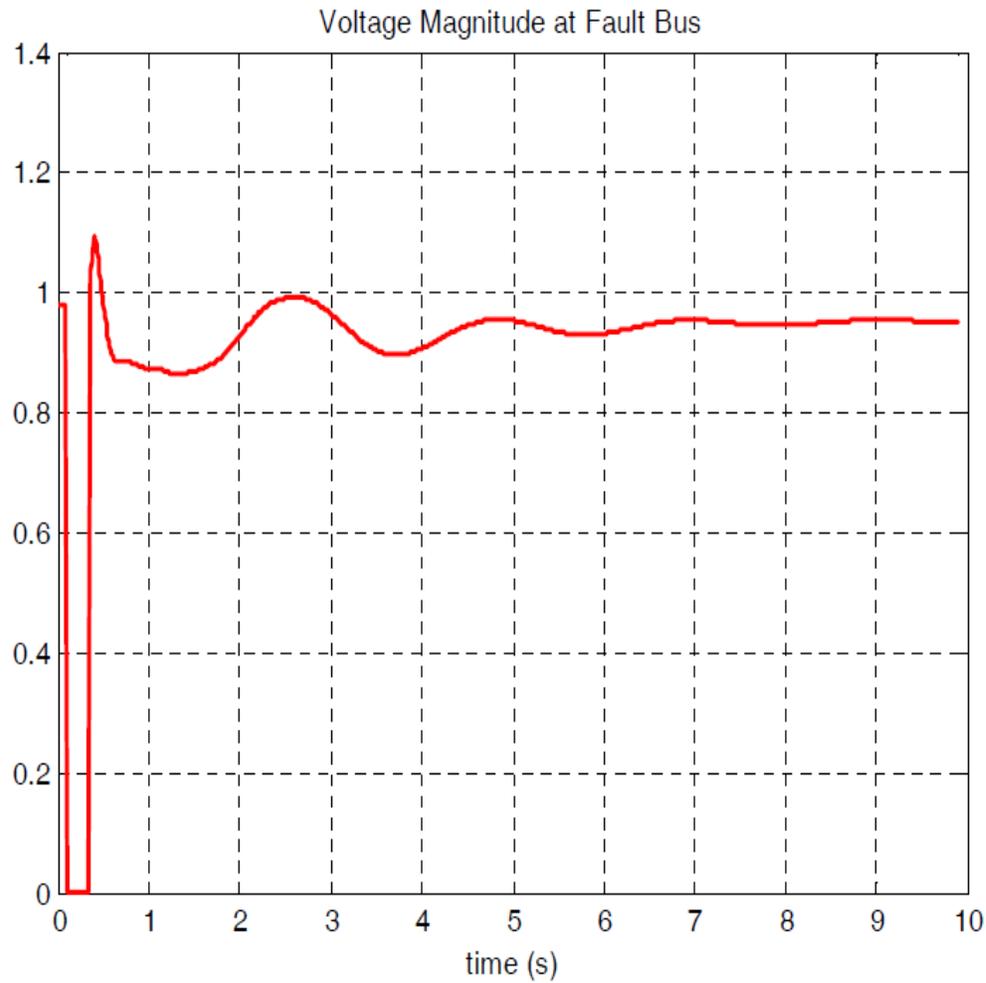
Case study: Two-area system



As the simulation progresses, the voltage at the fault bus (bus3) is plotted. The final response is shown below, critical clearing time $T_{crit} = 0.25$

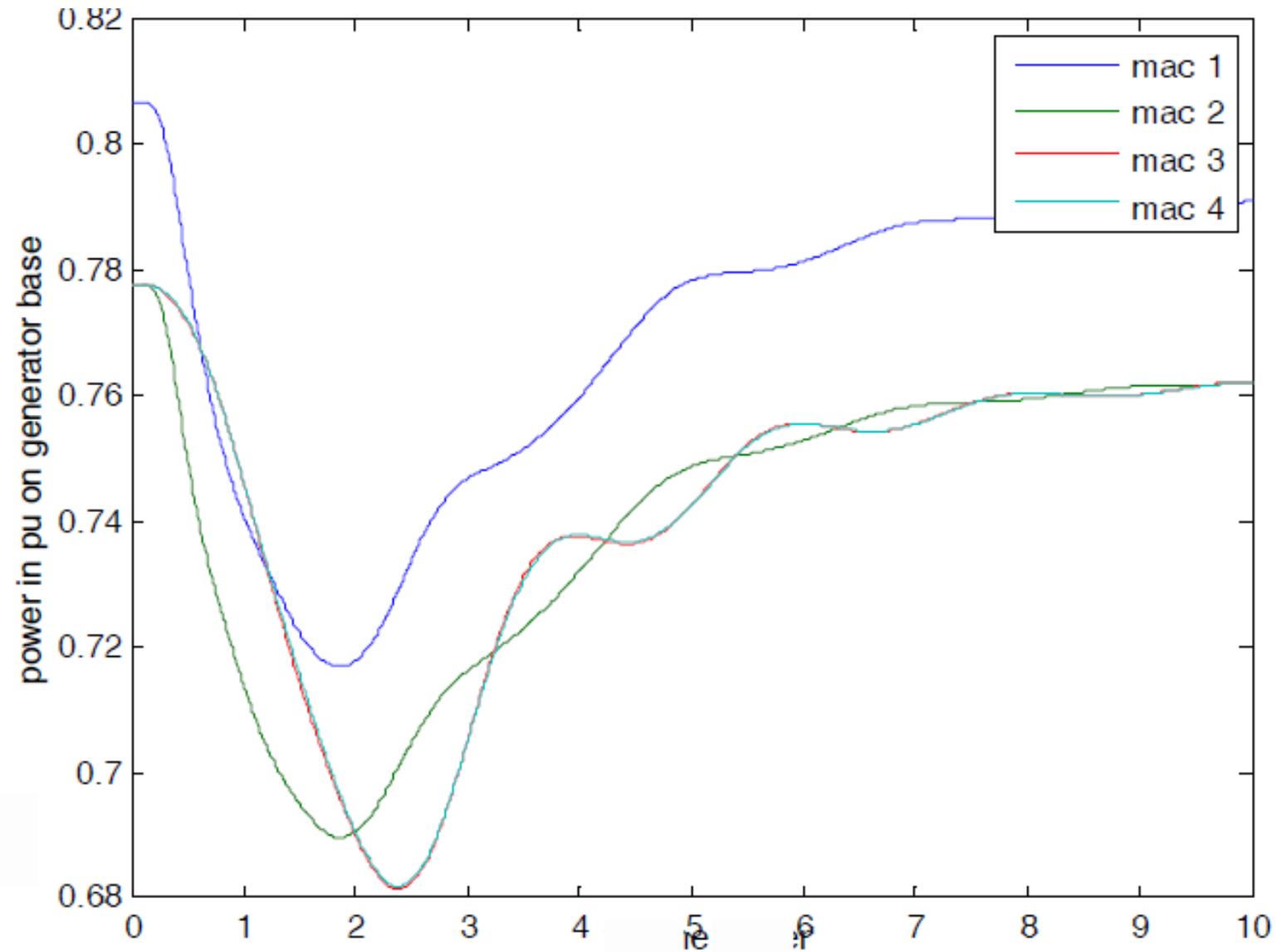


$$T_{crit} = 0.25 \text{ s}$$



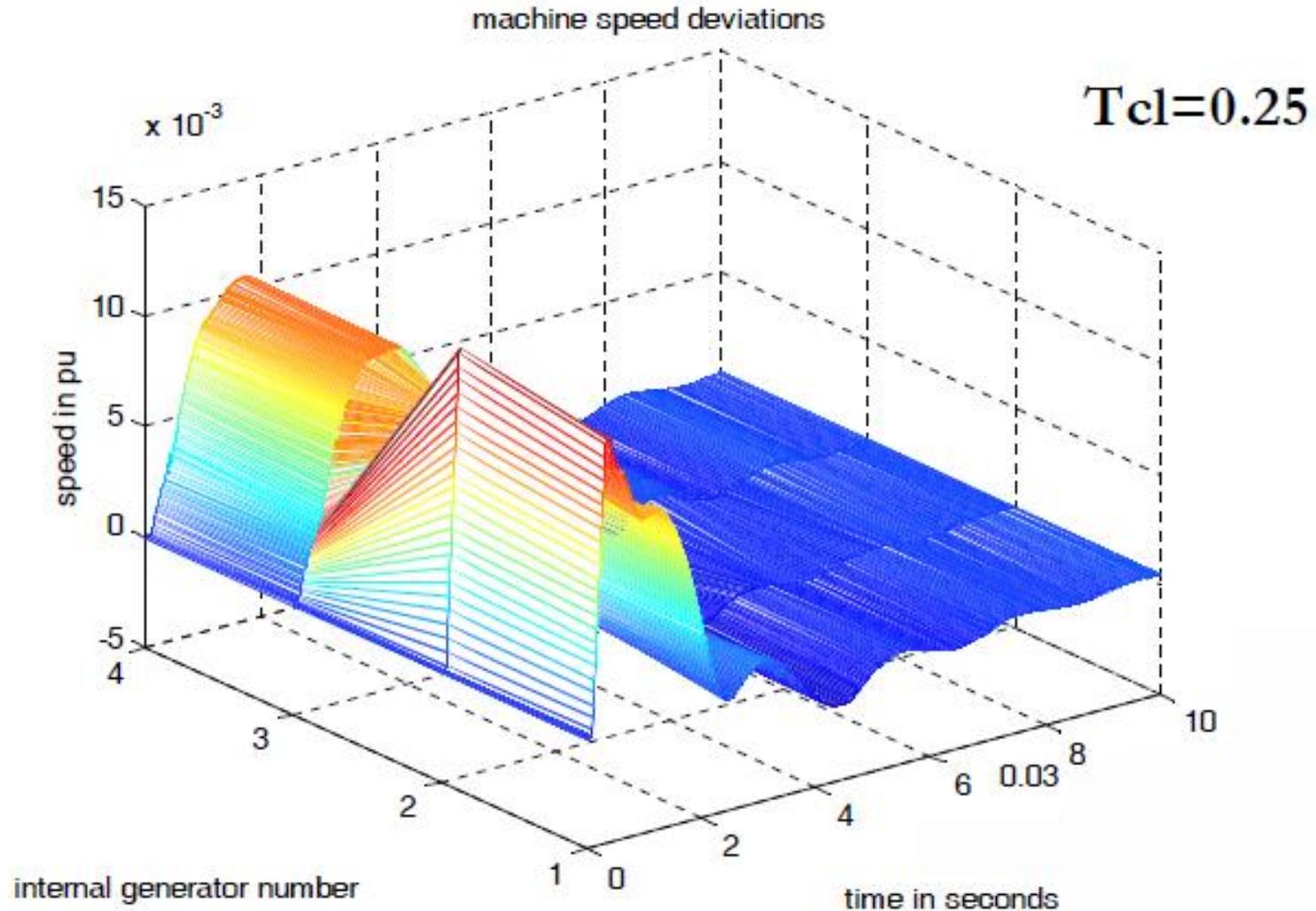


Turbine Power



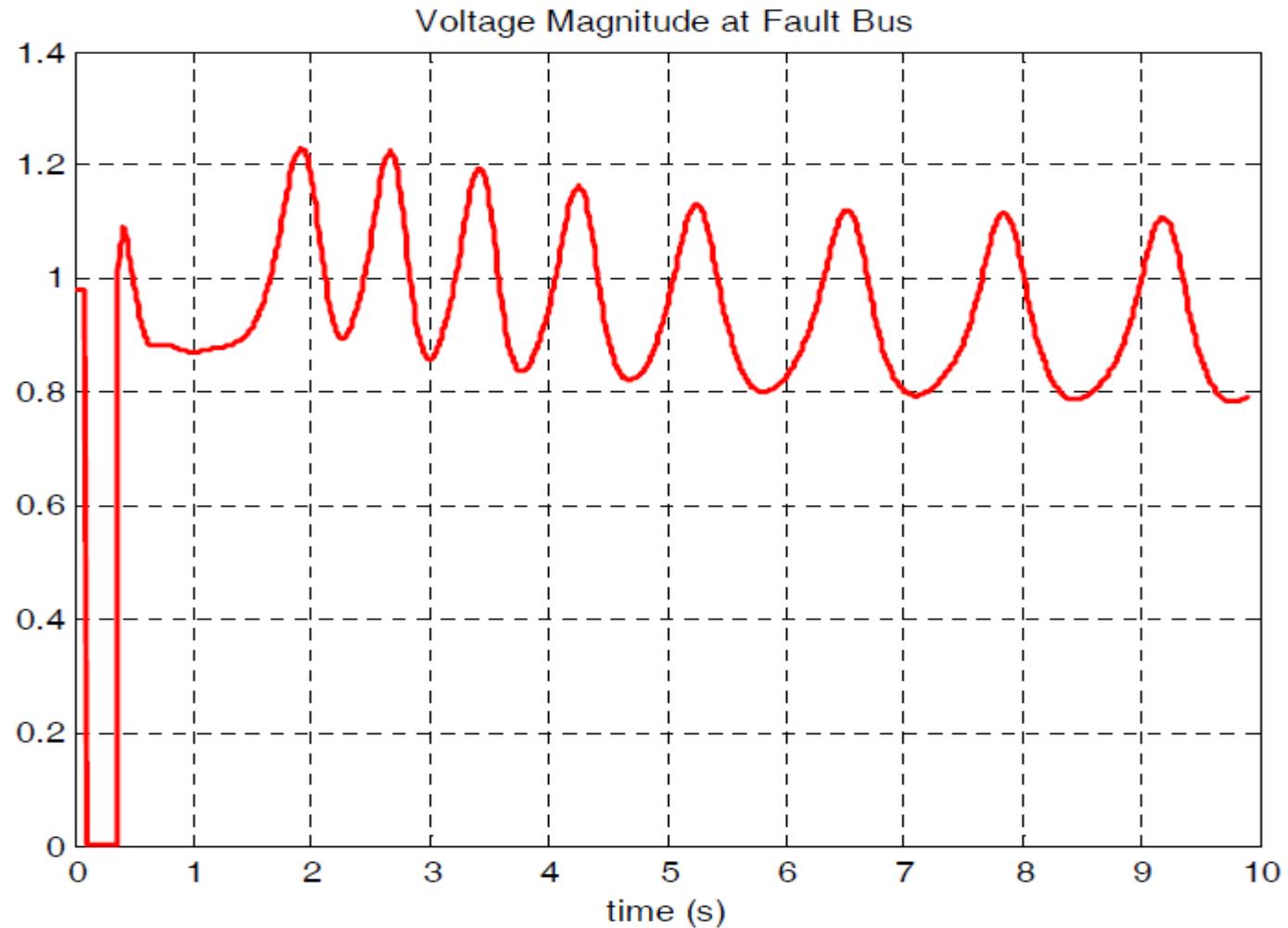


Machine Speed Variation



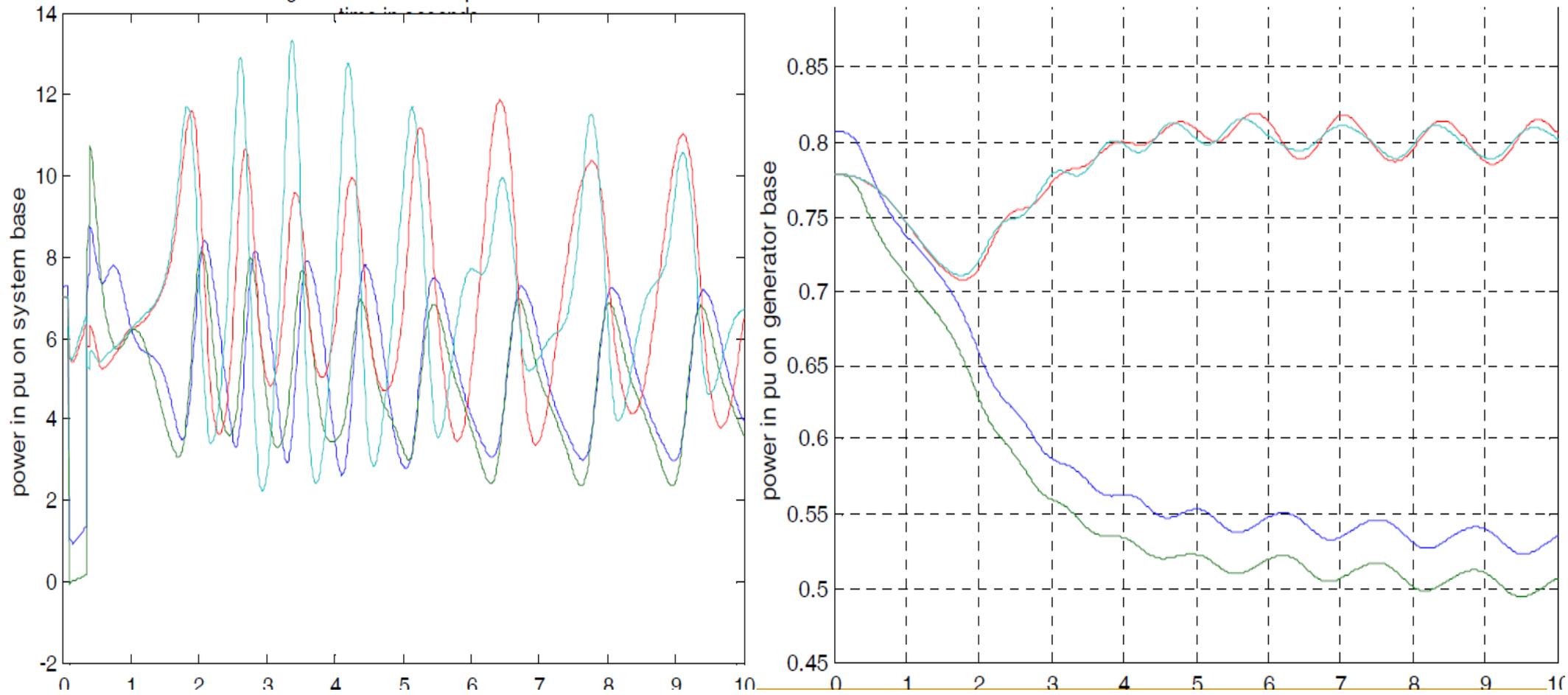


$$T_{\text{crit}} = 0.26 \text{ s}$$



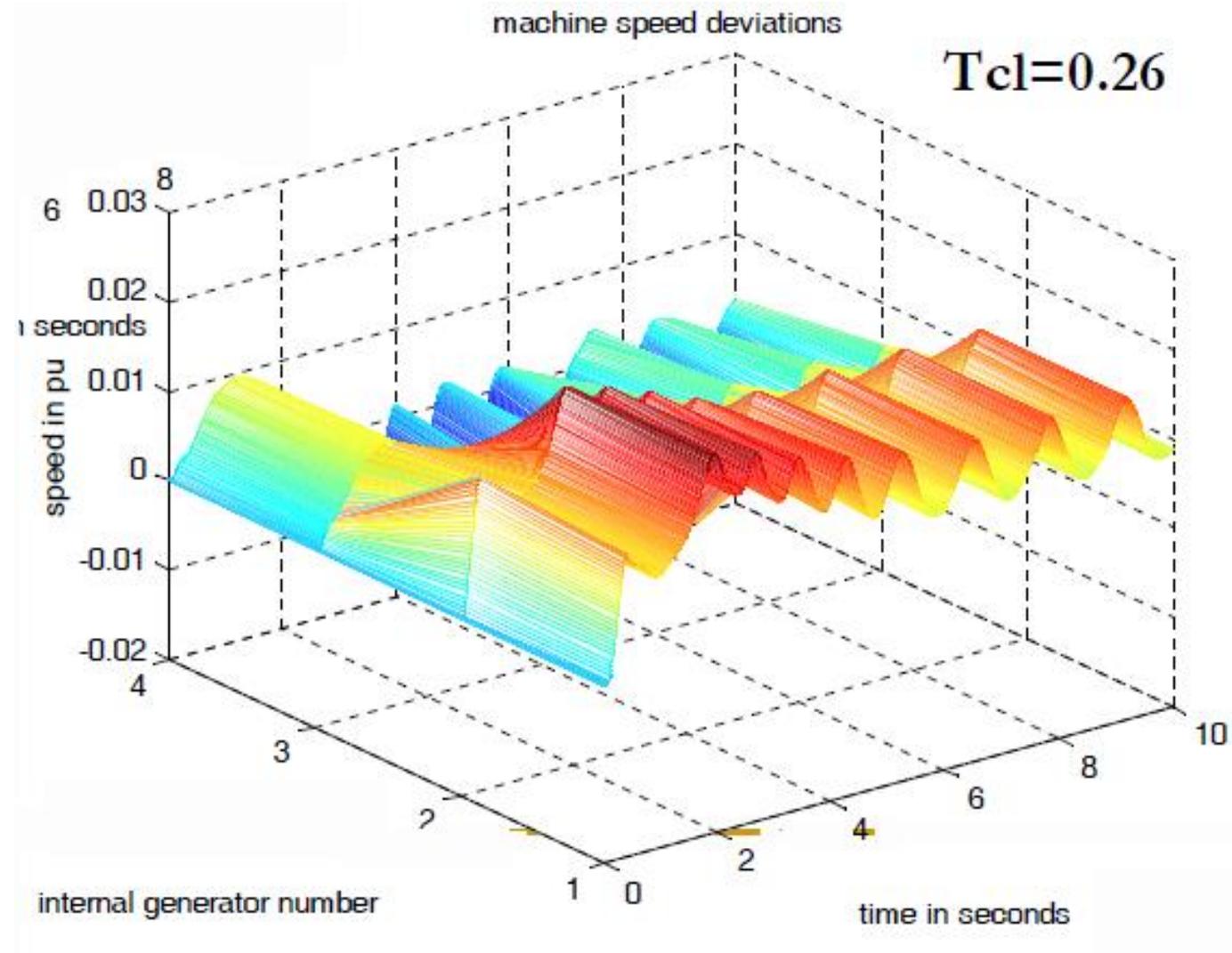


$$T_{\text{crit}} = 0.26 \text{ s}$$

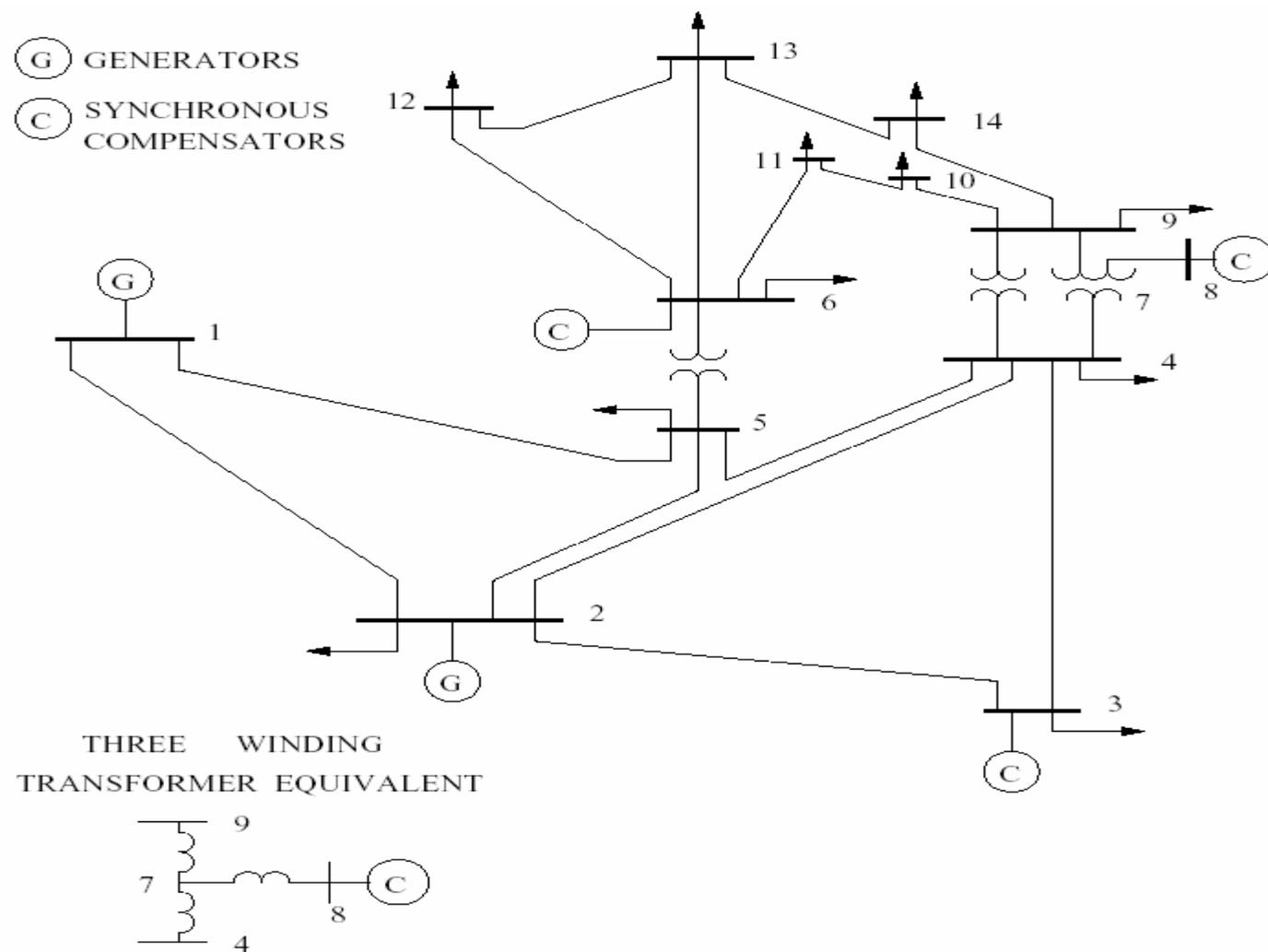




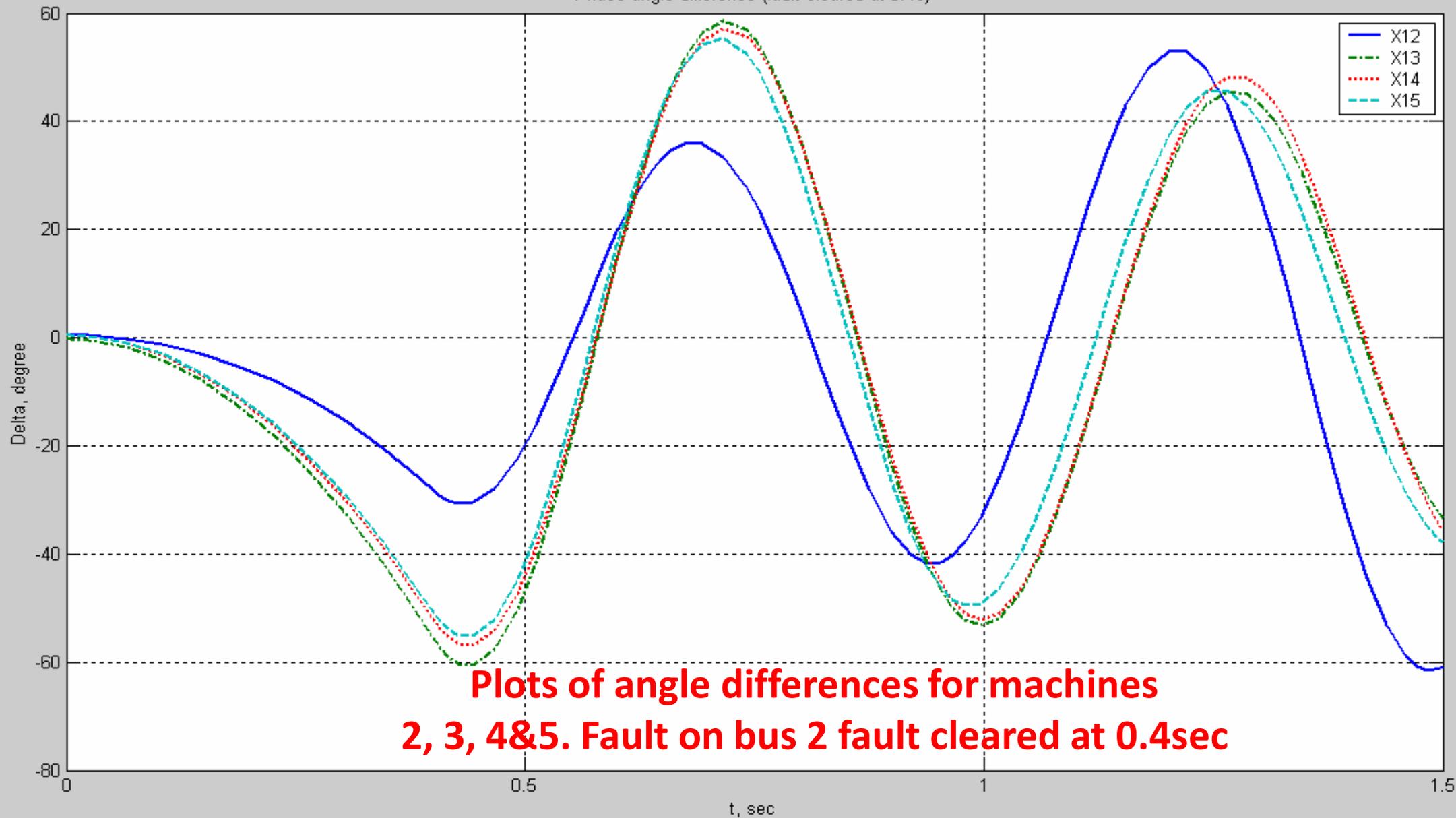
Machine Speed Variation



Case Study: IEEE 14 Bus System

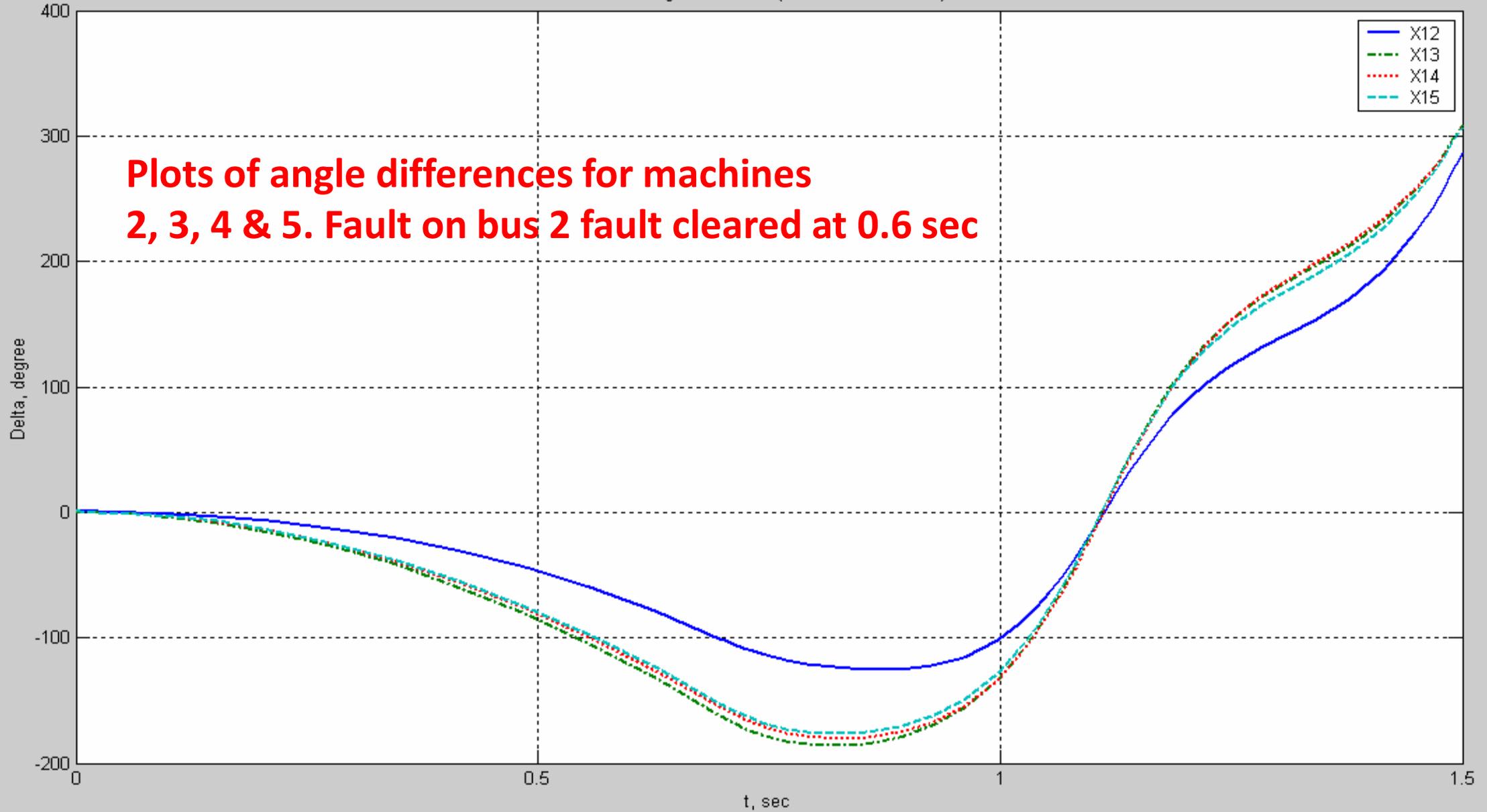


Phase angle difference (fault cleared at 0.4s)



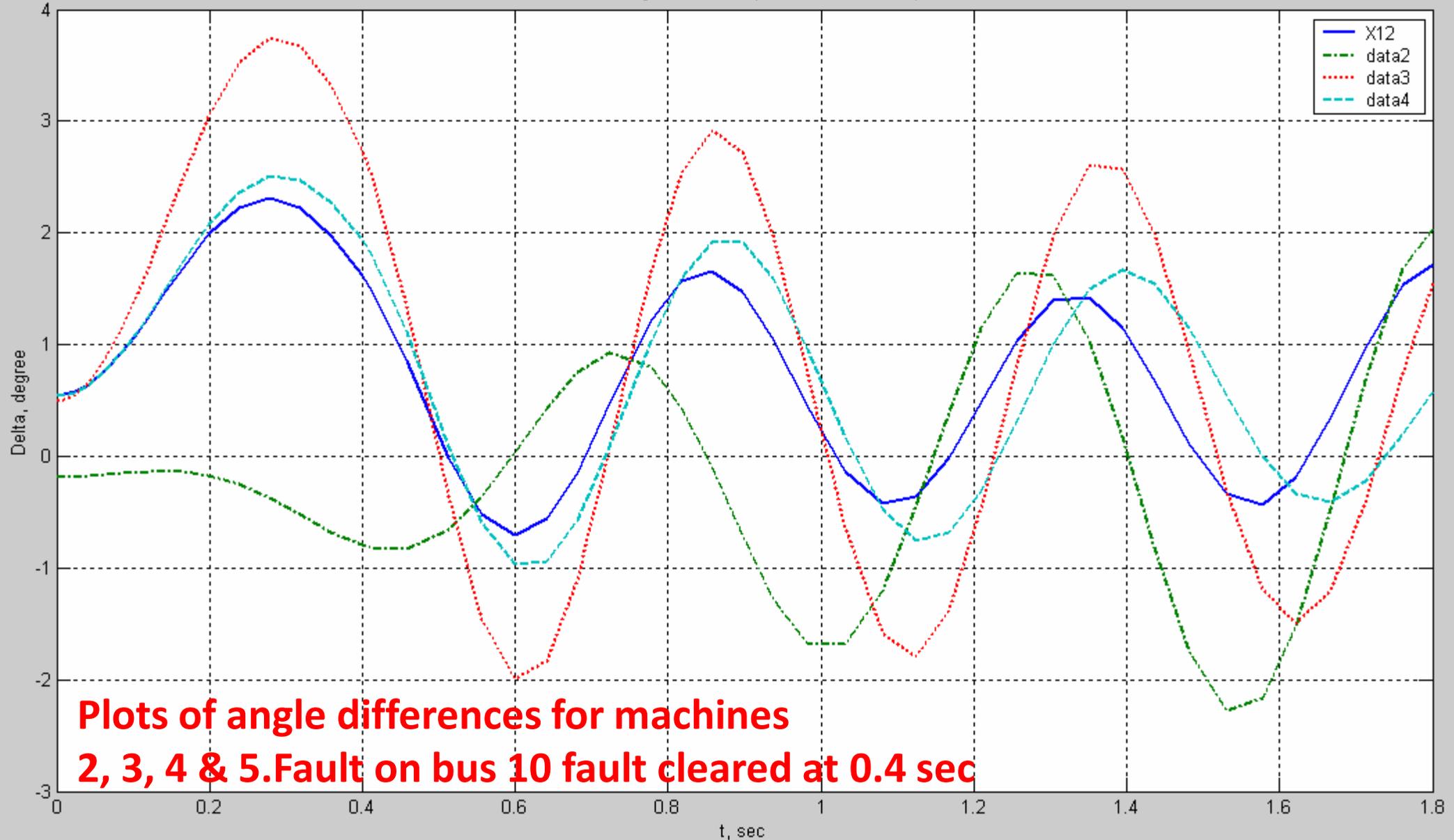
**Plots of angle differences for machines
2, 3, 4&5. Fault on bus 2 fault cleared at 0.4sec**

Phase angle difference (fault cleared at 0.7s)



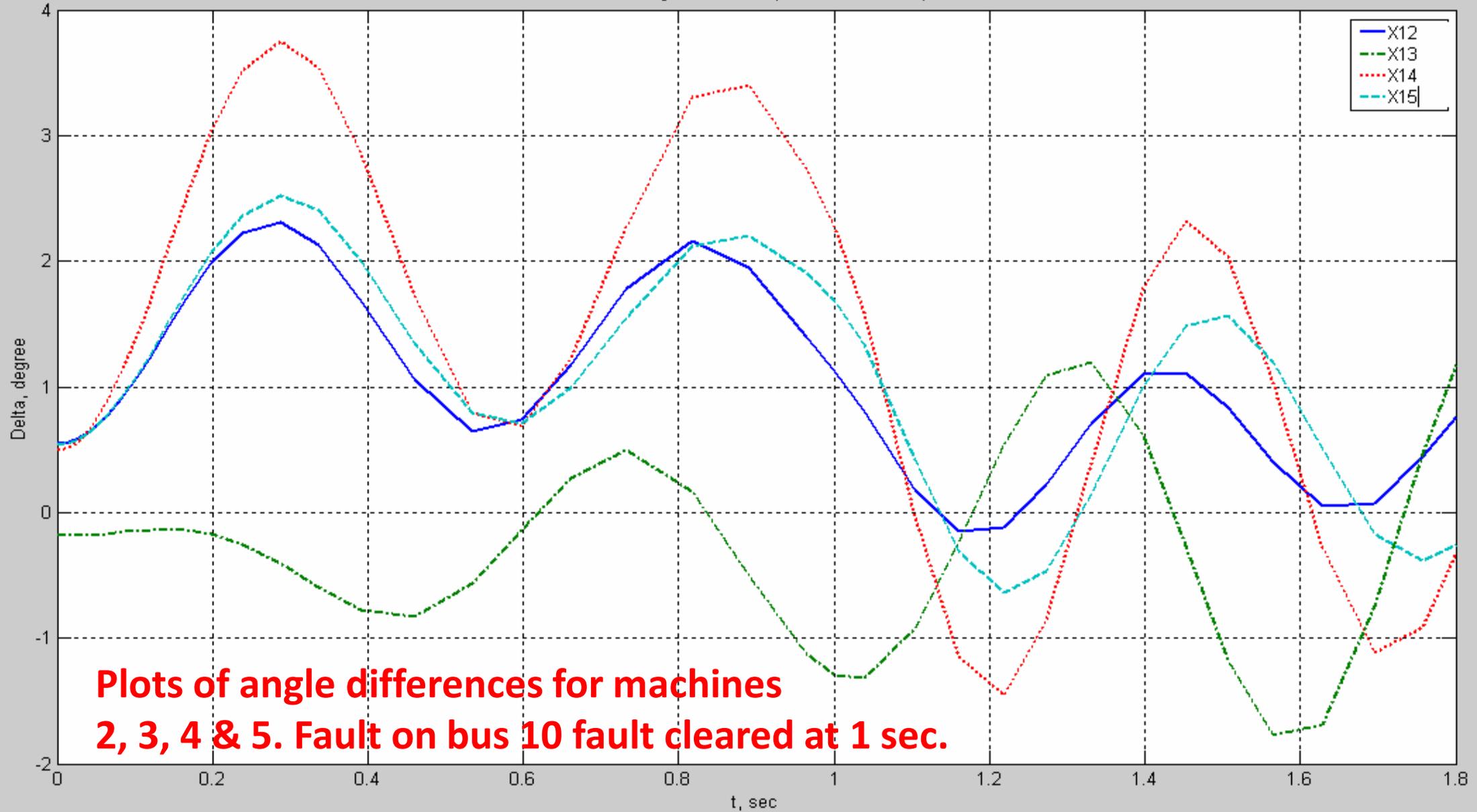
**Plots of angle differences for machines
2, 3, 4 & 5. Fault on bus 2 fault cleared at 0.6 sec**

Phase angle difference (fault cleared at 0.4s)



**Plots of angle differences for machines
2, 3, 4 & 5. Fault on bus 10 fault cleared at 0.4 sec**

Phase angle difference (fault cleared at 1s)



Renewable Integration- Stability Issues

Frequency stability

- Renewable energy sources are often connected via a converter interface and have no inertia (as seen from the grid)
- Replacing synchronous generators with sources using a converter interface reduces total system inertia and is more sensitive to frequency deviations
- Thermal generators may run under minimum load if displaced by renewable energy sources

Renewable Integration- Stability Issues

Voltage stability

- Renewable energy sources with limited or no reactive power control (e.g. fixed-speed induction wind turbines, household-scale PV inverters) will decrease voltage stability
- Integrating renewable energy sources into weak parts of the grid can actually improve voltage stability



Bifurcation

- Some mechanisms behind instability incidents of power system can be explained well using bifurcation theories
- Bifurcation generally means “intersection of two or more branches with opposite stability”
- There are variety of bifurcation points identified in dynamical systems, including power systems and some are being researched
- Among these bifurcation points, **Saddle-node (SNB), Limit-induced, and Hopf bifurcation (HB)** can be well associated with some power system instability



Introduction (cont.)

- In the case of saddle-node bifurcation, a singularity of a system Jacobian and/or state matrix results in the disappearance of steady state solutions
- Limit-induced bifurcations, on the other hand, occur due to the lack of steady state solutions arising from systems controls reaching limits (e.g. generator control limits)
- Both of these bifurcation modes typically lead to voltage collapses



Introduction (cont.)

- HBs produce limit cycles (periodic orbits) that may lead the system to oscillatory instabilities. Also, they have been detected in a variety of power system models and are observed in practice

Example

Western Electricity Coordinating Council (WECC) disturbance in 1996.



Introduction (cont.)

- A bifurcation point is defined as the intersection point of two or more branches with opposite stability
- At the bifurcation point, a qualitative and quantitative change in the solution occurs (i.e. from stable to unstable, from stable to oscillatory, or etc)
- In general, bifurcations can be classified into **“Local” and “Global” bifurcations**



Introduction (cont.)

Local Bifurcation

- Bifurcation of a fixed point
- Determined by observing the behavior of model in a small neighborhood of an equilibrium point

Examples

- Saddle-node
- Pitch Fork
- Transcritical
- Hopf Bifurcation



Introduction (cont.)

Global Bifurcation

- Emerges out of a period orbit
- Qualitative changes in phase portrait are not restricted to the small neighborhood of an equilibrium

Examples

- Period doubling
- Cycle fold and chaos



Saddle-Node Bifurcation

- Saddle-node bifurcation is associated with the disappearance of the steady state solution or a loss of equilibrium. Hence, it is referred to as “static bifurcation”
- In order to explain the basic theory behind the saddle-node bifurcation, recall the power system model of DAEs (4.18):

$$\dot{x} = f(x, y, l, p)$$

$$0 = g(x, y, l, p)$$



Saddle-Node Bifurcation

- The system state matrix or the Jacobian matrix is given by:
$$\Delta\dot{x} = (J_1 - J_2 J_4^{-1} J_3) \Delta x = A \Delta x$$
- Saddle-node bifurcations (SNBs) occur at the equilibrium point, where the Jacobian has a simple and unique zero eigenvalue, with nonzero right and left eigenvectors v and w , respectively
- In a power system, however, we will look at only the power flow Jacobian for studying saddle-node bifurcations



Hopf Bifurcations

- Consider a dynamical system modeled by a set of ordinary differential equations (ODEs):

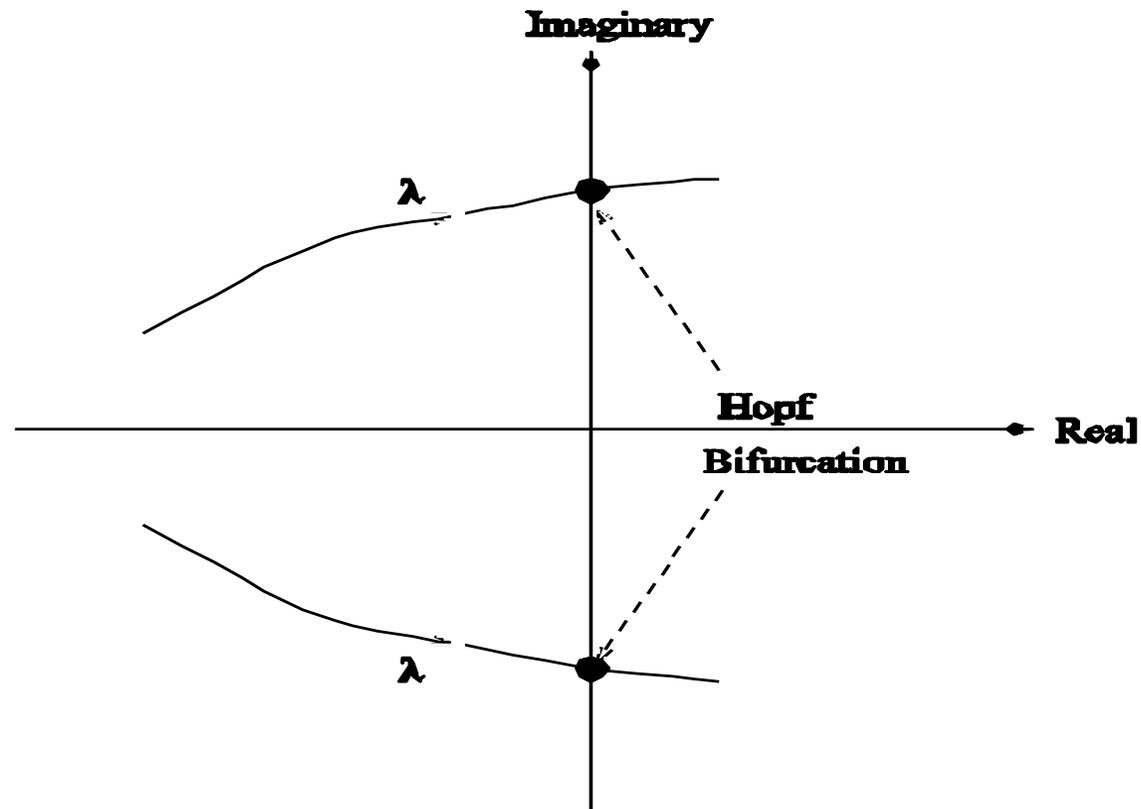
$$\dot{x} = f(x, \lambda)$$

- When the parameter λ varies, the equilibrium points x_0 of the system change and so do the eigenvalues i.e.) As the parameter λ changes, the eigenvalues associated with the corresponding equilibrium point change as well



Hopf Bifurcations

The point where a complex conjugate pair of eigenvalues reach the imaginary axis with changes in λ , say (x_0, λ_0) , is known as the Hopf Bifurcation point.





Hopf Bifurcations

At a Hopf bifurcation point (x_0, λ_0) , the following conditions are satisfied:

1. $f(x_0, \lambda_0) = 0$ (the point is an equilibrium point)
2. The Jacobian evaluated at $f(x_0, \lambda_0)$ should only have a simple pair of purely imaginary eigenvalues
3. The rate of change of the real part of the purely imaginary eigenvalues with respect to the parameter change λ should be non zero

$$\frac{d \operatorname{Re}\{\mu\}}{d\lambda} \neq 0$$



Hopf Bifurcations

These conditions basically state..

- A Hopf Bifurcation corresponds to a system equilibrium with a pair of purely imaginary eigenvalues with all other eigenvalues having non-zero real parts
- The pair of Bifurcation or critical eigenvalues cross the imaginary axis with nonzero “speed”



Hopf Bifurcations

Example

Lorenz's equation

$$\dot{y}_1 = P(y_2 - y_1)$$

$$\dot{y}_2 = -y_1 y_3 + R y_1 - y_2$$

$$\dot{y}_3 = y_1 y_2 - b y_3$$

Where,

b and P are positive constants

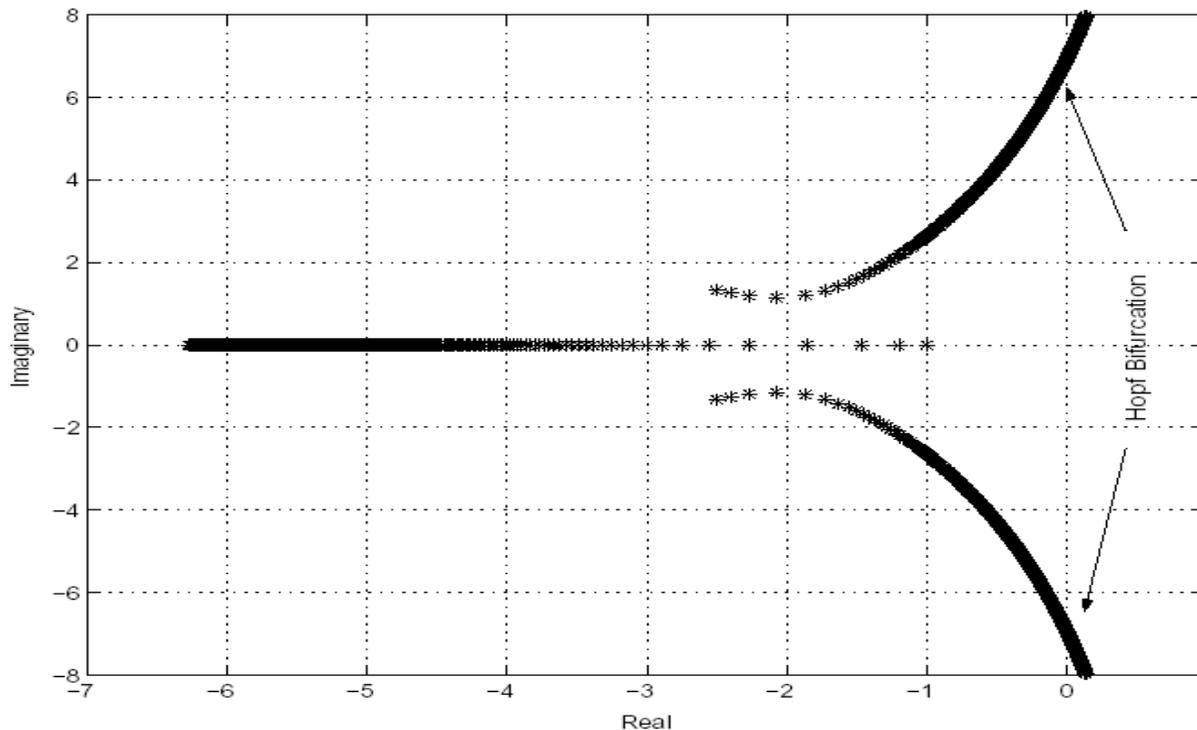
R is the bifurcation parameter



Hopf Bifurcations

For $R \geq 1$, the system presents the following equilibrium point: $(\pm \sqrt{R-1}, \pm \sqrt{R-1}, R-1)$

The system has a Hopf Bifurcation for $b=1$, $P=4$ and a given value of R

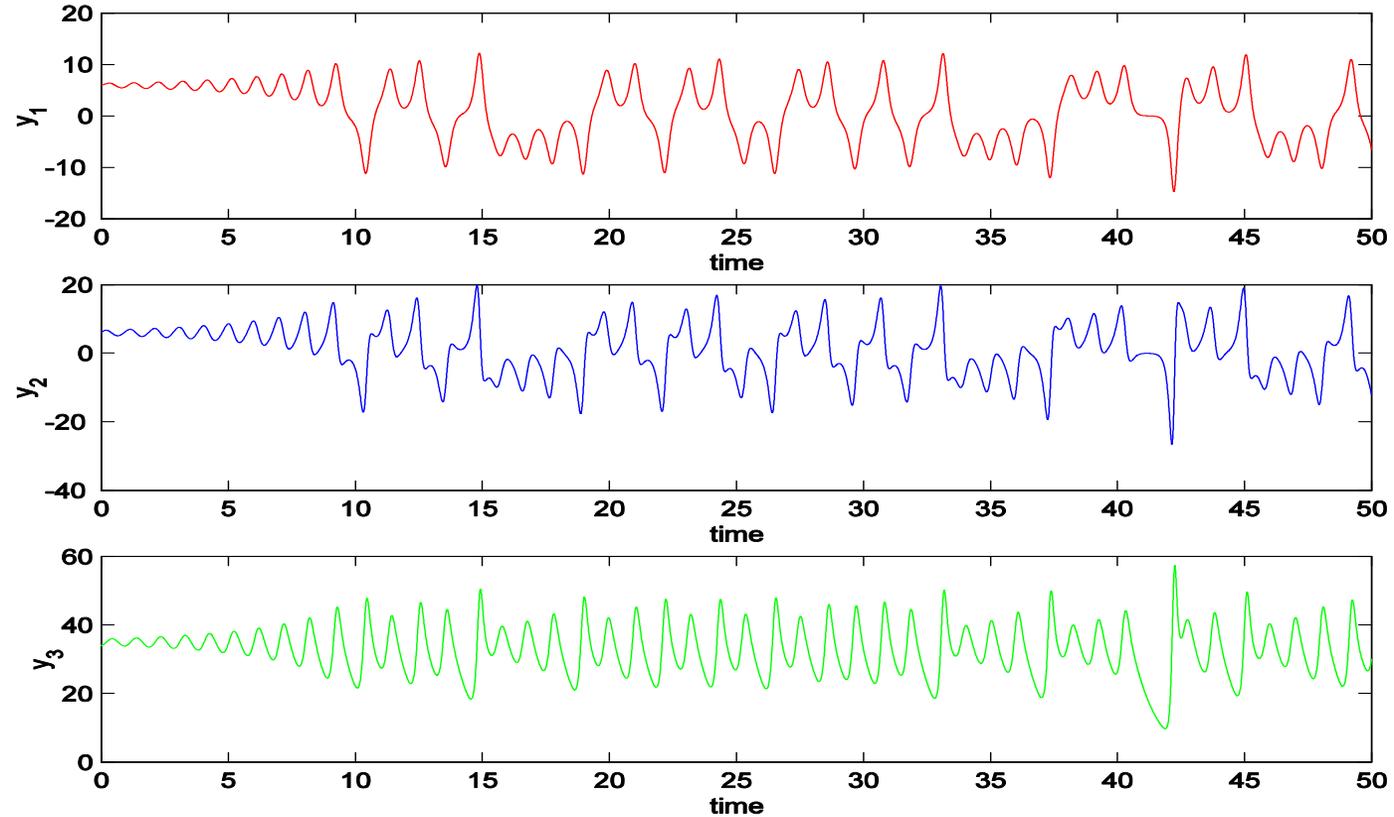


Eigenvalues of the system for $R=1$ to 40

The system has a pair of complex eigenvalues at $R=36$



Hopf Bifurcations



Time domain simulation of the system for small disturbance around the Hopf Bifurcation Point



Hopf Bifurcations in Power Systems

- Power systems are modeled by a set of highly nonlinear differential and algebraic equations
- This includes a variety of independent and control parameter
- In critical circumstances, variations of a parameter may cause unstable oscillation in the system
- This type of behavior in a power system can be analyzed well using the **Hopf Bifurcation** theory.
- The theory behind Hopf Bifurcation can also be conveniently used to predict or detect an oscillatory problem in power system.



Online Resources

1. <http://www.eagle.ca/~cherry>
2. http://www.mathworks.com/products/connections/product_main.html
3. http://www.ece.mtu.edu/faculty/ljbohman/peec/Dig_Rsor.htm
4. <http://ieeexplore.ieee.org/iel5/6472/17304/00798273.pdf>
5. <http://ieeexplore.ieee.org/iel5/10730/33853/01611822.pdf>
6. <http://www.eng.fsu.edu/~tbaldwin/eel6252/public/index.html>
7. <http://www.phys.ufl.edu/docs/matlab/toolbox/powersys/powergui.htm>
8. http://www.utexas.edu/research/cem/Electric_Ship_Hamid.html
9. <http://www.ecse.rpi.edu/Courses/F06/EPOW4010-F06/>
10. <http://www.ida.liu.se/~pelab/realsim/library/ObjectStab/objectstab.pdf>
11. <http://delivery.acm.org/10.1145/1170000/1165552/p36-doolla.pdf?key1=1165552&key2=9348120711&coll=&dl=acm&CFID=1>



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Smart Grid – Modeling and Control

Questions?



REDLAB

Renewable Energy Design Laboratory

<http://manoa.hawaii.edu/me/redlab>

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