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Smart Grid – Modeling and Control

Power Flow

University of Hawaii's Renewable Energy Design Laboratory (REDLab)
in collaboration with Powersim Inc. and MyWay

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Introduction

Load flow or power flow analysis is the determination of current, voltage, active power, and reactive VA at various points in a power system operating under normal steady - state or static conditions.

Objectives of Load Flow Analysis

- To plan the best **operation and control** for the existing system and the future expansion to keep up with the pace of the load growth
- To help ascertain the **effects of new loads**, new generating stations, new lines, and new interconnections before they are installed
- To **minimize the system losses** and provide a check on the system stability

Load Flow Research

- Load flow research calculates…
  - Magnitude and phase angles of voltages at the buses
  - Active power and reactive VA flow for the given terminal or bus conditions.
- Each bus is characterized by the following variables:

$$V_i : Voltage\ Magnitude$$

$$\partial_i : Phase\ \ Angle$$

$$Pi : Active\ Power$$

$$Qi : \mathrm{Re}\,active\ Power$$

Buses in Load Flow Research

Three types of buses or nodes can be identified in a power system network for load flow studies.

- In each bus, two variables are known (specified), and two are to be determined.

(a) Swing bus/ Reference bus/ Slack bus

- Voltage magnitude and phase angle are specified.
- This bus will respond first to a changing load condition.
- Solution: active and reactive power injections

Load Flow Research

(b) Generator bus or voltage – controlled bus or P-Vbus

- Models generation – station buses
- Voltage magnitude and real power are specified.
- Solution: reactive power injection and voltage angle

Load Flow Research

(c) Load Bus/ P-Q bus

- Models load - centre buses
- Active and reactive powers are specified.
- If any bus in a power system network has both load and generator, then load is generally treated as negative generation.
- Solution: voltage magnitude and angle

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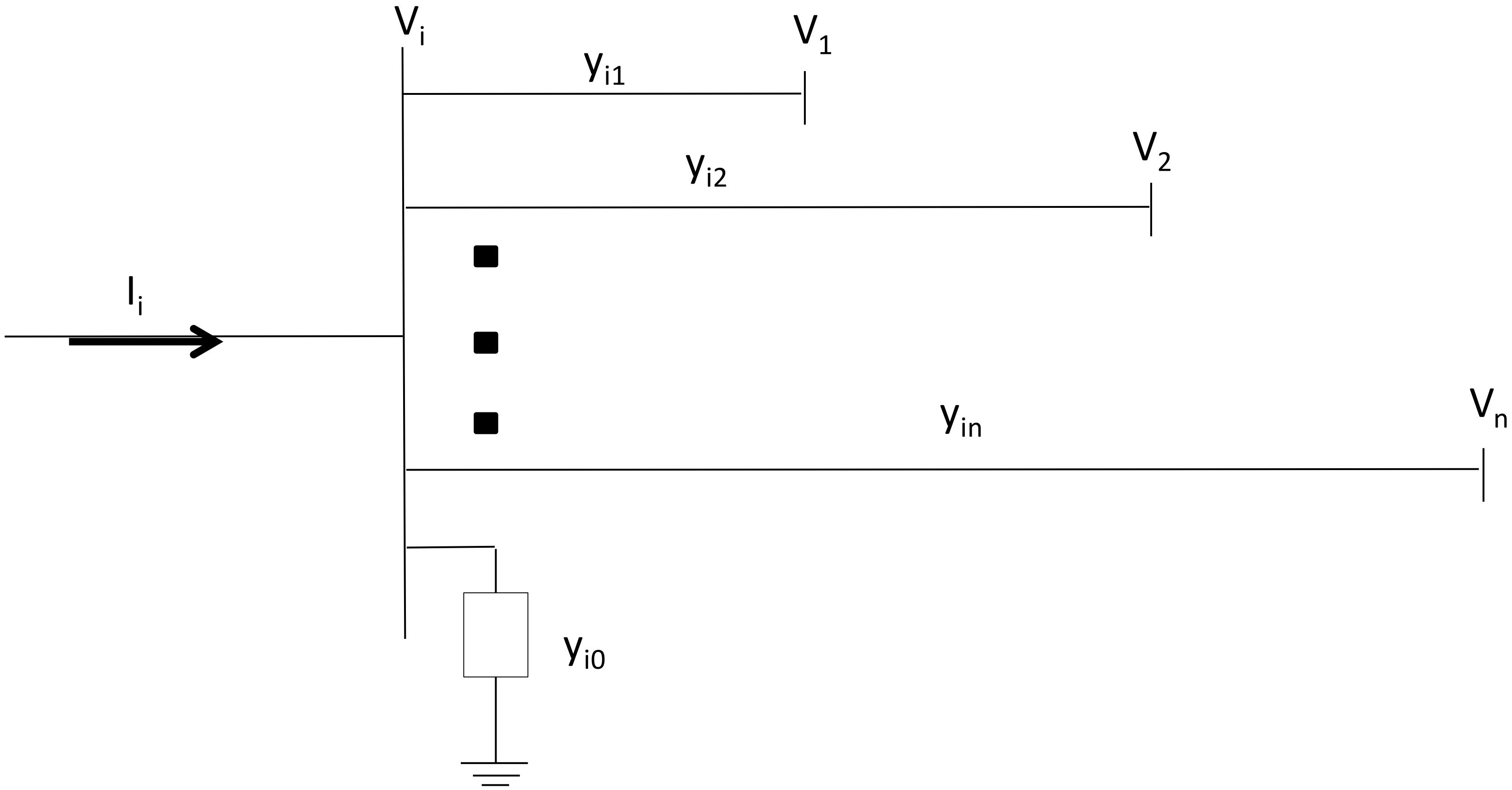
Buses in Load Flow Research

Bus Type	Specified Variables	Unknown Variables
Reference bus or slack bus	V_i, ∂_i	P_i, Q_i
Generator bus or voltage controlled bus or PV bus	P_i, V_i	Q_i, ∂_i
Load bus or PQ bus	P_i, Q_i	V_i, ∂_i

Power Flow Equation

Consider

A simple power system network as given below



Applying Kirchhoff's Law

$$I_i = y_{io}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + ... + y_{in}(V_i - V_n)$$
$$= (y_{io} + y_{i1} + y_{i2} + ... + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - ..... - y_{in}V_n$$

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad \mathrm{j \neq i}$$

The real and reactive power at bus i is

$$P_i + jQ_i = V_i I_i^{\;*}$$

$$I_i = \frac{P_i - jQ_i}{V_i} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V \quad \mathrm{j \neq i}$$

Power Flow Requires Iterative Solutions

In the power flow we assume we know S_i and the $\mathbf{Y}_{bus}$. We would like to solve for the V's. The problem is the below equation has no closed form solution:

$$S_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

Rather, we must pursue an iterative approach.

Power Flow Solution Methods

1) Gauss-Seidel Power Flow Solution

2) Newton-Raphson Power Flow Solution

3) Fast Decoupled Power Flow Solution

Gauss Seidel Iteration

With the Gauss method we need to rewrite our equation in an implicit form: x = h(x)

To iterate we first make an initial guess of x, $\mathrm{x}^{(0)}$, and then iteratively solve $\mathrm{x}^{(v+1)} = h(x^{(v)})$ until we find a "fixed point", $\hat{\mathrm{x}}$, such that $\hat{\mathrm{x}} = h(\hat{\mathrm{x}})$.

Gauss Seidel Iteration Example

Example: Solve $x - \sqrt{x} - 1 = 0$

$$x^{(v+1)} = 1 + \sqrt{x^{(v)}}$$

Let k = 0 and arbitrarily guess $x^{(0)} = 1$ and solve

k	$x^{(v)}$
0	1
1	2
2	2.41421
3	2.55538
4	2.59805
5	2.61185
6	2.61612
7	2.61744
8	2.61785
9	2.61798

Stopping Criteria

A key problem to address is when to stop the iteration. With the Guass iteration we stop when

$$\left|\Delta x^{(v)}\right| < \varepsilon \qquad \text{with } \Delta x^{(v)} \triangleq x^{(v+1)} - x^{(v)}$$

If x is a scalar this is clear, but if x is a vector we need to generalize the absolute value by using a norm

$$\left\|\Delta x^{(v)}\right\|_j < \varepsilon$$

Two common norms are the Euclidean & infinity

$$\left\|\Delta \mathbf{x}\right\|_2 = \sqrt{\sum_{i=1}^{n} \Delta x_i^2} \qquad \left\|\Delta \mathbf{x}\right\|_\infty = \max_i \left|\Delta \mathrm{x}_\mathrm{i}\right|$$

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Gauss Power Flow

We first need to put the equation in the correct form

$$\mathrm{S}_\mathrm{i} = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

$$\mathrm{S}_i^* = V_i^* I_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$\frac{\mathrm{S}_i^*}{V_i^*} = \sum_{k=1}^{n} Y_{ik} V_k = Y_{ii} V_i + \sum_{k=1, k \neq i}^{n} Y_{ik} V_k$$

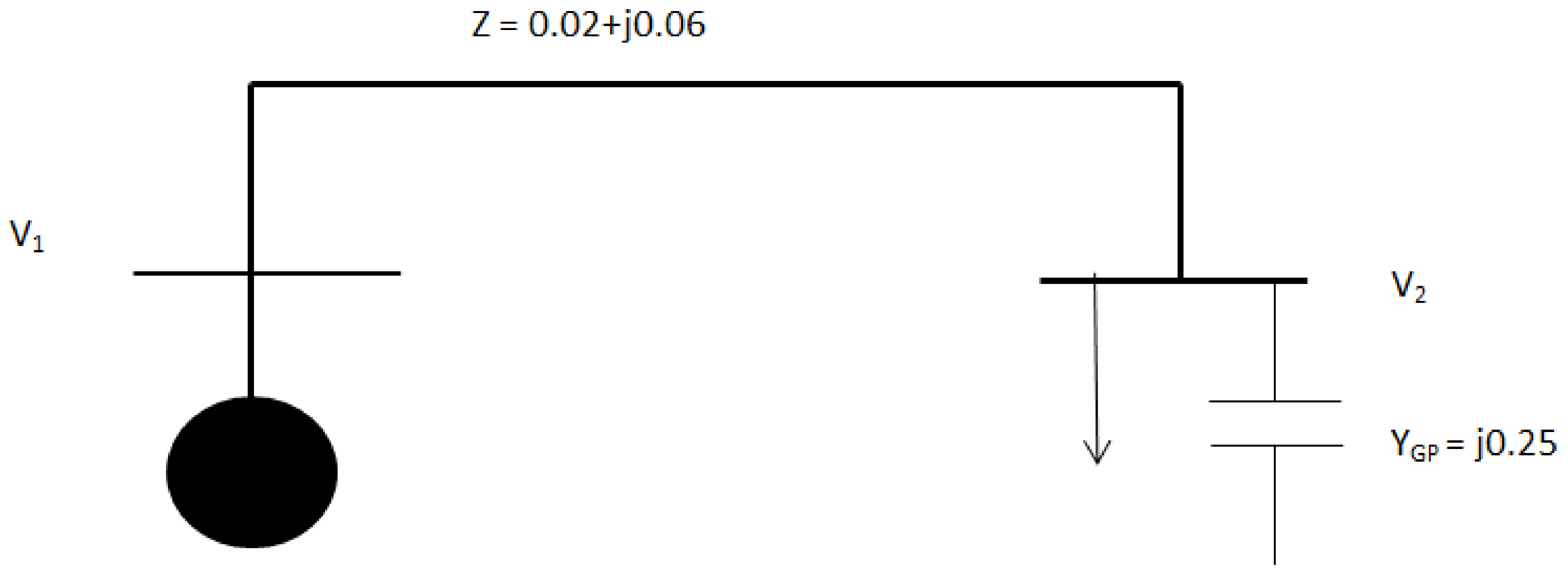
$$V_i = \frac{1}{Y_{ii}} \left(\frac{\mathrm{S}_i^*}{V_i^*} - \sum_{k=1, k \neq i}^{n} Y_{ik} V_k \right)$$

Gauss Two Bus Power Flow Example

A 100 MW, 50 Mvar load is connected to a generator through a line with z = 0.02 + j0.06 p.u. and line charging of 5 Mvar on each end (100 MVA base). Also, there is a 25 Mvar capacitor at bus 2.

If the generator voltage is 1.0 p.u., what is V_2?

Gauss Two Bus Example, cont'd



$$S_{Load} = 1.0 + j0.5 \text{ p.u.}$$

Gauss Two Bus Example, cont'd

The unknown is the complex load voltage, V_2.

To determine V_2 we need to know the $\mathbf{Y}_{\text{bus}}$.

$$\frac{1}{0.02 + j0.06} = 5 - j15$$

$$\text{Hence } \mathbf{Y}_{\text{bus}} = \begin{bmatrix} 5 - j14.95 & -5 + j15 \\ -5 + j15 & 5 - j14.70 \end{bmatrix}$$

$$(\text{Note } B_{22} = -j15 + j0.05 + j0.25)$$

Gauss Two Bus Example, cont'd

$$V_2 = \frac{1}{Y_{22}}\left(\frac{S_2^*}{V_2^*} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k\right)$$

$$V_2 = \frac{1}{5 - j14.70}\left(\frac{-1 + j0.5}{V_2^*} - (-5 + j15)(1.0\angle 0)\right)$$

Guess $V_2^{(0)} = 1.0\angle 0$ (this is known as a flat start)

v	$V_2^{(v)}$	v	$V_2^{(v)}$
0	$1.000 + j0.000$	3	$0.9622 - j0.0556$
1	$0.9671 - j0.0568$	4	$0.9622 - j0.0556$
2	$0.9624 - j0.0553$		

Gauss Two Bus Example, cont'd

$$V_2 = 0.9622 - j0.0556 = 0.9638\angle -3.3^\circ$$

Once the voltages are known all other values can be determined, such as the generator powers and the line flows

$$S_1^* = V_1^*(Y_{11}V_1 + Y_{12}V_2) = 1.023 - j0.239$$

In actual units $P_1 = 102.3$ MW, $Q_1 = 23.9$ Mvar

The capacitor is supplying $|V_2|^2\, 25 = 23.2$ Mvar

Slack Bus

In previous example, we specified S_2 and V_1 then solved for S_1 and V_2.

We can not arbitrarily specify S at all buses, because total generation must equal the total load and total losses.

We also need an angle reference bus.

To solve these problems we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, also a varying real/reactive power injection.

Gauss with Many Bus Systems

With multiple bus systems we could calculate new V_i's as follows:

$$V_i^{(v+1)} = \frac{1}{Y_{ii}}\left(\frac{S_i^*}{V_i^{(v)*}} - \sum_{k=1,k\neq i}^{n} Y_{ik}V_k^{(v)}\right)$$

$$= h_i(V_1^{(v)}, V_2^{(v)}, \ldots, V_n^{(v)})$$

But after we've determined $V_i^{(v+1)}$ we have a better estimate of its voltage , so it makes sense to use this new value. This approach is known as the Gauss-Seidel iteration.

Gauss-Seidel Iteration

Immediately use the new voltage estimates:

$$
\begin{aligned}
V_2^{(v+1)} &= h_2(V_1, V_2^{(v)}, V_3^{(v)}, \ldots, V_n^{(v)}) \\
V_3^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v)}, \ldots, V_n^{(v)}) \\
V_4^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v+1)}, V_4^{(v)} \ldots, V_n^{(v)}) \\
&\vdots \\
V_n^{(v+1)} &= h_2(V_1, V_2^{(v+1)}, V_3^{(v+1)}, V_4^{(v+1)} \ldots, V_n^{(v)})
\end{aligned}
$$

The Gauss-Seidel works better than the Gauss, and is actually easier to implement. It is used instead of Gauss.

Gauss-Seidel Advantages

- Each iteration is relatively fast.

Computational Order =

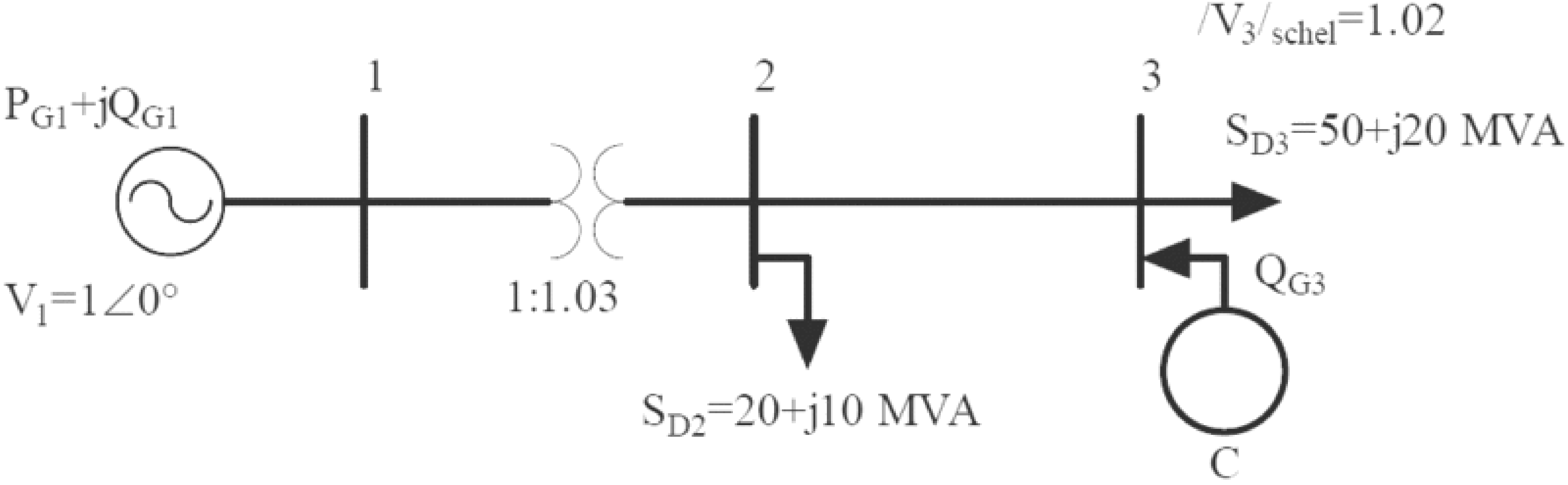
Number of Branches + Number of Buses in the system

- Relatively easy to program

Gauss-Seidel Disadvantages

- Tends to converge relatively slowly, although this can be improved with acceleration
- Has tendency to miss solutions, particularly in large systems
- Tends to diverge in cases with negative branch reactances (common with compensated lines)
- Need to program using complex numbers

Assignment #1



Choose bus 1 as slack bus, bus 2 as PQ, and bus 3 as PV bus then show the first iteration of the power flow solution for this system using the Gauss-Seidel method.

Assignment #1 (Hint)

Once substituting the values, we get the following network.

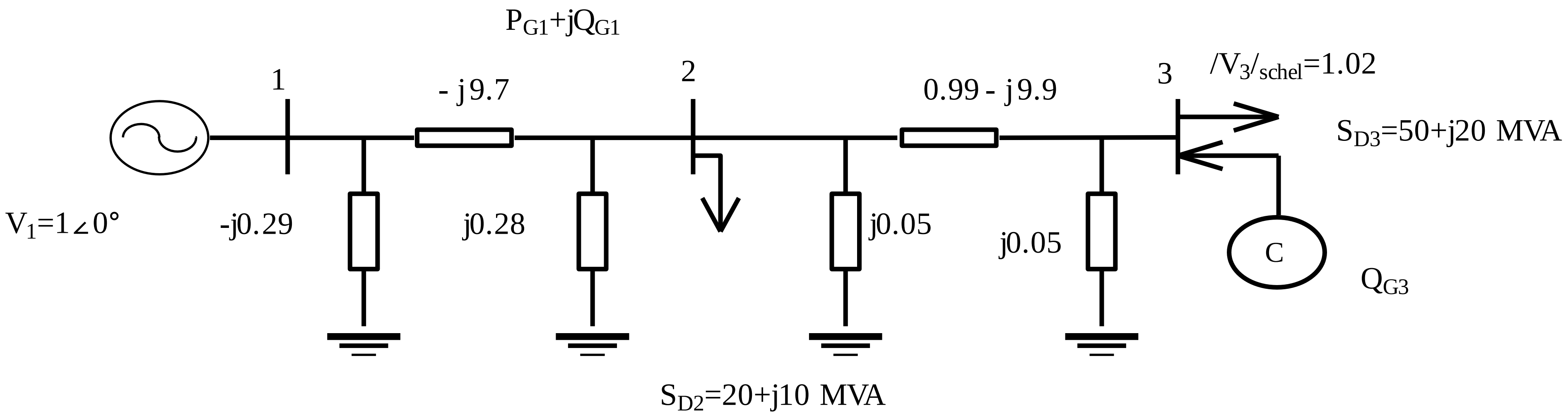


Figure 3 Equivalent Network Diagram

y_{12} = 1/j0.1 = -j10

y_{23} = 1/0.01+j0.1 = 0.990099-9.90099j

y_{pq}(1-1/a)= - j0.29126

y_{pq}/a= -j9.7087

y_{pq}1/a(1/a-1) = j0.28278

Assignment #1 (Hint)

We can construct Y bus matrix as follows

$$Y_{bus} = \begin{bmatrix} -j10 & j9.7087 & 0 \\ j9.7087 & 0.99099 - j19.27691 & -0.99099 + j9.90099 \\ 0 & -0.99099 + j9.90099 & 0.99099 - j9.85099 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 10\angle 90^\circ & 9.7\angle 90^\circ & 0 \\ 9.7\angle 90^\circ & 19.3\angle -87.6^\circ & 9.95\angle 95.71^\circ \\ 0 & 9.95\angle 95.71^\circ & 9.9\angle -84.26^\circ \end{bmatrix}$$

In Bus 1 (slack bus), IV_1I=1, δ_1 =0 ; but P_{G1} and Q_{G1} are unknown.

Bus 2 is PQ bus, P_{D2} = 0.2, Q_{D2} = 0.1 ; but $|V_2|$ and δ_2 are unknown.

Bus 3 is PV bus, P_{G3} = 0, P_{D3} = 0.5, Q_{D3} = 0.2, but Q_{G3} and δ_3 are unknown.

Newton-Raphson Method

To solve for x from f(x) = K

$f_1(x_1,x_2) = K_1$

$f_2(x_1,x_2) = K_2$

You are given two functions (f_1 and f_2) and asked to find x_1 and x_2.

$K_1 = f_1(x_1,x_2) = f_1(x_1^{(0)}+\Delta x_1^{(0)},x_2^{(0)}+\Delta x_2^{(0)})$

$K_2 = f_2(x_1,x_2) = f_2(x_1^{(0)}+\Delta x_1^{(0)},x_2^{(0)}+\Delta x_2^{(0)})$

You guess x_1,
then you need to find Δx_1.

By Taylor's Expansion, you get:

$K_1 = f_1(x_1,x_2) = f_1(x_1^{(0)},x_2^{(0)})+\Delta x_1^{(0)}[\partial f_1/\partial x_1]^{(0)}+ \Delta x_2^{(0)}[\partial f_1/\partial x_2]^{(0)}$

$K_2 = f_2(x_1,x_2) = f_2(x_1^{(0)},x_2^{(0)})+\Delta x_1^{(0)}[\partial f_2/\partial x_1]^{(0)}+ \Delta x_2^{(0)}[\partial f_2/\partial x_2]^{(0)}$

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Newton-Raphson Method

To solve for x from f(x) = K:

Reform into matrix form, you get :

$$\begin{pmatrix} K_1 - f_1(x_1^{(0)},x_2^{(0)}) \\ K_2 - f_2(x_1^{(0)},x_2^{(0)}) \end{pmatrix} = \begin{pmatrix} [\partial f_1/\partial x_1]^{(0)} & [\partial f_1/\partial x_2]^{(0)} \\ [\partial f_2/\partial x_1]^{(0)} & [\partial f_2/\partial x_2]^{(0)} \end{pmatrix} \begin{pmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{pmatrix}$$

$$\begin{pmatrix} \Delta K_1^{(0)} \\ \Delta K_2^{(0)} \end{pmatrix} = \mathbf{J}^{(0)} \begin{pmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{pmatrix} \Rightarrow \begin{pmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{pmatrix} = \underset{\text{known}}{\mathbf{J}^{\mathbf{-1}(0)}} \underset{\text{known}}{\begin{pmatrix} \Delta K_1^{(0)} \\ \Delta K_2^{(0)} \end{pmatrix}}$$

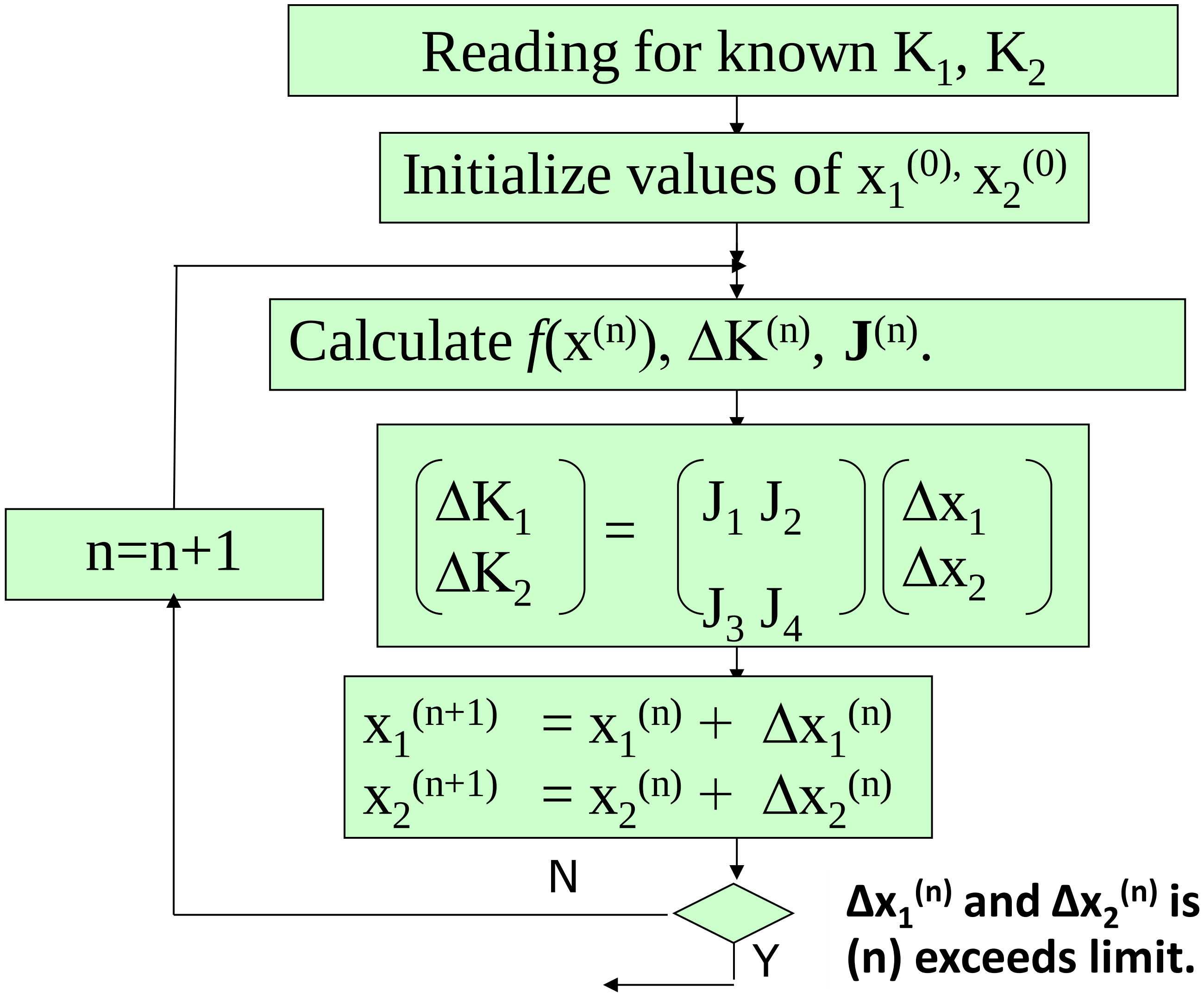
$$x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)}$$

$$x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)}$$

In general,

$$\boxed{\begin{aligned} \mathbf{x_1^{(n+1)}} &= \mathbf{x_1^{(n)}} + \mathbf{\Delta x_1^{(n)}} \\ \mathbf{x_2^{(n+1)}} &= \mathbf{x_2^{(n)}} + \mathbf{\Delta x_2^{(n)}} \end{aligned}}$$

Newton-Raphson Method



Reading for known K_1, K_2

Initialize values of $x_1^{(0),}$ $x_2^{(0)}$

Calculate $f(x^{(n)})$, $\Delta K^{(n)}$, $\mathbf{J}^{(n)}$.

$$\begin{pmatrix} \Delta K_1 \\ \Delta K_2 \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix}$$

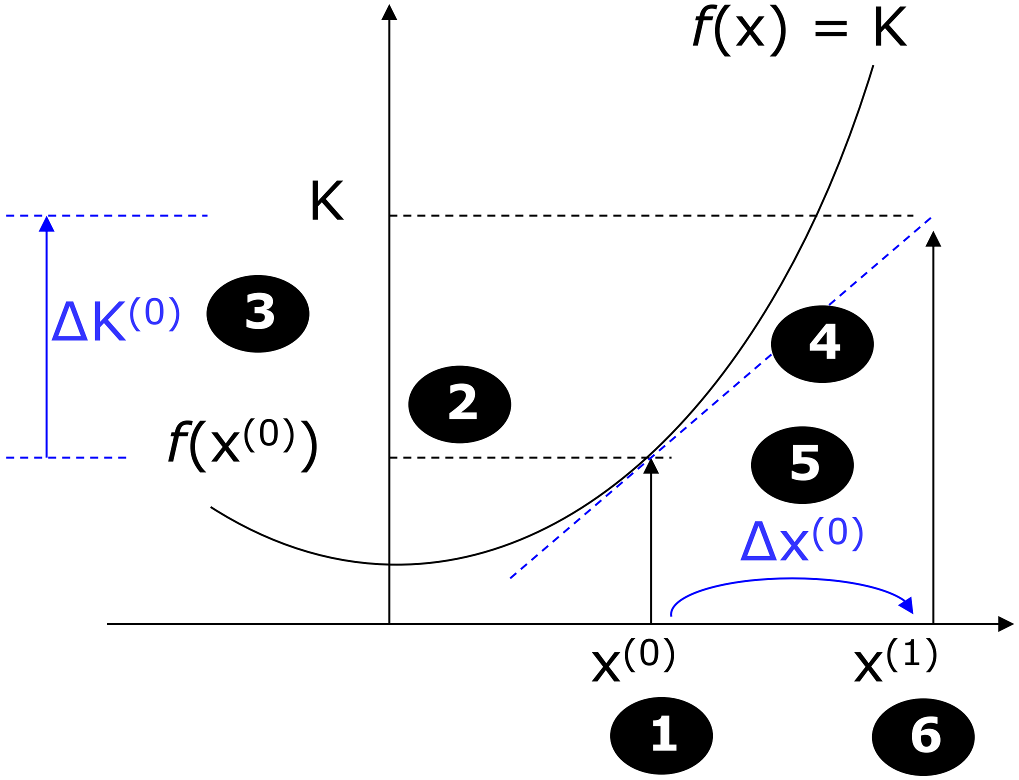
$$x_1^{(n+1)} = x_1^{(n)} + \Delta x_1^{(n)}$$
$$x_2^{(n+1)} = x_2^{(n)} + \Delta x_2^{(n)}$$

n=n+1

$\Delta x_1^{(n)}$ and $\Delta x_2^{(n)}$ is very small or iteration (n) exceeds limit.

Newton-Raphson Method

To solve for x from f(x) = K



- ❶ Arbitrarily set $x^{(0)}$
- ❷ Calculate $f(x^{(0)})$
- ❸ Calculate $\Delta K^{(0)} = K - f(x^{(0)})$
- ❹ Calculate $\mathbf{J}^{(0)} = [\partial f/\partial x]^{(0)}$
- ❺ Calculate $\Delta x^{(0)} = \mathbf{J}^{-1(0)} \Delta K^{(0)}$
- ❻ Calculate $x^{(1)} = x^{(0)} + \Delta x^{(0)}$

- ❷ Calculate $f(x^{(1)})$ …

Assignment #2

Create the shortest M-file (or Sci-file) to construct a Y-Matrix from line data given: 6-bus, 11-line system.

Line No.	From Bus	To Bus	R (p.u.)	X (p.u.)	½B (p.u.)
1	1	2	0.10	0.20	0.02
2	1	4	0.05	0.20	0.02
3	1	5	0.08	0.30	0.03
4	2	3	0.05	0.25	0.03
5	2	4	0.05	0.10	0.01
6	2	5	0.10	0.30	0.02
7	2	6	0.07	0.20	0.025
8	3	5	0.12	0.26	0.025
9	3	6	0.02	0.10	0.01
10	4	5	0.20	0.40	0.04
11	5	6	0.10	0.30	0.03

Assignment #3

Using the system given in Assignment (2), how would you place a capacitor at bus 6? Assume that the capacitor has susceptance of 0.1 p.u.

How does it affect the diagram and the Y-matrix? And what do you do on the Y-matrix if a line between buses 2 and 6 (line no.7) is switched off?

Please provide your **descriptive comments** on this inquiry. Providing the resulting values of Y-matrix or a programming source code.

Assignment #4

Write a M-file or Sci-file to solve the below problem by using Newton-Raphson method.

F1(X) = $3X_1 + 4X_1X_2 + 5X_3 =$ 70

F2(X) = $4X_1 + 7X_2 + 3X_3 =$ 51

F3(X) = $5X_1 + 6X_2 + 5X_3 =$ 61

NR Application to Power Flow

We first need to rewrite complex power equations as equations with real coefficients

$$\mathrm{S_i} = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

These can be derived by defining

$$Y_{ik} \triangleq G_{ik} + jB_{ik}$$

$$V_i \triangleq |V_i| e^{j\partial_i} = |V_i| \angle \partial_i$$

$$\partial_{ik} \triangleq \partial_i - \partial_k$$

Recall $\mathrm{e}^{\mathrm{j}\partial} = \cos\partial + j\sin\partial$

Real Power Balance Equations

$$\mathrm{S_i} = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} = \sum_{k=1}^{n} |V_i||V_k| \, e^{j\partial_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\partial_{ik} + j\sin\partial_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\partial_{ik} + B_{ik}\sin\partial_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\partial_{ik} - B_{ik}\cos\partial_{ik}) = Q_{Gi} - Q_{Di}$$

Newton-Raphson Power Flow

In the Newton-Raphson power flow we use Newton's method to determine the voltage magnitude and angle at each bus in the power system.

We need to solve the power balance equations

$$\mathrm{P_i} \;=\; \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\partial_{ik} + B_{ik}\sin\partial_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} \;=\; \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\partial_{ik} - B_{ik}\cos\partial_{ik}) = Q_{Gi} - Q_{Di}$$

Power Flow Variables

Assume the slack bus is the first bus (with a fixed voltage angle/magnitude). We then need to determine the voltage angle/magnitude at the other buses.

$$\mathbf{x} = \begin{bmatrix} \partial_2 \\ \vdots \\ \partial_{\mathrm{n}} \\ |V_2| \\ \vdots \\ |V_n| \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_{G2} + P_{D2} \\ \vdots \\ P_n(\mathbf{x}) - P_{Gn} + P_{Dn} \\ Q_2(\mathbf{x}) - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n(\mathbf{x}) - Q_{Gn} + Q_{Dn} \end{bmatrix}$$

N-R Power Flow Solution

The power flow is solved using the same procedure discussed last time:

```
Set $v = 0$; make an initial guess of $\mathbf{x}$, $\mathbf{x}^{(v)}$
While $\left\| \mathbf{f}(\mathbf{x}^{(v)}) \right\| > \varepsilon$ Do
    $\mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} - \mathbf{J}(\mathbf{x}^{(v)})^{-1}\mathbf{f}(\mathbf{x}^{(v)})$
    $v = v+1$
End While
```

Power Flow Jacobian Matrix

The most difficult part of the algorithm is determining and inverting the n by n Jacobian matrix, $\mathbf{J}(\mathbf{x})$

$$\mathbf{J}(\mathbf{x}) \quad = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \frac{\partial f_n(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}$$

Power Flow Jacobian Matrix, cont'd

Jacobian elements are calculated by differentiating each function, $\mathrm{f_i}(\mathbf{x})$, with respect to each variable. For example, if $\mathrm{f_i}(\mathbf{x})$ is the bus i real power equation

$$\mathrm{f_i}(x) = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\partial_{ik} + B_{ik}\sin\partial_{ik}) - P_{Gi} + P_{Di}$$

$$\frac{\partial \mathrm{f_i}(x)}{\partial \theta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(-G_{ik}\sin\partial_{ik} + B_{ik}\cos\partial_{ik})$$

$$\frac{\partial \mathrm{f_i}(x)}{\partial \theta_j} = |V_i||V_j|(G_{ik}\sin\partial_{ik} - B_{ik}\cos\partial_{ik}) \quad (j \neq i)$$

Flow Chart

Construct Y-Matrix

↓

Specify Bus Type

↓

Construct the structure of:

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix} \begin{pmatrix} \Delta\delta \\ \Delta|V| \end{pmatrix}$$

With eliminating the known $|V|, \delta$ variables

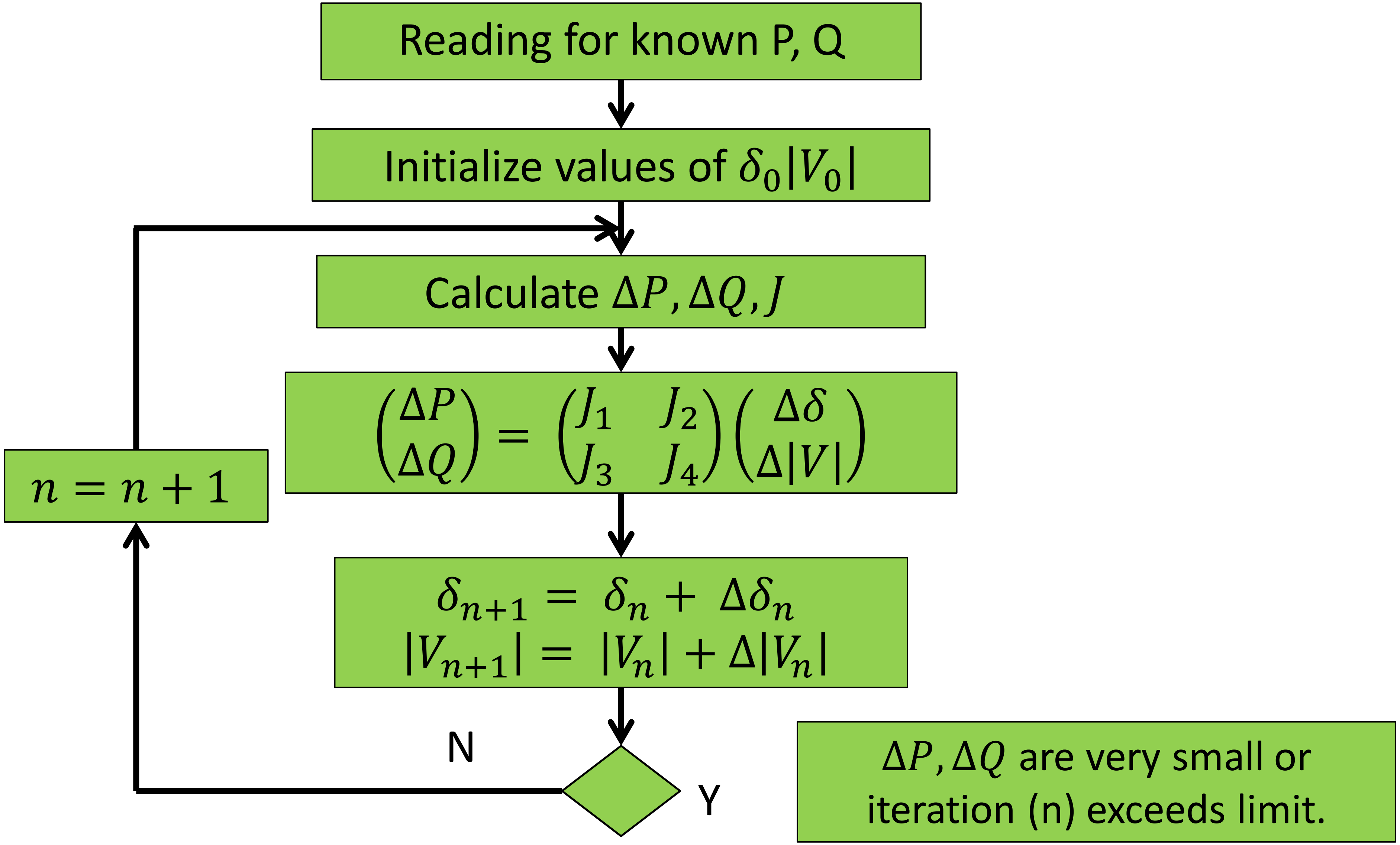
↓

Solve for the unknown $\delta, |V|$

↓

Calculate for unknown P and Q, Line Flow, etc.

Flow Chart



Reading for known P, Q

Initialize values of $\delta_0 |V_0|$

Calculate $\Delta P, \Delta Q, J$

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix} \begin{pmatrix} \Delta \delta \\ \Delta |V| \end{pmatrix}$$

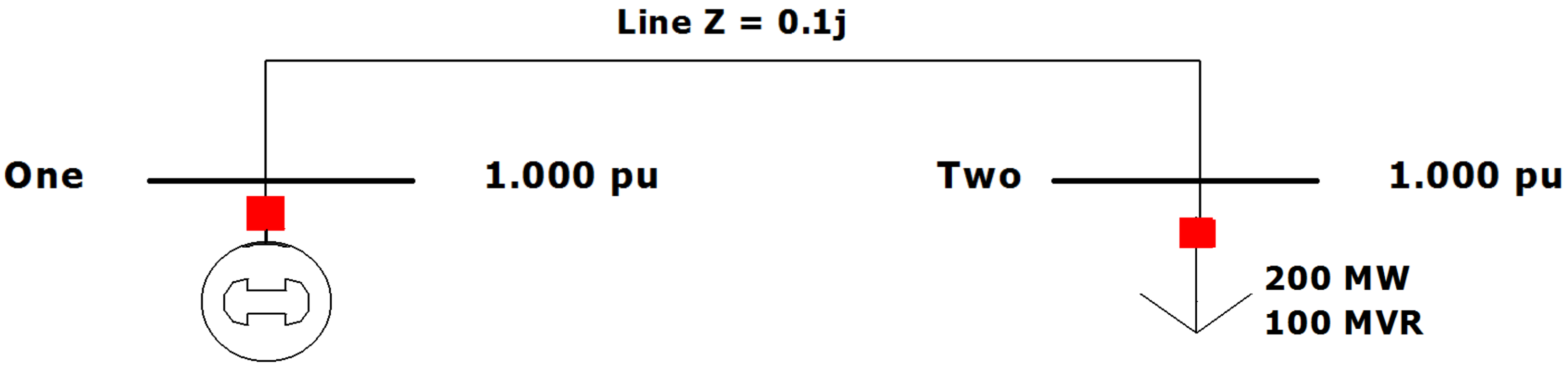
$$\delta_{n+1} = \delta_n + \Delta\delta_n$$
$$|V_{n+1}| = |V_n| + \Delta|V_n|$$

$\Delta P, \Delta Q$ are very small or iteration (n) exceeds limit.

$n = n + 1$

Two Bus Newton-Raphson Example

For the two bus power system shown below, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two. Assume that bus one is the slack and S_{Base} = 100 MVA.



$$\mathbf{x} = \begin{bmatrix} \partial_2 \\ |V_2| \end{bmatrix} \qquad \mathbf{Y}_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

Two Bus Example, cont'd

General power balance equations

$$\mathrm{P_i} \;=\; \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\partial_{ik} + B_{ik}\sin\partial_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} \;=\; \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\partial_{ik} - B_{ik}\cos\partial_{ik}) = Q_{Gi} - Q_{Di}$$

Bus two power balance equations

$$|V_2||V_1|(10\sin\partial_2) + 2.0 \;=\; 0$$

$$|V_2||V_1|(-10\cos\partial_2) + |V_2|^2(10) + 1.0 = 0$$

Two Bus Example, cont'd

$$P_2(\mathbf{x}) = |V_2|(10\sin\partial_2) + 2.0 = 0$$

$$Q_2(\mathbf{x}) = |V_2|(-10\cos\partial_2) + |V_2|^2(10) + 1.0 = 0$$

Now calculate the power flow Jacobian

$$J(\mathbf{x}) = \begin{bmatrix} \dfrac{\partial P_2(\mathbf{x})}{\partial\partial_2} & \dfrac{\partial P_2(\mathbf{x})}{\partial|V|_2} \\ \dfrac{\partial Q_2(\mathbf{x})}{\partial\partial_2} & \dfrac{\partial Q_2(\mathbf{x})}{\partial|V|_2} \end{bmatrix}$$

$$= \begin{bmatrix} 10|V_2|\cos\partial_2 & 10\sin\partial_2 \\ 10|V_2|\sin\partial_2 & -10\cos\partial_2 + 20|V_2| \end{bmatrix}$$

Smart Grid – Modeling and Control

Two Bus Example, First Iteration

$$\text{Set } v = 0, \text{ guess } \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Calculate

$$\text{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\partial_2) + 2.0 \\ |V_2|(-10\cos\partial_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos\partial_2 & 10\sin\partial_2 \\ 10|V_2|\sin\partial_2 & -10\cos\partial_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$\text{Solve } \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$$

Two Bus Example, Next Iterations

$$\mathrm{f}(\mathbf{x}^{(1)}) = \begin{bmatrix} 0.9(10\sin(-0.2)) + 2.0 \\ 0.9(-10\cos(-0.2)) + 0.9^2 \times 10 + 1.0 \end{bmatrix} = \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix}$$

$$\mathbf{J}(\mathbf{x}^{(1)}) = \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}$$

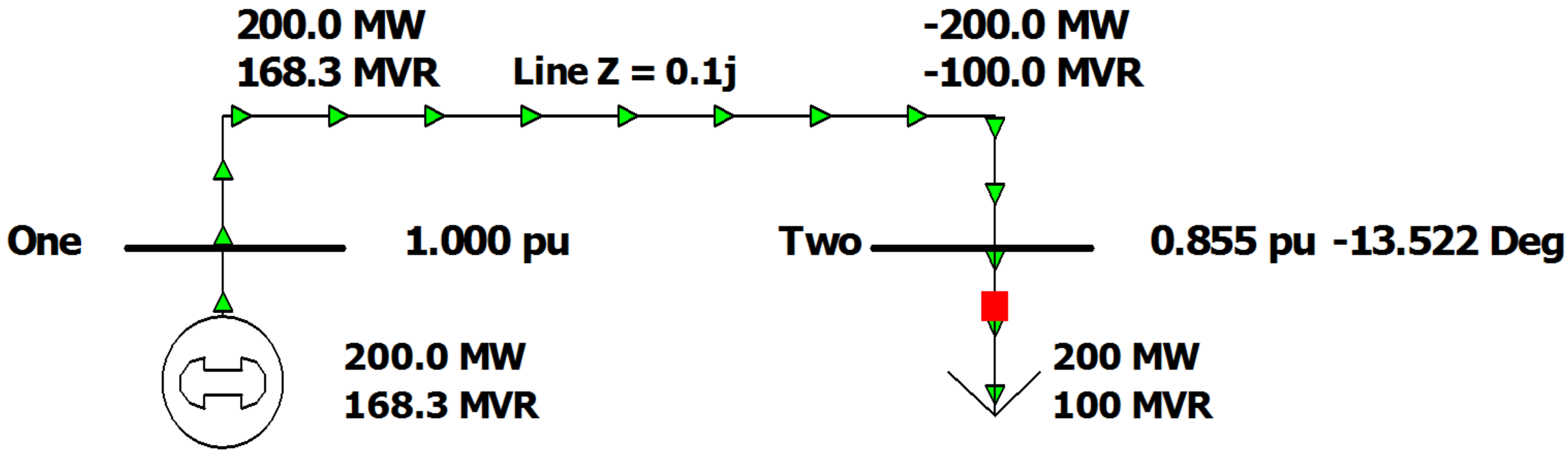
$$\mathbf{x}^{(2)} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}^{-1} \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix} = \begin{bmatrix} -0.233 \\ 0.8586 \end{bmatrix}$$

$$\mathrm{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.0145 \\ 0.0190 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} -0.236 \\ 0.8554 \end{bmatrix}$$

$$\mathrm{f}(\mathbf{x}^{(3)}) = \begin{bmatrix} 0.0000906 \\ 0.0001175 \end{bmatrix} \qquad \text{Done!} \qquad V_2 = 0.8554\angle -13.52°$$

Two Bus Solved Values

Once the voltage angle and magnitude at bus 2 are known, we can calculate all the other system values, such as the line flows and the generator reactive power output.



Two Bus Case Low Voltage Solution

This case actually has two solutions! The second "low voltage" is found by using a low initial guess.

$$\text{Set } v = 0, \text{ guess } \mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$$

Calculate

$$\mathrm{f}(\mathbf{x}^{(0)}) = \begin{bmatrix} |V_2|(10\sin\partial_2) + 2.0 \\ |V_2|(-10\cos\partial_2) + |V_2|^2(10) + 1.0 \end{bmatrix} = \begin{bmatrix} 2 \\ -0.875 \end{bmatrix}$$

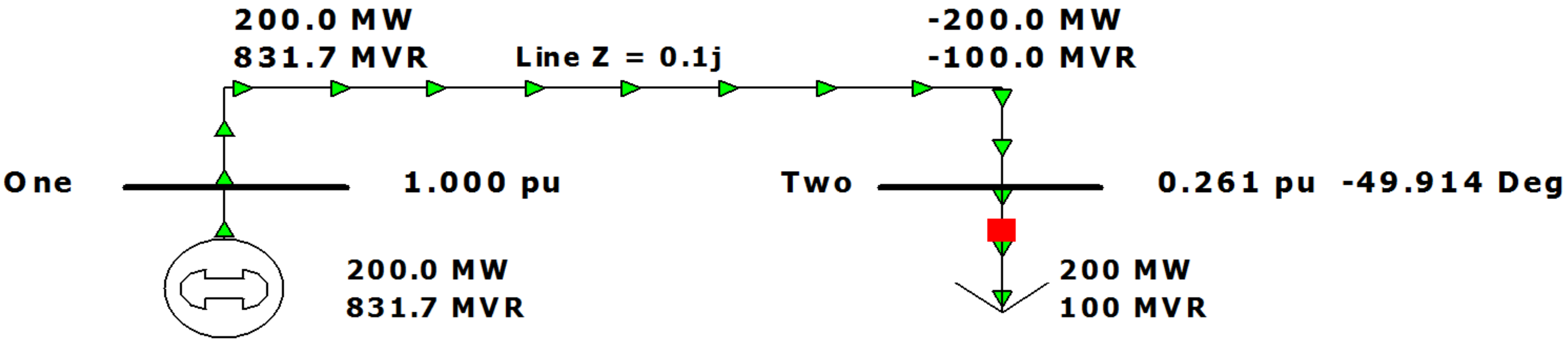
$$\mathbf{J}(\mathbf{x}^{(0)}) = \begin{bmatrix} 10|V_2|\cos\partial_2 & 10\sin\partial_2 \\ 10|V_2|\sin\partial_2 & -10\cos\partial_2 + 20|V_2| \end{bmatrix} = \begin{bmatrix} 2.5 & 0 \\ 0 & -5 \end{bmatrix}$$

Low Voltage Solution, cont'd

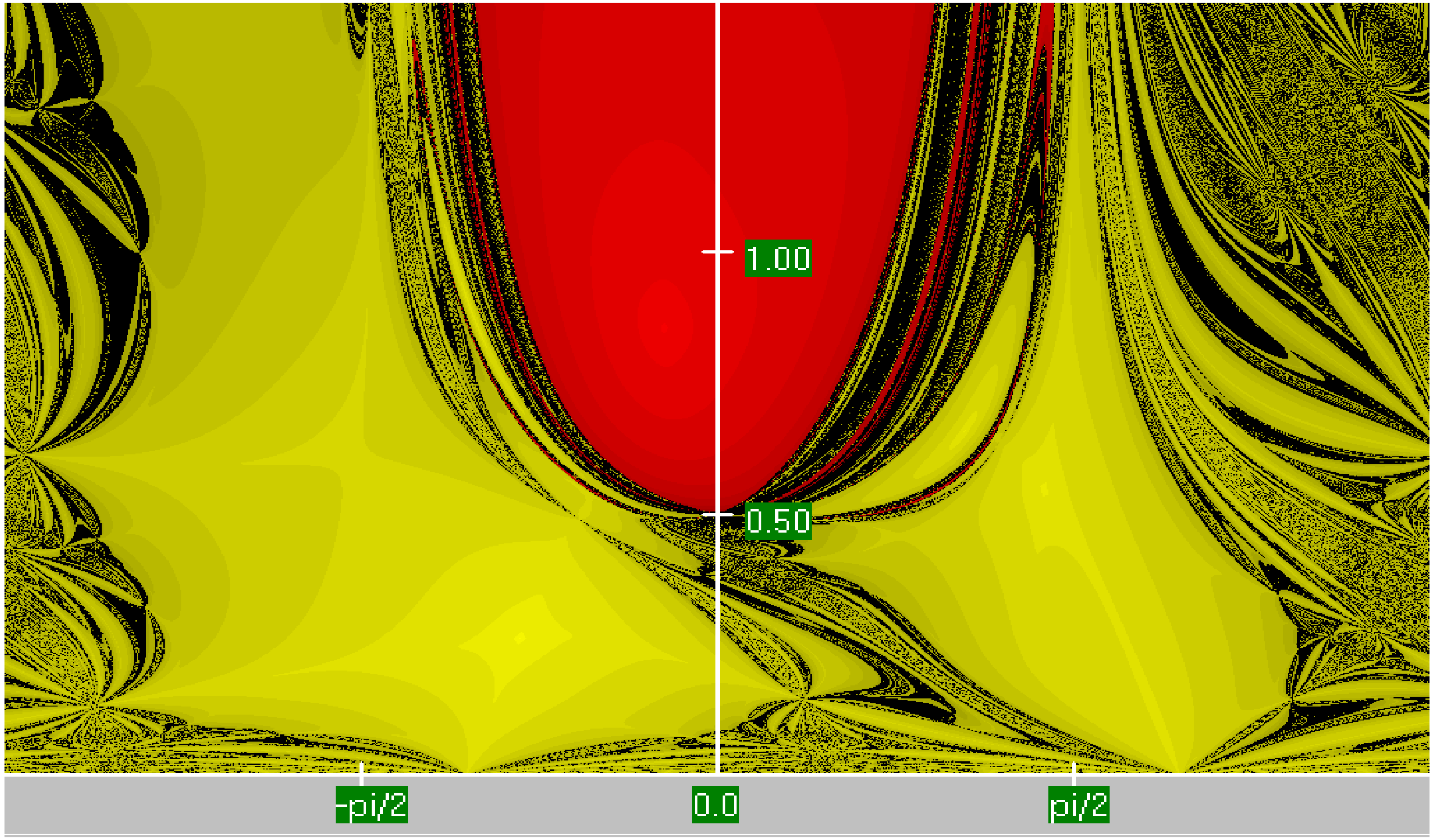
$$\text{Solve} \quad \mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix} - \begin{bmatrix} 2.5 & 0 \\ 0 & -5 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -0.875 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0.075 \end{bmatrix}$$

$$\mathbf{f}(\mathbf{x}^{(2)}) = \begin{bmatrix} 1.462 \\ 0.534 \end{bmatrix} \quad \mathbf{x}^{(2)} = \begin{bmatrix} -1.42 \\ 0.2336 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} -0.921 \\ 0.220 \end{bmatrix}$$

Low voltage solution



Two Bus Region of Convergence



Red region converges to the high voltage solution, while the yellow region converges to the low voltage solution

Figure 1.) Image shows the region of convergence for different initial guesses of bus 2 angle (x-axis) and magnitude (y-axis)

Smart Grid – Modeling and Control

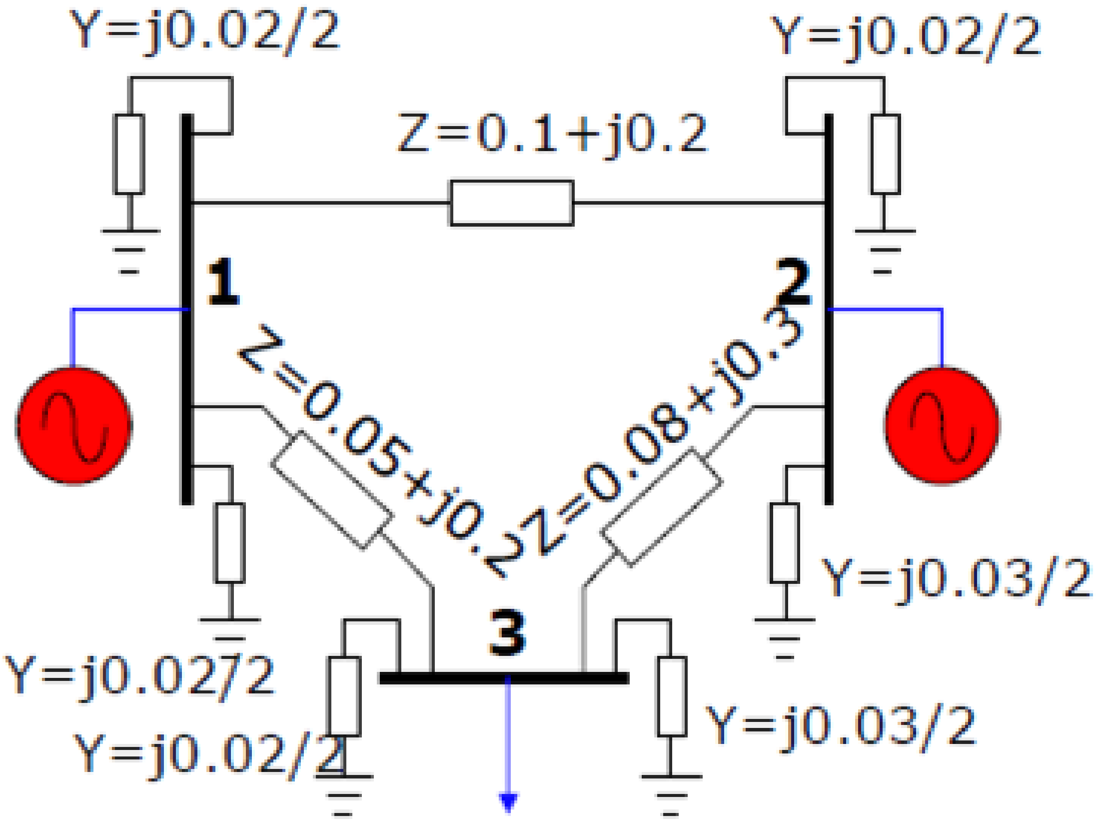
Example of a Three Bus

Line Data

Line No.	From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)
1	1	2	0.1	0.2	0.02
2	1	3	0.05	0.2	0.02
3	2	3	0.08	0.3	0.03

Bus Data

Bus No.	Bus Type	V	δ	P_G	Q_G	P_L	Q_L
1	Slack	1.05	0	-	-	0.1	0.1
2	PV	1.03	-	0.4	-	0.2	0.1
3	PQ	-	-	0	0	0.7	0.4



Y Bus Formulation

$$
\mathbf{Y}=\begin{pmatrix}
\frac{1}{0.1+j0.2}+\frac{1}{0.05+j0.2}+j0.02/2+j0.02/2 & -\frac{1}{0.1+j0.2} & -\frac{1}{0.05+j0.2} \\
-\frac{1}{0.1+j0.2} & \frac{1}{0.1+j0.2}+\frac{1}{0.08+j0.3}+j0.02/2+j0.03/2 & -\frac{1}{0.08+j0.3} \\
-\frac{1}{0.05+j0.2} & -\frac{1}{0.08+j0.3} & \frac{1}{0.05+j0.2}+\frac{1}{0.08+j0.3}+j0.02/2+j0.03/2
\end{pmatrix}
$$

$$
=\begin{pmatrix}
3.1765-8.6859i & -2.0000+4.0000i & -1.1765+4.7059i \\
-2.0000+4.0000i & 2.8299-7.0920i & -0.8299+3.1120i \\
-1.1765+4.7059i & -0.8299+3.1120i & 2.0063-7.7979i
\end{pmatrix}
$$

$$
=\begin{pmatrix}
9.2485\,/-69.91^{\circ} & 4.4721\,/116.57^{\circ} & 4.8507\,/104.04^{\circ} \\
4.4721\,/116.57^{\circ} & 7.6358\,/-68.25^{\circ} & 3.2208\,/104.93^{\circ} \\
4.8507\,/104.04^{\circ} & 3.2208\,/104.93^{\circ} & 8.0519\,/75.57^{\circ}
\end{pmatrix}
$$

Updating Voltage and Angle

- Structure of Equation

$$\begin{pmatrix} \Delta\delta \\ \Delta|V| \end{pmatrix}^{n} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix}^{-1n} \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}^{n}$$

$$\begin{pmatrix} \Delta\delta \\ \Delta|V| \end{pmatrix}^{n} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix}^{-1n} \begin{pmatrix} P_{schedule} & - P(\delta^{n}, |V^{n}| \\ Q_{schedule} & - Q(\delta^{n}, |V^{n}| \end{pmatrix}$$

- Updating the Value of Variables

$$\delta^{(n+1)} = \delta^{(n)} + \Delta\delta^{(n)}$$

$$\left|V^{(n+1)}\right| = \left|V^{(n)}\right| + \Delta\left|V^{(n)}\right|$$

Smart Grid – Modeling and Control

Jacobian Structure (1)

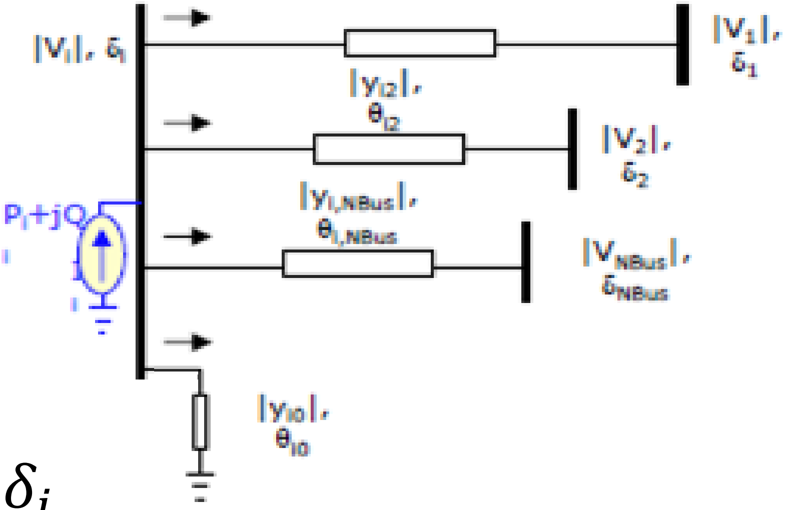
Reviewing Power Flow Equation

$$P_i - jQ_i = {S_i}^* = V_i^* I_i = V_i^* \sum_{j=1}^{NBus} Y_{ij} V_j$$

$$V_i^* \sum_{j=1}^{NBus} Y_{ij} V_j = |V_i| \angle -\ \delta_i \sum_{j=1}^{NBus} |Y_{ij}||V_j| \angle \theta_{ij} + \delta_j$$

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$



$$V_i = |V_i| \angle \partial_i$$

$$V_j = |V_j| \angle \partial_j$$

$$Yij = |Y_{ij}| \angle \theta_{ij}$$

Jacobian Structure (2)

$$P_i = \sum_{j=1}^{n}|V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix}\begin{pmatrix} \Delta\delta \\ \Delta|V| \end{pmatrix}$$

$$If\ J_1 = \frac{\partial P}{\partial \delta}$$

(On diagonal) $\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{n}|V_i||V_j||Y_{ij}|\sin\left(\theta_{ij} - \delta_i + \delta_j\right)$

(Off diagonal) $\frac{\partial P_i}{\partial \delta_i} = -|V_i||V_j||Y_{ij}|\sin\left(\theta_{ij} - \delta_i + \delta_j\right), \mathrm{i} \neq j$

$$If\ J_2 = \frac{\partial P}{\partial |V|}$$

(On diagonal)

$$\frac{\partial P_i}{\partial |V|_i} = 2|V_i||Y_{ii}|\cos\theta_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n}|V_j||Y_{ij}|\,cos\left(\theta_{ij} - \delta_i + \delta_j\right)$$

(Off diagonal) $\frac{\partial P_i}{\partial |V|_i} = |V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j), i \neq j$

$$V_i = |V_i|\angle\partial_i$$

$$V_j = |V_j|\angle\partial_j$$

$$Yij = |Y_{ij}|\angle\theta_{ij}$$

Jacobian Structure (3)

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix} \begin{pmatrix} \Delta\delta \\ \Delta|V| \end{pmatrix}$$

$$If\ J_1 = \frac{\partial Q}{\partial \delta}$$

(On diagonal) $\frac{\partial Q_i}{\partial \delta_i} = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$

(Off diagonal) $\frac{\partial Q_i}{\partial \delta_i} = -|V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)\ , \mathrm{i} \neq j$

$$V_i = |V_i| \angle \partial_i$$

$$V_j = |V_j| \angle \partial_j$$

$$Yij = |Y_{ij}| \angle \theta_{ij}$$

$$If\ J_2 = \frac{\partial Q}{\partial |V|}$$

(On diagonal)

$$\frac{\partial Q_i}{\partial |V|_i} = 2|V_i||Y_{ii}| \sin \theta_{ii} - \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_j||Y_{ij}|\, sin(\theta_{ij} - \delta_i + \delta_j)$$

(Off diagonal) $\frac{\partial Q_i}{\partial |V|_i} = |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)\ , i \neq j$

Jacobian Structure (4)

$$\left(\begin{array}{c} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \hline \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{array}\right) = \left(\begin{array}{ccc|ccc} \boxed{\frac{\partial P_1}{\partial \delta_1}} & \frac{\partial P_1}{\partial \delta_2} & \frac{\partial P_1}{\partial \delta_3} & \boxed{\frac{\partial P_1}{\partial |V_1|}} & \frac{\partial P_1}{\partial |V_2|} & \frac{\partial P_1}{\partial |V_3|} \\ \frac{\partial P_2}{\partial \delta_1} & \boxed{\frac{\partial P_2}{\partial \delta_2}} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_1|} & \boxed{\frac{\partial P_2}{\partial |V_2|}} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_1} & \frac{\partial P_3}{\partial \delta_2} & \boxed{\frac{\partial P_3}{\partial \delta_3}} & \frac{\partial P_3}{\partial |V_1|} & \frac{\partial P_3}{\partial |V_2|} & \boxed{\frac{\partial P_3}{\partial |V_3|}} \\ \hline \boxed{\frac{\partial Q_1}{\partial \delta_1}} & \frac{\partial Q_1}{\partial \delta_2} & \frac{\partial Q_1}{\partial \delta_3} & \boxed{\frac{\partial Q_1}{\partial |V_1|}} & \frac{\partial Q_1}{\partial |V_2|} & \frac{\partial Q_1}{\partial |V_3|} \\ \frac{\partial Q_2}{\partial \delta_1} & \boxed{\frac{\partial Q_2}{\partial \delta_2}} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_1|} & \boxed{\frac{\partial Q_2}{\partial |V_2|}} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_1} & \frac{\partial Q_3}{\partial \delta_2} & \boxed{\frac{\partial Q_3}{\partial \delta_3}} & \frac{\partial Q_3}{\partial |V_1|} & \frac{\partial Q_3}{\partial |V_2|} & \boxed{\frac{\partial Q_3}{\partial |V_3|}} \end{array}\right) \left(\begin{array}{c} \Delta \delta_1 \\ \Delta \delta_2 \\ \Delta \delta_3 \\ \hline \Delta |V_1| \\ \Delta |V_2| \\ \Delta |V_3| \end{array}\right)$$

Consider:

- **Each J_1, J_2, J_3, and J_4 element**
- **On diagonal and off-diagonal elements**

Jacobian Structure (5)

$$
\begin{pmatrix} \cancel{\Delta P_1} \\ \Delta P_2 \\ \Delta P_3 \\ \hdashline \cancel{\Delta Q_1} \\ \cancel{\Delta Q_2} \\ \Delta Q_3 \end{pmatrix}
=
\left(\begin{array}{ccc:ccc}
\cancel{\frac{\partial P_1}{\partial \delta_1}} & \cancel{\frac{\partial P_1}{\partial \delta_2}} & \cancel{\frac{\partial P_1}{\partial \delta_3}} & \cancel{\frac{\partial P_1}{\partial |V_1|}} & \cancel{\frac{\partial P_1}{\partial |V_2|}} & \cancel{\frac{\partial P_1}{\partial |V_3|}} \\
\cancel{\frac{\partial P_2}{\partial \delta_1}} & \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \cancel{\frac{\partial P_2}{\partial |V_1|}} & \cancel{\frac{\partial P_2}{\partial |V_2|}} & \frac{\partial P_2}{\partial |V_3|} \\
\cancel{\frac{\partial P_3}{\partial \delta_1}} & \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cancel{\frac{\partial P_3}{\partial |V_1|}} & \cancel{\frac{\partial P_3}{\partial |V_2|}} & \frac{\partial P_3}{\partial |V_3|} \\
\hdashline
\cancel{\frac{\partial Q_1}{\partial \delta_1}} & \cancel{\frac{\partial Q_1}{\partial \delta_2}} & \cancel{\frac{\partial Q_1}{\partial \delta_3}} & \cancel{\frac{\partial Q_1}{\partial |V_1|}} & \cancel{\frac{\partial Q_1}{\partial |V_2|}} & \cancel{\frac{\partial Q_1}{\partial |V_3|}} \\
\cancel{\frac{\partial Q_2}{\partial \delta_1}} & \cancel{\frac{\partial Q_2}{\partial \delta_2}} & \cancel{\frac{\partial Q_2}{\partial \delta_3}} & \cancel{\frac{\partial Q_2}{\partial |V_1|}} & \cancel{\frac{\partial Q_2}{\partial |V_2|}} & \cancel{\frac{\partial Q_2}{\partial |V_3|}} \\
\cancel{\frac{\partial Q_3}{\partial \delta_1}} & \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \cancel{\frac{\partial Q_3}{\partial |V_1|}} & \cancel{\frac{\partial Q_3}{\partial |V_2|}} & \frac{\partial Q_3}{\partial |V_3|}
\end{array}\right)
\begin{pmatrix} \cancel{\Delta \delta_1} \\ \Delta \delta_2 \\ \Delta \delta_3 \\ \hdashline \cancel{\Delta |V_1|} \\ \cancel{\Delta |V_2|} \\ \Delta |V_3| \end{pmatrix}
$$

- Bus 1 is swing bus, δ_1 and $|V_1|$ are no longer variables.

- Bus 2 is PV bus, $|V_2|$ is no longer variable.

Jacobian Structure (6)

$$\left(\begin{array}{c} \Delta P_2 \\ \Delta P_3 \\ \hline \Delta Q_3 \end{array}\right) = \left(\begin{array}{cc|c} \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial \delta_3} & \dfrac{\partial P_2}{\partial |V_3|} \\ \dfrac{\partial P_3}{\partial \delta_2} & \dfrac{\partial P_3}{\partial \delta_3} & \dfrac{\partial P_3}{\partial |V_3|} \\ \hline \dfrac{\partial Q_3}{\partial \delta_2} & \dfrac{\partial Q_3}{\partial \delta_3} & \dfrac{\partial Q_3}{\partial |V_3|} \end{array}\right) \left(\begin{array}{c} \Delta \delta_2 \\ \Delta \delta_3 \\ \hline \Delta |V_3| \end{array}\right)$$

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix} \begin{pmatrix} \Delta \delta \\ \Delta |V| \end{pmatrix}$$

Jacobian Structure (7)

- With 3-Bus Example: Element Formulation

$$\begin{pmatrix}\Delta P_2\\ \Delta P_3\\ \Delta Q_3\end{pmatrix}=\left(\begin{array}{c:c}
\dfrac{\partial P_i}{\partial \delta_i}=\sum_{\substack{j=1\\ j\neq i}}^{n}|V_i||V_j||Y_{ij}|\sin\left(\theta_{ij}-\delta_i+\delta_j\right) & \dfrac{\partial P_i}{\partial |V|_i}=2|V_i||Y_{ii}|\cos\theta_{ii}+\sum_{\substack{j=1\\ j\neq i}}^{n}|V_j||Y_{ij}|\,cos\left(\theta_{ij}-\delta_i+\delta_j\right)\\
\dfrac{\partial P_i}{\partial \delta_i}=-|V_i||V_j||Y_{ij}|\sin(\theta_{ij}-\delta_i+\delta_j) & \dfrac{\partial P_i}{\partial |V|_i}=|V_j||Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j)\\
\hdashline
\dfrac{\partial Q_i}{\partial \delta_i}=\sum_{\substack{j=1\\ j\neq i}}^{n}|V_i||V_j||Y_{ij}|\cos\left(\theta_{ij}-\delta_i+\delta_j\right) & \dfrac{\partial Q_i}{\partial |V|_i}=-2|V_i||Y_{ii}|\cos\theta_{ii}-\sum_{\substack{j=1\\ j\neq i}}^{n}|V_j||Y_{ij}|\,sin\left(\theta_{ij}-\delta_i+\delta_j\right)\\
\dfrac{\partial Q_i}{\partial \delta_i}=-|V_i||V_j||Y_{ij}|\cos(\theta_{ij}-\delta_i+\delta_j) & \dfrac{\partial Q_i}{\partial |V|_i}=-|V_j||Y_{ij}|\sin(\theta_{ij}-\delta_i+\delta_j)
\end{array}\right)\begin{pmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta|V_3|\end{pmatrix}$$

- Bus 1 is swing bus, δ_1 and $|V_1|$ are no longer variables.

- Bus 2 is PV bus, $|V_2|$ is no longer variable.

Power Flow Analysis Solving

- δ_i, $|V_i|$ Initialization

  The δ_i of all buses shall be initialized as 0 degree.

  The $|V_i|$ of all buses shall be initialized as 1.0 p.u. except the swing and PV bus that they will be initialized with the specified values.

$$V_i = 1.0\angle 0^{\circ} \qquad ; i \in \text{all bus}$$

- 3-Bus Example Result

NRPF Result

Bus No.	V	δ (°)
1	1.0500	0
2	1.0000	0.408
3	0.9568	-4.056

Power Flow Analysis Solving

$$J=\left(\begin{array}{cc|c}\frac{\partial P_2}{\partial\delta_2} & \frac{\partial P_2}{\partial\delta_3} & \frac{\partial P_2}{\partial|V_3|}\\ \frac{\partial P_3}{\partial\delta_2} & \frac{\partial P_3}{\partial\delta_3} & \frac{\partial P_3}{\partial|V_3|}\\ \hline \frac{\partial Q_3}{\partial\delta_2} & \frac{\partial Q_3}{\partial\delta_3} & \frac{\partial Q_3}{\partial|V_3|}\end{array}\right)=\begin{pmatrix}7.2454 & -3.0305 & -0.5851\\ -2.9069 & 7.5393 & 1.1882\\ 1.0234 & -2.5369 & 7.0433\end{pmatrix}$$

$$\frac{\partial P_i}{\partial\delta_i}=\sum_{\substack{j=1\\ j\neq i}}|V_i||V_j||Y_{ij}|\sin(\theta_{ij}-\delta_i+\delta_j)|$$

$$\frac{\partial P_2}{\partial\delta_2}=|V_2||V_1||Y_{21}|\sin(\theta_{21}-\delta_2+\delta_1)+|V_2||V_3||Y_{23}|\sin(\theta_{23}-\delta_2+\delta_3)|$$

$$\frac{\partial P_2}{\partial\delta_2}=1.00\cdot1.05\cdot4.4721\cdot\sin(116.56°-0.4083+0)+1.00\cdot0.9568\cdot3.2208\cdot\sin(104.0352°-0.4083+(-4.0563))$$

$$\frac{\partial P_2}{\partial\delta_2}=7.2454$$

Assignment #5

Find the final value of the Jacobian of the below system.

Line Data

Line No.	From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)
1	1	2	0.01008	0.0504	0.1025
2	1	3	0.00744	0.0372	0.0775
3	2	4	0.00744	0.0372	0.0775
4	3	4	0.01272	0.0636	0.1275

Bus Data

Bus No.	Bus Type	V	δ	P_G (p.u.)	Q_G (p.u.)	P_L (p.u.)	Q_L (p.u.)
1	Slack	1.00	0	-	-	0.5	0.3099
2	PQ	-	-	-	-	1.7	1.0535
3	PQ	-	-	-	-	2.0	1.2394
4	PV	1.02	-	3.18	-	0.8	0.4958

NRPF Result

Bus No.	V	δ (°)
1	1.00	0
2	0.9824	-0.976
3	0.9690	-1.872
4	1.02	1.523

Either write in sci-file, MS. Excel, or any programmable tool; or, in case you calculate by hand, express how to calculate at least one element in each sub – matrix (J_1, J_2, J_3, J_4) of the Jacobian matrix.

PV Buses

Since the voltage magnitude at PV buses is fixed, there is no need to explicitly include these voltages in **x** or write the reactive power balance equations.

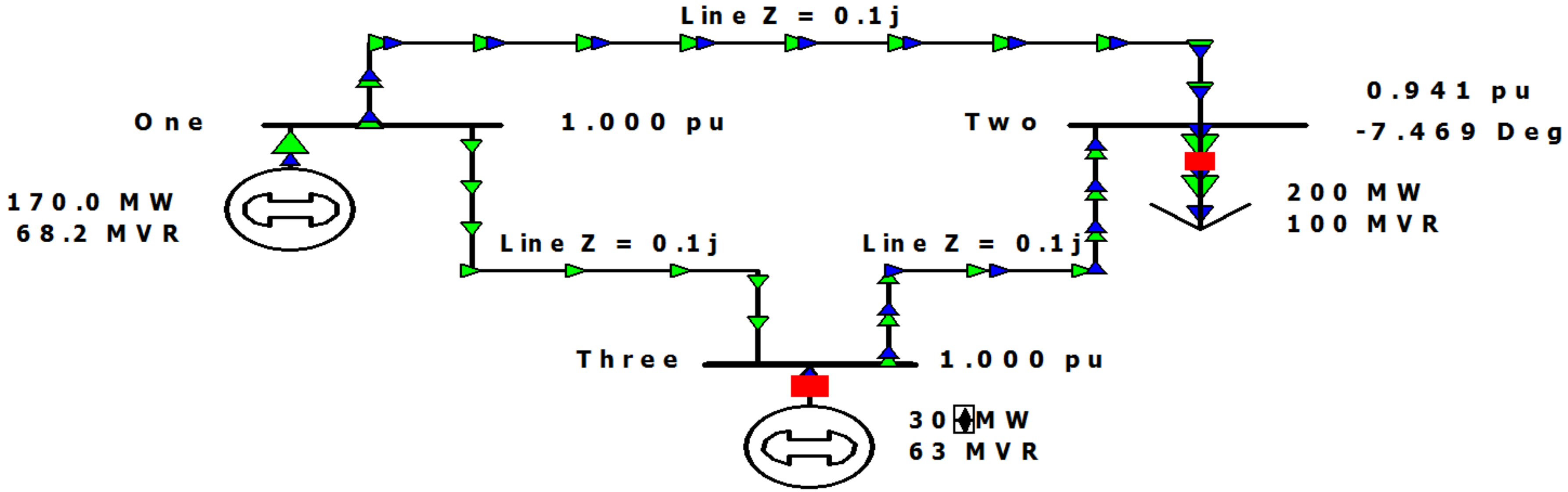
- The reactive power output of the generator varies to maintain the fixed terminal voltage (within limits).
- Optionally, these variations/equations can be included by just writing the explicit voltage constraint for the generator bus.

$| V_i | – V_{i \text{ set point}} = 0$

Three Bus PV Case Example

For this three bus case we have

$$\mathbf{x} = \begin{bmatrix} \partial_2 \\ \partial_3 \\ |V_2| \end{bmatrix} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_{G2} + P_{D2} \\ P_3(\mathbf{x}) - P_{G3} + P_{D3} \\ Q_2(\mathbf{x}) + Q_{D2} \end{bmatrix} = 0$$



The N-R Power Flow: 5-bus Example

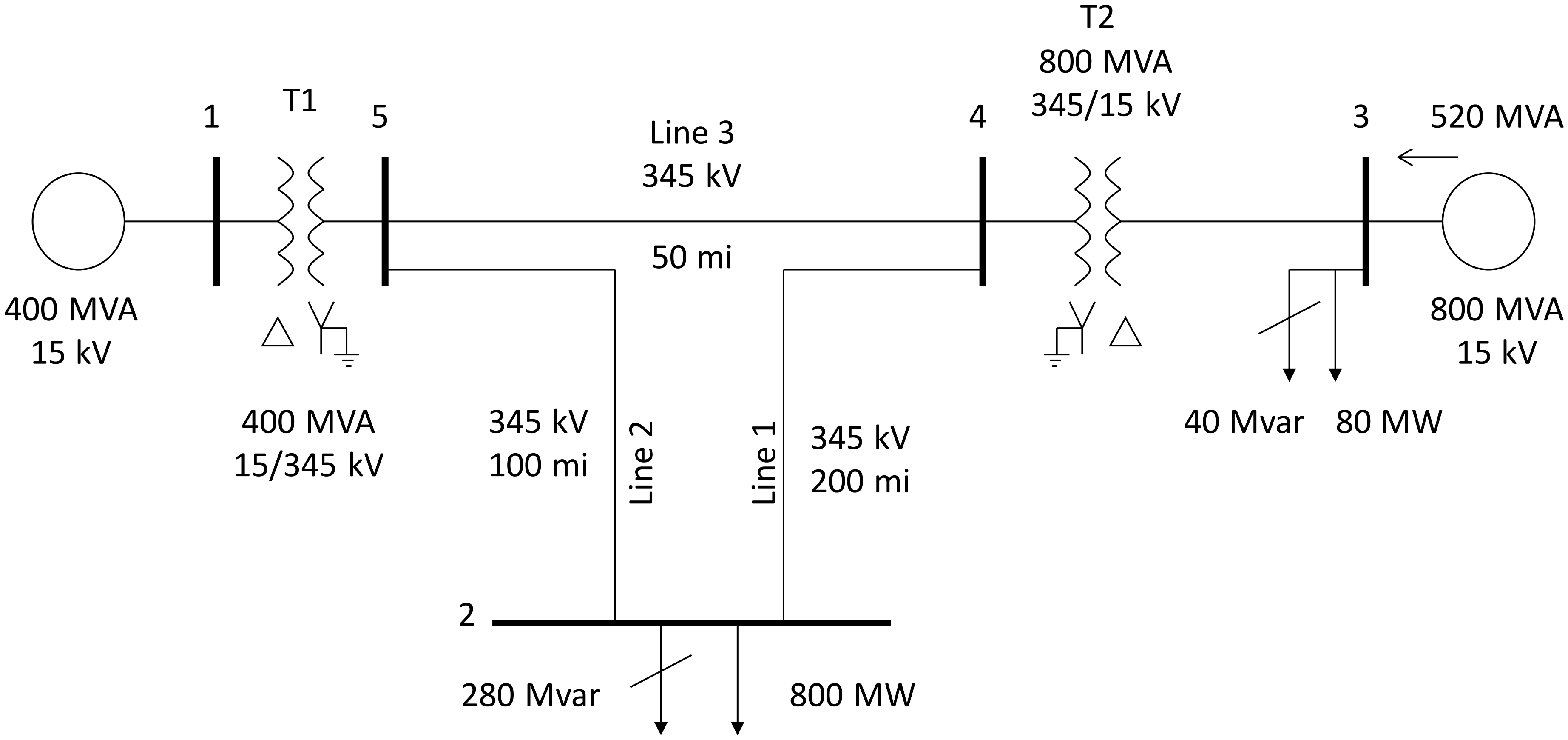


Figure 2.) Illustrates a single-line diagram.

The N-R Power Flow: 5-bus Example

Table 1.) Bus input data

Bus	Type	V/unit	δ degrees	P_G/unit	Q_G/unit	P_L/unit	Q_L/unit	Q_{Gmax}/unit	Q_{Gmin}/unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	8.0	2.8	—	—
3	Constant voltage	1.05	—	5.2	—	0.8	0.4	4.0	-2.8
4	Load	—	—	0	0	0	0	—	—
5	Load	—	—	0	0	0	0	—	—

Table 2.) Line input data

Bus-to-Bus	R' /unit	X'/unit	G'/unit	B'/unit	Maximum MVA/unit
2-4	0.0090	0.100	0	1.72	12.0
2-5	0.0045	0.050	0	0.88	12.0
4-5	0.00225	0.025	0	0.44	12.0

The N-R Power Flow: 5-bus Example

Table 3.) Transformer input data

Bus-to-Bus	R/unit	X/unit	G_c/unit	B_m/unit	Maximum MVA/unit	Maximum TAP Setting/unit
1-5	0.00150	0.02	0	0	6.0	—
3-4	0.00075	0.01	0	0	10.0	—

Table 4.) Input data and unknowns

Bus	Input Data	Unknowns
1	$V_1 = 1.0,\ \delta_1 = 0$	$P_1,\ Q_1$
2	$P_2 = P_{G2}\text{-}P_{L2} = -8$ $Q_2 = Q_{G2}\text{-}Q_{L2} = -2.8$	$V_2,\ \delta_2$
3	$V_3 = 1.05$ $P_3 = P_{G3}\text{-}P_{L3} = 4.4$	$Q_3,\ \delta_3$
4	$P_4 = 0,\ Q_4 = 0$	$V_4,\ \delta_4$
5	$P_5 = 0,\ Q_5 = 0$	$V_5,\ \delta_5$

Y bus Details

Elements of Y_{bus} connected to bus 2

$$Y_{21} = Y_{23} = 0$$

$$Y_{24} = \frac{-1}{R'_{24} + jX'_{24}} = \frac{-1}{0.009 + j0.1} = -0.89276 + j9.91964 \ per\ unit$$

$$Y_{25} = \frac{-1}{R'_{25} + jX'_{25}} = \frac{-1}{0.0045 + j0.05} = -1.78552 + j19.83932 \ per\ unit$$

$$\begin{aligned} Y_{22} &= \frac{1}{R'_{24} + jX'_{24}} + \frac{1}{R'_{25} + jX'_{25}} + j\frac{B'_{24}}{2} + j\frac{B'_{25}}{2} \\ &= (0.89276 - j9.91964) + (1.78552 - j19.83932) + j\frac{1.72}{2} + j\frac{0.88}{2} \\ &= 2.67828 - j28.4590 = 28.5847\angle -84.624^\circ \ per\ unit \end{aligned}$$

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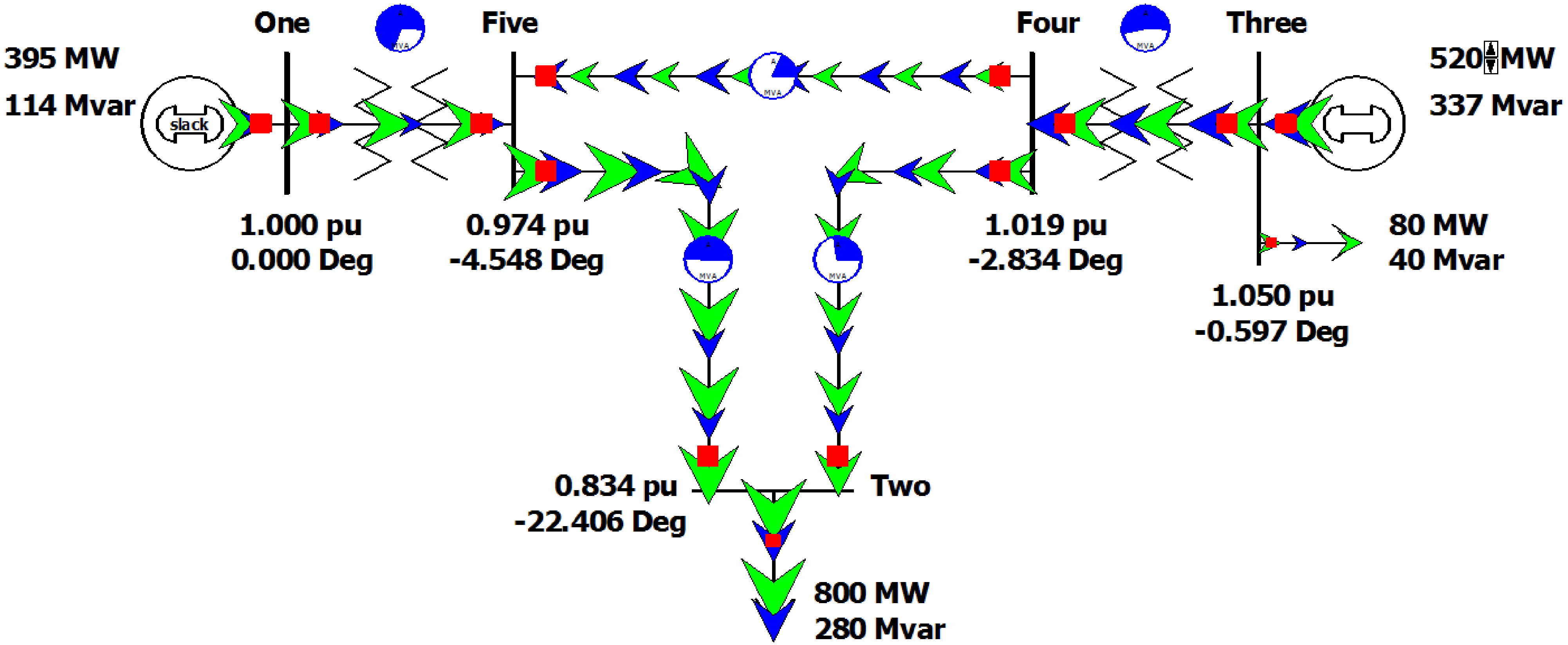
Hand Calculation Details

$$\begin{aligned}\Delta P_2(0) = P_2 - P_2(x) = P_2 - V_2(0)\{&Y_{21}V_1\cos[\delta_2(0)-\delta_1(0)-\theta_{21}]\\ &+Y_{22}V_2\cos[-\theta_{22}]+Y_{23}V_3\cos[\delta_2(0)-\delta_3(0)-\theta_{23}]\\ &+Y_{24}V_4\cos[\delta_2(0)-\delta_4(0)-\theta_{24}]\\ &+Y_{25}V_5\cos[\delta_2(0)-\delta_5(0)-\theta_{25}]\}\\ =-8.0-1.0\{&28.5847(1.0)\cos(84.624°)\\ &+9.95972(1.0)\cos(-95.143°)\\ &+19.9159(1.0)\cos(-95.143°)\}\\ =-8.0-(&-2.89\times10^{-4}) = -7.99972 \ \ per\,unit\end{aligned}$$

$$\begin{aligned}J1_{24}(0) &= V_2(0)Y_{24}V_4(0)\sin[\delta_2(0)-\delta_4(0)-\theta_{24}]\\ &=(1.0)(9.95972)(1.0)\sin[-95.143°]\\ &=-9.91964 \ \ per\,unit\end{aligned}$$

Five Bus Power System Solved



Solving Large Power Systems

The most difficult computational task is inverting the Jacobian matrix.

- The amount of computation can be decreased substantially by recognizing that the Y_{bus} is a sparse matrix, thus the Jacobian is also a sparse matrix.
- This substantially saves time when solving systems with tens of thousands of buses.

Newton-Raphson Power Flow

Advantages

- Fast convergence if the initial guess is close to the solution
- Large region of convergence

Disadvantages

- Each iteration takes much longer than a Gauss-Seidel iteration
- More complicated to code, particularly when implementing sparse matrix algorithms

Fast Decoupled Power Flow

- By further approximating the Jacobian, we obtain a typically reasonable approximation that is independent of the voltage magnitudes/angles.
- This means that the Jacobian needs only be built and factorized once.
- This approach is known as the fast decoupled power flow (FDPF).
- FDPF uses the same mismatch equations as standard power flow so it should have same solution if it converges.
- The FDPF is widely used, particularly when we only need an approximate solution.

FDPF Approximations

The FDPF makes the following approximations:

1. $\left|G_{ij}\right| = 0$
2. $\left|V_i\right| \quad = \ 1$ (for some occurrences),
3. $\sin \partial_{ij} = 0 \qquad \cos \partial_{ij} = 1$

Then: $\Delta\boldsymbol{\theta}^{(v)} = \mathbf{B}^{-1} diag\{|\mathbf{V}|^{(v)}\}^{-1} \Delta\mathbf{P}(\mathbf{x}^{(v)}),$

$$\Delta\left|\mathbf{V}\right|^{(v)} = \mathbf{B}^{-1} diag\{|\mathbf{V}|^{(v)}\}^{-1} \Delta\mathbf{Q}(\mathbf{x}^{(v)})$$

Where $\mathbf{B}$ is just the imaginary part of the $\mathbf{Y}_{\text{bus}} = \mathbf{G}_{\text{bus}} + j\mathbf{B}_{\text{bus}}$, except the slack bus row/column are omitted. That is,

$\mathbf{B}$ is $\mathbf{B}_{\text{bus}}$, but with the slack bus row and column deleted.

Sometimes approximate $diag\{|\mathbf{V}|^{(v)}\}$ by identity.

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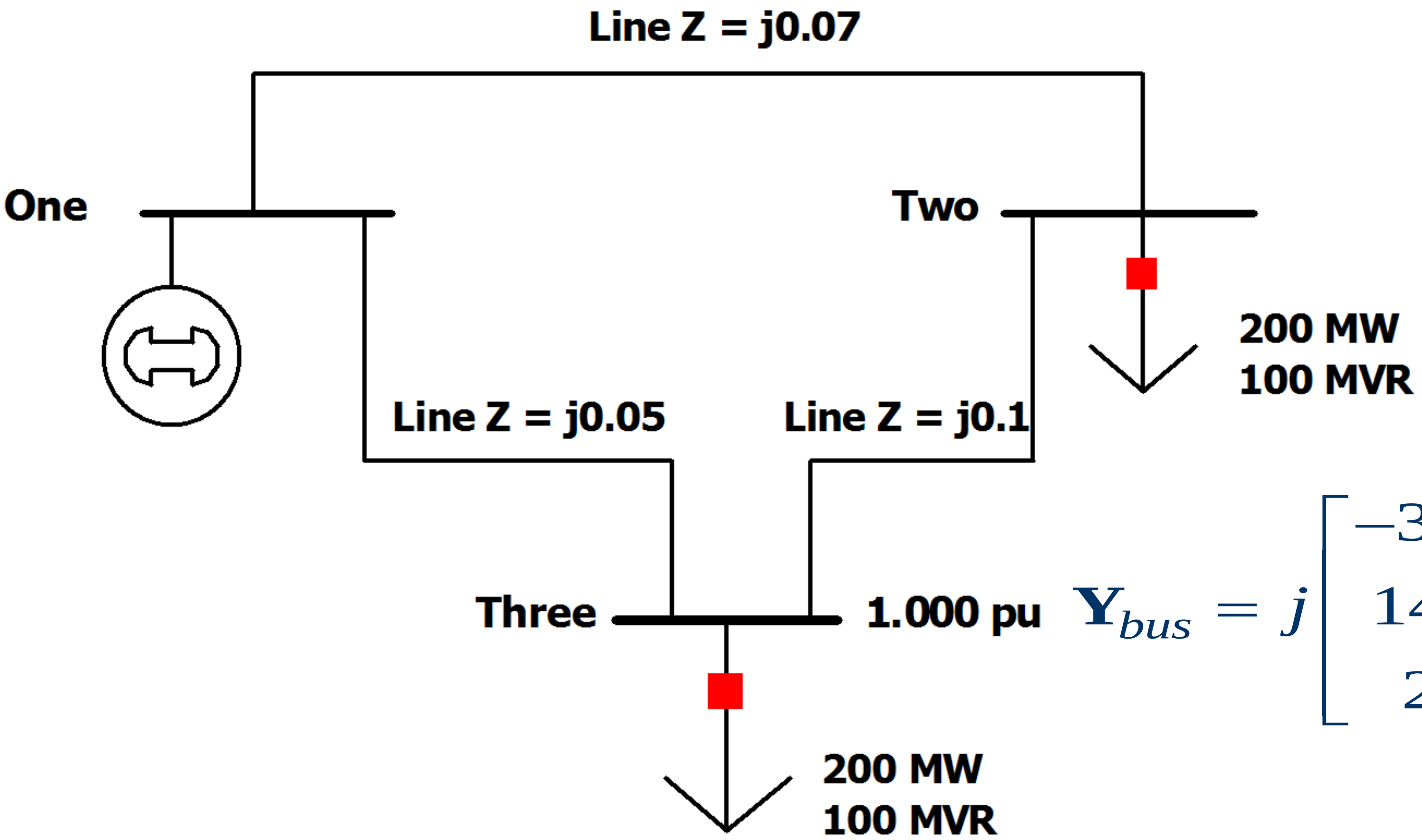
FDPF Three Bus Example

Use the FDPF to solve the following three bus system.



$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix}$$

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FDPF Three Bus Example, cont’d

$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix} \Rightarrow \mathbf{B} = \begin{bmatrix} -24.3 & 10 \\ 10 & -30 \end{bmatrix}$$

$$\mathbf{B}^{-1} = \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix}$$

Iteratively solve, starting with an initial voltage guess

$$\begin{bmatrix} \partial_2 \\ \partial_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \partial_2 \\ \partial_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix}$$

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FDPF Three Bus Example, cont'd

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9364 \\ 0.9455 \end{bmatrix}$$

$$\frac{\Delta P_i(x)}{|V_i|} = \sum_{k=1}^{n} |V_k| (G_{ik} \cos \partial_{ik} + B_{ik} \sin \partial_{ik}) + \frac{P_{Di} - P_{Gi}}{|V_i|}$$

$$\begin{bmatrix} \partial_2 \\ \partial_3 \end{bmatrix}^{(2)} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 0.151 \\ 0.107 \end{bmatrix} = \begin{bmatrix} -0.1361 \\ -0.1156 \end{bmatrix}$$

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(2)} = \begin{bmatrix} 0.924 \\ 0.936 \end{bmatrix}$$

$$\text{Actual solution: } \boldsymbol{\theta} = \begin{bmatrix} -0.1384 \\ -0.1171 \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} 0.9224 \\ 0.9338 \end{bmatrix}$$

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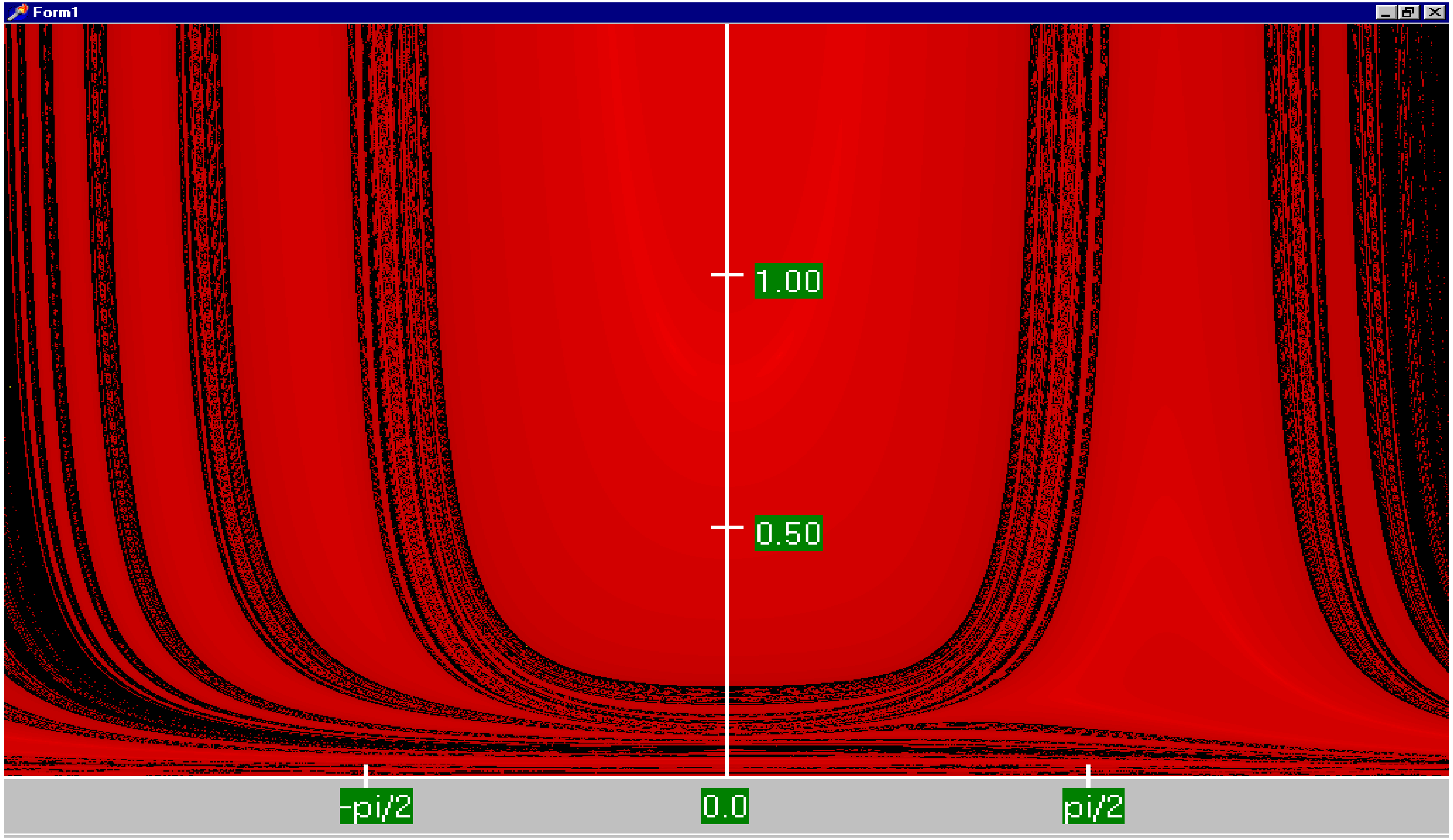
FDPF Region of Convergence



Questions?

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