Financial Literacy with Mr. 401(k)
['FinLit with Mr. 401(k)']
Winter Term 2023-2024
January 31, 2024

Time Value of Money

Class 21: Compound Interest & Lump Sum Formulas

Practical Application

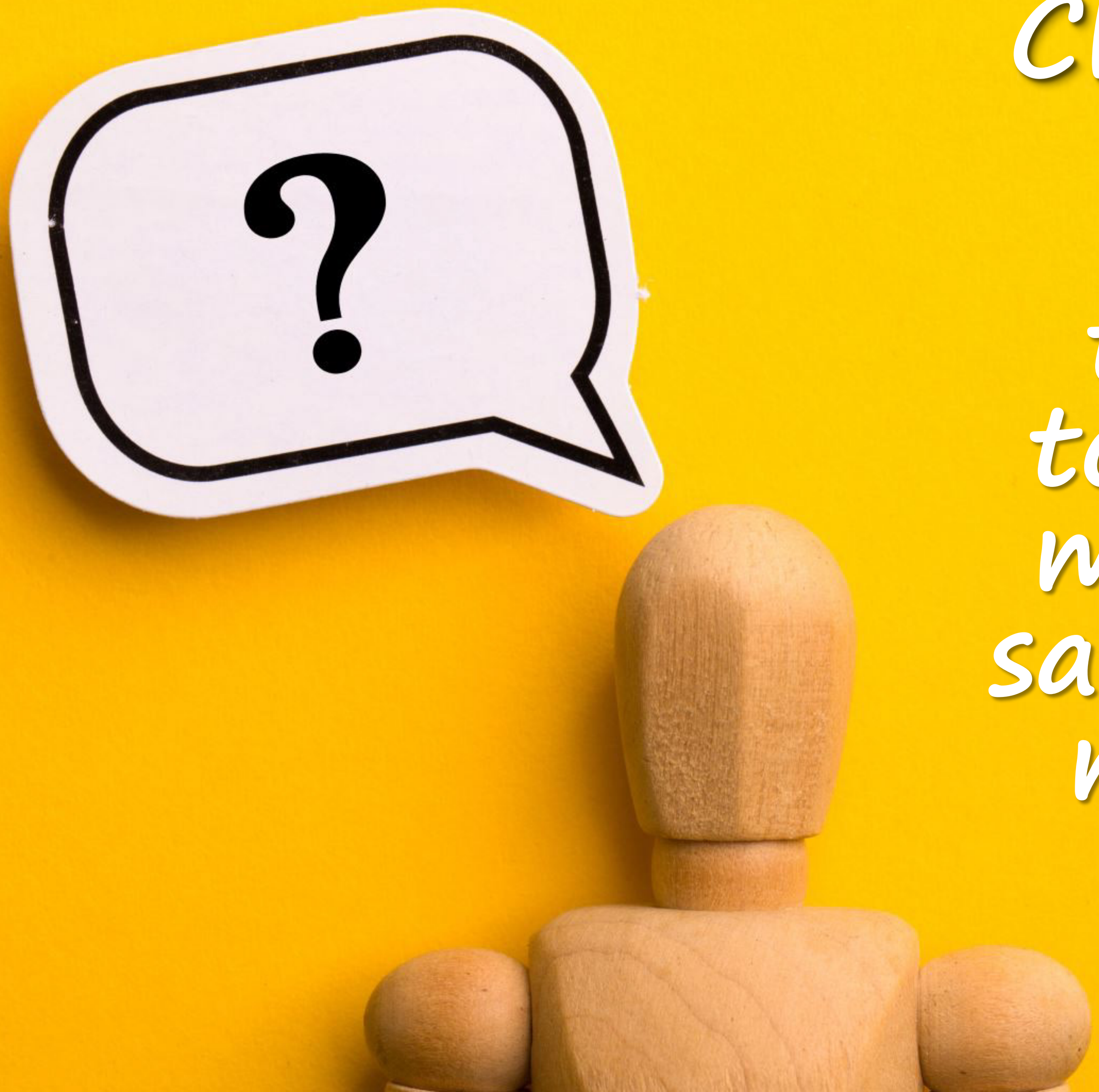
Jane lends $100 to John for exactly one month. At the end of one month, John repays Jane exactly $100.

Who benefited most from this trade? Why?

Money today is always worth more than the same amount of money in the future.*

Time Value of Money

* Presuming an inflating monetary unit of account



Class Discussion

Why do you think money today is worth more than the same amount of money in the future?

The Time Value of Money is based on the effects of...

Inflation

Causes the prices of goods and services to increase, which reduces the value of money by reducing its purchasing power.

Interest Rates

Causes the value of money in investments or interest-bearing accounts to increase.

Warren Buffett at Berkshire Hathaway Annual General Meeting 1994

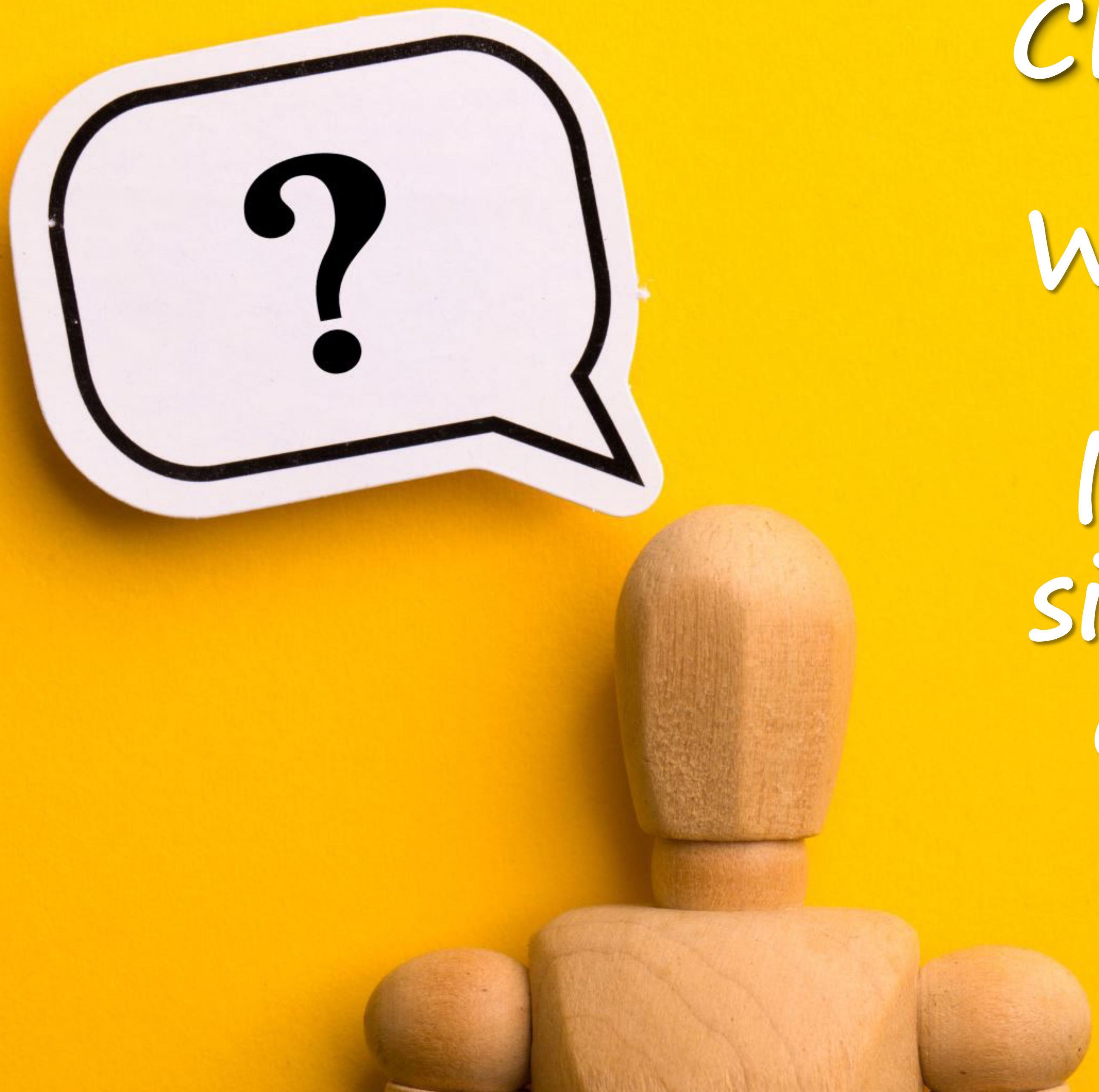
Watch Video Clip on YouTube

Source: https://www.youtube.com/watch?v=kgKEiP6L9EQ&t=7504s

“The value of every business, the value of a farm, the value of an apartment house, the value of any economic asset, is 100% sensitive to interest rates because **all you are doing in investing is transferring some money to somebody now in exchange for what you expect the stream of money to be,** to come in over a period of time, and **the higher interest rates are the less that present value is going to be**.” – Warren Buffett

Recap: Simple Interest Formula

Simple Interest Amount = Principal Amount × Interest Rate × Time in Years



Class Discussion

What might be some of the limitations of simple interest calculations?

Simple Interest Limitations

Calculation Basis

Calculated only on the original principal of a loan or deposit

Prior Time Periods

Does not consider the interest on an asset or liability in previous time periods

Compounding is a method of calculating the total interest on the principal, where the interest is reinvested. Think of this as "interest on interest."

Compound Interest

Simple Interest vs. Compound Interest calculation on a **$1,000** investment earning **10% annual interest** for **7 years.**

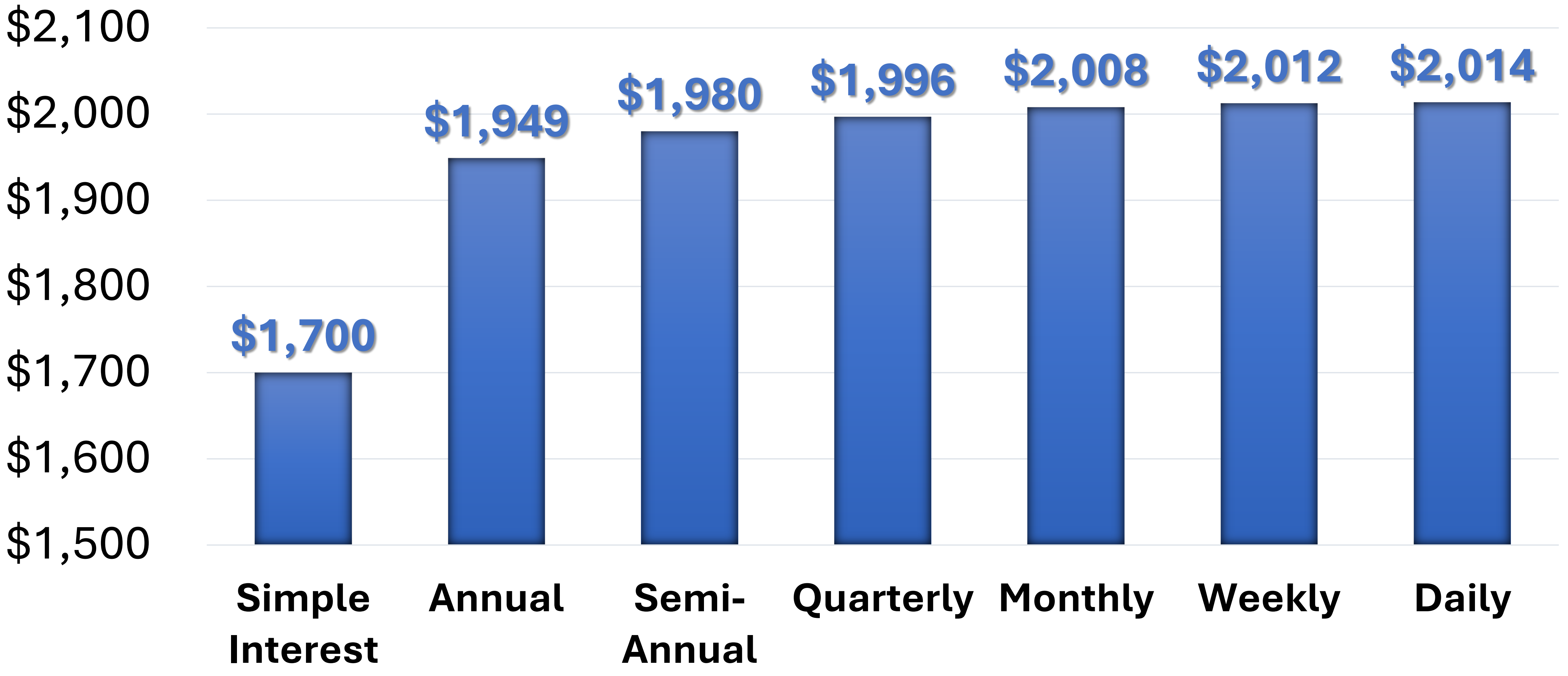
Annual Simple Interest

End of Year…	Interest Income	Balance
1	$100	$1,100
2	$100	$1,200
3	$100	$1,300
4	$100	$1,400
5	$100	$1,500
6	$100	$1,600
7	$100	$1,700

Annual Compound Interest

End of Year…	Interest Income	Balance
1	$100	$1,100
2	$110	$1,210
3	$121	$1,331
4	$133	$1,464
5	$146	$1,611
6	$161	$1,772
7	$177	$1,949

$1,000 invested at a **10% annual interest** rate for **7 years** under simple interest and select compound interest frequencies



The **frequency of compounding** affects investment returns

Interest accumulates faster when interest is compounded more frequently. An investment that compounds every quarter accumulates more interest than the same investment compounded annually. **Continuous compound interest has the highest returns of all.**

Time Value of Money Lump Sum Formulas Can Calculate the...

Future Value

Helps you to know how much the money you have today can grow to, if you save or invest it

Present Value

Helps you to know how much money you need now, to reach a certain amount of money in the future

Interest Rate

Helps you to know how fast your money will grow, or the interest rate you earn

Time

Helps you to know how long it will take to reach a specific savings goal or investment outcome

Time Value of Money: Future Value of a Lump Sum Formula

The value of **n** depends on the number of times the interest compounds or is reinvested.

- n = 1, compounds **annually**
- n = 2, compounds **semi-annually**.
- n = 4, compounds **quarterly**.
- n = 12, compounds **monthly**.
- n = 52, compounds **weekly**.
- n = 365, compounds **daily**.

$$FV = PV \times (1 + {}^{r}/_{n})^{nt}$$

Where:

- FV = Future Value
- PV = Present Value
- r = Interest rate or growth rate as a percentage
- n = Number of times the interest compounds annually
- t = Time in years

Practical Application

John saved $1,000 from summer yard care work. John deposits the $1,000 into a high yield savings account. The high yield savings account pays 4.5% annual interest, compounded monthly. Let's calculate what the value of this would be after 3 years.

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$$\boldsymbol{FV = \$1,144.25}$$

Time Value of Money: Lump Sum Formulas

Where:

- FV = Future Value
- PV = Present Value
- r = Interest rate or growth rate as a percentage
- n = Number of times the interest compounds annually
- t = Time in years

$$FV = PV \times (1 + {}^{r}/_{n})^{nt}$$

$$PV = \frac{FV}{(1 + {}^{r}/_{n})^{nt}}$$

$$r = n\left[\left({}^{FV}/_{PV}\right)^{({}^{1}/_{nt})-1}\right]$$

$$t = \frac{\ln\left({}^{FV}/_{PV}\right)}{n[\ln(1 + {}^{r}/_{n})]}$$

Practical Application

Jane wants to purchase a new bicycle in 4 years. Jane expects the bicycle will cost about $800. Jane wants to invest money today toward the purchase. If Jane expects to earn 10% compounded annually on a stock portfolio, how much does Jane need to invest today?

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$$PV = \frac{FV}{(1 + {}^{r}/_{n})^{nt}} = \frac{800}{\left(1 + {}^{10\%}/_{1}\right)^{1*4}} = \$546.41$$

Practical Application

Parents make a deal with their child who is student starting high school as a freshman. If the student earns an **A**, **A-**, or **B+** in every class, then the parents will give the student $2,000 at graduation. The student wants to calculate the present value of that money, assuming a 4% annual discount rate.

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$$PV = \frac{FV}{\left(1 + {}^{r}/_{n}\right)^{nt}} = \frac{2,000}{\left(1 + {}^{4\%}/_{1}\right)^{1*4}}$$

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$$PV = \frac{FV}{(1 + {}^{r}/_{n})^{nt}} = \frac{2{,}000}{(1 + {}^{4\%}/_{1})^{1*4}} = \$1{,}709.61$$

Money Mavericks

Objective: Work within your Money Mavericks Workgroups to solve each problem on the following Practical Applications Slide.

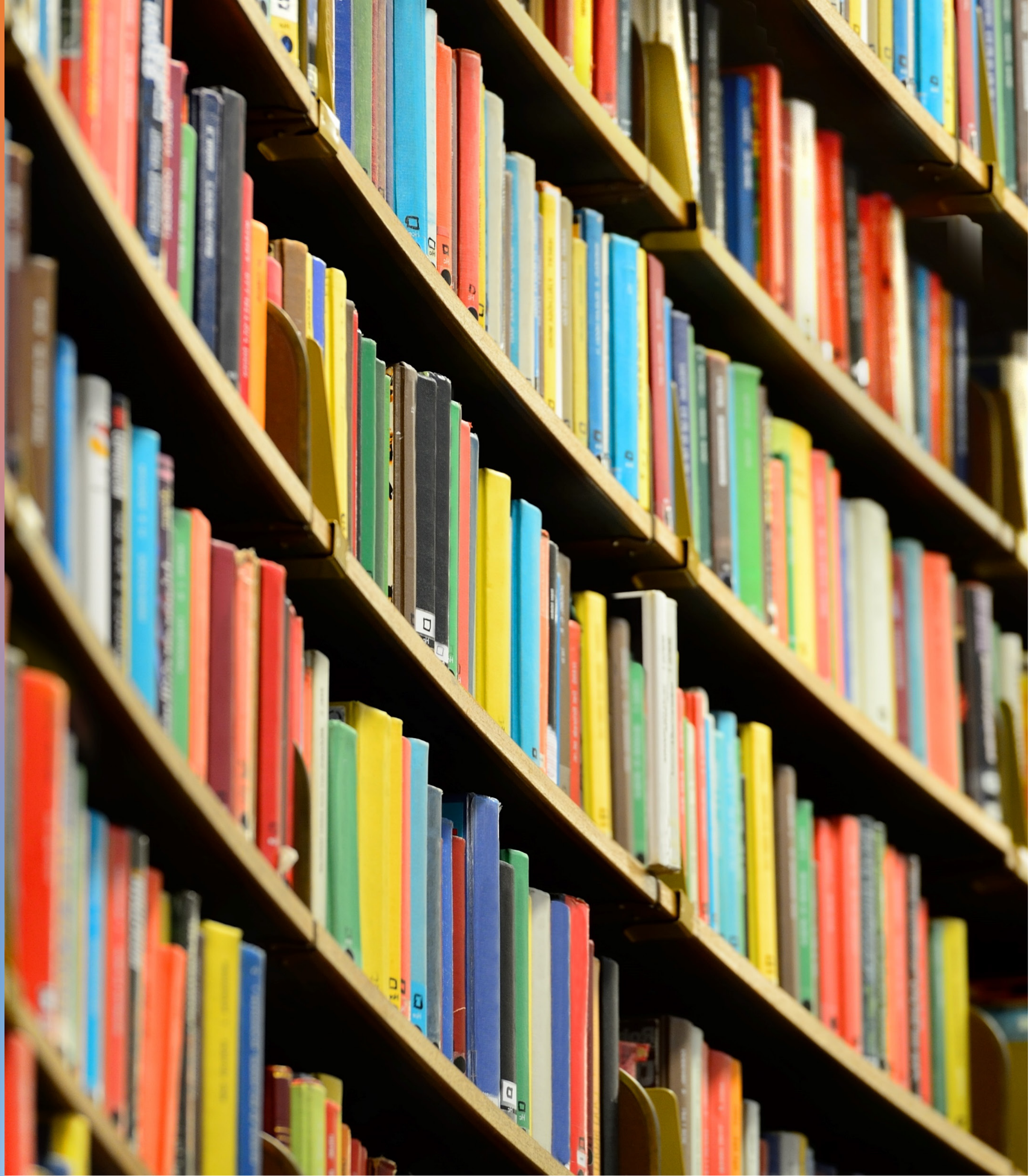
Practical Application

1. Jane invested $1,000 for 1 year at a 10% annual interest rate compounded annually. How much did Jane have after 1 year?
2. John invested $1,000 for 1 year at a 10% interest rate compounded **monthly**. How much did John have after 1 year?
3. Jane invested $1,000 for **10 years** at a 10% interest rate compounded monthly. How much did Jane have after 10 years?
4. John invested $1,000 for 10 years at a 10% interest rate compounded **daily**. How much did John have after 10 years?
5. Jane invested **$5,000** for **25 years** at a 10% interest rate compounded **monthly**. How much did Jane have after 25 years?
6. John invested **$100,000** for **30 years** at a 10% interest rate compounded **monthly**. How much did John have after 30 years?

$$FV = PV \times (1 + {}^{r}/_{n})^{nt}$$

Three Key Takeaways

1. Money today is worth more than the same amount of money in the future because of inflation and interest rates.
2. Simple interest considers only interest on the principal amount whereas compound interest calculates the interest on interest.
3. One of the most important formulas in finance is $\boldsymbol{FV = PV \times (1 + r/n)^{nt}}$



Where to Learn More

- Time Value of Money Explained with Formula and Examples by Jason Fernado via Investopedia
- Understanding the Time Value of Money by Shauna Carther Heyford via Investopedia
- Video: Time Value of Money, by Khan Academy

Money Mavericks Workgroups Answer Key
Replicate in Microsoft Excel =FV(rate,nper,pmt,[pv],[type])

1. Jane invested $1,000 for 1 year at a 10% annual interest rate compounded annually. How much did Jane have after 1 year? =FV(10%/1,1*1,0,-1000,0) = $1,100
2. John invested $1,000 for 1 year at a 10% interest rate compounded **monthly**. How much did John have after 1 year? =FV(10%/12,1*12,0,-1000,0) = $1,105
3. Jane invested $1,000 for **10 years** at a 10% interest rate compounded monthly. How much did Jane have after 10 years? =FV(10%/12,10*12,0,-1000,0) = $2,707
4. John invested $1,000 for 10 years at a 10% interest rate compounded **daily**. How much did John have after 10 years? =FV(10%/365,10*365,0,-1000,0) = $2,718
5. Jane invested **$5,000** for **25 years** at a 10% interest rate compounded **monthly**. How much did Jane have after 25 years? =FV(10%/12,25*12,0,-5000,0) = $60,285
6. John invested **$100,000** for **30 years** at a 10% interest rate compounded **monthly**. How much did John have after 30 years? =FV(10%/12,30*12,0,-100000,0) = $1,983,740