Financial Literacy with Mr. 401(k)
['FinLit with Mr. 401(k)']
Winter Term 2023-2024
February 1, 2024

Time Value of Money
Class 22:
Annuity Basics
& Business Plan
Project

Practical Application

Jane earns a $25 weekly allowance if she completes her chores and homework.

Remembering to pay the allowance at the end of each week was a nuisance for Jane's parents.

Jane's parents offer to pay her an allowance of $100 after every 4-week period, instead of $25 at the end of each week.

Should Jane accept the offer? Why or why not?

Recap: Money today is always worth more than the same amount of money in the future because of the effects of inflation and interest rates.*

Time Value of Money

* Presuming an inflationary monetary system

Recap: Time Value of Money **Lump Sum** Formulas

Where:
- FV = Future Value
- PV = Present Value
- r = Interest rate or growth rate as a percentage
- n = Number of times the interest compounds annually
- t = Time in years

$$FV = PV \times \left(1 + \frac{r}{n}\right)^{nt}$$

$$PV = \frac{FV}{\left(1 + \frac{r}{n}\right)^{nt}}$$

$$r = n\left[\left(\frac{FV}{PV}\right)^{\left(\frac{1}{nt}\right)-1}\right]$$
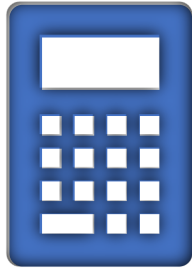
$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{n\left[\ln\left(1 + \frac{r}{n}\right)\right]}$$
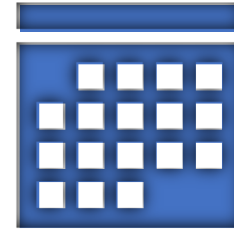
Class Discussion

Do you see any limitations in the time value of money lump sum formulas?

Time Value of Money Lump Sum Formula Limitations

Lump Sum Basis

Formulas calculate values based only on lump sums of money

Ignores Periodic Cashflows

Does not consider the effect of regular periodic cashflows or payments

Class Discussion

Can you think of any examples of regular periodic cashflows or payments over time?

Examples of Periodic Cashflows or Payments

Earn a Weekly Allowance

A weekly allowance for completing chores and doing homework

Fund a Retirement Plan

Contribute 15% of semi-monthly payroll into a 401(k) or other retirement plan

Invest Childcare Income

Invest $100 each month of the earned income from babysitting jobs

Make Auto Loan Payments

Pay $750 per month towards an automobile loan

Pay Online Streaming Services

Pay a monthly payment for subscriptions to online streaming service platforms

Series of regular periodic cashflows or payments for a fixed period of time

Annuity

Future Value of an Ordinary Annuity Formula

$$FV = PMT \cdot \left[\frac{\left(1 + \frac{i}{n}\right)^{n \cdot t} - 1}{\frac{i}{n}} \right]$$

In this context, i/n represents the interest rate per period (assuming n periods per year), and $n \cdot t$ calculates the total number of periods over the life of the annuity.

To clarify the notations:
- i is the annual interest rate, but when we divide i by n, we are adjusting it to reflect the rate for each compounding period within a year.
- n in the context of the FV formula is used to denote the number of compounding periods per year, and when used in $n \cdot t$, it calculates the total number of payment periods.
- t represents the duration of the annuity payments in years.

Where:
- FV = Future Value of an ordinary annuity
- PMT = Payment or cashflow amount per period
- i = Annual interest rate or growth rate as a percentage
- n = Number of compounding periods per year (e.g., 12 for monthly payments)
- t = Time in years

Present Value of an Ordinary Annuity Formula

In this context, i/n represents the interest rate per period (assuming n periods per year), and $n \cdot t$ calculates the total number of periods over the life of the annuity.

To clarify the notations:
- i is the annual interest rate, but when we divide i by n, we are adjusting it to reflect the rate for each compounding period within a year.
- n in the context of the PV formula is used to denote the number of compounding periods per year, and when used in $n \cdot t$, it calculates the total number of payment periods.
- t represents the duration of the annuity payments in years.

$$PV = PMT \cdot \left[\frac{1-\left(1+\dfrac{i}{n}\right)^{-n \cdot t}}{\dfrac{i}{n}} \right]$$

Where:
- PV = Present Value of the ordinary annuity
- PMT = Payment or cashflow amount per period
- i = Annual interest rate or growth rate as a percentage
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Number of Payments for the Future Value of an Ordinary Annuity Formula

This formula assumes that the payments are made at the end of each period and the compounding occurs at the same frequency as the payments. If the payment frequency and the compounding frequency do not match, the formula would need to be adjusted.

$$N = \frac{\ln\left(\frac{FV \cdot \frac{i}{n} + PMT}{PMT}\right)}{\ln\left(1 + \frac{i}{n}\right)}$$

Where:

- N = Total number of payments
- PV = Present Value of the ordinary annuity
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$$N = \frac{\ln\left(\frac{PMT}{PMT - PV \cdot \frac{i}{n}}\right)}{\ln\left(1 + \frac{i}{n}\right)}$$

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Practical Application

John is 13 years old. He earns a weekly allowance of $30 if he does his chores and homework. John wants to invest $15 per week in a stock portfolio. John expects to earn 8% annually on the stock portfolio. John wants to estimate how much the portfolio will be worth in 5 years when he graduates high school.

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$$FV = PMT \cdot \left[\frac{\left(1 + \frac{i}{n}\right)^{n \cdot t} - 1}{\frac{i}{n}} \right] = 15 \cdot \left[\frac{\left(1 + \frac{8\%}{52}\right)^{52 \cdot 5} - 1}{\frac{8\%}{52}} \right]$$

Practical Application

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Practical Application

Jane borrowed $100,000 in student loans to pay for college education. Jane recently graduated and now has a full-time job. Jane has budgeted $500 per month towards her student loan debt. The student loan is at a fixed annual interest rate of 5%. How long will it take Jane to payoff the loan?

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Jane borrowed $100,000 in student loans to pay for college education. Jane recently graduated and now has a full-time job. Jane has budgeted $500 per month towards her student loan debt. The student loan is at a fixed annual interest rate of 5%. How long will it take Jane to payoff the loan?

$$N = \frac{\ln\left(\dfrac{PMT}{PMT - PV \cdot \frac{i}{n}}\right)}{\ln\left(1 + \frac{i}{n}\right)} = \frac{\ln\left(\dfrac{500}{500 - 100{,}000 \cdot \frac{5\%}{12}}\right)}{\ln\left(1 + \frac{5\%}{12}\right)}$$

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Practical Application

If Jane graduated college at the age of 22, she would not pay off her student loans until she was 58 years old! What if Jane increased her student loan payments from $500 to $1,000?
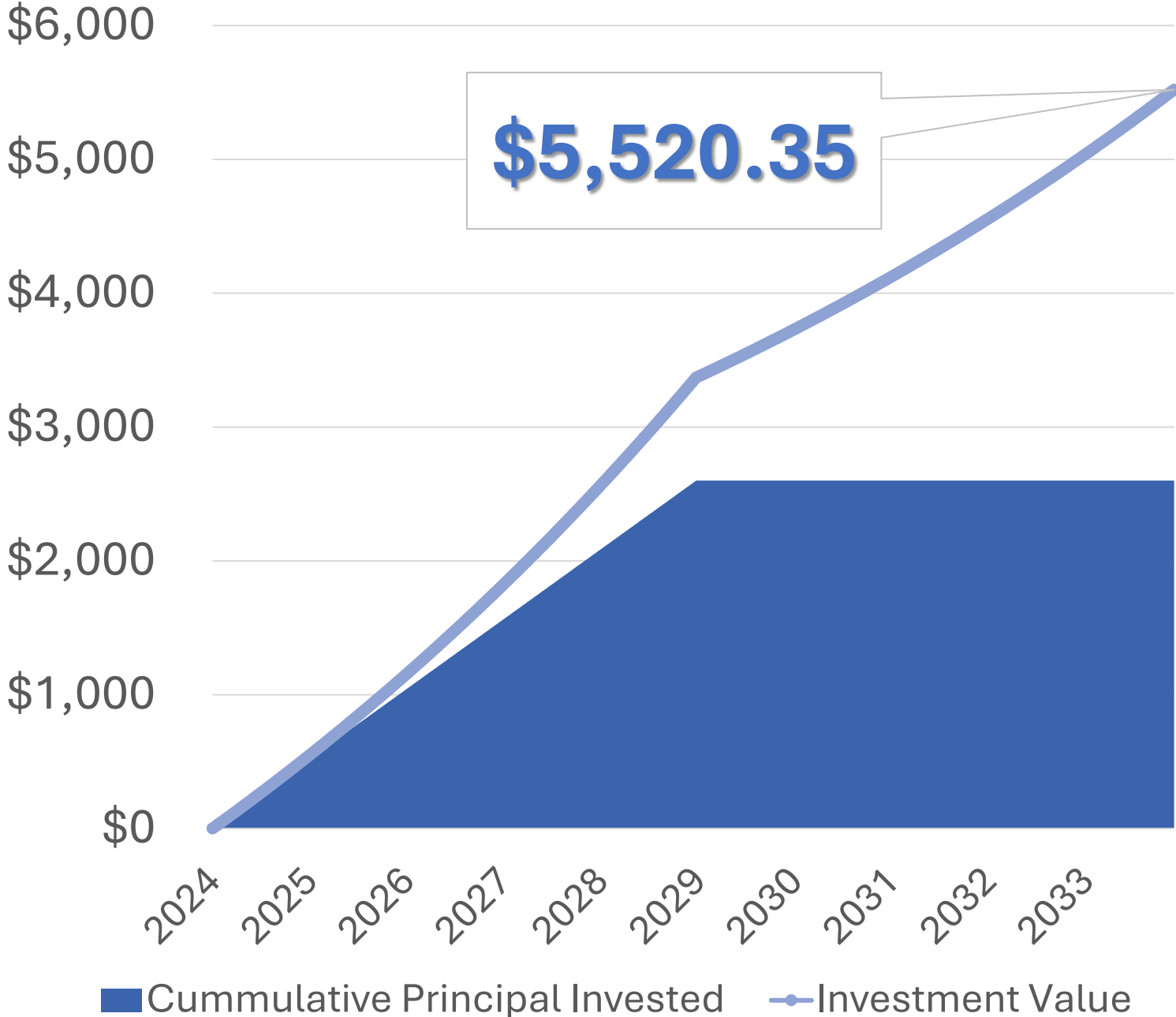
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130 monthly payments

$\cong$ **11 years**

# Future Value of an Ordinary Annuity Formula & Future Value of a Lump Sum Example

From age 13 to age 18, Jane invests $10 per week from her allowance, in an investment account earning 10% annual investment returns. Jane stops funding her investment account while she attends college. How large does it grow?

$6,000

$5,000

$5,520.35

$4,000

$3,000

$2,000

$1,000

$0

2024 2025 2026 2027 2028 2029 2030 2031 2032 2033

Cummulative Principal Invested   Investment Value

Three Key Takeaways

1. Annuities are regular periodic cashflows or payments over time.

2. Regular periodic cashflows into investment vehicles can grow significantly over time through cashflows and the power of compound interest.

3. You can use annuity formulas to plan better for your future.
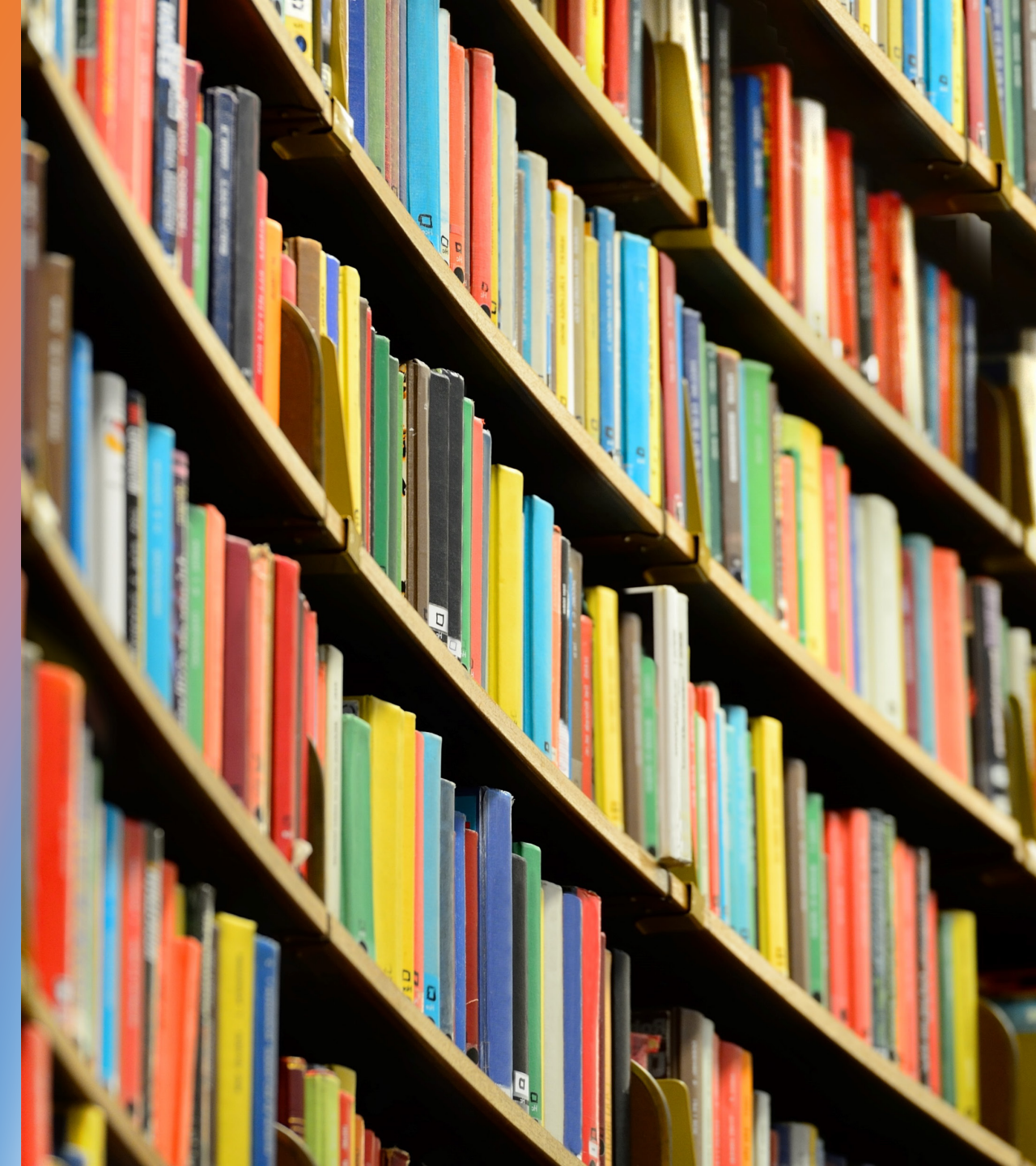
Where to Learn More

- [Time Value of Money Explained with Formula and Examples](#) by Jason Fernado via Investopedia

- [Understanding the Time Value of Money](#) by Shauna Carther Heyford via Investopedia

- Video: [Time Value of Money](#), by Khan Academy

Business Idea & Business Plan Project within Money Mavericks Groups

Develop Business Idea

What problem does your business solve? Who are the business's target customers? **[Completed]**

Write Business Plans

How does the business earn money? What are the ongoing costs? How will it differentiate?

Present Your Business Plans

Each Money Mavericks Group gives a 10-minute presentation of their business plan to class

Business Ideas by Money Mavericks Workgroup

Alpha
Babysitting / Childcare Services

Beta
App that Recommends and Markets Books to Users

Gamma
Online Sales of Sporting Apparel & Goods, Focusing on Footwear

Delta
Digital Content Creator AI Support

Money Mavericks

Objective: Start writing your business plans within your Money Mavericks Workgroups. As a guide to help write your business plans, focus on answering the questions on the following practical applications slide. I suggest assigning each question to a person in your workgroups.

Practical Application

- What is your business name? Feel free to design a logo for your business.

- Who are your business's target customers?

- How does your business earn money?

- What are your business's products and/or services?

- What are the costs to start your business?

- Where will your business's startup costs come from?

- What are your business's ongoing expenses?

- What are the prices for your business's products and/or services? How did you determine those prices?

- How will your business promote its products and/or services?

- Write a "pitch" that describes your business to a target customer.

- Who else competes with your business?

- How will your business differentiate itself from competitors?