Money Now or Money Later?

Class 19: Time Value of Money

NITWOM

News In The World Of Money

01/19/2023: The United States hit its debt limit today. The debit limit is the maximum amount the U.S. Government can borrow to pay financial obligations. U.S. Treasury Secretary, Janet Yellen announced extraordinary measures to avoid a debt default. The extraordinary measures include suspending new investments in the Civil Service Retirement and Disability Fund and the Postal Service Retiree Health Benefits Fund. Yellen believes the extraordinary measures could allow the government to pay its obligations until early June 2023.

Who has heard of this phrase, “a penny saved is a penny earned?”

What does this phrase mean to you?

Money today is worth more than the same amount of money tomorrow.

Why?

1

Inflation

causes the value of money to decrease by losing purchasing power

2

Interest rates

causes the value of money in investments or interest-bearing accounts to increase

1994 Berkshire Hathaway Annual General Meeting Video Clip

If the video does not launch, copy/paste the following URL directly into your web browser:

https://www.youtube.com/watch?v=kgKEiP6L9EQ&t=7504s

Warren Buffett, Chairman and CEO, Berkshire Hathaway

"The value of every business, the value of a farm, the value of an apartment house, the value of any economic asset, is 100% sensitive to interest rates because **all you are doing in investing is transferring some money to somebody now in exchange for what you expect the stream of money to be, to come in over a period of time**, and **the higher interest rates are the less that present value is going to be**."

Simple interest is a method of calculating the interest on an asset or liability where the interest is **calculated only on the principal amount.**

Asset = What you **own**

Liability = What you **owe**

Simple Interest Formula

Simple Interest Amount = Principal Amount × Interest Rate × Time in Years

Simple Interest Example

Mia makes a \$1,000 loan to a Mathias at a 10% annual simple interest rate. Mathias pays the loan principal + interest after exactly 1-year.

1. **How much interest did Mathias pay to Mia?**

Now, suppose Mathias paid the loan principal + interest at the end of the 3rd year.

2. **How much interest did Mathias pay to Mia?**

Simple Interest Limitations

- Calculated only on the original principal of a loan or deposit
- Does not consider the interest on an asset or liability in previous time periods

Compound Interest

Compounding is a method of calculating total interest on the principal, **where the interest is reinvested**.

Think of this as <u>interest on interest</u>.

Simple Interest

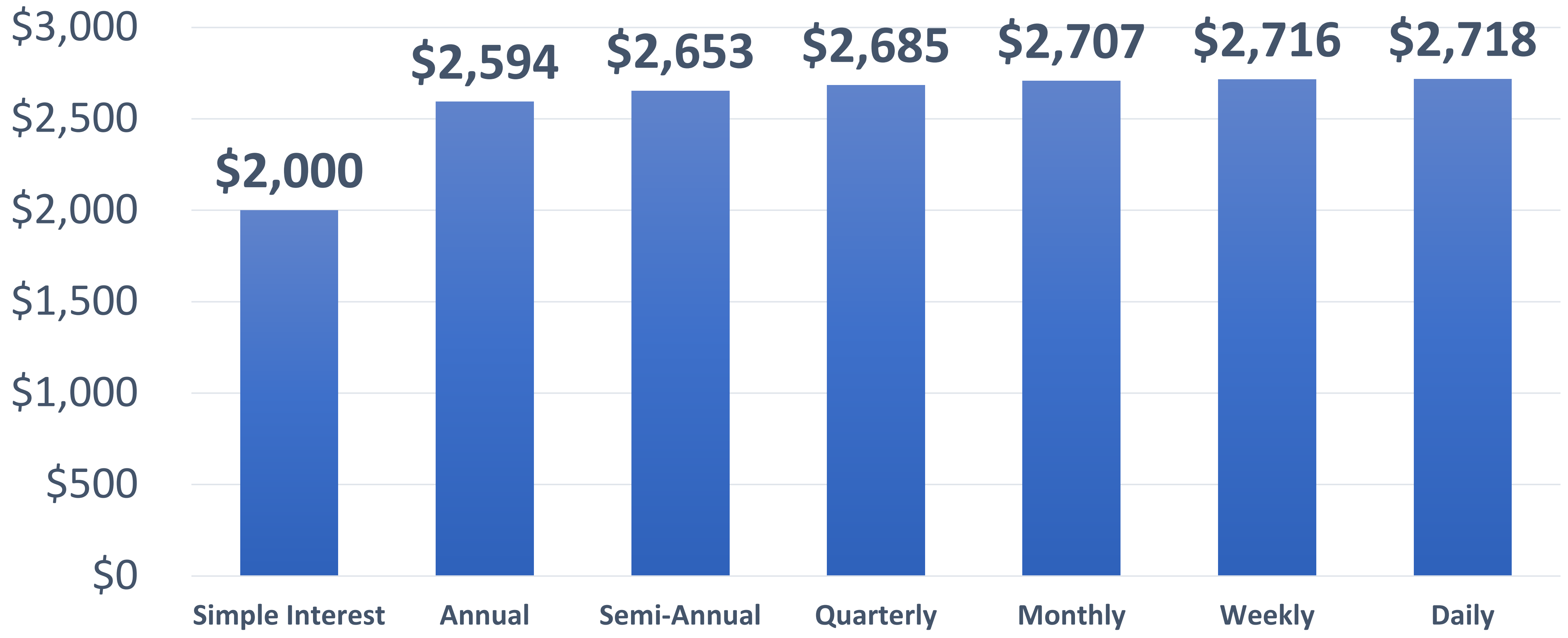
End of Year...	Interest Income	Balance
1	$100	$1,100
2	$100	$1,200
3	$100	$1,300
4	$100	$1,400
5	$100	$1,500
6	$100	$1,600
7	$100	$1,700
8	$100	**$1,800**

Compound Interest (Annual)

End of Year...	Interest Income	Balance
1	$100	$1,100
2	$110	$1,210
3	$121	$1,331
4	$133	$1,464
5	$146	$1,611
6	$161	$1,772
7	$177	$1,949
8	$195	**$2,144**

Simple Interest vs. Compound Interest on a **$1,000** investment earning **10% annual interest**

$1,000 invested at a **10% annual interest** rate for **10 years** under simple interest and select compound interest frequencies



The **frequency of compounding** affects investment returns

Increasing the compounding frequency (ex. from annual to daily) increases the amount of interest, but the rate of interest increase is reduced as the compounding frequency increases

Time Value of Money Formula

- $FV = PV \times (1 + {}^{r}/_{n})^{nt}$
- Where:
  - FV = Future Value
  - PV = Present Value
  - r = Interest rate or growth rate as percentage
  - n = Number of times the interest compounds annually
  - t = Time in years

The value of n varies depending on the number of times the amount is compounding, which is simply the number of times the interest is being reinvested.

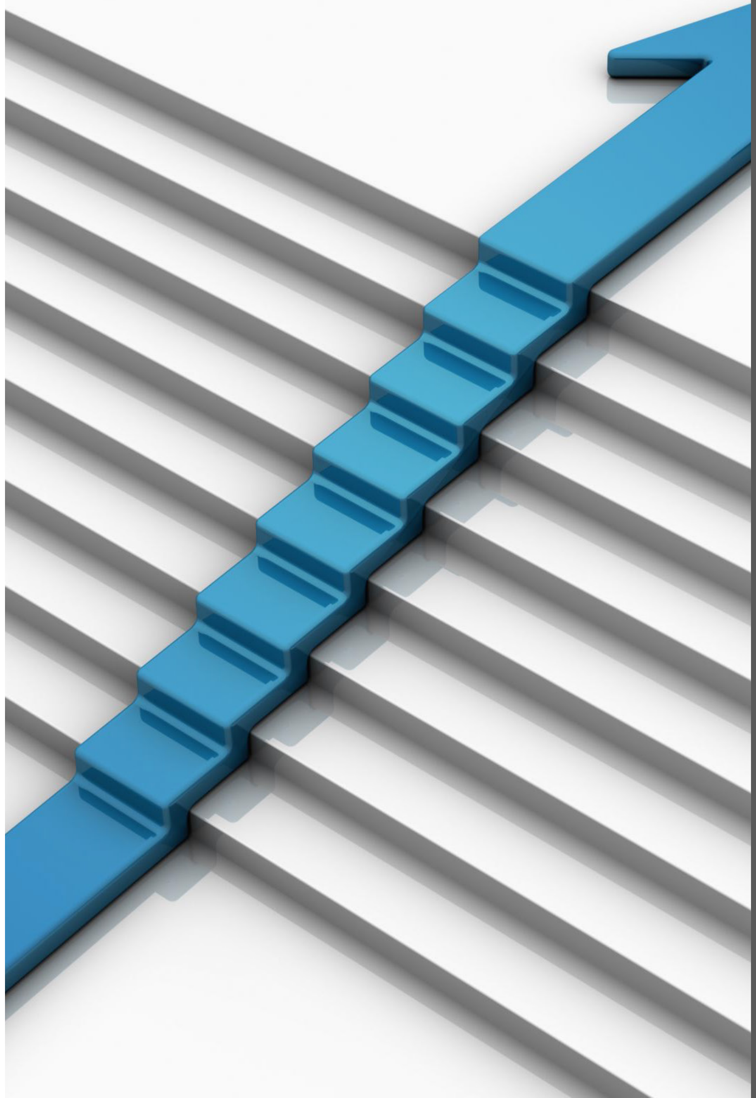
- n = 1, amount is compounded **annually**
- n = 2, amount is compounded s**emi-annually**.
- n = 4, amount is compounded **quarterly**.
- n = 12, amount is compounded **monthly**.
- n = 52, amount is compounded **weekly**.
- n = 365, amount is compounded **daily**.

Small Group Exercise

Work within your groups to solve each problem

$$FV = PV \times (1 + {}^{r}/_{n})^{nt}$$

1. Mia invested $1,000 for 1 year at a 10% annual interest rate compounded annually. How much did Mia have after 1 year?
2. Mia invested $1,000 for 1 year at a 10% interest rate compounded **monthly**. How much did Mia have after 1 year?
3. Mia invested $1,000 for **10 years** at a 10% interest rate compounded monthly. How much did Mia have after 10 years?
4. Mia invested $1,000 for 10 years at a 10% interest rate compounded **daily**. How much did Mia have after 10 years?
5. Mia invested **$5,000** for **25 years** at a 10% interest rate compounded **monthly**. How much did Mia have after 25 years?
6. Mia invested **$100,000** for **30 years** at a 10% interest rate compounded **monthly**. How much did Mia have after 30 years?



Rewriting the Time Value of Money Formula

- $FV = PV \times (1 + {}^{r}/_{n})^{nt}$
- $PV = \dfrac{FV}{(1 + {}^{r}/_{n})^{nt}}$
- $r = n\left[({}^{FV}/_{PV})^{({}^{1}/_{nt})-1}\right]$
- $t = \dfrac{\ln({}^{FV}/_{PV})}{n[\ln(1+{}^{r}/_{n})]}$

- Where:
  - FV = Future Value
  - PV = Present Value
  - r = Interest Rate or Growth Rate as %
  - n = Number of times the amount is compounding
  - t = Time in Years

The "Rule of 72" - a way to keep it simple

- A quick formula to **estimate**
  - the number of years required to double an investment based on investment return, or
  - the investment return required to double an investment over a period of years.
- **Estimate** the number of years required to double an investment
  - Divide 72 by the interest rate or the expected rate of return
- **Estimate** the investment return required to double an investment over a period of years
  - Divide 72 by the period of years

- Prove it using \$1 & 10% Interest Rate
  - Rule of 72
    - 72 / 10 = 7.2 **estimated** years to double
  - Time Value of Money, Solve for t
    - $t = \frac{\ln(2/1)}{1[\ln(1+.10/1)]}$ = 7.27 **actual** years to double
- Prove it using \$1 & 10 Years
  - Rule of 72
    - 72 / 10 Years = 7.2% **estimated** annual interest rate to double
  - Time Value of Money, Solve for r
    - $r = 1\left[(2/1)^{\left(1/(1*10)\right)-1}\right] \times 100 = 7.18\%$ **actual** interest rate to double

Key Takeaways

- Money today is always worth more than the same amount of money tomorrow
- The most important formula in finance is $\boldsymbol{FV = PV \times (1 + {}^{r}/_{n})^{nt}}$
- The **Rule of 72** estimates the:
  - number of years required to double an investment based on investment return, or
  - investment return required to double an investment over a period of years.

Answer Key =FV(rate,nper,pmt,[pv],[type])

1. Mia invested $1,000 for 1 year at a 10% annual interest rate compounded annually. How much did Mia have after 1 year? FV(10%/1,1*1,0,-1000,0) = $1,100
2. Mia invested $1,000 for 1 year at a 10% interest rate compounded **monthly**. How much did Mia have after 1 year? FV(10%/12,1*12,0,-1000,0) = $1,105
3. Mia invested $1,000 for **10 years** at a 10% interest rate compounded monthly. How much did Mia have after 10 years? FV(10%/12,10*12,0,-1000,0) = $2,707
4. Mia invested $1,000 for 10 years at a 10% interest rate compounded **daily**. How much did Mia have after 10 years? FV(10%/365,10*365,0,-1000,0) = $2,718
5. Mia invested **$5,000** for **25 years** at a 10% interest rate compounded **monthly**. How much did Mia have after 25 years? FV(10%/12,25*12,0,-5000,0) = $60,285
6. Mia invested **$100,000** for **30 years** at a 10% interest rate compounded **monthly**. How much did Mia have after 30 years? FV(10%/12,30*12,0,-100000,0) = $1,983,740