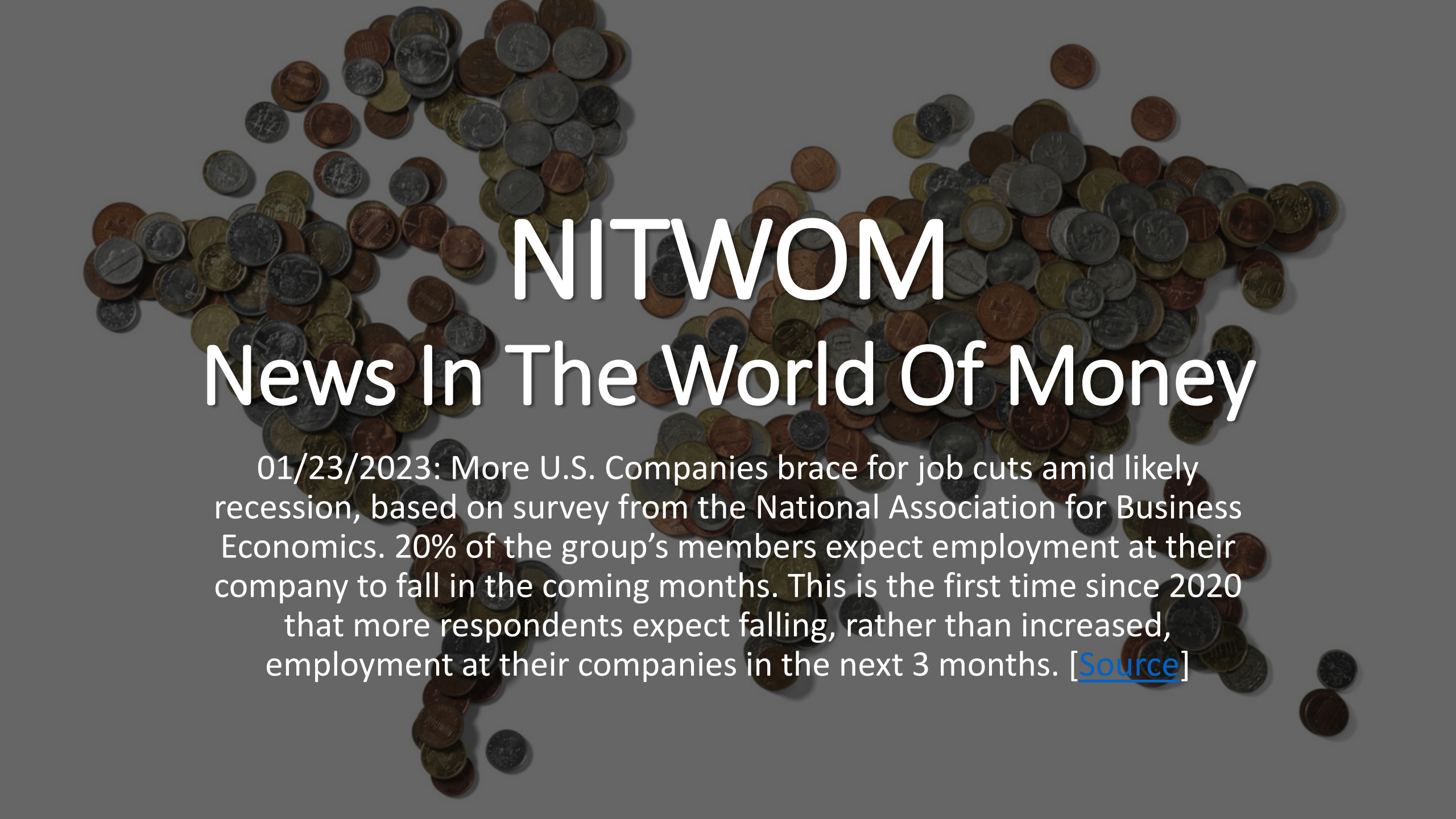
Money Now or Money Later?

Class 20: Time Value of Money II



NITWOM
News In The World Of Money

01/23/2023: More U.S. Companies brace for job cuts amid likely recession, based on survey from the National Association for Business Economics. 20% of the group's members expect employment at their company to fall in the coming months. This is the first time since 2020 that more respondents expect falling, rather than increased, employment at their companies in the next 3 months. [Source]

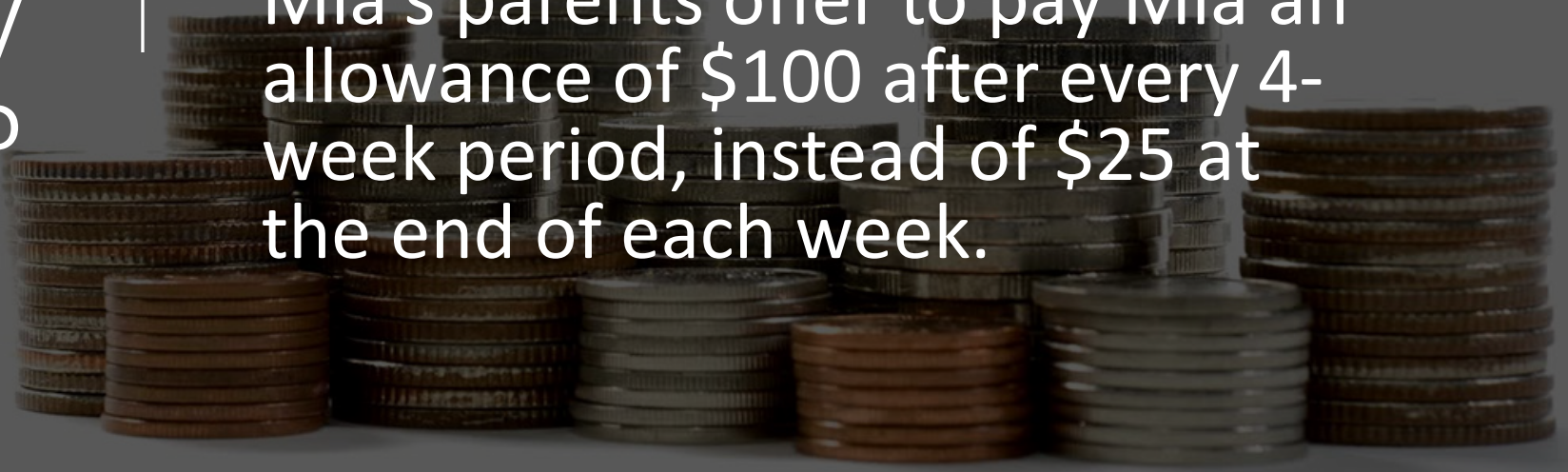
Should Mia Accept the Offer?

Why or Why Not?

Mia earns a $25 weekly allowance if she completes her chores and homework.

Remembering to pay the allowance at the end of each week was a nuisance for Mia's parents.

Mia's parents offer to pay Mia an allowance of $100 after every 4-week period, instead of $25 at the end of each week.



Mr. 401(k) recently renewed University of Washington Football season tickets. The ticket office gave Mr. 401(k) two renewal options:

1) Pay 100% of the renewal cost at the time of renewal
2) Pay 25% of the renewal cost at the time of renewal + the exact same amount for each of the next 3 months

Which did Mr. 401(k) choose? Why?

Recap of the Time Value of Money Future Value Formula

- $FV = PV \times (1 + {}^{r}/_{n})^{nt}$
- Where:
  - FV = Future Value
  - PV = Present Value
  - r = Interest rate or growth rate as percentage
  - n = Number of times the interest compounds annually
  - t = Time in years

The value of **n** varies depending on the number of times the amount is compounding, which is simply the number of times the interest is being reinvested.

- n = 1, compounded **annually**
- n = 2, compounded **semi-annually**.
- n = 4, compounded **quarterly**.
- n = 12, compounded **monthly**.
- n = 52, compounded **weekly**.
- n = 365, compounded **daily**.

The **Time Value of Money Future Value Formula** calculates the effect of compound interest on principal over time, but it has limitations.

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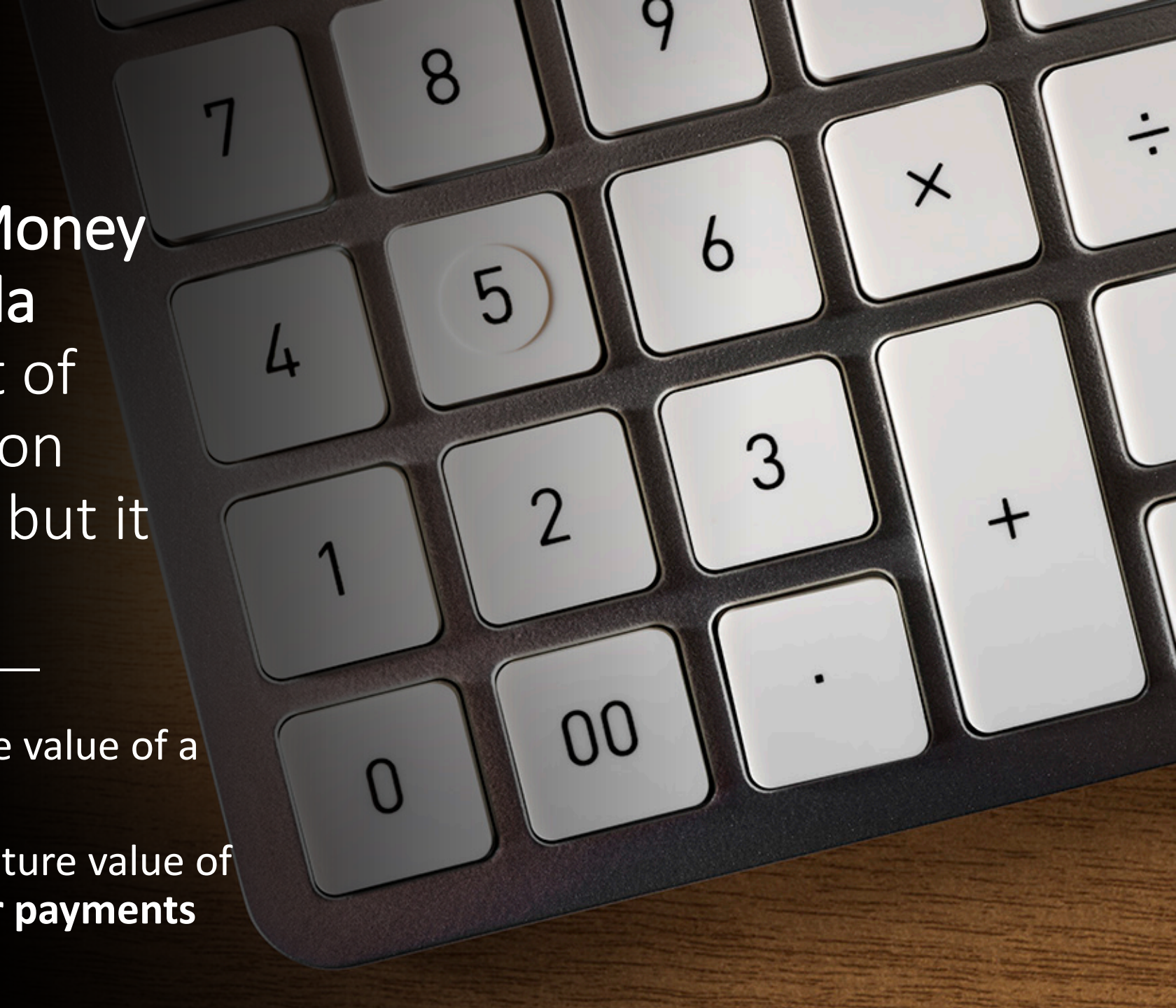
1) It **only** calculates the future value of a **lump sum of money**

2) It **does not** calculate the future value of **regular periodic cashflows or payments**



Can you think of any examples of regular periodic cashflows or payments over time?

Regular periodic cashflows or payments over time examples

- Contributing **12% of pay <u>each pay period</u>** to a retirement account, like a 401(k) plan or individual retirement account.
- Investing **$10** of a **<u>weekly</u>** allowance.
- Investing **$2,500** earned from a summer job **<u>every September</u>**.
- Making coffee at home and investing the **$5 <u>per day</u>** that would have otherwise been spent at the coffee shop.
- Saving **$50 <u>per week</u>** in an investment account toward college tuition, like a 529 Plan.
- Transferring **$500** on the **<u>last day of each month</u>** from a savings account into long-term investments.

Series of regular periodic cashflows or payments

Annuity

- A series of regular cashflows or payments **for a fixed period**
- Very common

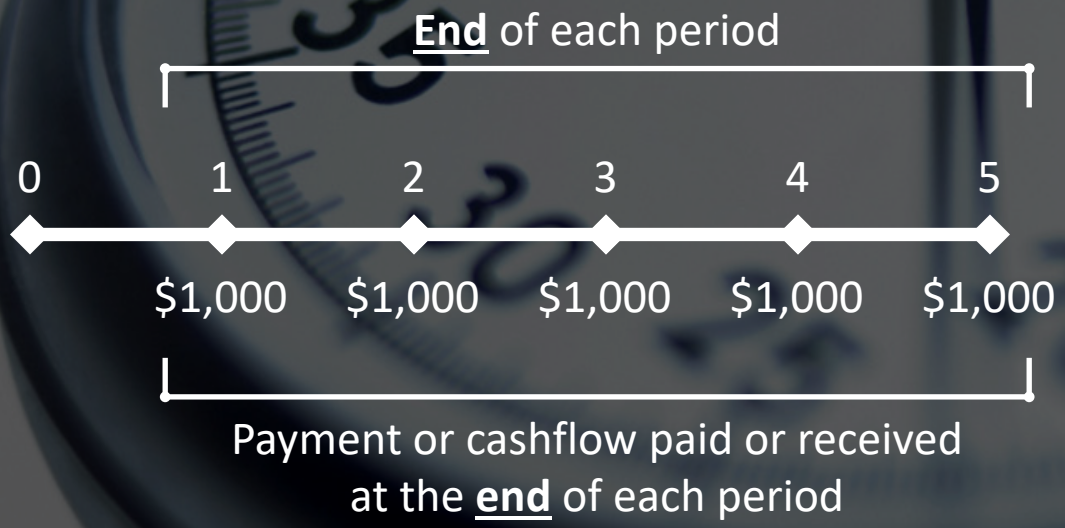
Perpetuity

- A series of regular cashflows or payments **with no end date**
- Very rare

Annuities also are insurance products sold to investors, where an insurance company guarantees a series of payments to an investor, which are backed by the general assets of the insurance company. Annuity insurance products are out of scope for finlit.

Ordinary Annuities

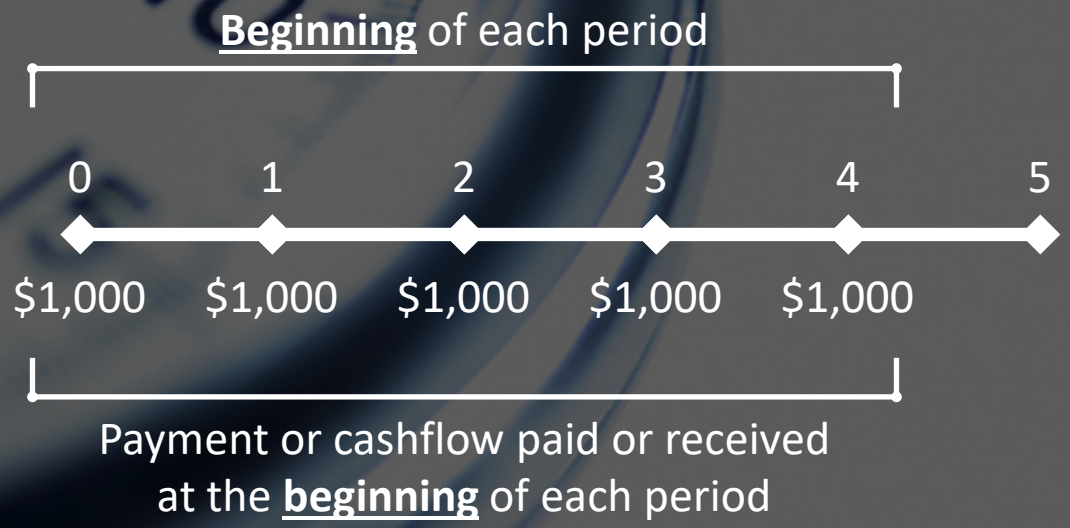
Makes or requires payments at the **end** of each period. What it looks like for a $1,000 payment:



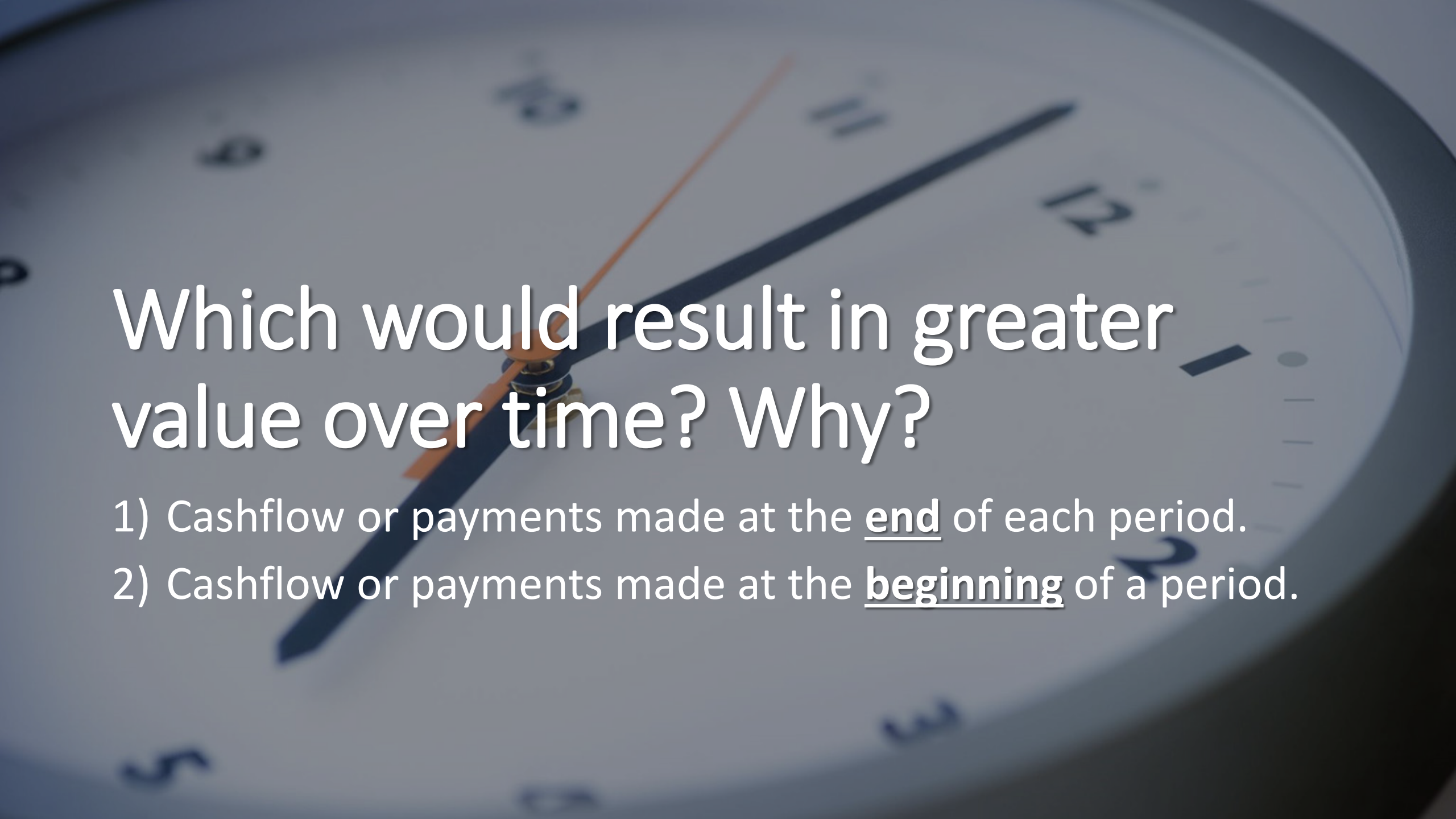
Real World Example: earning an allowance at the end of each week

Annuities Due

Makes or requires payments at the **beginning** of each period. What it looks like for a $1,000 payment:



Real World Example: paying for a streaming video subscription in advance -- 1st day of each month



Which would result in greater value over time? Why?

1) Cashflow or payments made at the **end** of each period.
2) Cashflow or payments made at the **beginning** of a period.

What is the future value of $1,000 invested at the end of each year, for 5-years, at a 10% annual interest rate?

We know from the time value of money that **money now is worth more than the same amount of money in the future** because of the effects of inflation and interest. Therefore, the amount invested at the end of Year 1 is more valuable than the same amount invested at the end of Year 5. We could use the **time value of money future value formula** for each amount invested.

$$FV = PV \times (1 + {}^{r}/_{n})^{nt}$$

Year-End	Amount invested	Future Value Formula	Future Value
1	$1,000	$1{,}000 * (1 + 10\%/1)^{1*4}$ =	$1,464
2	$1,000	$1{,}000 * (1 + 10\%/1)^{1*3}$ =	+ $1,331
3	$1,000	$1{,}000 * (1 + 10\%/1)^{1*2}$ =	+ $1,210
4	$1,000	$1{,}000 * (1 + 10\%/1)^{1*1}$ =	+ $1,100
5	$1,000	$1{,}000 * (1 + 10\%/1)^{1*0}$ =	+ $1,000
		Total:	= **$6,105**

Valuing an Annuity

- **Future Value** of an Annuity
  - The **future monetary value** of a series of cashflows or payments based on a **specific interest rate or growth rate**
- **Use Case**
  - Projecting the future value of investing $10 from a weekly allowance from age 13 to age 18

- **Present Value** of an Annuity
  - The **current monetary value** of all **future** cashflows or payments impacted by a specific **discount** rate.
- **Use Case**
  - Projecting the present value of a $25 weekly allowance from age 13 to age 18

Time Value of Money Future Value of an Ordinary Annuity Formula

- $FV = PMT \times \left[\frac{(1+i)^n - 1}{i}\right]$
- Where:
  - FV = Future Value
  - PMT = Payment or cashflow amount **per period**
  - i = Effective interest rate or growth rate as a percentage = annual interest rate / number of payments or cashflows **per year**
  - n = Number of payments or cashflows

Time Value of Money Present Value of an Ordinary Annuity Formula

- $PV = PMT \times \left[\frac{1-(1+i)^{-n}}{i}\right]$
- Where:
  - PV = Present Value
  - PMT = Payment or cashflow amount **per period**
  - i = Effective interest rate or growth rate as a percentage = annual interest rate / number of payments or cashflows **per year**
  - n = Number of payments or cashflows

Time Value of Money Future Value of an Annuity Due Formula

- $FV = PMT \times \left[\frac{(1+i)^n - 1}{i}\right] \times (1+i)$
- Where:
  - FV = Future Value
  - PMT = Payment or cashflow amount **per period**
  - i = Effective interest rate or growth rate as a percentage = annual interest rate / number of payments or cashflows **per year**
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Time Value of Money Present Value of an Annuity Due Formula

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- Where:
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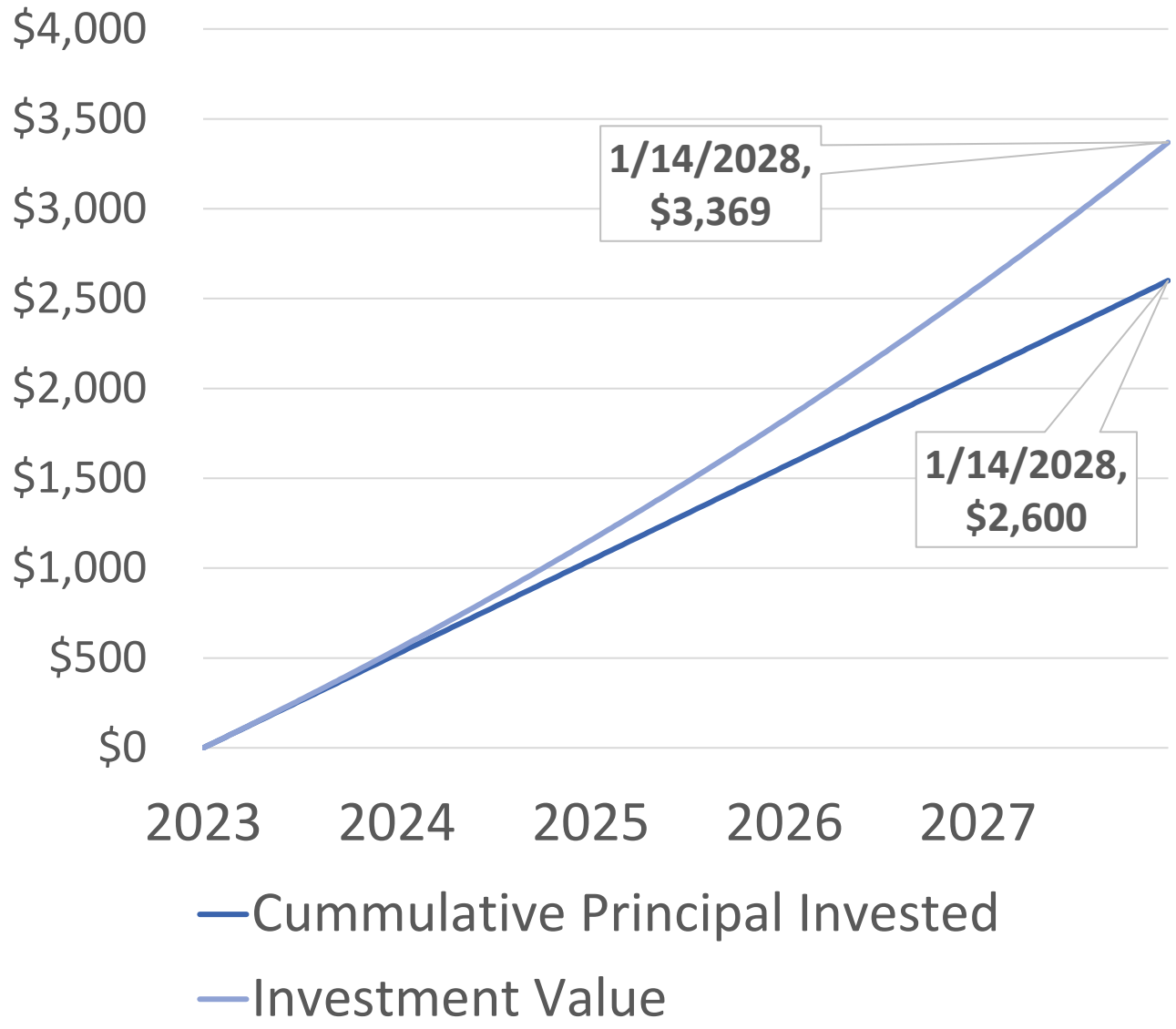
  - i = Effective interest rate or growth rate as a percentage = annual interest rate / number of payments or cashflows **per year**
  - n = Number of payments or cashflows

Mia invests $10 per week from her allowance, from age 13 to age 18, in an investment account earning 10% annual investment returns.

$$FV = PMT \times \left[\frac{(1 + i)^n - 1}{i}\right]$$

$$FV = \$10 \times \left[\frac{\left(1 + \left({}^{10\%}/_{52}\right)\right)^{260} - 1}{\left({}^{10\%}/_{52}\right)}\right]$$

$$\boldsymbol{FV = \$3,369}$$

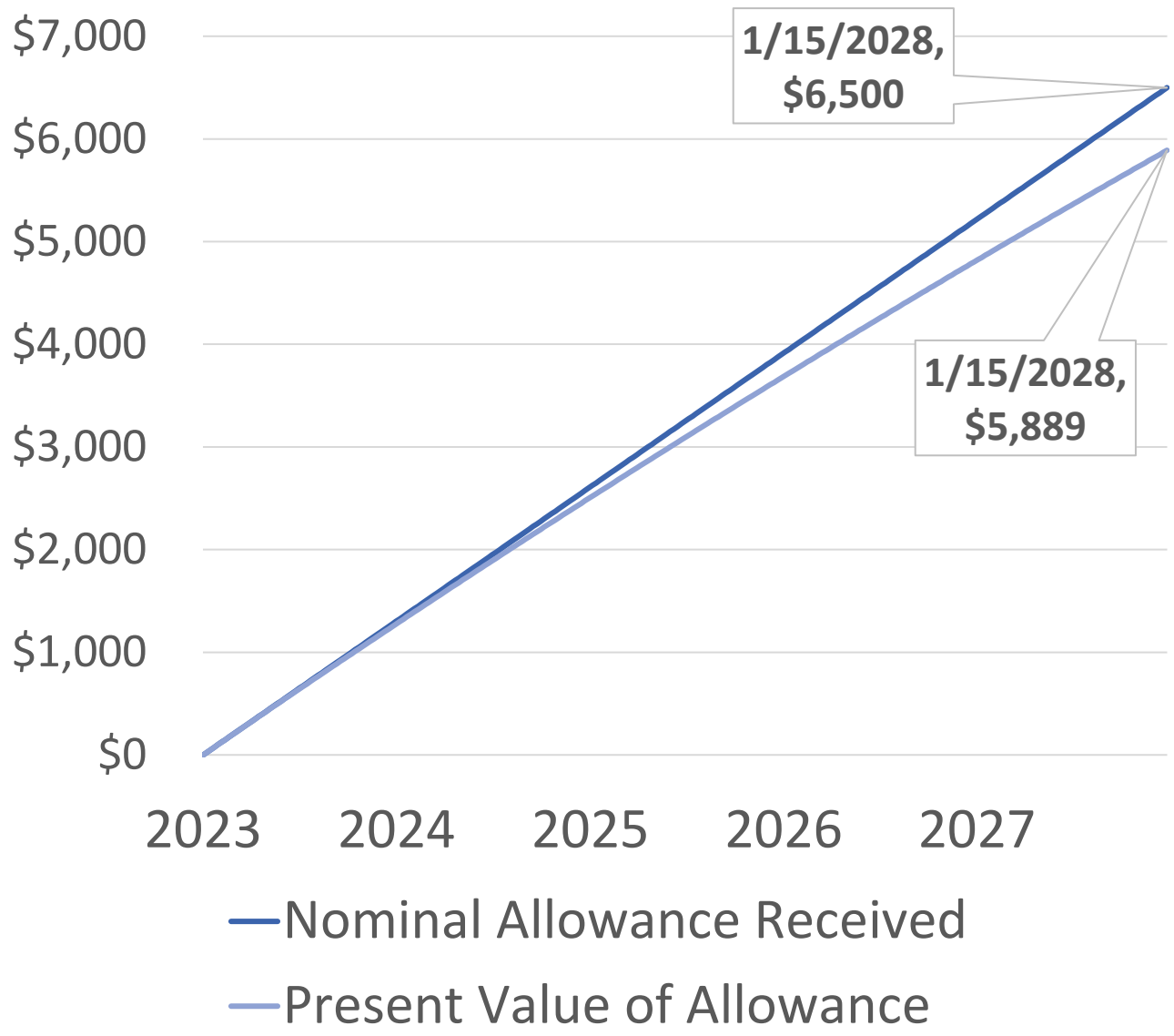


Mia wants to know the present value of her $25 per week allowance from age 13 to age 18, based on a 4% annual discount rate – an estimate of inflation.

$$PV = PMT \times \left[\frac{1 - (1 + i)^{-n}}{i}\right]$$

$$PV = \$25 \times \left[\frac{1 - \left(1 + \left({}^{4\%}/_{52}\right)\right)^{-260}}{\left({}^{4\%}/_{52}\right)}\right]$$

$$\boldsymbol{PV = \$5,889}$$

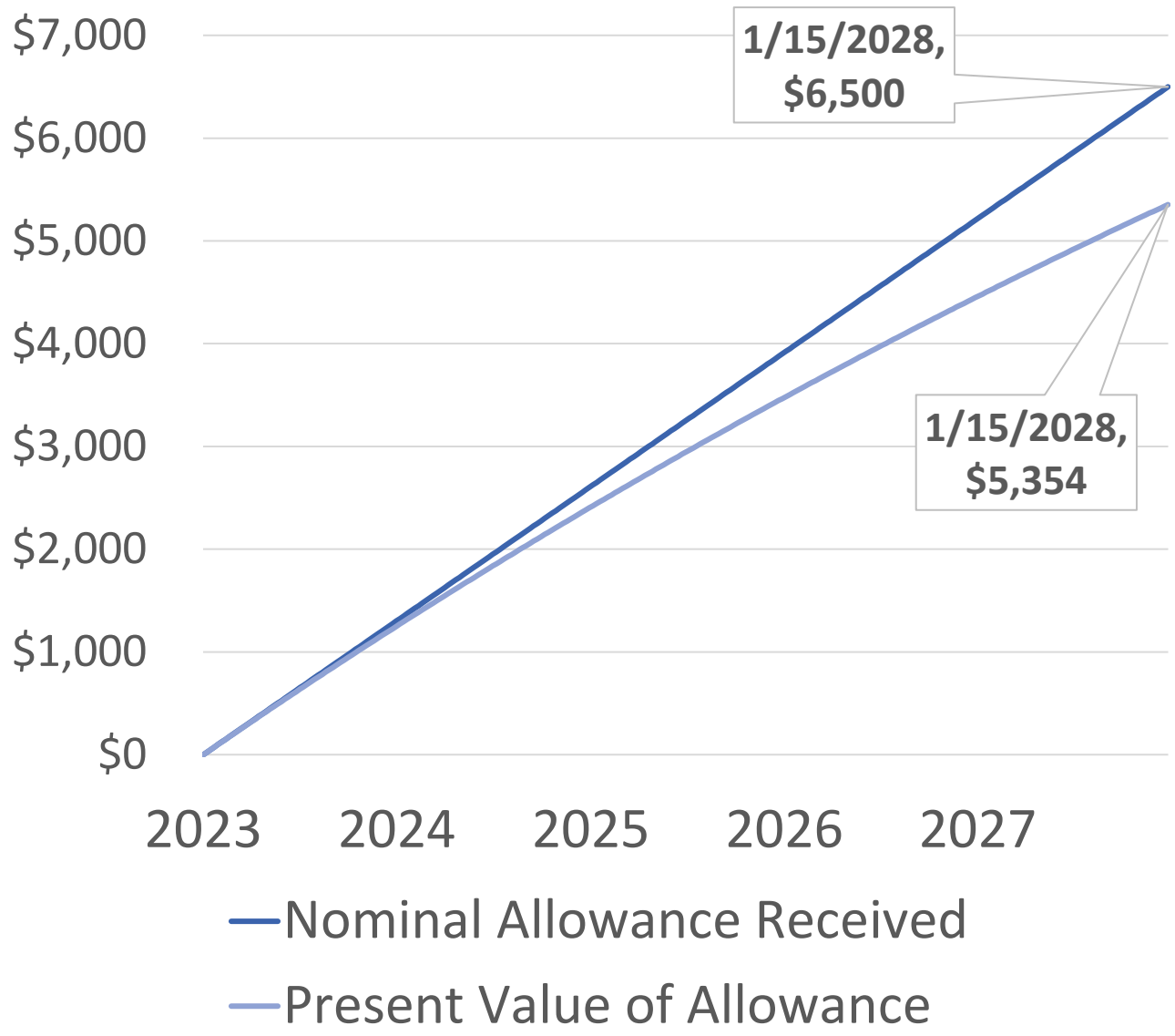


What if inflation was much higher and Mia assumes an 8% discount rate instead of 4% in her present value calculation?

$$PV = PMT \times \left[\frac{1 - (1 + i)^{-n}}{i}\right]$$

$$PV = \$25 \times \left[\frac{1 - \left(1 + \left({}^{8\%}/_{52}\right)\right)^{-260}}{\left({}^{8\%}/_{52}\right)}\right]$$

$$\boldsymbol{PV = \$5,354}$$

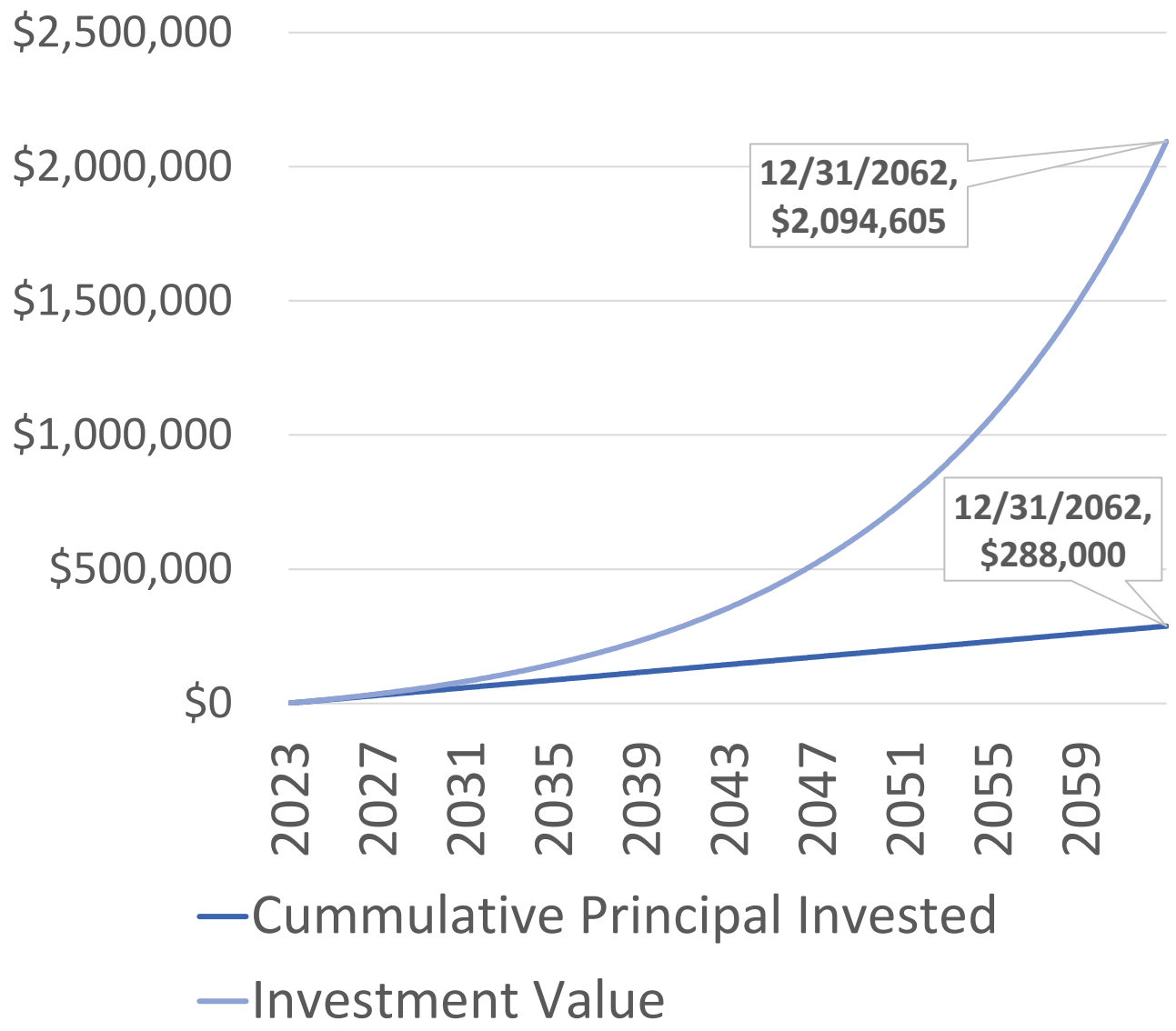


Mathias contributes $600 per month to a retirement account, from age 22 to age 62, earning 8% annual investment returns.

$$FV = PMT \times \left[\frac{(1 + i)^n - 1}{i}\right]$$

$$FV = \$600 \times \left[\frac{\left(1 + \left({}^{8\%}/_{12}\right)\right)^{480} - 1}{\left({}^{8\%}/_{12}\right)}\right]$$

$$\boldsymbol{FV = \$2,094,605}$$

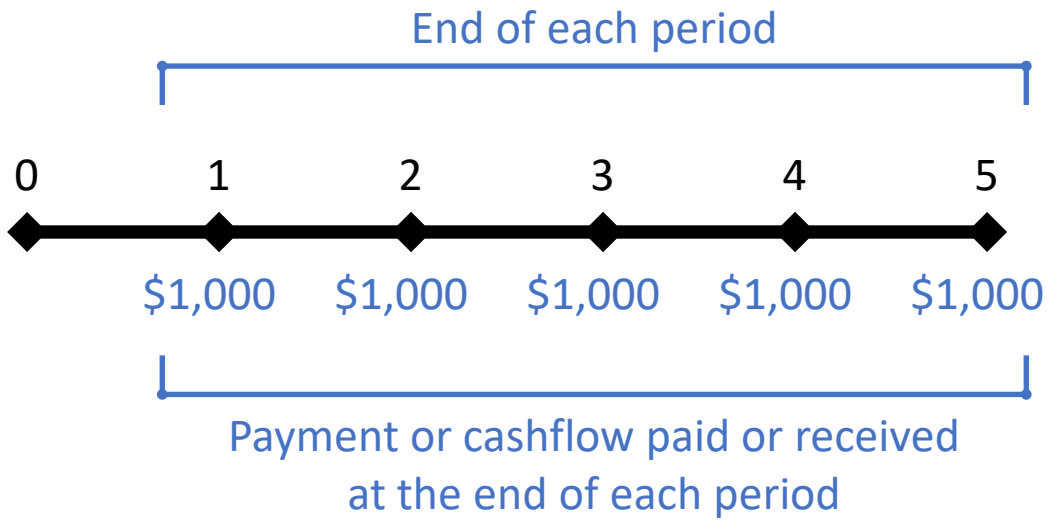


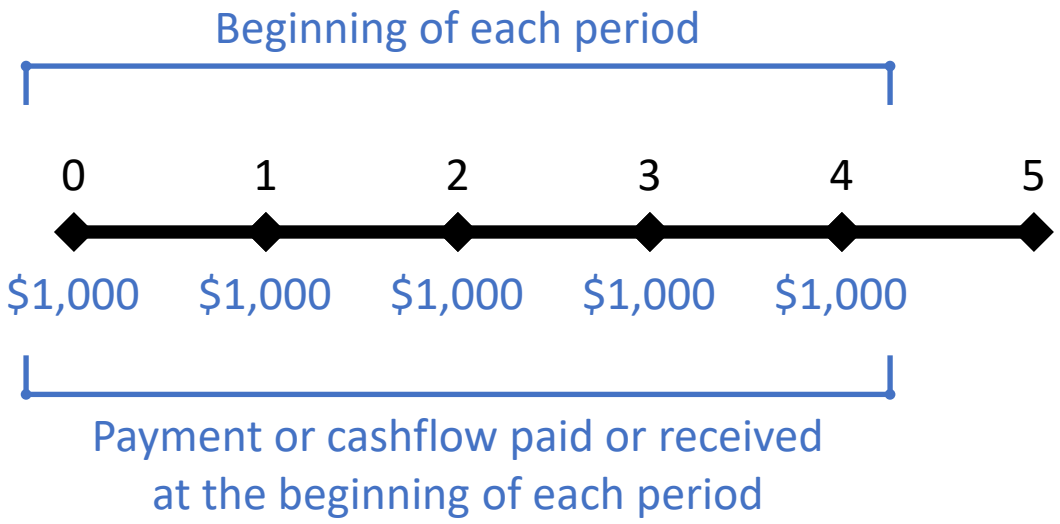
Key Takeaways

- An annuity is series of regular cashflows or payments for a fixed period
- The time value of money future value and present value of annuity functions calculate the value of a series of cashflows or payments over time
- The power of compound interest is very significant over long time periods

Backup

Supplemental material and references


End of each period
0
1
2
3
4
5
$1,000
$1,000
$1,000
$1,000
$1,000
Payment or cashflow paid or received
at the end of each period


Beginning of each period
0
1
2
3
4
5
$1,000
$1,000
$1,000
$1,000
$1,000
Payment or cashflow paid or received
at the beginning of each period