PRIMER: RIGHT TRIANGLE TRIGONOMETRY FOR EARTH SYSTEMS SCIENCE

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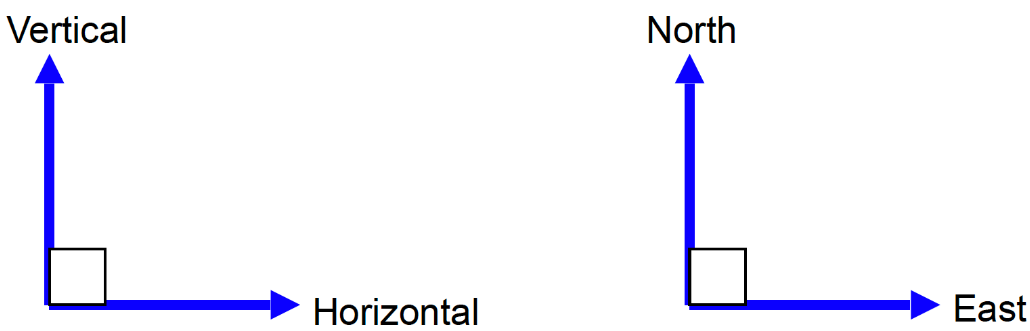
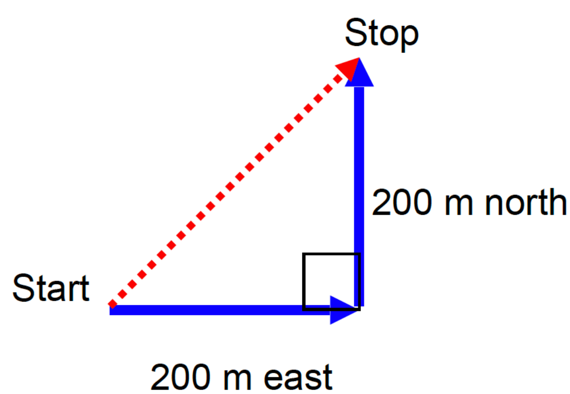
The *Need* for Trigonometry in Science

Earth-based directional axes are perpendicular to each other. Therefore, right angle trigonometry is used to relate distances (and many other variables) in these directions. Several common two-dimensional axes are:



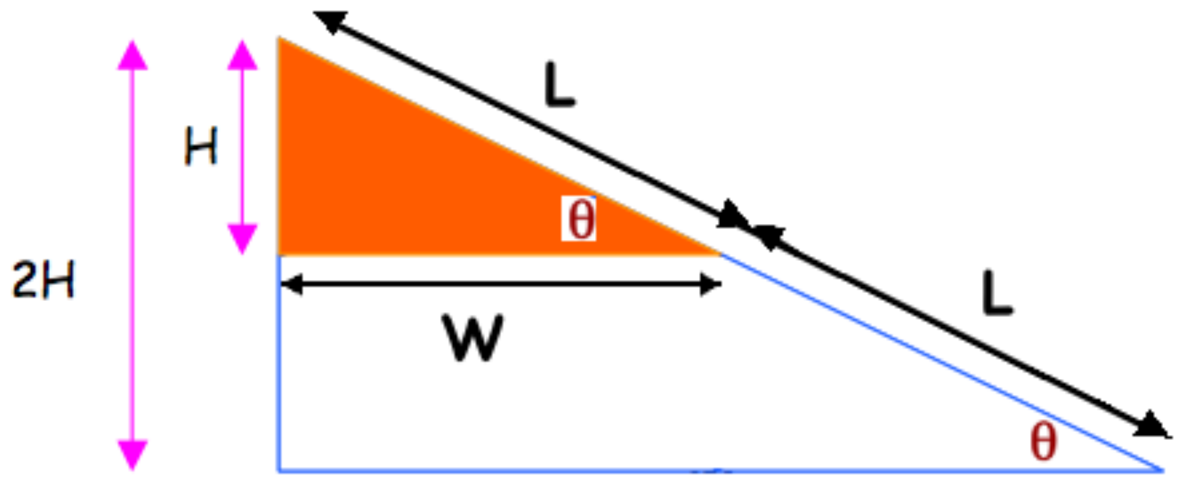
The symbol for a *right* angle (90°) is a *square* at the intersection of the two lines. Since distance is not limited to one dimension, we can use right angle trigonometry to isolate lengths within each dimension or combine these components of distance.

Example: Rosco the dog follows a scent 200 meters east and then 200 meters north. The following sections will illustrate how we describe this change in position. Notice we are dealing with a *right* triangle (A triangle with one of the interior angles equal to 90°).



Relation of H/W/L to Trigonometric Functions

When analyzing variables associated with right triangles, the ratio, H/L, keeps popping up again and again. For instance, consider the large right triangle with *hypotenuse* 2L and vertical side length 2H. Insert a similar right triangle with an H/L ratio of ½. This means that its hypotenuse is only L and it's vertical side is only H. But of course, both triangles have the same angle of inclination, θ (θ is the Greek letter theta). Both H/L and θ indicate the pitch or slope of the ramp.



$$H / L \;=\; 2H / 2L \;=\; \text{something to do with } \theta$$

When a quantity keeps appearing in a calculation's final form, scientists pay attention and give that quantity its own name. H/L has its own name, "sine of θ", where θ is the angle that the incline makes with the horizontal.

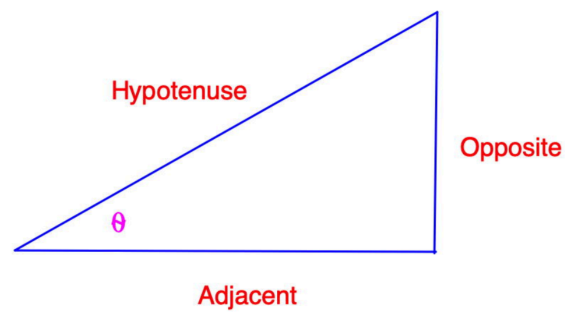
$$\text{sine}(\theta) = \sin(\theta) = H / L$$

The base of the triangle, W, is also significant in other physical phenomena. W is the side lying along the x-axis in the previous scenarios. The ratio W/L is the cosine of θ:

$$\text{cosine}(\theta) = \cos(\theta) = W / L$$

Right Triangle Trigonometry: Trigonometry is Greek for triangle metrics, or the measures of triangles.

With many direction problems, there is an angle of interest or one that is measured. The first step is to choose and label the angle as "θ". We can then uniquely label the three sides of the right triangle for this choice of θ. As the picture to right illustrates, the side opposite the right angle is the ***hypotenuse***. The other two sides are either ***adjacent to*** angle θ or opposite to angle θ. The side that touches the angle θ is considered to be adjacent to θ, and the side that is far away from angle θ is ***opposite to*** angle θ.



If you selected the other angle as θ, would your labels change? Which would stay the same.

Rewriting the definition for sine and cosine of an angle becomes:

$$\sin(\theta) = \text{opposite} / \text{hypotenuse}$$

$$\cos(\theta) = \text{adjacent} / \text{hypotenuse}$$

In addition, there is a function that involves both the sin and cos functions. The **tangent** of θ, $\tan(\theta)$, is defined as:

$$\tan(\theta) = \sin(\theta) / \cos(\theta) =$$
$$(\text{opposite}/\text{hypotenuse}) / (\text{adjacent}/\text{hypotenuse}) =$$
$$\text{opposite} / \text{adjacent}.$$

Trigonometric Inverse Functions

At every step in learning mathematical operations, we learn how to “undo” it at the same time. Addition was taught with subtraction, multiplication with division, square root with square, etc. So we need to learn how to undo each of the trig functions so we can calculate the angle once we have the ratio of the sides of a right triangle.

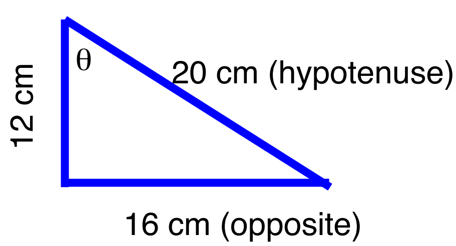
For the trig functions, the inverse functions are identified with a $^{-1}$ superscript, such that:

$\sin^{-1}(\sin(\theta)) = \theta$ and $\sin(\sin^{-1}(\theta)) = \theta$

$\cos^{-1}(\cos(\theta)) = \theta$ and $\cos(\cos^{-1}(\theta)) = \theta \tan^{-1}(\tan(\theta)) = \theta$ and $\tan(\tan^{-1}(\theta)) = \theta$

Meaning, each inverse function undoes the function, and the function undoes the inverse.

Sometimes in navigation calculations, one knows the sine of an angle, but desires to find the angle. For example, calculate θ in the following triangle:



$\sin(\theta)$ = opposite / hypotenuse = 16 / 20 = 0.8 or $\theta = \sin^{-1}(0.8) = 53.13°$

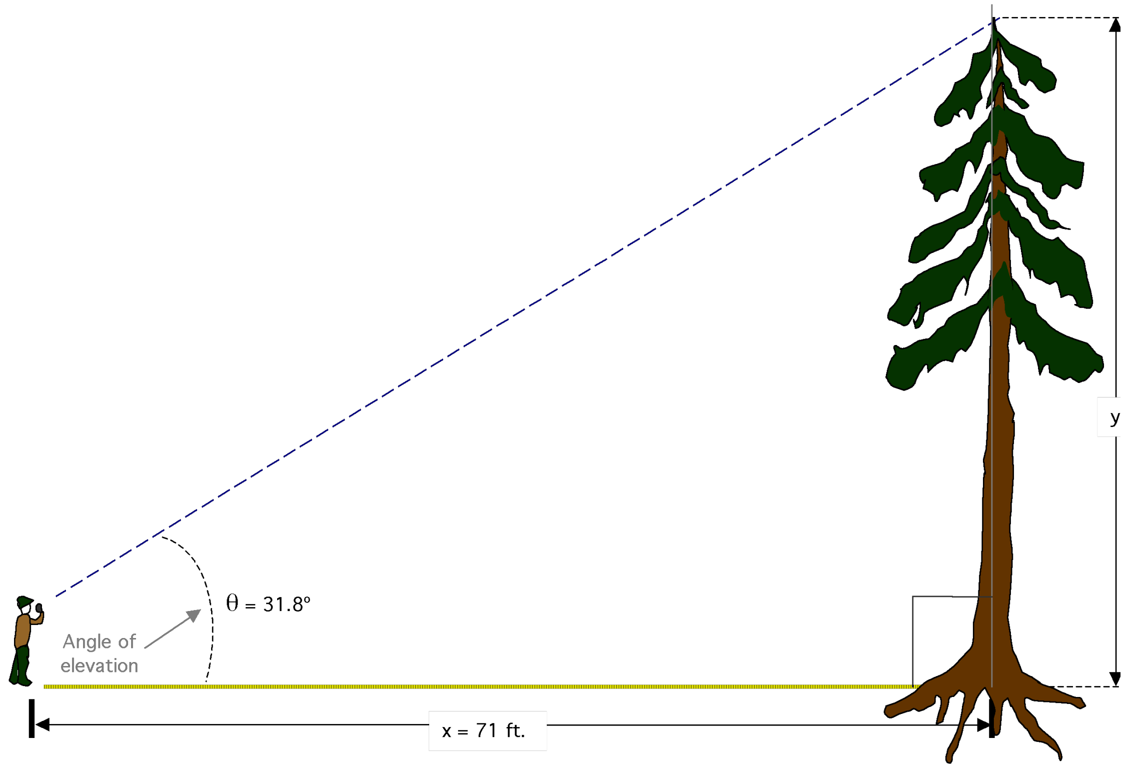
$\cos(\theta)$ = adjacent / hypotenuse = 12 / 20 = 0.6 or $\theta = \cos^{-1}(0.6) = 53.13°$

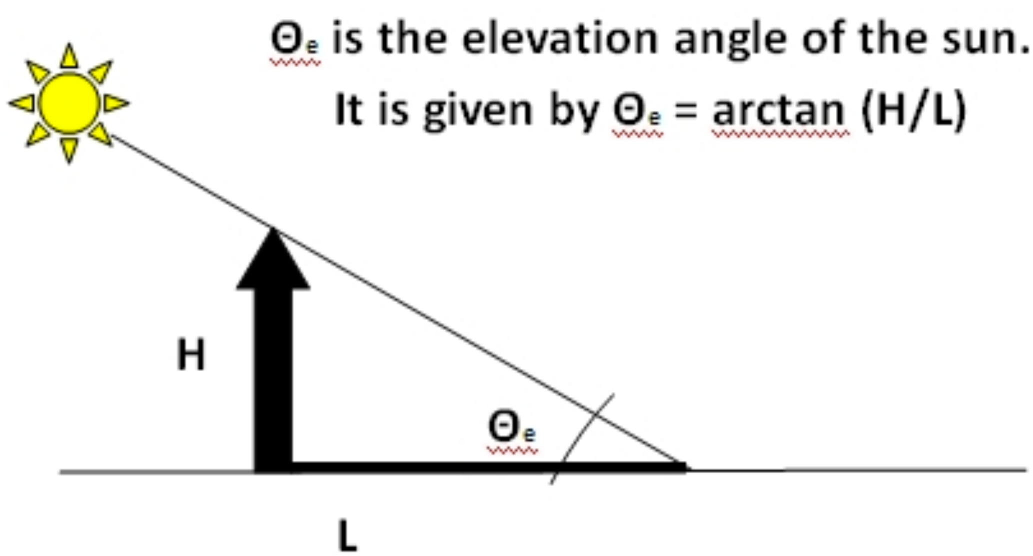
$\tan(\theta)$ = opposite / adjacent = 16 / 12 = 1.333 or $\theta = \tan^{-1}(1.333) = 53.13°$

If $y = \sin(\theta)$ then $\theta = \sin^{-1}(y)$. On your calculator this is $\sin^{-1}$, *inv sin*, arcsin, or “asin”. Similarly, $y = \cos(\theta)$ then $\theta = \cos^{-1}(y)$ which is $\cos^{-1}$, *inv cos*, arcos, or “acos”. Finally, $y = \tan(\theta)$ then $\theta = \tan^{-1}(y)$ which is $\tan^{-1}$, *inv tan*, arctan, or “atan”.

Examples of Right Triangles in Earth Science

Calculating height of tree (or any object, such as a mountain). From http://www.tiem.utk.edu/~gross/bioed/bealsmodules/triangle3.gif

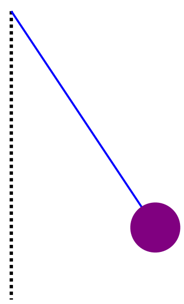




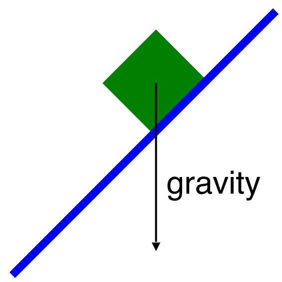
Calculating the angle of sun above the horizon (an essential measurement for navigation). The inverse of the sine function is covered in a later section. Image from http://www.forensicgenealogy.info/shadows.html.

Finding Right Triangles - Essential in Solving Physics Problems

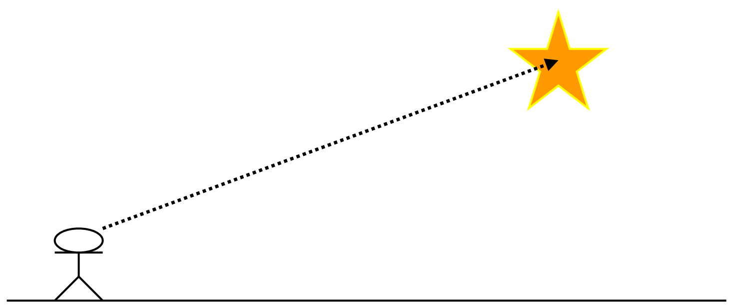
Identify the right triangles in the diagrams below.



Pendulum



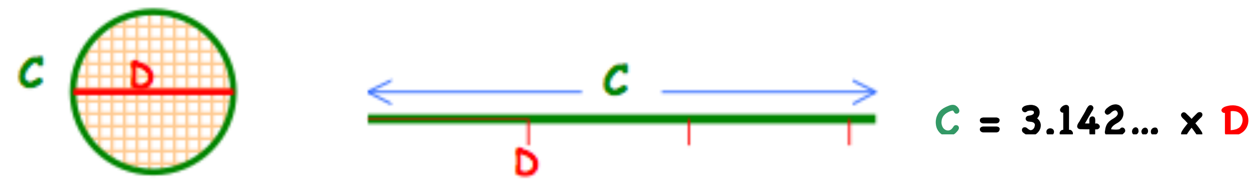
Box on an incline

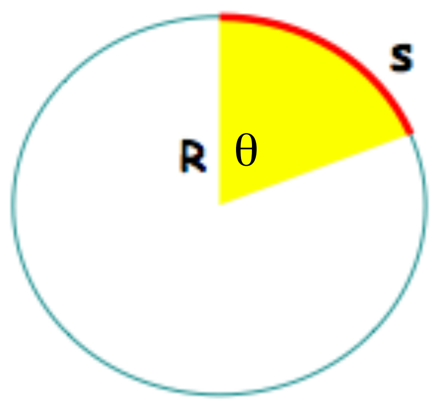


Star above horizon

Units of Angular Measurement: Degrees versus Radians

In plane geometry, a circle contains 360º. In the analysis of the angles associated with circles, there exist a more sophisticated unit called *radians (rad)*. For instance, the Greeks had discovered that the ratio of the circumference of a circle to its diameter was always 3.14159265.... If you “unrolled” the circumference and stretched it out:





They named this ratio π (Greek pi pronounced pie). The degree unit probably came about because there are approximately 360 days in a year, the time it takes for the Earth to make a complete revolution around the Sun. However, the natural unit to describe angles is the radian (abbreviated as rad).

Arc-length and angle: In radians, the length of an arc of a circle, s, is: $s = R\theta$ where R is the radius of the circle and θ is the angle in radians.

If the value of θ = 1 rad then s = R. In that instance, the length of the arc is the same as the length of the radius.

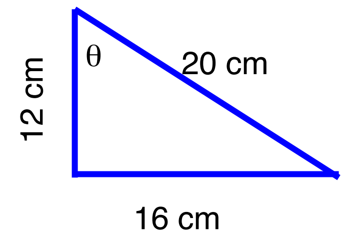
Since 2 π rad = 360º, then 1 rad = 180º/π degrees and 1 degree = π/180º rad.
If an angle is given without units, then it is understood to be radians, not degrees. Also be sure to set your calculator in the correct mode, RAD (radians) or DEG (degrees).
Note: sin(4.0 rad) does not equal sin(4.0 degree)!!!

Sample Sin, Cos, and Tan Problems

1. For the following right triangle and the angle shown,

    (a) Which side is opposite θ? Which is adjacent?

    (b) Evaluate $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.



2. Using your calculator, evaluate the following:

    (a) sin(30.0º) (b) cos(4.36) (c) sin(37.0º) (d) cos(π)

3. Convert from degrees to radians:

    (a) 120º (b) 45º (c) 90º (d) 75º (e) 330º

4. Convert from radians to degrees:

    (a) 6 (b) 1.5 (c) 3 (d) 7 (e) 2

5. On your calculator, find the following tangents, and note if any are undefined.

    (a) tan(45º) (b) tan(-2.34) (c) tan(48.0 rad) (d) tan(90º)

    (e) tan(0.893) (f) tan(135º) (g) tan(270º) (h) tan(2π)

5. (a) For which angles does the cosine of the angle equal 0? Notice that when the cosine of the angle is 0, the tangent of that angle is undefined.

    (b) For which angles does the sine of the angle equal 0?

    (c) For which angles does $\sin(\theta) = \cos(\theta)$?

Sample Inverse Trig Problems

(a) Inverse tangent of 3.75

(b) Inverse sine of 0.500

(c) Inverse cosine of -0.7071

(d) Inverse sine of -0.600

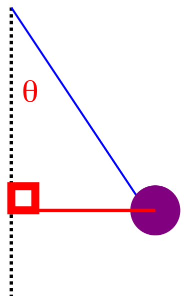
(e) Inverse tangent of 1.732

(f) Inverse sine of 2.00

(g) Inverse cosine of -0.3333

Answers to Examples

Finding Right Triangles - Essentials in Solving Science Problems



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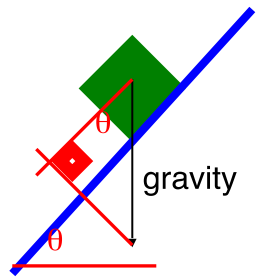
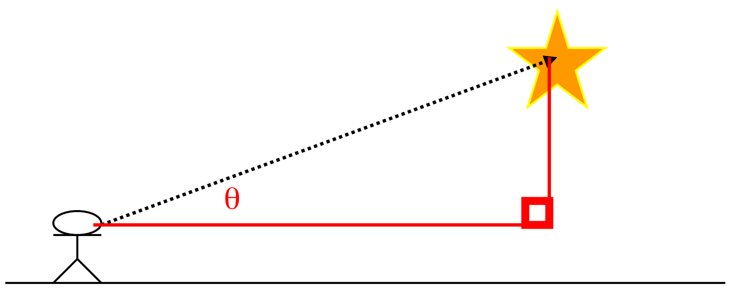


Box on an incline



Star above horizon

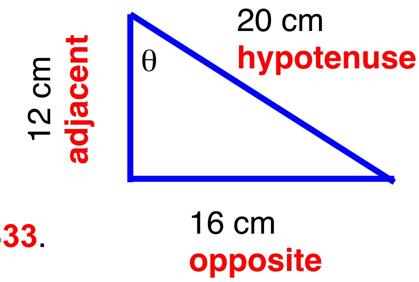
Sample Sin, Cos, and Tan Problems

1. For the following right triangle and the angle shown,

   (a) Which side is opposite θ? Which is adjacent?

   (b) Evaluate $\sin(\theta)$ **= 16/20 = 0.8**,

   $\cos(\theta)$ **= 12/20 = 0.6**, and $\tan(\theta)$ **= 16/12 = 1.333**.



2. Using your calculator, evaluate the following:

   (a) sin(30.0º) **= 0.5** (b) cos(4.36) **= -0.345** (c) sin(37.0º) **= 0.602** (d) cos(π) **= -1**

3. Convert from degrees to radians **(multiply degrees by π / 180 rad/degrees)**:

   (a) 120º **= 2.09** (b) 45º **= 0.785** (c) 90º **= 1.57** (d) 75º **= 1.31** (e) 330º **= 5.76**

4. Convert from radians to degrees **(multiply radians by 180 / π degrees/rad)**:

   (a) 6 **= 343.8º** (b) 1.5 **= 85.9º** (c) 3 **= 171.9º** (d) 7 **= 401.1º** (e) 2 **= 114.6º**

5. On your calculator, find the following tangents, and note if any are undefined.

   (a) tan(45º) **= 1** (b) tan(-2.34) **= 1.03** (c) tan(48.0) **= 1.20** (d) tan(90º) **= error = 1/0**

   (e) tan(0.893) **= 1.24** (f) tan(135º) **= -1** (g) tan(270º) **= error = 1/0** (h) tan(2π) **= 0**

5. (a) For which angles does the cosine of the angle equal 0? Notice that when the cosine of the angle is 0, the tangent of that angle is undefined.
   90º, 270º, 450º, 630º, … or π/2, 3π/2, 5π/2, 7π/2, … radians.

   (b) For which angles does the sine of the angle equal 0? **0º, 180º, 360º, 540º, …**
   or 0, π, 2π, 3π, 4π, … radians.

   (c) For which angles does $\sin(\theta) = \cos(\theta)$? **45º, 225º, 405º, 585º, …**
   or π/4, 5π/4, 9π/4, 13π/4, … radians.

Sample Inverse Trig Problems

(a) Inverse tangent of 3.75 **= 75.1º = 1.31 rad**

(b) Inverse sine of 0.500 **= 30º = 0.524 rad or π / 6 rad**

(c) Inverse cosine of -0.7071 **= 135º = 2.356 rad or 3 π / 4 rad**

(d) Inverse sine of -0.600 **= 36.9º = -0.644 rad**

(e) Inverse tangent of 1.732 **= 60º = 1.047 rad or π / 3 rad**

(f) Inverse sine of 2.00 **= error (note: sine of any angle falls between 1 & -1)**

(g) Inverse cosine of -0.3333 **= 109.5º = 1.911 rad**