Topic: Several complex variables and holomorphic dynamics.
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On the automorphism group of bounded pseudoconvex domains. *Pseudoconvexity* is the complex-analytic analogue of convexity, which is preserved under biholomorphism of $\mathbb{C}^n$. In particular, a bounded domain $\Omega \subsetneq \mathbb{C}^n$ with $\mathcal{C}^2$-smooth boundary with a defining function ρ of Ω, is pseudoconvex if for $p \in \partial\Omega$,

$$\sum_{j=1}^{n} \frac{\partial^2 \rho}{\partial z_j \bar{z}_k}(p) v_j \bar{v}_k \geq 0 \text{ where } v = (v_1, \cdots, v_n) \in H_p(\partial\Omega) := T_p(\partial\Omega) \cap iT_p(\partial\Omega).$$

If the above inequality is strict for all $p \in \partial\Omega$ and $v \in H_p(\partial\Omega)$, Ω is said to be *strongly pseudoconvex.* For Ω pseudoconvex, the classification of the automorphism group $\mathrm{Aut}(\Omega)$ is of great interest in several complex variables. $\mathrm{Aut}(\Omega)$ is a Lie group and acts properly on Ω. When $\partial\Omega$ is "sufficiently nice", it has been believed that there are very few domains with "large" automorphism group. For instance, a seminal result by Wong and Rosay says that given a bounded strongly pseudoconvex domain $\Omega \subsetneq \mathbb{C}^n$, $\mathrm{Aut}(\Omega)$ is non-compact if and only if Ω is biholomorphic to the unit ball in $\mathbb{C}^n$. When Ω is not strongly pseudoconvex, the notion of D'Angelo type — a parameter that measures the order of contact of holomorphic curves with $\partial\Omega$ — plays an important role. In a series of papers, Bedford–Pinchuk established a method for classifications of smoothly bounded pseudoconvex domains with finite D'Angelo type with non-compact automorphism groups. Motivated by these works, recently, Zhang studied this classification problem on smoothly bounded domains with finite D'Angelo type such that up to a holomorphic change of coordinates, near each point $p \in \partial\Omega$, $\partial\Omega$ has some special geometric structure, namely, Ω is *decoupled near each* $p \in \partial\Omega$. For such Ω, Zhang showed that $\mathrm{Aut}(\Omega)$ is non-compact if and only if Ω is biholomorphic to the complex ellipsoid

$$\{(w, z_1, \cdots, z_{n-1}) \in \mathbb{C} \times \mathbb{C}^{n-1} : |w|^2 + |z_1|^{2m_1} + \cdots + |z_{n-1}|^{2m_{n-1}} < 1\},$$

for some $m_1, \cdots, m_{n-1} \in \mathbb{Z}_+$. The aim of this project is to understand the underlying ideas of the techniques that have been used so far, which may lead to new classification problems of $\mathrm{Aut}(\Omega)$ for smoothly bounded pseudoconvex domains of finite D'Angelo type, whose boundaries satisfy certain geometric conditions.

On the holomorphic dynamics of matrix-valued maps. The goal of this project is to generalise the Fatou-Julia theory from the dynamics of polynomial maps on the Riemann sphere for matrix valued maps. To recall, *Fatou set* is the largest open set where the family $\{p^{\circ n}\}_{n \geq 1}$ is normal and *Julia set* is the complement of Fatou set in $\mathbb{C}$. In particular, we consider a complex polynomial $p(z) = \sum_{i=0}^{d} a_i z^i$ of degree $d \geq 2$ and consider the corresponding map $P_p : \mathcal{M}(n; \mathbb{C}) \to \mathcal{M}(n; \mathbb{C})$ defined as

$$P_p(M) = \sum_{i=0}^{d} a_i M^i \text{ for every } M \in \mathcal{M}(n; \mathbb{C}).$$

Our aim is to develop an analog Fatou-Julia theory on $\mathcal{M}(n; \mathbb{C}), n \geq 2$, where the normality criteria is given Aladro–Krantz. This had been earlier studied for $n = 2$ in Pal and Cerveau-Déserti. The general approach is to learn the basics of holomorphic dynamics of polynomial maps (or rational maps) in $\mathbb{C}$ from the standard references and then delve in to the theory of matrices. In particular, give a proper analytic description of the Julia set and Fatou set and analyse the rigidity of these sets by constructing (dynamical) Greens function and other dynamical invariants, in this set up.

On the ubiquitous solenoids. The goal of this project is to understand the construction of solenoids (Smale attractors) from the dynamics of self maps of the filled torus, i.e., $\mathbb{D} \times S^1$ from Katok-Hasselblatt. To mention here, the solenoids arise naturally in the description of Julia sets (hyperbolic) complex Hénon maps in $\mathbb{C}^2$, due to work of Hubbard–Oberste-Vorth. Finally, we will attempt to connect the above theory to certain class of polynomial automorphisms of $\mathbb{C}^n, n \geq 3$.