

Topic: Classical and finite multiple zeta values

Organizer: Sanoli Gun and Masanobu Kaneko

Multiple zeta values are real numbers defined by simple infinite series obtained as a multi-variable generalization of the values of the Riemann zeta function at positive integers. Research on these values dates back to Euler, and since the 1990s their study has advanced explosively. Connections have emerged not only with number theory and arithmetic geometry, but also with knot theory and quantum field theory in physics and many more. Even now the subject continues to develop rapidly. Project will start with a series of lectures. First half of the lectures and project will cover the basic aspects of these *classical* multiple zeta values. One of the reasons why multiple zeta values form such a rich subject is that they possess nice integral representations, from which a wealth of relations can be derived. Starting from their basic definition as series, the first part of these lectures introduces, in detail, the family of relations known as the *double shuffle relations*, arising from the interplay between the series and integral representations. The second half concerns the finite analogues of multiple zeta values. These begin from the simple idea of taking truncated versions of the original series and considering their residues modulo primes. Surprisingly, these objects contain, or at least are believed to contain, information comparable to that of their real counterparts, and exhibit striking parallel phenomena with the so-called *symmetric multiple zeta values*, which are regarded as the corresponding objects in the real setting. The **Main Conjecture**, which formalizes these observations, and the two families of objects, – *finite multiple zeta values* and *symmetric multiple zeta values* – will be explained in detail.

Some advanced topics such as the relation between double zeta values and modular forms, and the recent work of Hirose and Sato on a family of relations called the *confluence relations* will be investigated in this project. According to Furusho, these relations are equivalent to the renowned *associator relations*, which have deep connections to various areas of mathematics and physics.