



Introduction to Trees

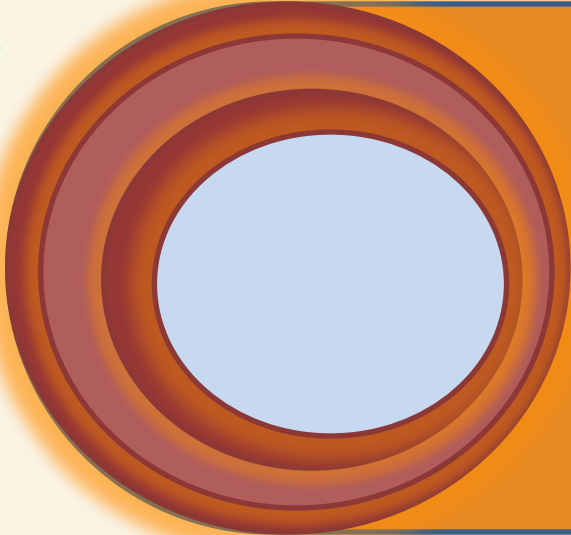
By

Dr. Yogesh M. Rajput

Assistant Professor

Dr. G. Y. Pathrikar College of Computer Science and Information Technology,

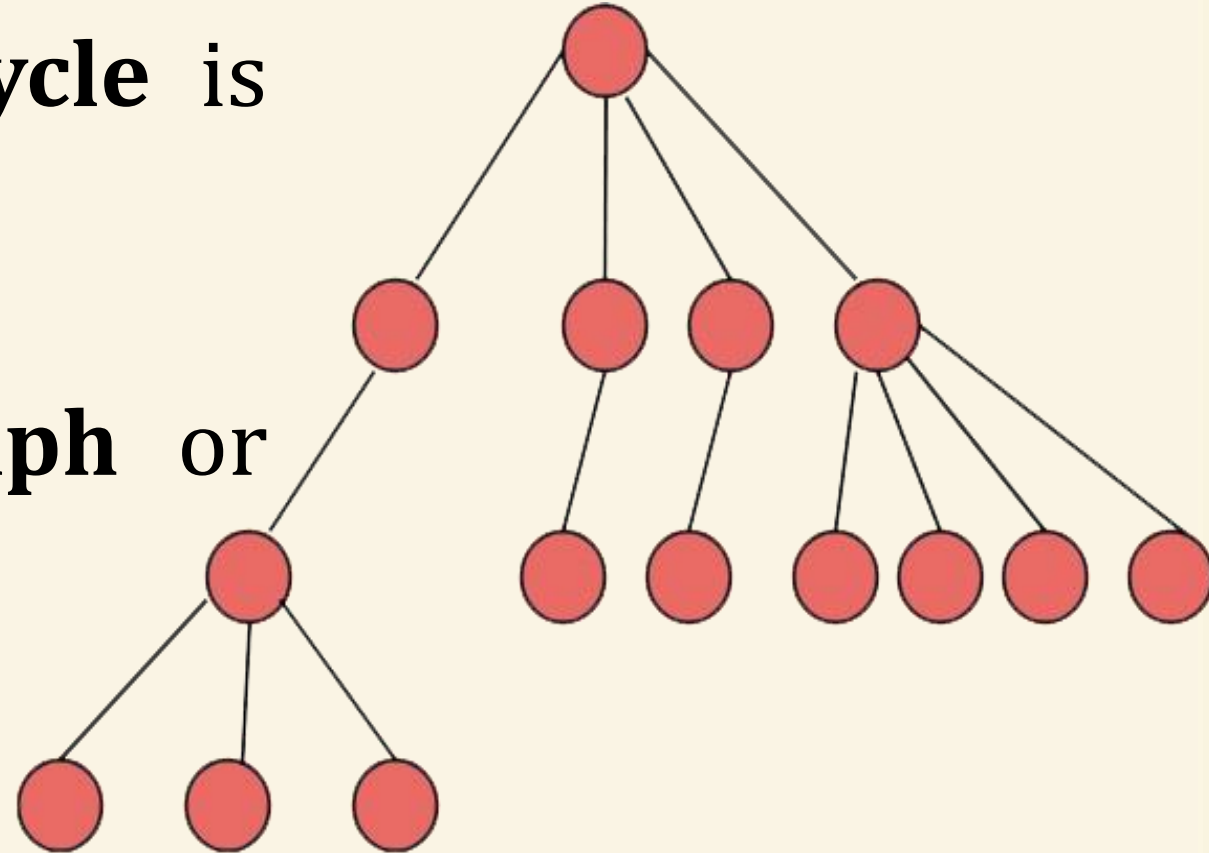
MGM University, Aurangabad.

A decorative graphic consisting of concentric circles. The innermost circle is light blue, followed by a dark red ring, and then a wider orange ring. The entire graphic is set against a light beige background.

Introduction to Trees

Introduction to Trees

- A **graph** which has **no cycle** is called an **acyclic graph**.
- A **tree** is an **acyclic graph** or **graph having no cycles**.

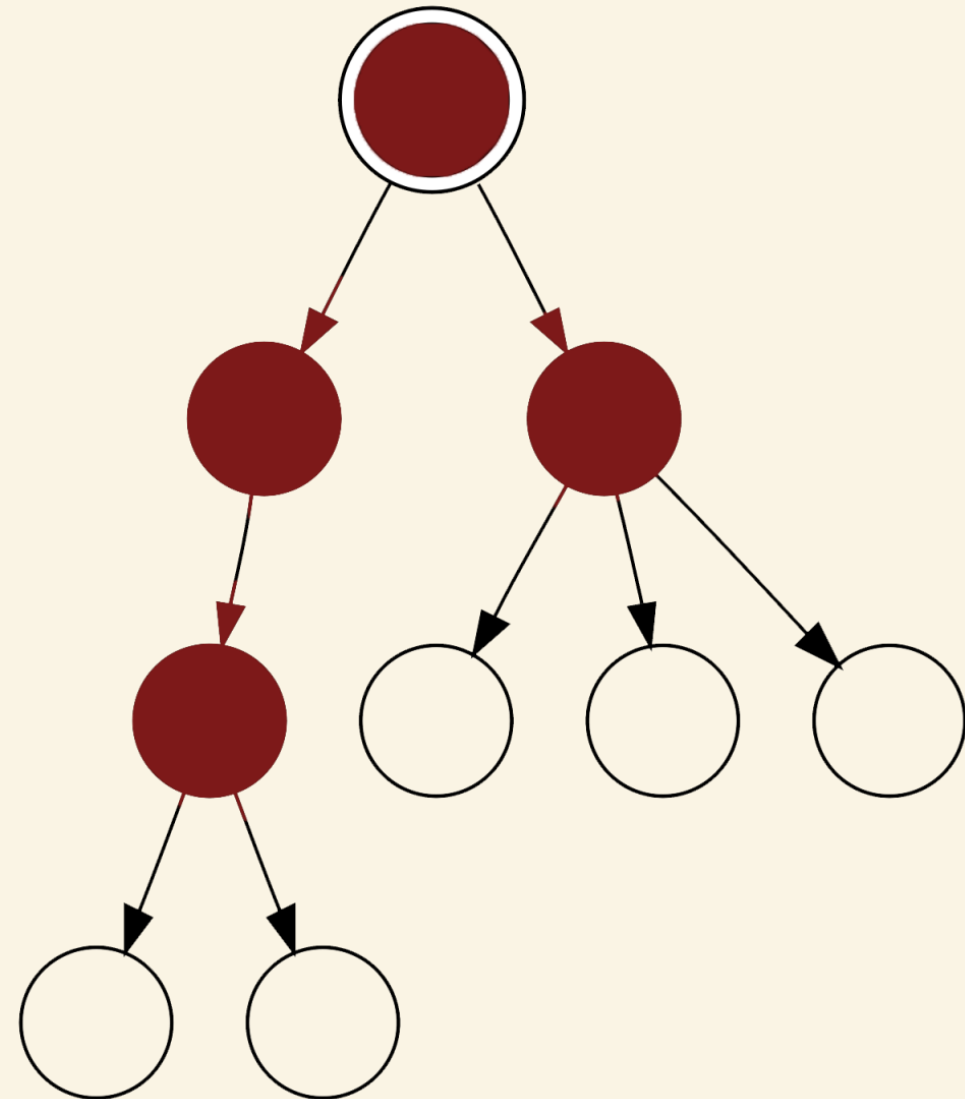


Introduction to Trees

- A **tree** or **general trees** is defined as a **non-empty finite set** of elements called **vertices** or **nodes** having the property that each **node** can have minimum **degree 1** and maximum **degree n**.

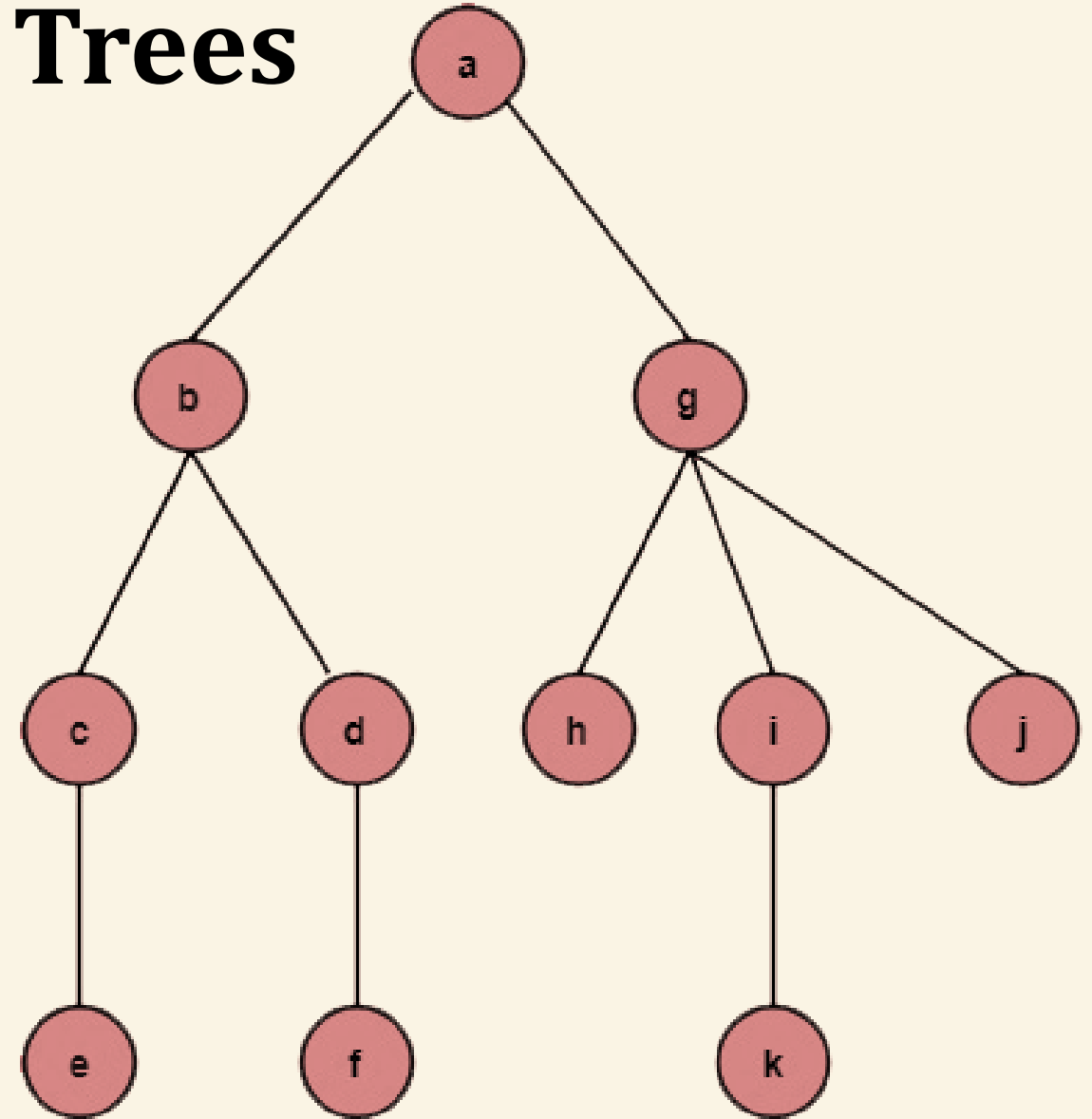
Directed Trees

- A **directed tree** is an **acyclic directed graph**.



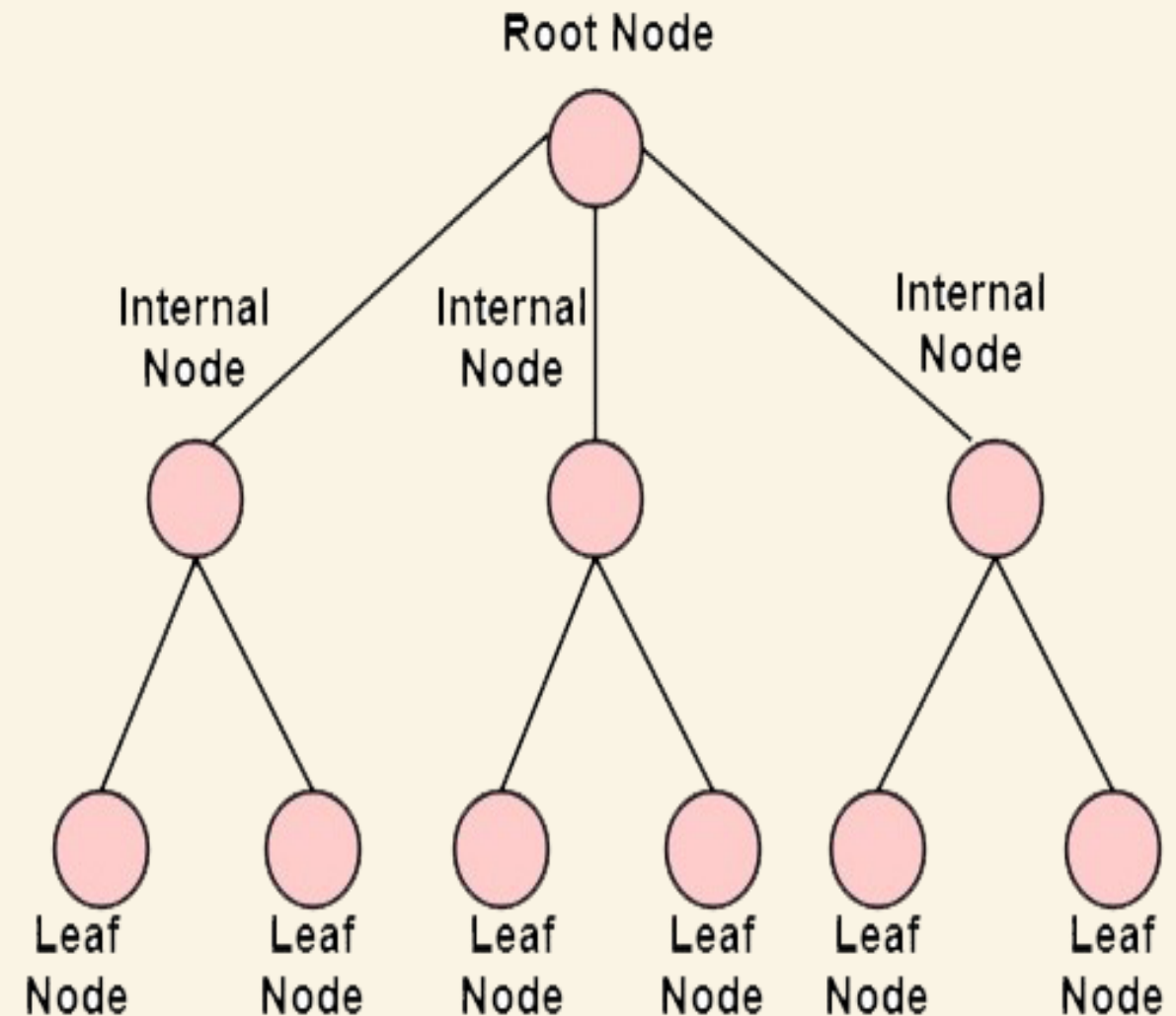
Ordered Trees

- If in a tree at **each level**, an **ordering** is **defined**, then such a **tree** is called an **ordered tree**.



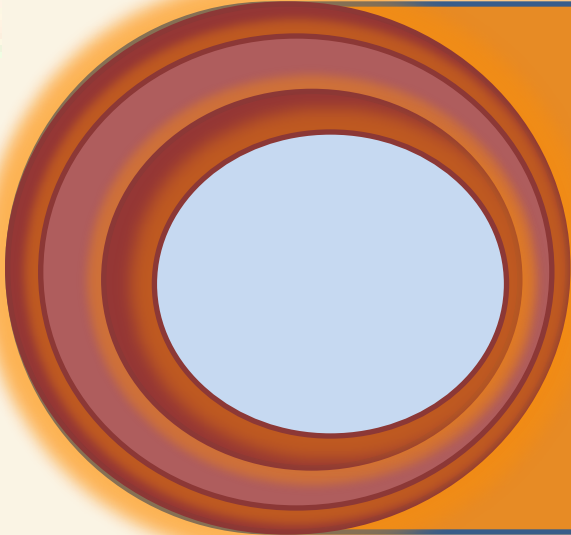
Rooted Trees

- A **rooted tree G** is a connected **acyclic** graph with a special node that is called the **root** of the tree and every edge directly or indirectly **originates** from the **root**.



Properties of Trees

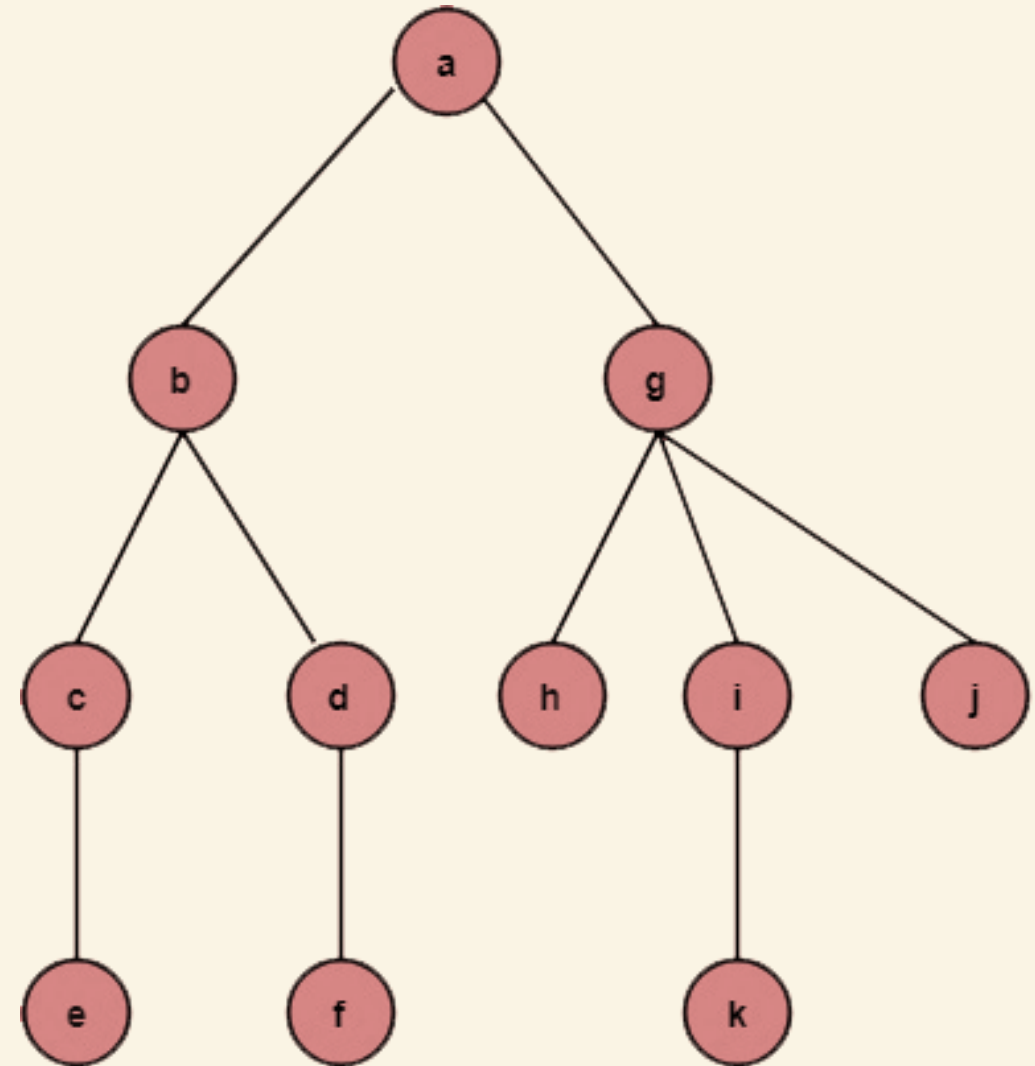
1. There is only **one path** between each pair of vertices of a **tree**.
2. If a graph G there is **one and only one path** between each pair of vertices G is a **tree**.
3. A **tree** T with n vertices has **$n-1$ edges**.
4. A **graph** is a tree if and only if it a **minimal connected**.

A decorative graphic consisting of three concentric circles. The innermost circle is light blue, the middle ring is a darker blue, and the outermost ring is a dark brown color.

Pendant Vertices In A Tree

Pendant Vertices In A Tree

- A **pendant vertex** was defined as a vertex of **degree one**.
- The reason is that in a tree of **n** vertices we have **$n-1$** edges.

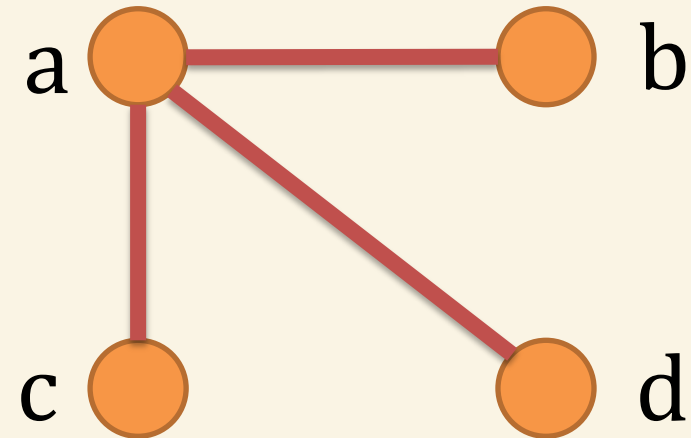


Question

Q1. Prove that any connected **graph G** with **n vertices** and **(n-1)** edges is a tree.

Vertices (n) = 4

Edges (n-1) = 3

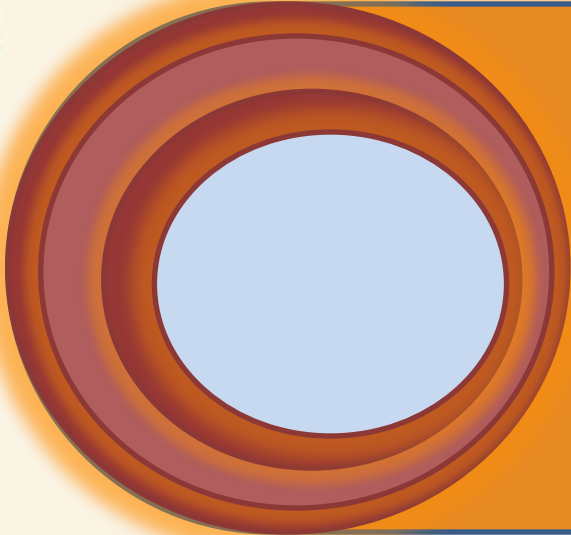


Question

Q2. Prove that any connected **graph G** with **n vertices** and **(n-1)** edges is a tree.

Vertices (n) = 5

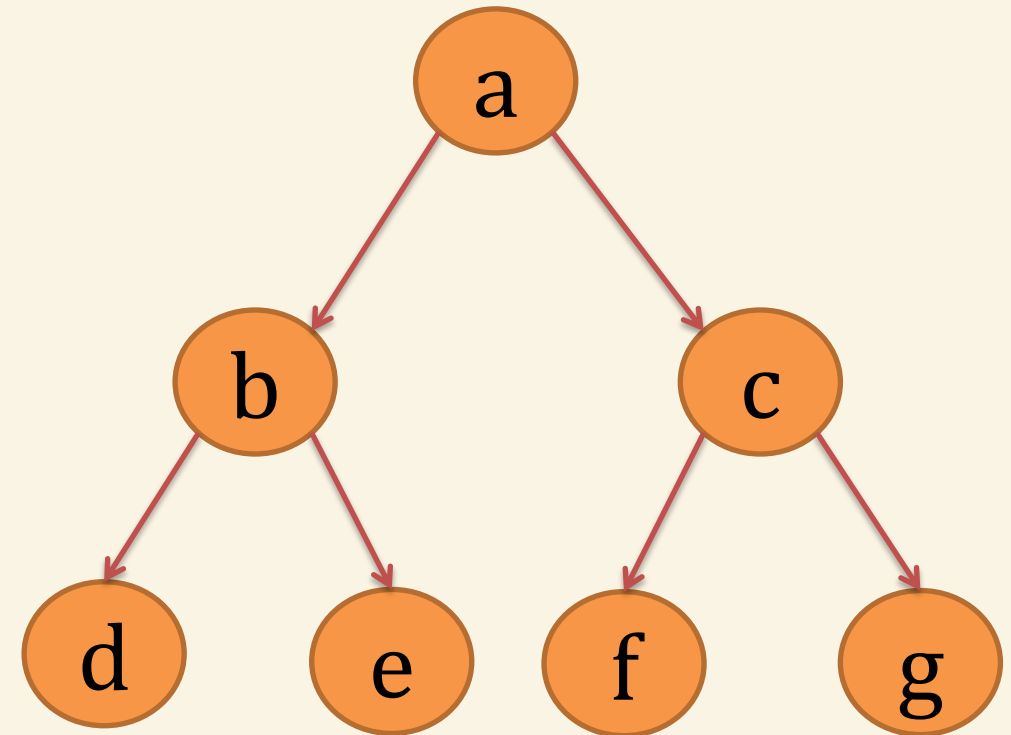
Edges (n-1) = 4

A decorative graphic on the left side of the slide, consisting of a light blue circle surrounded by several concentric rings in shades of brown and orange, all contained within a larger orange rounded rectangle.

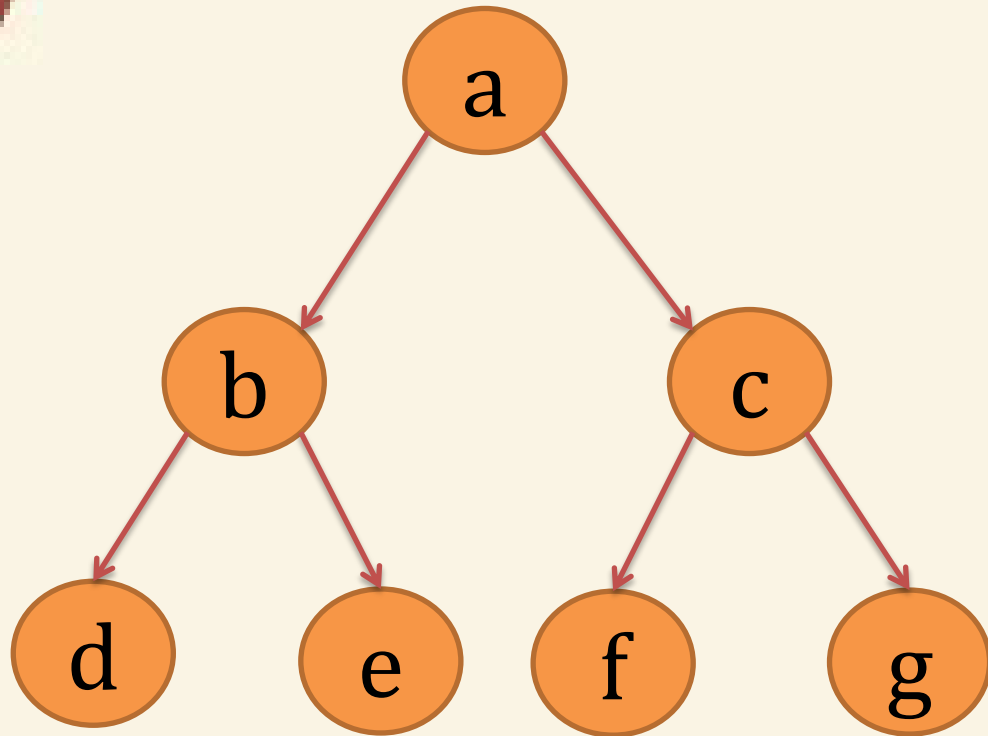
Binary Trees

Binary Trees

- If the **outdegree** of every node is less than or equal to **2**, in a **directed tree** than the tree is called a binary tree.



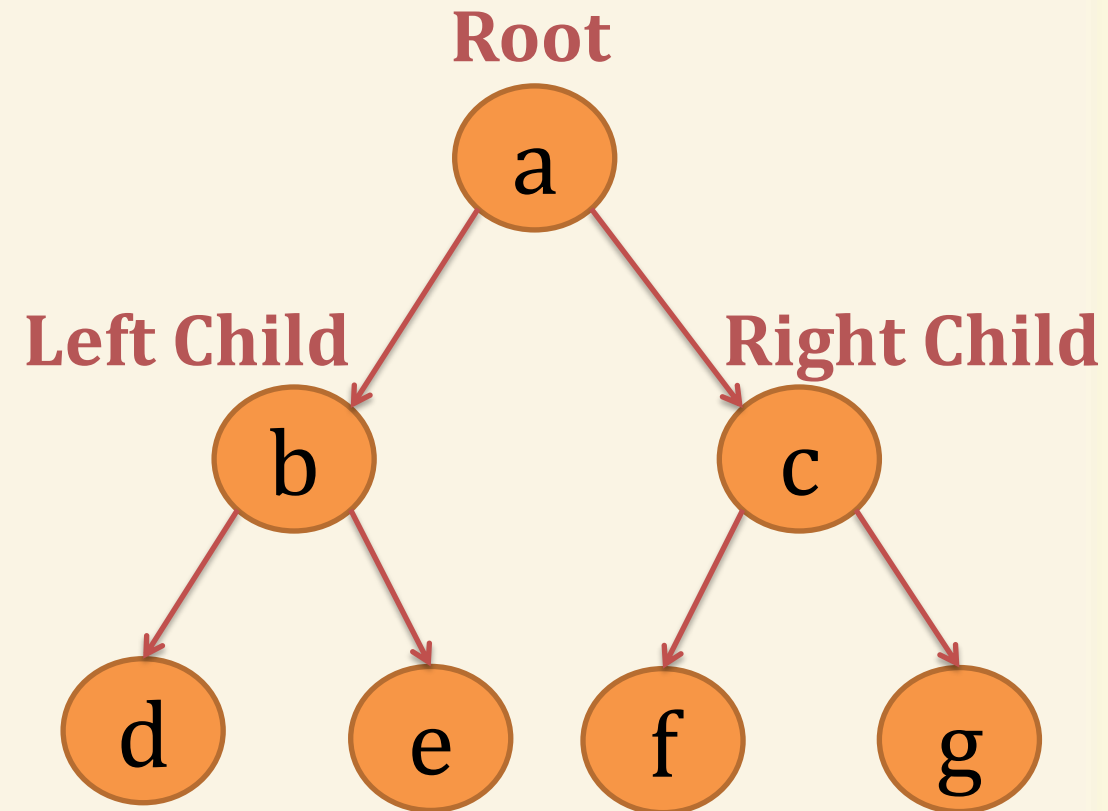
Binary Trees



Vertex	Indegree	Outdegree
a	0	2
b	0	2
c	0	2
d	0	0
e	0	0
f	0	0
g	0	0

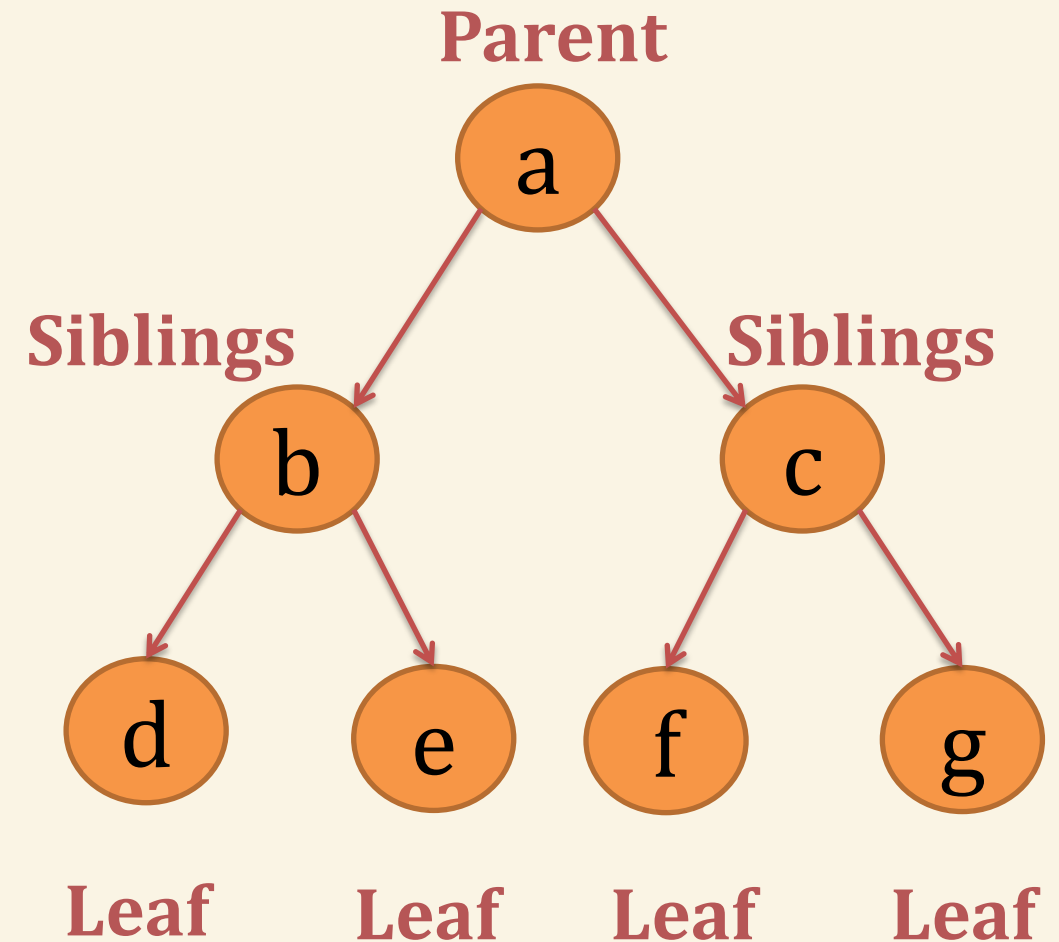
Basic Terminology

- **Root:** A binary tree has a unique node called the root of the tree.
- **Left Child:** The node to the left of the root is called its left child.
- **Right Child:** The node to the right of the root is called its right child.



Basic Terminology

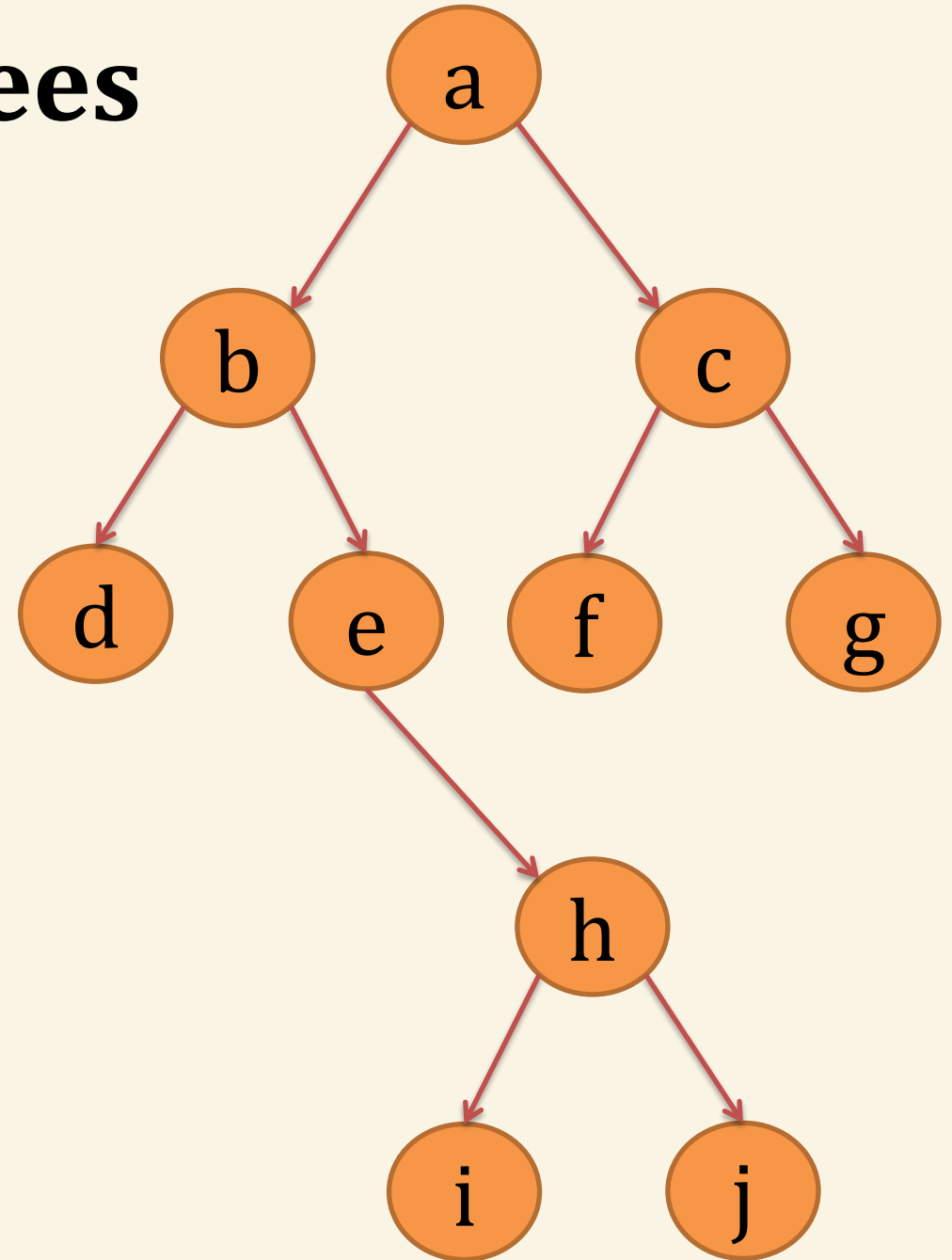
- **Parent:** A node having a left child or right child or both are called the parent of the nodes.
- **Siblings:** Two nodes having the same parent are called siblings.
- **Leaf:** A node with no children is called a leaf.



Binary Trees

Example: For the tree as shown in fig:

- Which node is the root?
- Which nodes are leaves?
- Name the parent node of each node.



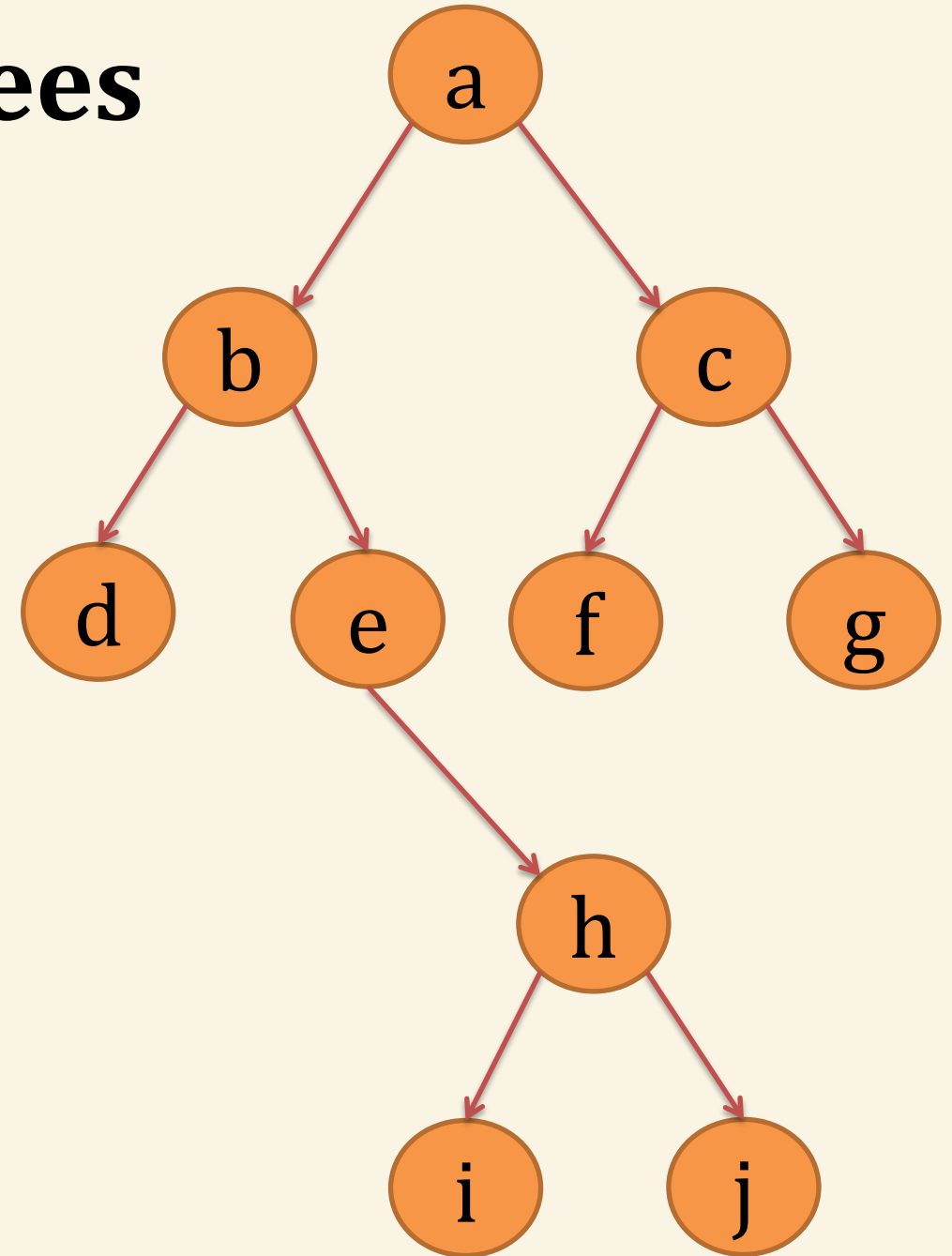
Binary Trees

Solution:

Q1. Which node is the root?

Ans. The node A is the **root** node.

i. The nodes **d, f, g, h, i and j** are **leaves**.

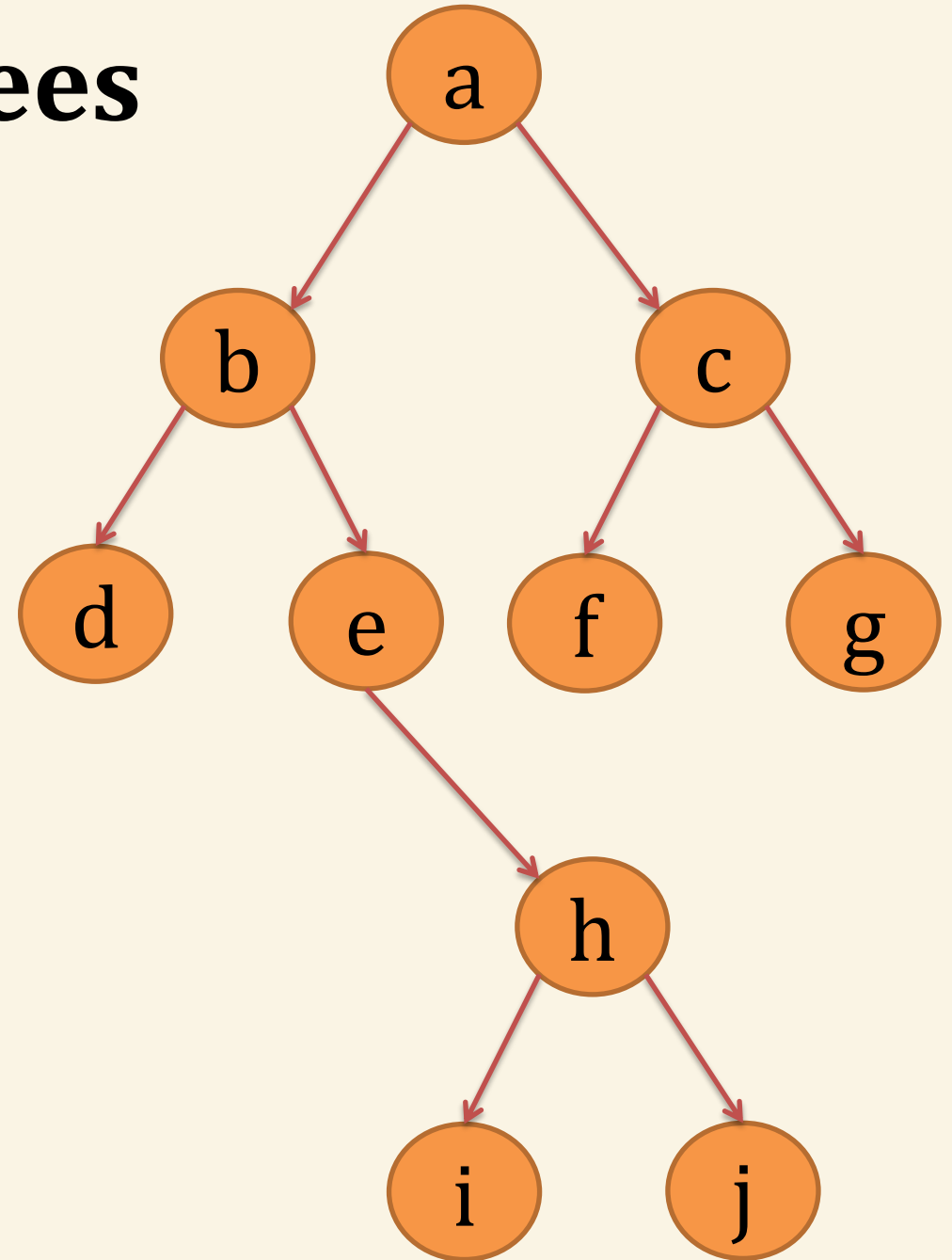


Binary Trees

Solution:

Q2. Which nodes are leaves?

Ans. The nodes **d, f, g, i and j** are **leaves**.



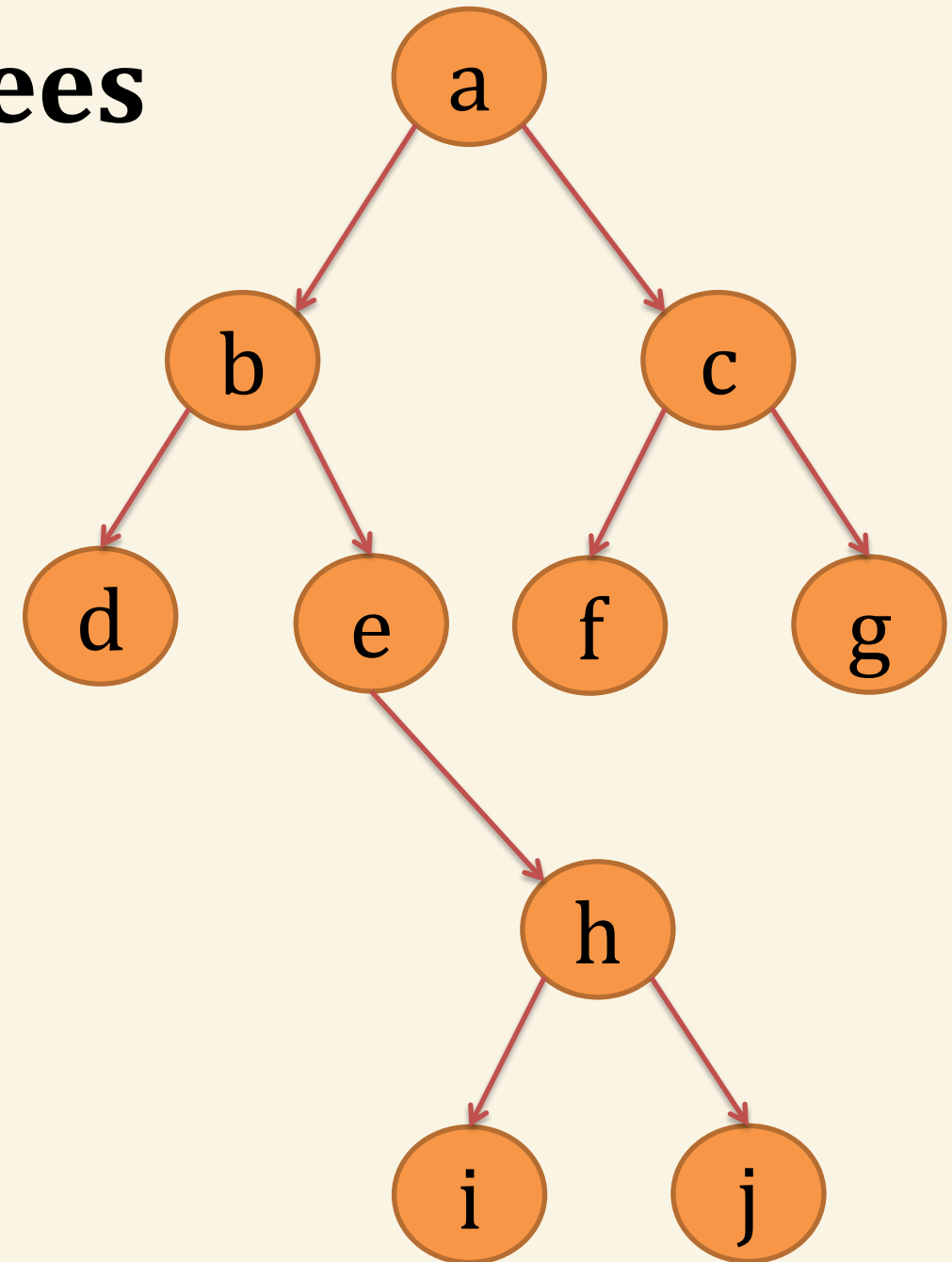
Binary Trees

Solution:

Q3. Name the parent node of each node.

Ans.

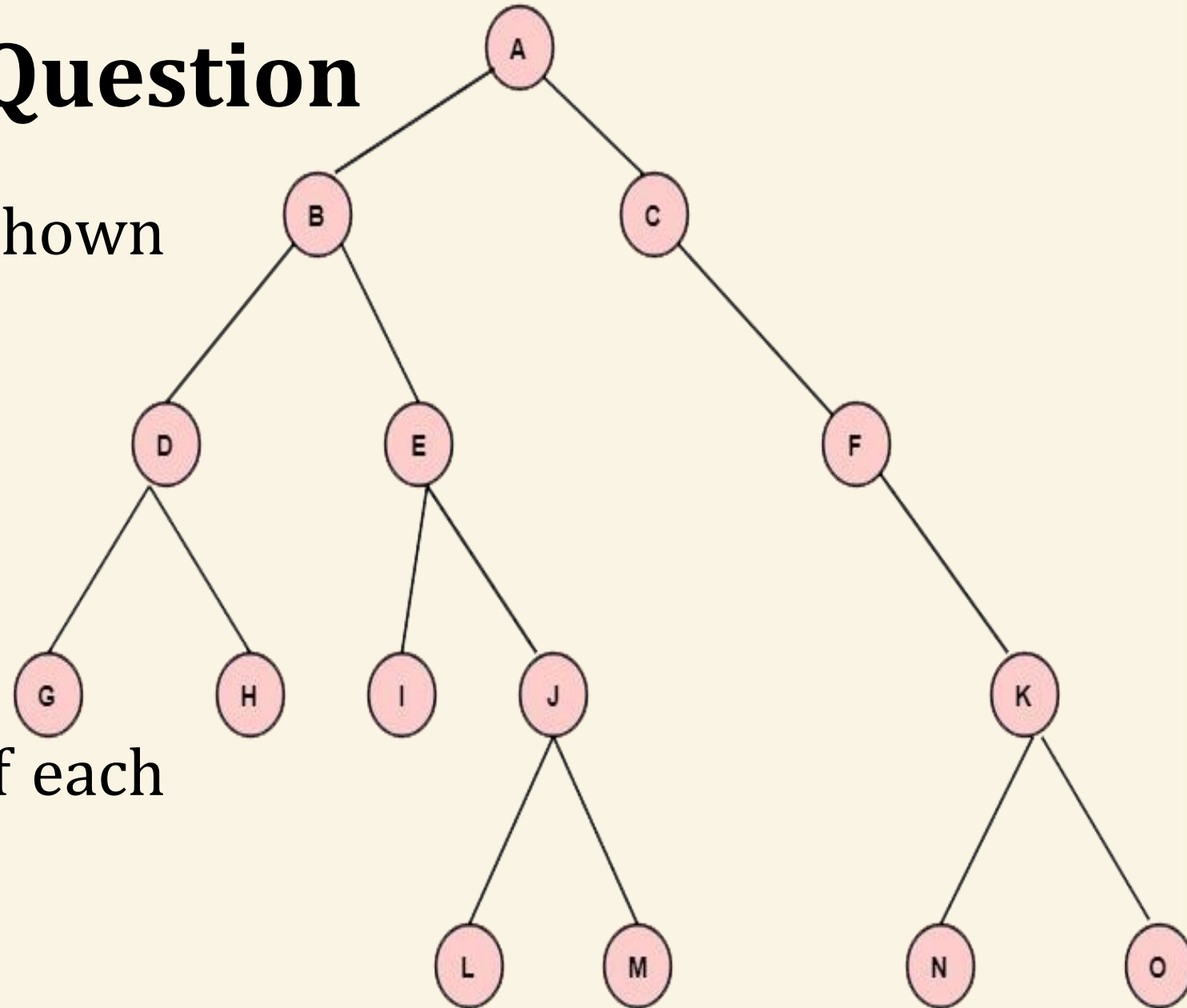
Nodes	Parent
b, c	a
d, e	b
f, g	c
i, j	h
h	e

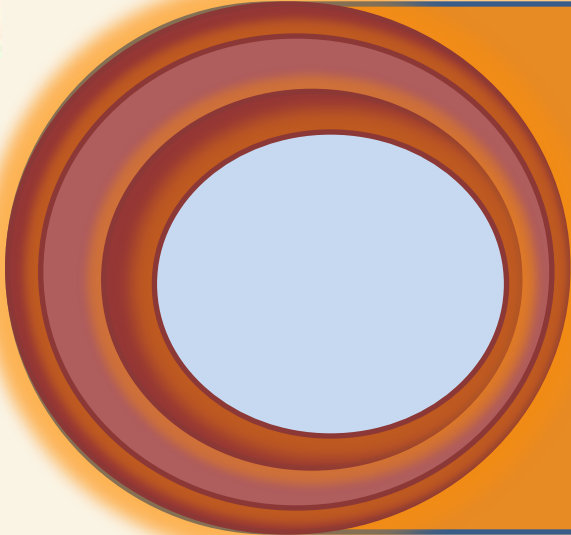


Question

Example: For the tree as shown
in fig:

- Which node is the root?
- Which nodes are leaves?
- Name the parent node of each node.



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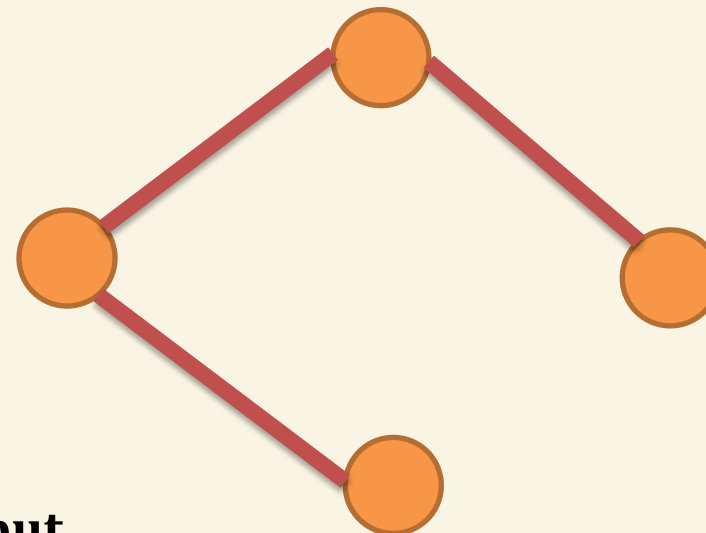
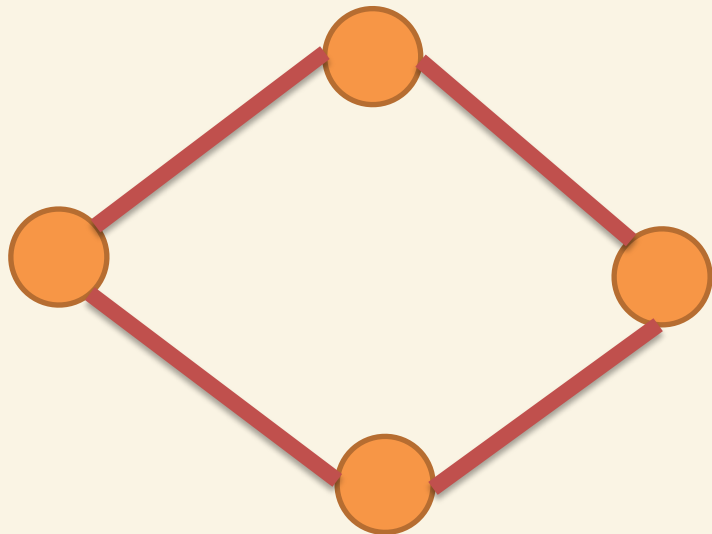
Spanning Trees

Spanning Tree

- A **spanning tree** for a **graph G** is a **subgraph** of G that contains **all vertices** of G and is a **tree**. Every connected graph G has a **spanning tree**.
- A **spanning tree** of a connected undirected graph G is a tree that minimally includes all of the vertices of G .
- A **graph** may have many **spanning trees**.

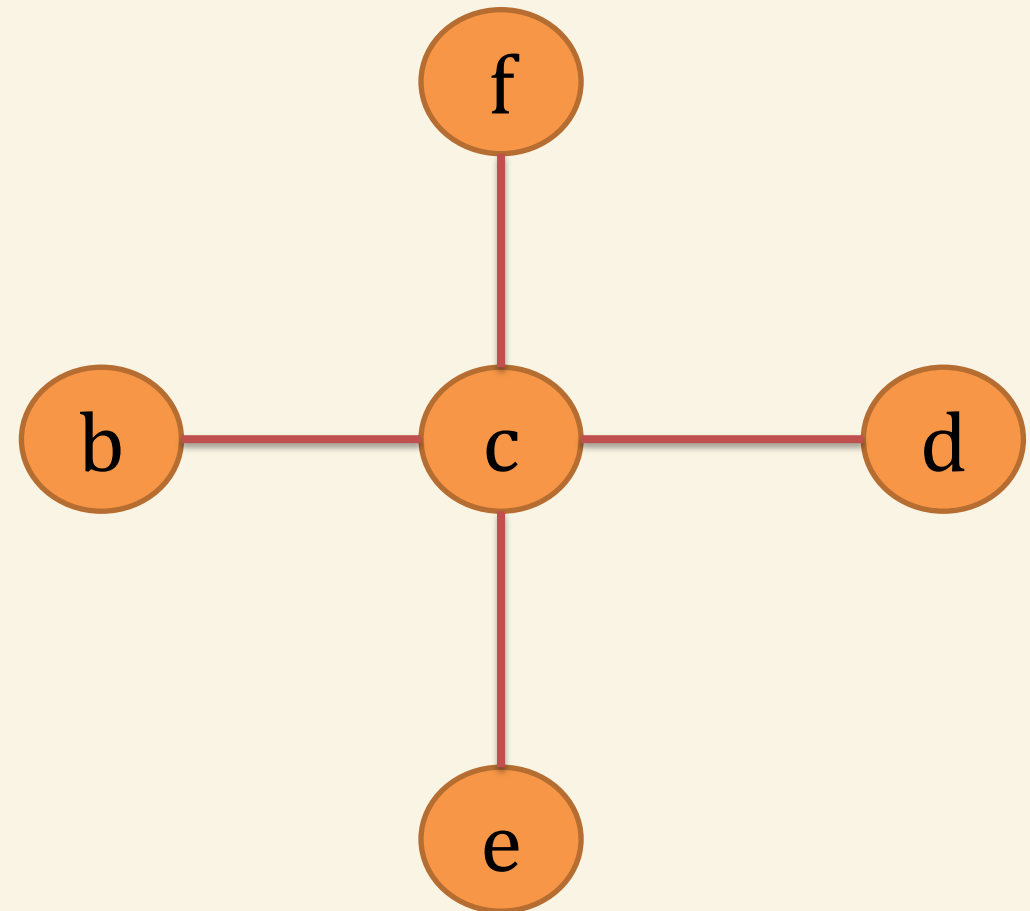
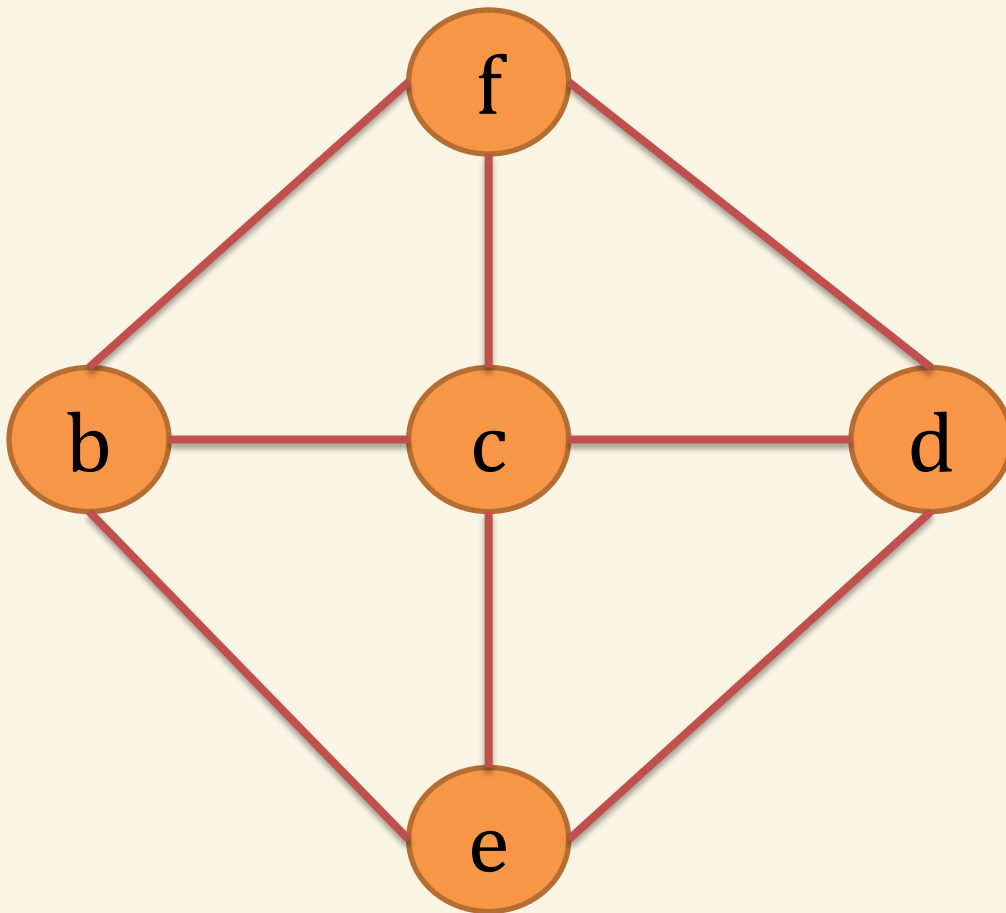
Spanning Tree

- **In fact**, if one keeps breaking any remaining **significant circuit** of G , or the **in-between** subgraphs after such steps, by removing an **edge** from the **circuit**, then the final resulting **subgraph** will be a **spanning tree**.



Spanning Tree

- Example:



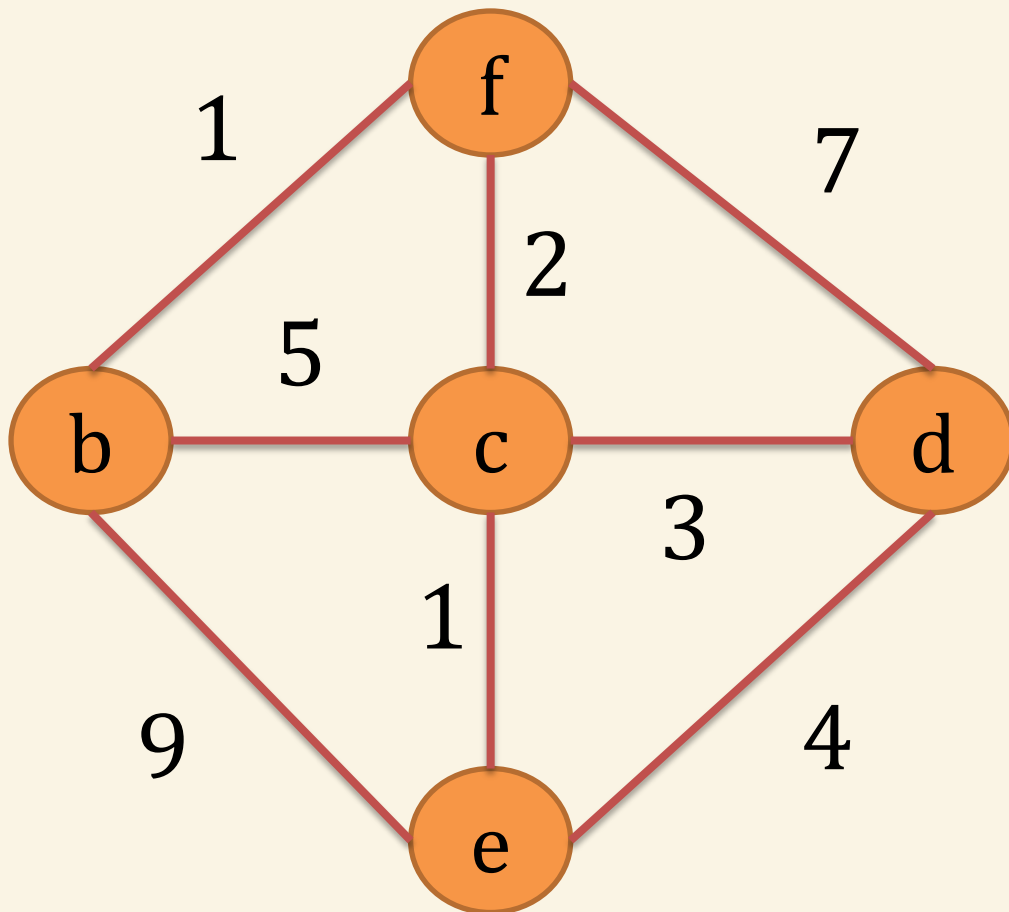
Minimum Spanning Tree

- A **spanning tree** with assigned **weight** less than or equal to the **weight** of every possible **spanning tree** of a weighted, connected and **undirected graph** G , it is called **minimum spanning tree (MST)**.
- The **weight** of a **spanning tree** is the **sum** of all the **weights** assigned to **each edge** of the **spanning tree**.

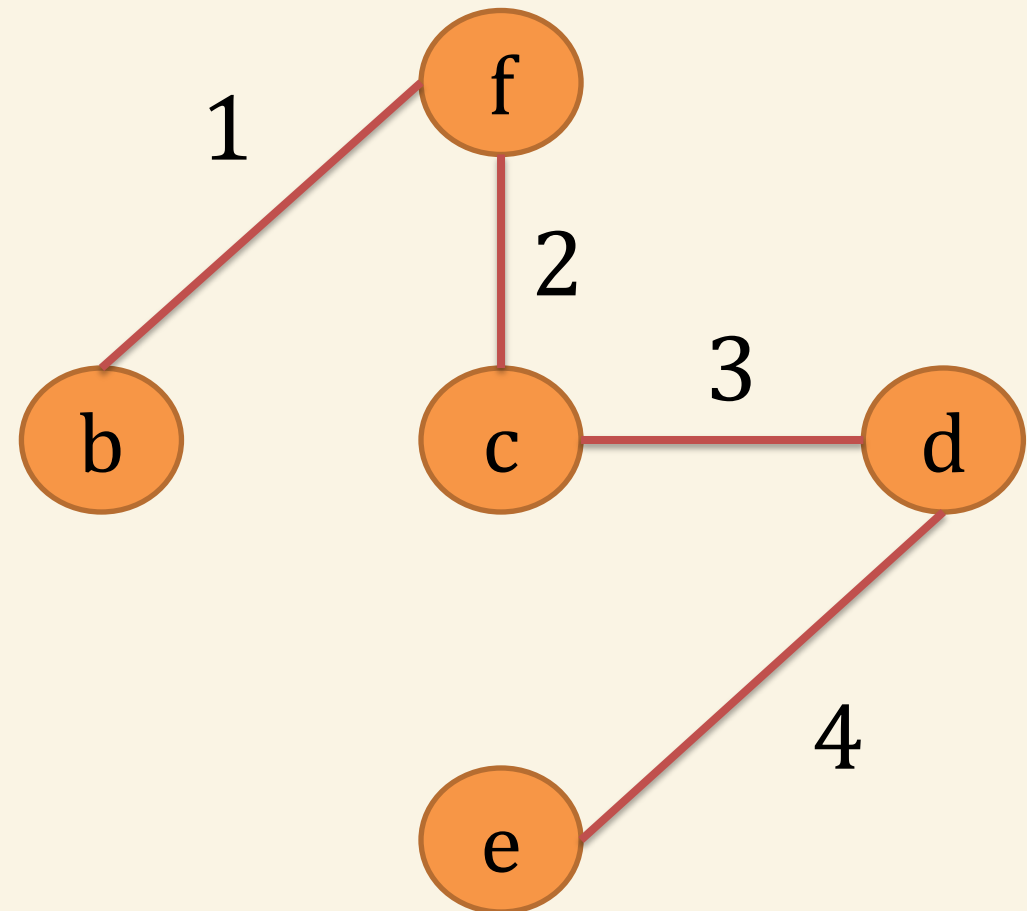
Minimum Spanning Tree

- Example:

$$1+7+4+9+1+5+2+3 = 32$$

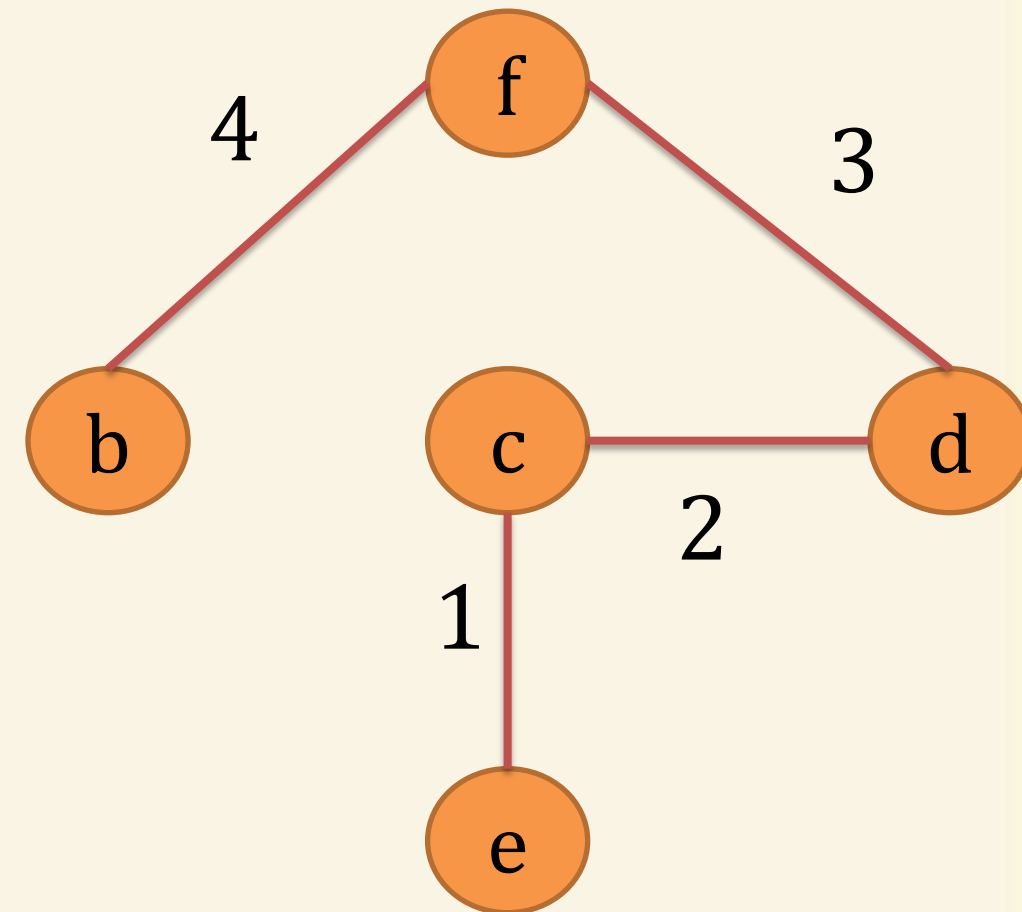
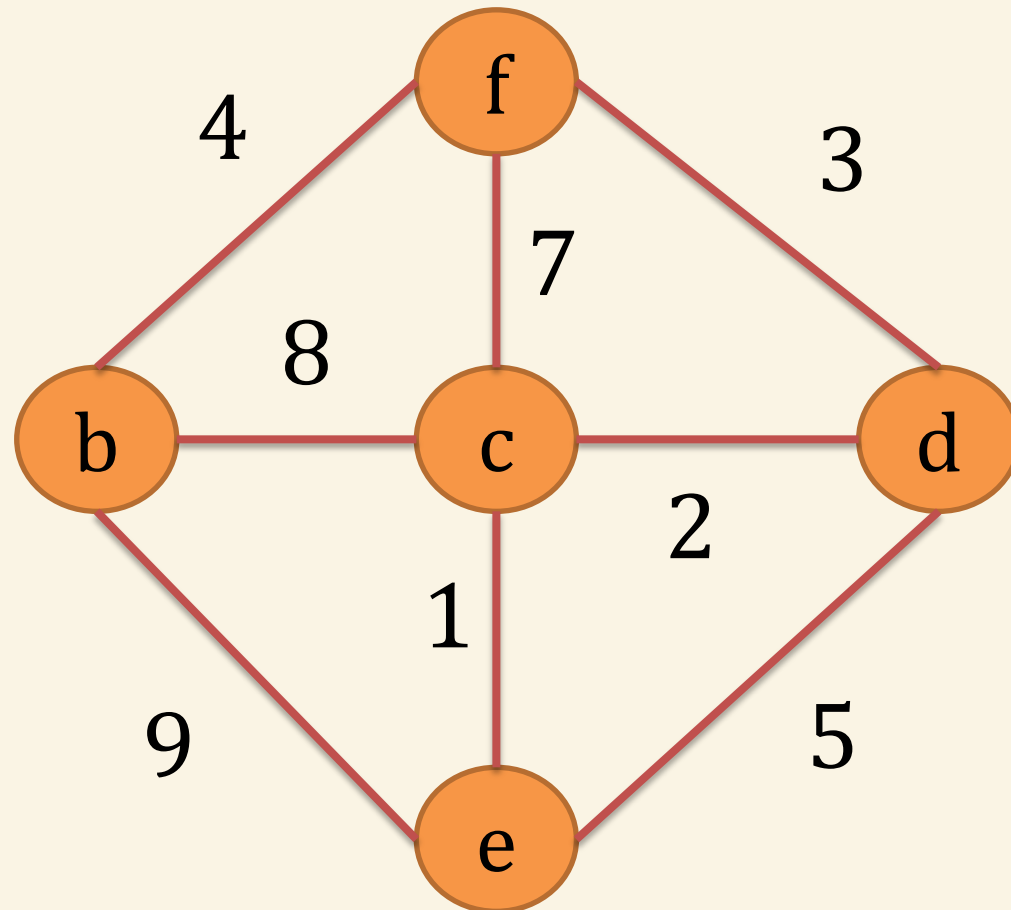


$$1+2+3+4 = 10$$



Question

Q1. Compute the minimum spanning tree for the following graph. Calculate the weight.



Thank You