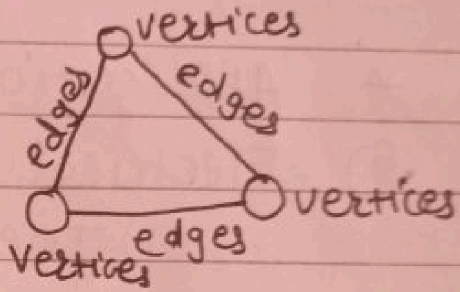
Graph Theory and Tree

* Introduction to Graph

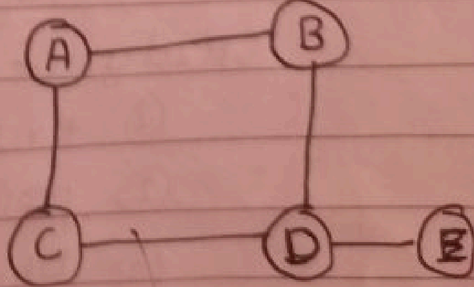
In the domain of mathematics and computer science, graph theory is the study of graphs that concerns with the relationship among edges and vertices.



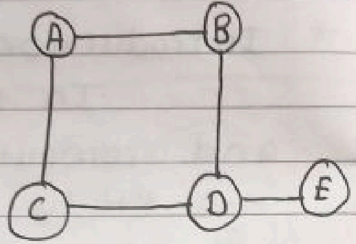
* A graph is pictorial representation of a set of objects where some pairs of objects are connected by links.

* The interconnected objects are represented by points termed as <u>vertices</u> and the links that connect the vertices are called <u>edges</u>

* formally, a graph is a pair of sets (V, E), where V is the set of vertices and E is the set of Edges, connecting the pairs of vertices



* In the graph



V = { A, B, C, D, E }
E = { AB, AC, ~~BD~~, ~~BE~~ CD, DE, BD }

* Applications of Graph

① Electrical Engineering

i) The concepts of graph theory is used extensively in designing circuit connections.

ii) The types or organization of connections are named as topologies

iii) Some example for topologies are star, bridge, series and parallel topologies.

② Computer Science.

Graph theory is used for the study of algorithms. for example:

① Kruskal's Algorithm
② prim's Algorithm
③ Dijkstra's Algorithm.

③ Computer Network

The relationships among interconnected computers in the network follows the principles of graph theory.

④ Science :

The molecular structure and chemical structure of a substance, the DNA structure of an organism etc. are represented by graphs.

⑤ General :

Routes between the cities can be represented using ~~gr~~ graphs.

Depicting hierarchical ordered information such as family tree can be used as a special type of graph called tree.

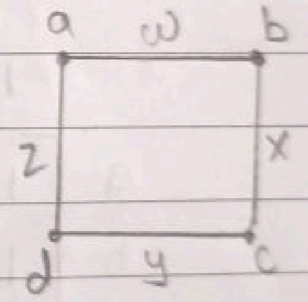
* Matrix Representation of graph
Graph is a set of edges and vertices. graph can be represented in the form of matrix.
Two matrix that can be form
① Incidence matrix.
② Adjancy matrix.

① Incidence Matrix. [A_{ij}] :
let G be a graph with 'n' vertices 'm' edges without selfloo

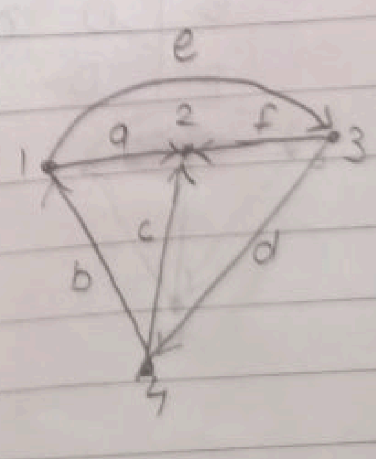
The incidence matrix A of G is an 'n x m' matrix. whose n rows correspond to the n vertices and m colums correspond to the m edges.

a) Directed graph :

$$A_{ij} = \begin{cases} 1 & \text{- if edge strats from vertex} \\ -1 & \text{- if edge ends with vertex} \\ 0 & \text{- otherwise.} \end{cases}$$



$$\begin{array}{c c} & \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \end{array}$$



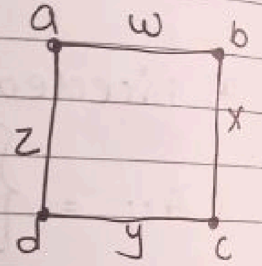
$$\begin{array}{c c} & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

b) Undirected Graph

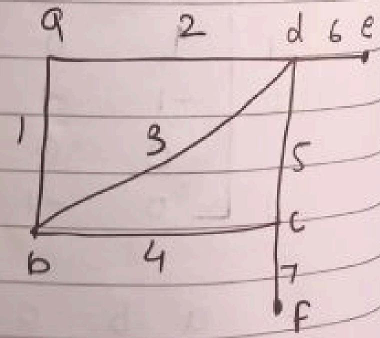
Incident of matrix $[A_{ij}]$

$$A_{ij} = \begin{cases} 1 - \text{If edge is connected to vertex} \\ 0 - \text{otherwise} \end{cases}$$

$$\begin{array}{c|cccc} & w & x & y & z \\ \hline a & 1 & 0 & 0 & 1 \\ b & 1 & 1 & 0 & 0 \\ c & 0 & 1 & 1 & 0 \\ d & 0 & 0 & 1 & 1 \end{array}$$



$$\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline a & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ d & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ e & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ F & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$



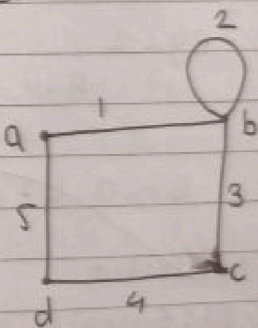
② Adjacency matrix:

i) If two vertices are connected by a single edge then they are known as adjacent vertices.

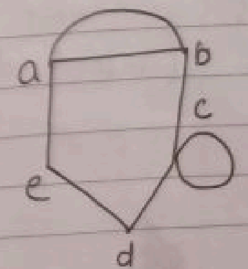
ii) If the vertex is connected to itself (loop) then the vertex is said to be adjecent to itself.

$$\text{Adjecency matrix} = \begin{cases} 2 - \text{It self loop} \\ 1 - \text{If vertex is conneed to other vertex} \\ 0 - \text{otherwise.} \end{cases}$$

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 1 \\ b & 1 & 2 & 1 & 0 \\ c & 0 & 1 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$



$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 0 & 0 & 1 \\ b & 1 & 0 & 1 & 0 & 0 \\ c & 0 & 1 & 2 & 1 & 0 \\ d & 0 & 0 & 1 & 0 & 1 \\ e & 1 & 0 & 0 & 1 & 0 \end{array}$$

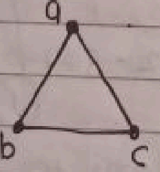


* operation on graph

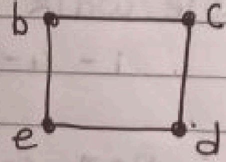
① union of ~~sets~~ graph:

Union of ~~sets~~ graph A and B is defined to be the graph of all those ~~elements~~ vertices & edges which belong to G_1 or G_2 or both and is denoted by $A \cup B$

eg:



(G_1)



(G_2)

$V(G_1) = \{a, b, c\}$ $V(G_2) = \{b, c, e, d\}$

$E(G_1) = \{ab, bc, ac\}$ $E(G_2) = \{bc, be, bd, ed\}$

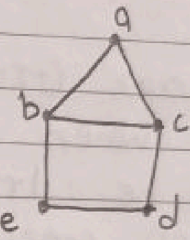
union of vertex

$V(G_1) \cup V(G_2) = \{a, b, c, d, e\}$

union of edge

$E(G_1) \cup E(G_2) = \{ab, ac, bc, be, de, dc\}$

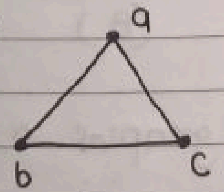
union of graph.



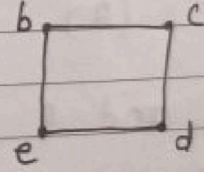
② Intersection of graph:

Intersection of two graph G_1 and G_2 is the graph of all those vertices and ~~eg~~ edges ~~of~~ which belong to both G_1 and G_2 and is denoted by $A \cap B$

eg:



(G_1)



(G_2)

$V(G_1) = \{a, b, c\}$ $V(G_2) = \{b, c, e, d\}$

$E(G_1) = \{ab, bc, ac\}$ $E(G_2) = \{bc, cd, ed, \ldots$

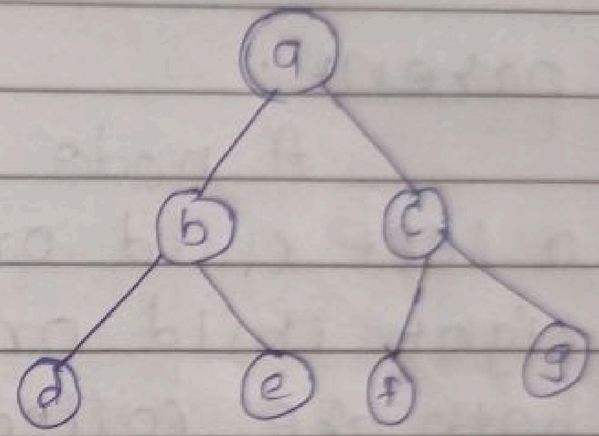
$V(G_1) \cap V(G_2) = \{b, c\}$

$E(G_1) \cap E(G_2) = \{bc\}$

intersection of graph

* Binary Trees

If the outdegree of every node is less than or equal to 2, in a directed tree than the tree is called a binary tree.



eg :

vertese	Indegree	outdegree
a	0	2
b	0	2
c	0	2
d	0	0
e	0	0
f	0	0
g	0	0

* Path and circuit (Eluer's path & ciruit)

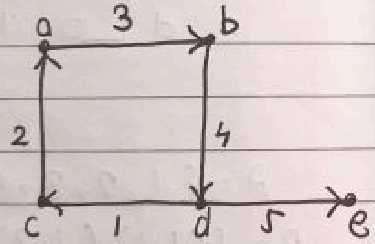
Path contains each edge exactly once and each vertex atleast once

circuit : If starting vertex is same as ending vertex, then it is called circuit

eg 1:

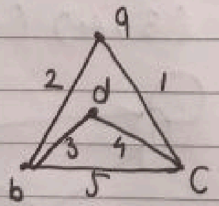
Euler's path

d-c-a-b-d-e



eg 2:

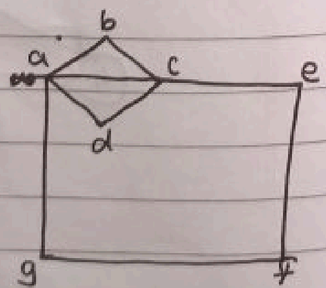
8-c-d-b-a-c



ex for circuit

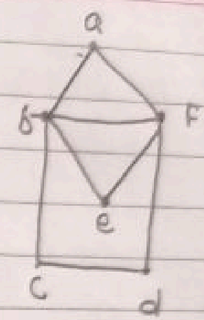
ex 1:

(a-b-c-d-a-,g-f-e-c-a)

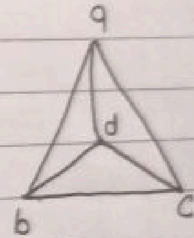


eg 2:

(b-a-f-e-b-c-d-f-b)



* compute path and circuite of following graph



1) circuit :

(a-b-d-a-c-b-d-c-a)

2) path -

The path is not satisfied the properties of path.

* Isomorphic Graph:

Two graphs G_1 and G_2 are said to be Isomorphic if

① If there no. of componant (vertices and edges) are same

② There connectivity is fixed.

NOTE : Inshort, out of the two isomorphic graph, one is twisted version of the other.

③ An labeled graph also can be isomorphic graph.

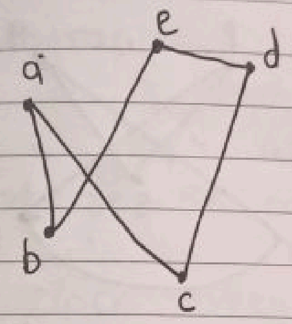
* If $G_1 = G_2$ then

1. $|V(G_1)| = |V(G_2)|$
2. $|E(G_1)| = |E(G_2)|$
3. Degree sequence of G_1 and G_2 are same

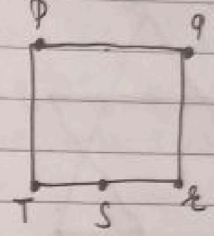
* example 1 :

Which of the following is Isomorphic

Good



(G1)



(G_2)

$V = \{a, b, c, d, e\}$

$E = \{ab, be, ed, cd, ac\}$

$V = \{P, Q, R, S, T\}$

$E = \{PQ, Qr, rS, ST, TP\}$

① $|V(G_1)| = 5$

② $|E(G_2)| = 5$

③ sequence

① $|V(G_2)| = 5$

② $|E(G_2)| = 5$

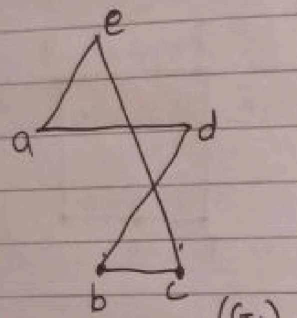
③ Degree sequence

vertex	connected vertex	degree	vertex	connected vertex	degree
a	c, b	2	p	q, T	2
b	a, e	2	q	r, P	2
c	a, d	2	r	q, S	2
d	e, c	2	S	T r	2
e	b, d	2	T	P, S.	2

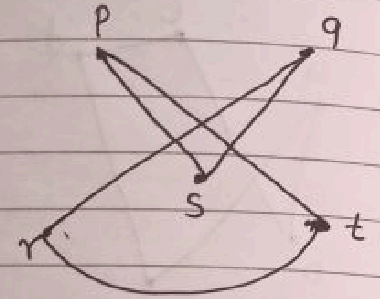
∴ Both graphs are isomorphic graph.

Bec. Both graph satisfy isomorphic property.

* example :2



(G₁)



(G_2)

$|V(G_1)| = 5$ $\qquad$ $|V(G_1)| = 5$

$|E(G_2)| = 5$ $\qquad$ $|E(G_2)| = 5$

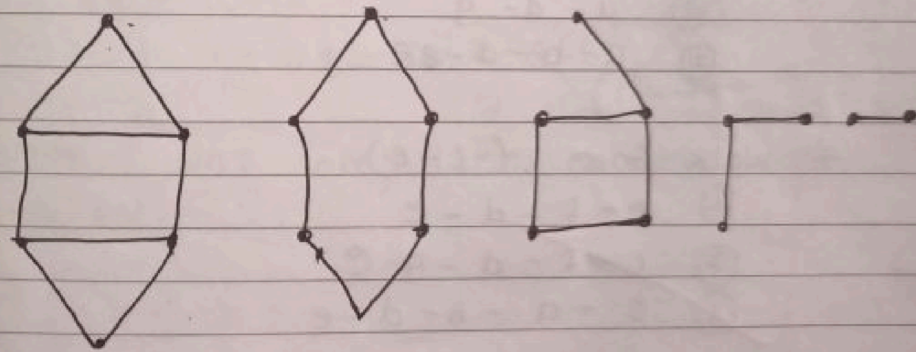
vertex	connected vertex	degree	vertex	connected vertex	degree
A	e, b	2	p	s, t	2
b	d, c	2	q	s, r	2
c	e, b	2	r	q t	2
d	a, b	2	s	p, q	2
e	a, c	2	t	p r	2

∴ Graph G_1 and G_2 both are isomorphic

* Subgraph.

A ~~graph~~ subgraph 's' of graph 'G' is a graph whose set of vertices and set of edges are all subsets of 'G'

Since every set is a subset of itself, so, every graph is subgraph of itself.

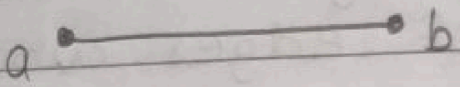


(G) $\qquad$ (S_1) $\qquad$ (S_2) $\qquad$ (S_3) $\qquad$ (S_4)

* Walk :

A walk is an alterernating sequence of vertices and connecting edges. A walk is consider to be close if the starting vertex is same as the ending vertex. otherwise consider as open walk.

* Pendent vertex :
By using degree of vertex we have two special types of vertices. A vertex with degree one is called pendent vertex



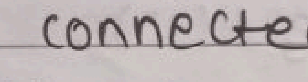
In this figure vertex a and vertex b have a connected edge ab

deg (a) = 1

deg (b) = 1

Hence ~~a~~ vertex a and b are pendent vertex.

* Isolated vertex :
A vertex with degree 0(zero) is called Isolated vertex



Here, vertex a & vertex b has no connectivity between each other

deg (a) = 0 , deg(b) = 0

Hence vertex a & vertex b is called as isolated vertex.

* Spanning Tree

① A spanning tree for a graph G is a subgraph of G that contains all vertices of G and is a tree. Every connected graph G has spanning tree

② A Spanning tree of a connected undirected graph G is a tree that minimally includes all of the vertices of G.

③ A graph may have many spannings trees

④ In fact, if one keep breaking any remaining significant circuit of G, or the in- between subgraphs after such steps by removing an edge from the circuit, then the final resulting subgraph will be a Spaning tree

eg: