Descriptions of Work

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6. *Supercritical and Robust Trade-offs for Resolution Depth Versus Width and Weisfeiler–Leman*

   With Duri Janett and Jakob Norström. In preparation

   Description. We present the first robust resolution tradeoffs where low width implies that depth must be superlinear in formula size. We give analogous results for the Weisfeiler–Leman algorithm, which also translate into tradeoffs between number of variables and quantifier depth in first-order logic.

   The technical contribution is a new compression scheme and analysis of the compressed Cop-Robber game introduced by [Grohe-Lichter-Neuen-Schweitzer 2023].

5. *SoS lower bounds for Non-Gaussian Component Analysis*

   With Ilias Diakonikolas, Sushrut Karmalkar, and Aaron Potechin

   To appear at FOCS 2024

   Description. Non-Gaussian Component Analysis (NGCA) is the statistical task of finding a non-Gaussian direction in a high-dimensional dataset. Specifically, given i.i.d. samples from a distribution P_v on $\mathbb{R}^n$ that behaves like a known distribution A in a hidden direction v and like a standard Gaussian in $v^{\perp}$, the goal is to approximate the hidden direction. If A matches low-order moments with Gaussian and satisfies some mild conditions, it is known learning algorithms such as Statistical Query and low-degree polynomial tests require a large number of samples.

This work studies the complexity of NGCA in the Sum-of-Squares (SoS) algorithmic framework. Our main result is the first super-constant degree SoS lower bound, which translates to super-polynomial time for the corresponding SoS algorithms. Specifically, we show that if distribution A matches the first $(k-1)$ moments of $\mathcal{N}(0,1)$ and satisfies mild conditions, then with high probability given $< n^{\frac{(1-\epsilon)k}{2}}$ many Gaussian samples, degree $\widetilde{o}(\sqrt{\log n})$ SoS fails to refute the existence of direction v. This significantly strengthens prior work by establishing a super-polynomial information-computation tradeoff against a broader family of algorithms. As a corollary, we obtain SoS lower bounds for several problems in robust statistics and learning of mixture models.

The lower bound proof introduces a new technique, along with a few improvements of the previous ones. As in previous works, we use the framework of [Barak et al. 2016] where, given the moment matrix M, we find an approximate factorization $M \approx LQL^T$ using minimum vertex separators, and show that with high probability Q is PSD while error terms are small. What's new is the following. First, instead of the minimum weight vertex separator, we use the minimum square separator. Second, proving that the matrix Q is positive is challenging due to an intrinsic reason. In all prior works, the matrix Q modulo a negligible error was a constant matrix, whose entries are constants independent of the input. Here, however, it is (quite) a nontrivial linear combination of a class of non-constant, equally dominating matrices. To address this difficulty, we introduce an algebraic method that we believe is of more general interest. Specifically, we model the multiplications between the 'important' pseudo-random matrices by an $\mathbb{R}$-algebra, construct all irreducible representations of this algebra, and use the resulting Wedderburn-Artin decomposition to analyze the special element Q. Via this approach, we show that the PSDness of Q boils down to the multiplicative identities of Hermite polynomials.

4. *Graph Colouring Is Hard on Average for Polynomial Calculus and Nullstellensatz*
With Jonas Conneryd, Susanna de Rezende, Jakob Norström, Kilian Risse
FOCS 2023

Description. In this work, we prove that Polynomial Calculus, hence also

Nullstellensatz, requires linear degree to refute the 3-colorability of sparse random graphs and random regular graphs. An optimal, exponential size lower bound follows via the known size-degree relation.

The proof goes by constructing an Alekhnovich-Razborov pseudo-reduction operator, where we use the closure technique introduced by [Romero-Tunçel 2021] in the study of large-girth graphs. We extend this technique to get rid of the large-girth assumption and to deal with general sparse expanders.

3. *SoS Lower Bound for Exact Planted Clique*
   CCC 2021

   Description. This work proves Sum-of-Squares (SoS) degree lower bounds for the Exact Planted Clique problem on random graphs $G(n, 1/2)$. Here, the SoS algorithm wants to refute the existence of cliques of size ω in a sampled graph, where ω is so large that with probability $> 99.99\%$ the sample does not contain such a clique.

   The word "exact" in the title means that we allow SoS algorithms to use the full set of axioms for its reasoning—including the axiom on clique size $\sum_{i=1}^{n} x_i = \omega$, the 'global' axiom that the previous tight lower bound technique [Barak-Hopkins-Kelner-Kothari-Moitra-Potechin 2016] needs to weaken into a single objective function. In this work, we overcome this issue by proving a SoS degree lower bound $d = \Omega(\frac{\epsilon^2 \log n}{\log \log n})$ for $\omega = O(n^{\frac{1}{2}-\epsilon})$, almost optimal in d and ω.

   Another motivation is about further developing average-case SoS lower bound techniques. To deal with the global axiom, we design the 'pseudo-expectation operator' $\widetilde{E}(\cdot)$ differently from the pseudo-calibration framework [Barak et al. 2016]. Cost is, the moment matrix no longer has product-like entries, making it harder to analyze. To address this, we simplify the target matrix using an Hadamard factorization (where one factor is a Johnson scheme, inspired by [Feige-Krauthgamer 2003]) and a relativized matrix factorization (based on the binomial transform). The final positive semidefiniteness (PSD) proof relies on the analytical properties of a matrix family we call factorial Hankel matrices. In retrospect, the success of this approach is due to the 'correct' initial design of $\widetilde{E}(\cdot)$. Is there an a priori explanation for this design, which admittedly appears contingent?

2. *On CDCL-based Proof Systems with the Ordered Decision Strategy*
   With Nathan Mull and Alexander Razborov
   SAT 2020, SICOMP 2022

   Description. In this work, we prove that conflict-driven clause learning (CDCL) SAT-solvers with the *ordered decision strategy* and *DECISION learning scheme*, are equivalent to ordered resolution. We also prove that if replacing this learning scheme with its opposite, which stops backtracking right after the first non-conflict clause, then the solvers become equivalent to general resolution. This is among the first theoretical studies of the interplay between specific decision strategies and clause learning.

   For both results, we allow nondeterminism in the solver's ability to perform unit propagation, conflict analysis, and restarts, in a way similar to previous works in the literature. To aid the presentation of our results, and possibly future research, we define a model and language for discussing CDCL-based proof systems that allow for succinct and precise theorem statements.

1. *Large Clique Is Hard on Average for Resolution*
   CSR 2021

   Description. The main result of this paper is a $2^{\Omega\left(k^{1-o(1)}\right)}$ resolution size lower bound for the k-Clique problem on suitable random graphs, $k < n^{1/3}$. This complements the result in [Beame-Impagliazzo-Subharwal 2007] which was for $k > n^{5/6}$.

   Our proof uses the bottleneck counting framework based on a variant of clause width. The width is defined by thresholding the 'density' of the vertex sets associated with a clause, using the same notion of density as in previous works [Beyesdorff-Galesi-Lauria 2013, Atserias-Bonacina-de Rezende-Lauria-Nordström-Razborov 2018]. Additionally, we extend the $n^{\Omega(k)}$ regular resolution lower bound [Atserias et al. 2018] to a slightly stronger system that permits a certain degree of irregularity.