*Office Hours: MW 3-4 pm

1/20/17

*Review of Statistics

- Suppose we have the following data set:

  1, 2, 2, 3, 4, 4, 4, 5, 5, 6, 7, 8 (let units be in mm)

  - # of data points: $N = 12$

- Average = point that data points are centered around

  - Mean average:

$$\boxed{\mu = \frac{1}{N}\sum_{i=1}^{N} x_i}$$

~For our example dataset, $\mu = \frac{1}{12}(1+2+2+3+4+4+4+5+6+7+8) = \boxed{4.25\,mm = \mu}$

~Alternatively, say we are given the following information

$P(1\,mm) = \frac{1}{12}$, $P(2\,mm) = \frac{2}{12}$, $P(3\,mm) = \frac{1}{12}$, $P(4\,mm) = \frac{3}{12}$,

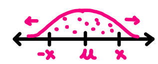
$P(5\,mm) = \frac{2}{12}$, $P(6\,mm) = \frac{1}{12}$, $P(7\,mm) = \frac{1}{12}$, $P(8\,mm) = \frac{1}{12}$

⇒ Can determine $\langle x \rangle$ using the following expression:

$$\boxed{\langle x \rangle = \sum_{i=1}^{N} x_i \cdot P(x_i)} \quad \text{(expected value)}$$

$$= 1P(1) + 2P(2) + 3P(3) + \ldots + 8P(8) = 1\left(\tfrac{1}{12}\right) + 2\left(\tfrac{2}{12}\right) + 3\left(\tfrac{1}{12}\right) + \ldots + 8\left(\tfrac{1}{12}\right) = 4.25\,mm \checkmark$$

- Variance = measures how far a set of data points tend to spread about the mean



$$\boxed{\sigma^2 = \langle (x-\mu)^2 \rangle} = \langle x^2 - 2x\mu + \mu^2 \rangle = \langle x^2 \rangle \overbrace{-2\underbrace{\mu\langle x \rangle}_{=2\mu^2 = 2\langle x \rangle^2} + \mu^2}^{=-\mu^2} \quad \boxed{= \langle x^2 \rangle - \mu^2}$$

- Aside:

$$\langle x^2 \rangle = \frac{1}{N}\sum_{i=1}^{N} x_i^2$$

- Variance of a data set given from a probability distribution

$$\langle x^2 \rangle = \sum_{j=1}^{m} x_j^2 P(x_j)$$ where m = # of possible values

j = index # for a given possible data value

~For our example data set,

$$\langle x^2 \rangle = 1^2 P(1) + 2^2 P(2) + 3^2 P(3) + \ldots + 8^2 P(8) = 22.08\,mm^2$$

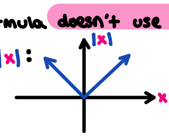
$$\Rightarrow \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = 22.08\,mm^2 - (4.25\,mm)^2 = 4.0208\,mm^2$$

* Note: variance has units² (relative to mean unit)

** ~Aside: The variance formula contains a squared exponent to account for negative data values ⇒ prevents things from cancelling out

- The variance formula doesn't use an absolute value b/c it doesn't always provide "good"/"well-behaved" answers

  - e.g., let $f(x) = |x|$:



(can cause problems)

Variance outputs a value w/ units² ⇒ Not very intuitive

~Monition:

⇒ we usually reconcile this by calculating a quantity called the standard deviation ⇒ gives a better intuition when interpreting variance

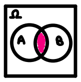
⇒ Standard deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 2.0052\,mm$

units aren't squared this time!!

~Take-Home Message: SD is better for gaining intuition about the spread of data than variance b/c it returns a value w/ units rather than units²

*Review of Probabilities

- Suppose A and B are independent events ⇒ whether B occurs is independent of whether A occurs and vice versa (e.g., coin flips)
- $\mathbb{P}$(A and B) is usually denoted $\mathbb{P}(A \cap B)$

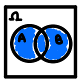


⇒ multiply probabilities : $\boxed{\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)}$

  - Ex: Probability of flipping 2 heads in a row if we flip a coin twice is...

    $\mathbb{P}$(H and H) $= \mathbb{P}(H)\mathbb{P}(H) = 0.5(0.5) = 0.25$

- $\mathbb{P}$(A or B) is usually denoted $\mathbb{P}(A \cup B)$



⇒ Add probabilities; subtract any double counting: $\boxed{\begin{aligned}\mathbb{P}(A \cup B) &= \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \text{ and } B)\\ &= \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A)\mathbb{P}(B)\end{aligned}}$

  - Ex: Roll 2 dice, what's the probability of rolling at least 1 even #?

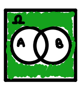
    ~ Let $\mathbb{P}(E_1) = 3/6 = 1/2$ be the probability of the first die giving an even #

    $\mathbb{P}(E_2) = 3/6 = 1/2$ be the probability of the second die giving an even #

    $\Rightarrow \mathbb{P}(E_1 \cup E_2) = \mathbb{P}(E_1) + \mathbb{P}(E_2) - \mathbb{P}(E_1 \cup E_2) = \mathbb{P}(E_1) + \mathbb{P}(E_2) - \mathbb{P}(E_1)\mathbb{P}(E_2) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$

- Alternatively, we may also encounter,

  $\mathbb{P}$(neither A nor B) $= \mathbb{P}$(not A) $\cdot \mathbb{P}$(not B) $= \mathbb{P}(A^c)\mathbb{P}(B^c)$



  - Note: $\mathbb{P}$(A or B) = 1 − (neither A nor B)

    ⇒ Sometimes, its convienent to find an answer by considering the opposite of what your trying to find and subtracting that probability from 1

    ~ e.g., $\mathbb{P}$(at least 1 die is even) = 1 − $\mathbb{P}$(both odd) = 1 − [$\mathbb{P}$(1st odd) $\mathbb{P}$(2nd odd)] $= 1 - (3/6)(3/6) = 1 - \frac{1}{4} = \frac{3}{4}$

    ⇒ $\boxed{\mathbb{P}(\text{at least 1 die is even}) = 0.75}$

\# Probability Examples:

1. Roll two dice. What is the probability one die is a 4 and the other die is odd? (Assume order doesn't matter)
   - Let event A be the event where a given die rolls a 4 $\Rightarrow p(A) = 1/6$
   - Let event B be the event where a given die rolls an odd number $\Rightarrow p(B) = 1/2$
   - Two possibilities that give $p(A \cap B)$ after rolling:

   ① "Die 1" gives event A and "die 2" gives event B

   $$\Rightarrow p(A_1 \cap B_2) = p(A_1)p(B_2) = p(A)p(B) = 1/6\,(1/2) = 1/12$$

   ② "Die 1" gives event B "die 2" gives event A

   $$\Rightarrow p(B_1 \cap A_2) = p(B_1)p(A_2) = p(B)p(A) = 1/2\,(1/6) = 1/12$$

   $$p(A \cap B) = p(A_1 \cap B_2) + p(B_1 \cap A_2) = 1/12 + 1/12 = 2/12 = 1/6 \Rightarrow \boxed{p(A \cap B) = 1/6}$$

2. Roll 2 dice. What is the probability neither die is a 6?
   - Define events:
     - Let event A_i correspond w/ "die i" NOT being a 6 $\Rightarrow p(A_i) = 5/6$
     - Note: Here, $i = 1,2$ and A_1 must occur before A_2 becomes relevant (order-dependent, kinda)
   - Solution:

   $$p(A_1 \cap A_2) = p(A_1)p(A_2) = 5/6\,(5/6) = 25/36 \Rightarrow \boxed{p(A_1 \cap A_2) = 25/36}$$

3. Roll 2 dice. What's the probability at least 1 is a 6?
   - Let event A_i, $i = 1,2$ correspond w/ "die i" being a 6 $\Rightarrow p(A_i) = 1/6$
   - Two Solutions:

   ① $1 - p(\text{neither is a } 6) = 1 - p(A_1^c \cap A_2^c) = 1 - 25/36 = 11/36 \Rightarrow \boxed{p(A_1 \cup A_2) = 11/36}$

   ② $p(A_1 \cup A_2) = p(A_1) + p(A_2) - p(A_1 \cap A_2) = 1/6 + 1/6 - (1/6)(1/6) = 2/6 - 1/36 = 1/3 - 1/36 = 12/36 - 1/36 = 11/36$ ✓

4. Flip 3 coins. Find the probability of getting 2 heads and 1 tail. (Assume order-independent)
   - Three possibilities:

   ① $p(HHT) = p(H)p(H)p(T) = 1/2\,(1/2)(1/2) = 1/8$

   ② $p(HTH) = p(H)p(T)p(H) = 1/2\,(1/2)(1/2) = 1/8$

   ③ $p(THH) = p(T)p(H)p(H) = 1/2\,(1/2)(1/2) = 1/8$

   $$p(2\text{ Heads}; 1\text{ Tail}) = p(HHT) + p(HTH) + p(THH) = 1/8 + 1/8 + 1/8 = 3/8$$

   $$\Rightarrow \boxed{p(2\text{ Heads}; 1\text{ Tail}) = 3/8}$$

** Always remember to account for the number of ways a particular event can occur

\# Announcement

- PPT from last class is on Blackboard
- Homework #1 is due next Friday