Introduction

The Common Core Institute is pleased to offer this grade-level tool for educators who are teaching with the Common Core State Standards.

The Common Core State Standards Deconstructed for Classroom Impact is designed for educators by educators as a two-pronged resource and tool 1) to help educators increase their depth of understanding of the Common Core Standards and 2) to enable teachers to plan College & Career Ready curriculum and classroom instruction that promotes inquiry and higher levels of cognitive demand.

What we have done is not all new. This work is a purposeful and thoughtful compilation of preexisting materials in the public domain, state department of education websites, and original work by the Center for College & Career Readiness. Among the works that have been compiled and/or referenced are the following: Common Core State Standards for Mathematics and the Appendix from the Common Core State Standards Initiative; Learning Progressions from The University of Arizona’s Institute for Mathematics and Education, chaired by Dr. William McCallum; the Arizona Academic Content Standards; the North Carolina Instructional Support Tools; and numerous math practitioners currently in the classroom.

We hope you will find the concentrated and consolidated resource of value in your own planning. We also hope you will use this resource to facilitate discussion with your colleagues and, perhaps, as a lever to help assess targeted professional learning opportunities.

Understanding the Organization

The Overview acts as a quick-reference table of contents as it shows you each of the domains and related clusters covered in this specific grade-level booklet. This can help serve as a reminder of what clusters are part of which domains and can reinforce the specific domains for each grade level.

Key Changes identifies what has been moved to and what has been moved from this particular grade level, as appropriate. This section also includes Critical Areas of Focus, which is designed to help you begin to approach how to examine your curriculum, resources, and instructional practices. A review of the Critical Areas of Focus might enable you to target specific areas of professional learning to refresh, as needed.

For each domain is the domain itself and the associated clusters. Within each domain are sections for each of the associated clusters. The cluster-specific content can take you to a deeper level of understanding. Perhaps most importantly, we include here the Learning Progressions. The Learning Progressions provide context for the current domain and its related standards. For any grade except Kindergarten, you will see the domain-specific standards for the current grade in the center column.

### Math Fluency Standards

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>Add/subtract within 10</td>
</tr>
</tbody>
</table>
| 2     | Add/subtract within 20  
|       | Add/subtract within 100 (pencil & paper) |
| 3     | Multiply/divide within 100  
|       | Add/subtract within 1000 |
| 4     | Add/subtract within 1,000,000 |
| 5     | Multi-digit multiplication |
| 6     | Multi-digit division  
|       | Multi-digit decimal operations |
| 7     | Solve px + q = r, p(x + q) = r |
| 8     | Solve simple 2 x 2 systems by inspection |
To the left are the domain-specific standards for the preceding grade and to the right are the domain-specific standards for the following grade. Combined with the Critical Areas of Focus, these Learning Progressions can assist you in focusing your planning.

For each cluster, we have included four key sections: Description, Big Idea, Academic Vocabulary, and Deconstructed Standard.

The cluster Description offers clarifying information, but also points to the Big Idea that can help you focus on that which is most important for this cluster within this domain. The Academic Vocabulary is derived from the cluster description and serves to remind you of potential challenges or barriers for your students.

Each standard specific to that cluster has been deconstructed. The Deconstructed Standard for each standard specific to that cluster and each Deconstructed Standard has its own subsections, which can provide you with additional guidance and insight as you plan. Note the deconstruction drills down to the substandards when appropriate. These subsections are:

- Standard Statement
- Standard Description
- Essential Question(s)
- Mathematical Practice(s)
- DOK Range Target for Learning and Assessment
- Instructional Targets
- Explanations and Examples

As noted, first are the Standard Statement and Standard Description, which are followed by the Essential Question(s) and the associated Mathematical Practices. The Essential Question(s) amplify the Big Idea, with the intent of taking you to a deeper level of understanding; they may also provide additional context for the Academic Vocabulary.

The DOK Range Target for Learning and Assessment remind you of the targeted level of cognitive demand. The Instructional Targets correlate to the DOK and express the student learning targets for student proficiency for KNOW, THINK, and DO, as appropriate. In some instances, there may be no learning targets for student proficiency for one or more of KNOW, THINK or DO. The learning targets are expressions of the deconstruction of the Standard as well as the alignment of the DOK with appropriate consideration of the Essential Questions.

The last subsection of the Deconstructed Standard includes Explanations and Examples. This subsection might be quite lengthy as it can include additional context for the standard itself as well as examples of what student work and student learning could look like. Explanations and Examples may offers ideas for instructional practice and lesson plans.
## Standards for Mathematical Practice in 8th Grade

Each of the explanations below articulates some of the knowledge and skills expected of students to demonstrate grade-level mathematical proficiency.

<table>
<thead>
<tr>
<th>PRACTICE</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MP.1 Make sense and persevere in problem solving.</strong></td>
<td>Students are able to solve problems, including real-world problems, through the application of appropriate math concepts and discuss how they solved the problems. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?” “Does this make sense?” and “Can I solve the problem in a different way?”</td>
</tr>
<tr>
<td><strong>MP.2 Reason abstractly and quantitatively.</strong></td>
<td>Students represent a wide variety of real-world contexts through mathematics and can contextualize and decontextualize as needed and appropriate as they work towards a solution.</td>
</tr>
<tr>
<td><strong>MP.3 Construct viable arguments and critique the reasoning of others.</strong></td>
<td>Students construct arguments using verbal or written explanations accompanied by appropriate mathematical language and tools. Students continue to refine their mathematical communication skills through discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions such as “How did you get that?” “Why is that true?” and “Does that always work?” They explain their thinking to others and respond to others’ thinking.</td>
</tr>
<tr>
<td><strong>MP.4 Model with mathematics.</strong></td>
<td>Students model problem situations appropriately and can create math models using expressions, equations, experiments, simulations, etc., as appropriate. Students may need several opportunities to connect and explain connections between different representations.</td>
</tr>
<tr>
<td><strong>MP.5 Use appropriate tools strategically.</strong></td>
<td>Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful.</td>
</tr>
<tr>
<td><strong>MP.6 Attend to precision.</strong></td>
<td>Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology in working through and explaining their thinking and solution.</td>
</tr>
<tr>
<td><strong>MP.7 Look for and make use of structure.</strong></td>
<td>Students routinely seek patterns or structures to model and solve problems.</td>
</tr>
<tr>
<td><strong>MP.8 Look for and express regularity in repeated reasoning.</strong></td>
<td>Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students formally begin to make connections showing the relationships between concepts and their solutions. Students may be willing to experiment with approaches for verification.</td>
</tr>
</tbody>
</table>
The Number System (NS)
- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations (EE)
- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions (F)
- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry (G)
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.

Theorem (TH)
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Statistics and Probability (SP)
- Investigate patterns of association in bivariate data.

Mathematical Practices (MP)
- **MB 1.** Make sense of problems and persevere in solving them.
- **MB 2.** Reason abstractly and quantitatively.
- **MB 3.** Construct viable arguments and critique the reasoning of others.
- **MB 4.** Model with mathematics.
- **MB 5.** Use appropriate tools strategically.
- **MB 6.** Attend to precision.
- **MB 7.** Look for and make use of structure.
- **MB 8.** Look for and express regularity in repeated reasoning.
### KEY CHANGES

<table>
<thead>
<tr>
<th>NEW TO 8TH GRADE</th>
<th>MOVED FROM 8TH GRADE</th>
</tr>
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<tbody>
<tr>
<td>- Integer exponents with numerical bases (8.EE.1)</td>
<td></td>
</tr>
<tr>
<td>- Scientific notation, including multiplication and division (8.EE.3 and 8.EE.4)</td>
<td></td>
</tr>
<tr>
<td>- Unit rate as slope (8.EE.5)</td>
<td></td>
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<tr>
<td>- Qualitative graphing (8.F.5)</td>
<td></td>
</tr>
<tr>
<td>- Transformations (8.G.1 and 8.G.3)</td>
<td></td>
</tr>
<tr>
<td>- Congruent and similar figures (characterized through transformations) (8.G.2 and 8.G.4)</td>
<td></td>
</tr>
<tr>
<td>- Angles (exterior angles, parallel cut by transversal, angle-angle criterion) (8.G.5)</td>
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<tr>
<td>- Finding diagonal distances on a coordinate plane using the Pythagorean Theorem (8.G.8)</td>
<td></td>
</tr>
<tr>
<td>- Volume of cones, cylinders, and spheres (8.G.9)</td>
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<tr>
<td>- Two-way tables (8.SP.4)</td>
<td></td>
</tr>
<tr>
<td>- Indirect measurement (embedded throughout)</td>
<td></td>
</tr>
<tr>
<td>- Linear inequalities (moved to high school)</td>
<td></td>
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<tr>
<td>- Effect of dimension changes (moved to high school)</td>
<td></td>
</tr>
<tr>
<td>- Misuses of data (embedded throughout)</td>
<td></td>
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<tr>
<td>- Function notation (moved to high school)</td>
<td></td>
</tr>
<tr>
<td>- Point-slope form (moved to high school) and standard form of a linear equation (not in CCSS)</td>
<td></td>
</tr>
</tbody>
</table>
1. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation and solving linear equations and systems of linear equations.

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions \( y/x = m \) or \( y = mx \) as special linear equations \( y = mx + b \), understanding that the constant of proportionality \( m \) is the slope, and the graphs are lines through the origin. They understand that the slope \( m \) of a line is a constant rate of change, so that if the input or \( x \)-coordinate changes by an amount \( A \), the output or \( y \)-coordinate changes by the amount \( m \cdot A \). Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and \( y \)-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Grasping the concept of a function and using functions to describe quantitative relationships.

Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.
DOMAIN:

THE NUMBER SYSTEM (NS)

EIGHTH GRADE
MATHEMATICS
## The Number System (NS)

### Clusters

1. Know that there are numbers that are not rational, and approximate them by rational numbers.

<table>
<thead>
<tr>
<th>SEVENTH</th>
<th>EIGHTH</th>
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<tbody>
<tr>
<td><strong>Integers, Number Lines, and Coordinate Planes</strong></td>
<td><strong>Integers, Number Lines, and Coordinate Planes</strong></td>
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<tr>
<td>Integers on the Number Line</td>
<td>Integers on the Number Line</td>
</tr>
<tr>
<td>8.EE.1.a Describe situations in which opposite quantities combine to make 0.</td>
<td>8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</td>
</tr>
<tr>
<td>8.NS.2.c Represent proportional relationships by equations.</td>
<td></td>
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</tbody>
</table>

### Rational and Irrational Numbers

<table>
<thead>
<tr>
<th>Irrational Numbers</th>
<th>Irrational Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.</td>
<td>8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., (\pi^2)).</td>
</tr>
</tbody>
</table>
Cluster 1. Know that there are numbers that are not rational, and approximate them by rational numbers.

All real numbers correspond to a unique point on the infinite number line.

Big Idea

- There are infinite ways to express a number or expression.

Academic Vocabulary

real numbers, irrational numbers, rational numbers, integers, whole numbers, natural numbers, radical, radicand, square roots, perfect squares, cube roots, terminating decimals, repeating decimals, truncate

Standard and Deconstruction

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers, show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Description

Students recognize that the decimal equivalent of a fraction will either terminate or repeat. Fractions that terminate will have denominators containing only prime factors of 2 and/or 5. This understanding builds on work in 7th grade when students used long division to distinguish between repeating and terminating decimals.

Students convert repeating decimals into their fraction equivalent using patterns or algebraic reasoning.

One method to find the fraction equivalent to a repeating decimal is shown below.

**Example 1:**

Change 0.4 to a fraction.

- Let \( x = 0.444444… \)
- Multiply both sides so that the repeating digits will be in front of the decimal. In this example, one digit repeats so both sides are multiplied by 10, giving \( 10x = 4.444444… \)
- Subtract the original equation from the new equation.
  \[
  10x = 4.444444… \\
  \quad - x = 0.444444… \\
  9x = 4
  \]
- Solve the equation to determine the equivalent fraction.
  \[
  \frac{9x}{9} = \frac{4}{9} \\
  x = \frac{4}{9}
  \]

Additionally, students can investigate repeating patterns that occur when fractions have denominators of 9, 99, or 11.
**ESSENTIAL QUESTION(S)**

How are rational and irrational numbers related to the decimal system?

**MATHEMATICAL PRACTICE(S)**

- 8.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

- ☒ 1
- ☐ 2
- ☐ 3
- ☐ 4

**Instructional Targets**

**Assessment Types**

- Tasks assessing concepts, skills, and procedures.
- Tasks assessing expressing mathematical reasoning.
- Tasks assessing modeling/applications.

**Students should be able to:**

- Define irrational numbers.
- Show that the decimal expansion of rational numbers repeats eventually.
- Convert a decimal expansion which repeats eventually into a rational number.
- Show informally that every number has a decimal expansion.

**EXPLANATIONS AND EXAMPLES**

Students can use graphic organizers to show the relationship between the subsets of the real number system.

**Real Numbers**

All real numbers are either rational or irrational.
## STANDARD AND DECONSTRUCTION

<table>
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<th>STANDARD AND DECONSTRUCTION</th>
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<tbody>
<tr>
<td>8.NS.2</td>
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</table>

## DESCRIPTION

Students locate rational and irrational numbers on the number line. Students compare and order rational and irrational numbers. Students also recognize that square roots may be negative and written as -\(\sqrt{28}\).

Example 1:
Compare \(\sqrt{2}\) and \(\sqrt{3}\).

**Solution:**
Statements for the comparison could include:
- \(\sqrt{2}\) and \(\sqrt{3}\) are between the whole numbers 1 and 2.
- \(\sqrt{3}\) is between 1.7 and 1.8.
- \(\sqrt{2}\) is less than \(\sqrt{3}\).

Additionally, students understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational.

Example 2:
Find an approximation of \(\sqrt{28}\).

- Determine the perfect squares \(\sqrt{28}\) is between, which would be 25 and 36.
- The square roots of 25 and 36 are 5 and 6 respectively, so we know that \(\sqrt{28}\) is between 5 and 6.
- Since 28 is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27.
- The estimate of \(\sqrt{28}\) would be 5.27 (the actual is 5.29).
**ESSENTIAL QUESTION(S)**
What are accurate strategies for comparing irrational numbers?

**MATHEMATICAL PRACTICE(S)**
- 8.MP.7. Look for and make use of structure.
- 8.MP.8. Look for and express regularity in repeated reasoning.

**DOK Range Target for Instruction & Assessment**

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
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<tbody>
<tr>
<td><strong>Assessment Types</strong></td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td><strong>Students should be able to:</strong></td>
<td>Approximate irrational numbers as rational numbers.</td>
<td>Compare the size of irrational numbers using rational approximations.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Approximately locate irrational numbers on a number line.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Estimate the value of expressions involving irrational numbers using rational approximations.</td>
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</tbody>
</table>

**EXPLANATIONS AND EXAMPLES**
Students can approximate square roots by iterative processes.

**Examples:**
Approximate the value of $\sqrt{2}$ to the nearest hundredth.

**Solution:**
Students start with a rough estimate based upon perfect squares. $\sqrt{2}$ falls between 2 and 3 because 5 falls between $2^2 = 4$ and $3^2 = 9$. The value will be closer to 2 than to 3. Students continue the iterative process with the tenths place value. $\sqrt{2}$ falls between 2.2 and 2.3 because 5 falls between $2.2^2 = 4.84$ and $2.3^2 = 5.29$. The value is closer to 2.2. Further iteration shows that the value of $\sqrt{2}$ is between 2.23 and 2.24 since $2.23^2$ is 4.9729 and $2.24^2$ is 5.0176.

Compare $\sqrt{2}$ and $\sqrt{3}$ by estimating their values, plotting them on a number line, and making comparative statements.

![Number Line]

**Solution:**
Statements for the comparison could include:
- $\sqrt{2}$ is approximately 0.3 less than $\sqrt{3}$.
- $\sqrt{2}$ is between the whole numbers 1 and 2.
- $\sqrt{3}$ is between 1.7 and 1.8.
EIGHTH GRADE

EXPRESSIONS AND EQUATIONS (EE)

EIGHTH GRADE
MATHEMATICS
**DOMAIN**  
Expressions and Equations (EE)

**CLUSTERS**
1. Work with radicals and integer exponents.  
2. Understand the connections between proportional relationships, lines, and linear equations.  
3. Analyze and solve linear equations and pairs of simultaneous linear equations.

### SEVENTH  
#### EARLY EQUATIONS AND EXPRESSIONS

**Working with Expressions**

- **7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**LINEAR EQUATIONS, INEQUALITIES, AND FUNCTIONS**

- **Solving Linear Equations and Inequalities**
  - **7.EE.4.a** Solve word problems leading to equations of the form \( px + q = r \) and \( p(x + q) = r \), where \( p, q, \) and \( r \) are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
  - **7.EE.4.b** Solve word problems leading to inequalities of the form \( px + q > r \) or \( px + q < r \), where \( p, q, \) and \( r \) are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

### EIGHTH  
#### EARLY EQUATIONS AND EXPRESSIONS

**Working with Expressions**

- **7.EE.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

**LINEAR EQUATIONS, INEQUALITIES, AND FUNCTIONS**

- **Solving Linear Equations and Inequalities**
  - **8.EE.7.b** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
  - **8.EE.7.a** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form \( x = a, a = a, \) or \( a = b \) results (where \( a \) and \( b \) are different numbers).

**Linear Functions**

- **8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- **8.F.3** Interpret the equation \( y = mx + b \) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x, y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
### Expressions and Equations (EE)

<table>
<thead>
<tr>
<th>Systems of Two Linear Equations</th>
<th>Systems of Two Linear Equations</th>
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</thead>
<tbody>
<tr>
<td>8.EE.8.a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</td>
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</tr>
<tr>
<td>8.EE.8.b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.</td>
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</tr>
<tr>
<td>8.EE.8.c Solve real-world and mathematical problems leading to two linear equations in two variables.</td>
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</table>
### EIGHTH GRADE

**LEXILE GRADE LEVEL BANDS: 1010L TO 1185L**

<table>
<thead>
<tr>
<th>CLUSTER</th>
<th>1. Work with radicals and integer exponents.</th>
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<tbody>
<tr>
<td></td>
<td>Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</td>
</tr>
<tr>
<td></td>
<td>Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.</td>
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<tr>
<td></td>
<td>Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$, and determine that the world population is more than 20 times larger.</td>
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<tr>
<td></td>
<td>Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</td>
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<table>
<thead>
<tr>
<th>BIG IDEA</th>
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<tbody>
<tr>
<td>• There are infinite ways to express a number or expression.</td>
</tr>
<tr>
<td>• Arithmetic and algebraic processes are governed by the rules of any number set.</td>
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<table>
<thead>
<tr>
<th>ACADEMIC VOCABULARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>real numbers, irrational numbers, rational numbers, integers, whole numbers, natural numbers, radical, radicand, square roots, perfect squares, cube roots, terminating decimals, repeating decimals, truncate</td>
</tr>
<tr>
<td>STANDARD AND DECONSTRUCTION</td>
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<td>-----------------------------</td>
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<tr>
<td>8.EE.1</td>
</tr>
</tbody>
</table>

**DESCRIPTION**

In 6th grade, students wrote and evaluated simple numerical expressions with whole number exponents (e.g., \(5^3 = 5 \times 5 \times 5 = 125\)). Integer (positive and negative) exponents are further developed to generate equivalent numerical expressions when multiplying, dividing or raising a power to a power. Using numerical bases and the laws of exponents, students generate equivalent expressions.

Students understand:

- Bases must be the same before exponents can be added, subtracted or multiplied. (Example 1)
- Exponents are subtracted when like bases are being divided. (Example 2)
- A number raised to the zero (0) power is equal to one. (Example 3)
- Negative exponents occur when there are more factors in the denominator. These exponents can be expressed as a positive if left in the denominator. (Example 4)
- Exponents are added when like bases are being multiplied. (Example 5)
- Exponents are multiplied when an exponent is raised to an exponent. (Example 6)
- Several properties may be used to simplify an expression. (Example 7)

<table>
<thead>
<tr>
<th>Example 1:</th>
<th>Example 2:</th>
<th>Example 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{2^3}{5^2} = \frac{8}{25})</td>
<td>(\frac{2^5}{2^4} = 2^{5-4} = 2^1 = \frac{1}{16})</td>
<td>(6^0 = 1)</td>
</tr>
</tbody>
</table>

Students understand this relationship from examples such as \(\frac{36}{36} = 1\). Using the laws of exponents this expression could also be written as \(6^{2-2} = 6^0\). Combining these gives \(6^0 = 1\).

<table>
<thead>
<tr>
<th>Example 4:</th>
<th>Example 5:</th>
<th>Example 6:</th>
<th>Example 7:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3^2}{2^4} = 3^{-2})</td>
<td>((3^2)(3^4) = (3^{2+4}) = 3^6 = 729)</td>
<td>((4^3)^2 = 4^{3 \times 2} = 4^6 = 4,096)</td>
<td>(\frac{(3^2)^4}{(3^3)(3^5)} = \frac{3^{2 \times 4}}{3^{2+3}} = \frac{3^8}{3^3} = 3^{8-3} = 3^5 = 27)</td>
</tr>
</tbody>
</table>
**EIGHTH GRADE**  
LEXILE GRADE LEVEL BANDS: 1010L TO 1185L

**ESSENTIAL QUESTION(S)**
Apply properties of integers to generate equivalent numerical expressions.

**MATHEMATICAL PRACTICE(S)**
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**
- ☒ 1
- ☐ 2
- ☐ 3
- ☐ 4

**Instructional Targets**

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<tr>
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<th>Know: Concepts/Skills</th>
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<tbody>
<tr>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
<td></td>
</tr>
</tbody>
</table>

**Students should be able to:**
- Explain the properties of integer exponents to generate equivalent numerical expressions.
- Apply the properties of integer exponents to produce equivalent numerical expressions.

**EXPLANATIONS AND EXAMPLES**

**Examples:**

- \[
\frac{4^3}{5^2} = \frac{64}{25}
\]
- \[
\frac{4^3}{4^7} = 4^{3-7} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}
\]
- \[
\frac{4^{-3}}{5^2} = 4^{-3} \times \frac{1}{5^2} = \frac{1}{4^3} \times \frac{1}{5^2} = \frac{1}{64} \times \frac{1}{25} = \frac{1}{16,000}
\]
**STANDARD AND DECONSTRUCTION**

<table>
<thead>
<tr>
<th>8.EE.2</th>
<th>Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that the square root of 2 is irrational.</th>
</tr>
</thead>
</table>

**DESCRIPTION**

Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational.

Students recognize that squaring a number and taking the square root $\sqrt{}$ of a number are inverse operations; likewise, cubing a number and taking the cube root $\sqrt[3]{\cdot}$ are inverse operations.

**Example 1:**

$4^2 = 16$ and $\sqrt{16} = \pm 4$

**Example 2:**

$$\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27} \text{ and } \sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$$

**Example 3:**

Solve: $x^2 = 25$

Solution: $\sqrt{x^2} = \pm\sqrt{25}$

$x = \pm 5$

**Example 4:**

Solve: $x^2 = \frac{4}{9}$

Solution: $\sqrt{x^2} = \pm\sqrt{\frac{4}{9}}$

$x = \pm\frac{2}{3}$

**Example 5:**

Solve: $x^3 = 27$

Solution: $\sqrt[3]{x} = \sqrt[3]{27}$

$x = 3$
Example 6:

Students understand that in geometry the square root of the area is the length of the side of a square, and a cube root of the volume is the length of the side of a cube. Students use this information to solve problems, such as finding the perimeter.

Example 7:

What is the side length of a square with an area of 49 ft²?

Solution:

\[ \sqrt{49} = 7 \text{ ft}. \] The length of one side is 7 ft.
Examples:

\[ 3^2 = 9 \quad \text{and} \quad \sqrt{9} = \pm 3 \]

\[ \left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27} \quad \text{and} \quad \sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3} \]

Solve: \[ x^2 = 9 \]

Solution: \[ x^2 = 9 \quad \sqrt{x^2} = \pm \sqrt{9} \quad x = \pm 3 \]

Solve: \[ x^3 = 8 \]

Solution: \[ x^3 = 8 \quad \sqrt[3]{x^3} = \sqrt[3]{8} \quad x = 2 \]
## STANDARD AND DECONSTRUCTION

### 8.EE.3

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$, and determine that the world population is more than 20 times larger.

### DESCRIPTION

Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that if the exponent increases by one, the value increases 10 times. Likewise, if the exponent decreases by one, the value decreases 10 times. Students solve problems using addition, subtraction, or multiplication, expressing the answer in scientific notation.

**Example 1:**
Write 75,000,000,000 in scientific notation.

**Solution:** $7.5 \times 10^{10}$

**Example 2:**
Write 0.0000429 in scientific notation.

**Solution:** $4.29 \times 10^{-5}$

**Example 3:**
Express $2.45 \times 10^5$ in standard form.

**Solution:** 245,000

**Example 4:**
How much larger is $6 \times 10^5$ compared to $2 \times 10^3$?

**Solution:** 300 times larger since 6 is 3 times larger than 2 and $10^5$ is 100 times larger than $10^3$.

**Example 5:**
Which is the larger value: $2 \times 10^6$ or $9 \times 10^5$?

**Solution:** $2 \times 10^6$ because the exponent is larger.
### Essential Question(s)

Why is it important to have an efficient way to express very large or very small numbers?

### Mathematical Practice(s)

- 8.MP.5. Use appropriate tools strategically.

### DOK Range Target for Instruction & Assessment

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</table>

### Instructional Targets

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<td>Express numbers as a single digit times an integer power of 10. Use scientific notation to estimate very large and/or very small quantities.</td>
<td>Compare quantities to express how much larger one is compared to the other.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
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<td></td>
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</table>

### Explanations and Examples

Students express numbers in scientific notation. Students will need to use single digit coefficients. They will need to convert between standard form and scientific notation, and vice versa. They should discover how to compare numbers while in scientific notation (move decimal point while multiplying or dividing by a power of 10). Estimating and understanding an increase or decrease of the coefficients and the powers of 10 are important. They will perform a mathematical operation with those increases or decreases.

**Example:**

An ant has a mass of approximately .004 grams and an elephant has a mass of approximately 8 metric tons.

a. How many ants does it take to have the same mass as an elephant?

b. An ant is 1 cm long. If you put all these ants from your answer to part a. in a line front to back, how long would the line be? Find two cities in the United States that are a similar distance apart to illustrate this length.

Note: 1 kg = 1000 grams, 1 metric ton = 1000 kg, 1 m = 100 cm, 1 km = 1000 m
### Eighth Grade

#### 8.EE.4

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

**Description**

Students understand scientific notation as generated on various calculators or other technology. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.

**Example 1:**

2.45E+23 is $2.45 \times 10^{23}$ and 3.5E-4 is $3.5 \times 10^{-4}$ (NOTE: There are other notations for scientific notation depending on the calculator being used.)

Students add and subtract with scientific notation.

**Example 2:**

In July 2010 there were approximately 500 million Facebook users. In July 2011 there were approximately 750 million Facebook users. How many more users were there in 2011? Write your answer in scientific notation.

**Solution:**

Subtract the two numbers: $750,000,000 - 500,000,000 = 250,000,000 \rightarrow 2.5 \times 10^8$

Students use laws of exponents to multiply or divide numbers written in scientific notation, writing the product or quotient in proper scientific notation.

**Example 3:**

$$(6.45 \times 10^{11})(3.2 \times 10^2) = (6.45 \times 3.2)(10^{11} \times 10^2)$$

Rearrange factors

$$= 20.64 \times 10^{13}$$

Add exponents when multiplying powers of 10

$$= 2.064 \times 10^{16}$$

Write in scientific notation

**Example 4:**

$$\frac{3.45 \times 10^7}{6.7 \times 10^2} = \frac{6.3}{1.6} \times 10^{7-2}$$

Subtract exponents when dividing powers of 10

$$= 0.515 \times 10^5$$

Write in scientific notation

**Example 5:**

$$0.0025(5.2 \times 10^5) = (2.5 \times 10^{-3})(5.2 \times 10^5)$$

Write factors in scientific notation

Rearrange factors

$$= (2.5 \times 5.2)(10^{-3} \times 10^5)$$

Add exponents when multiplying powers of 10

$$= 13 \times 10^2$$

Write in scientific notation
**ESSENTIAL QUESTION(S)**
What is an efficient strategy for expressing very large or very small numbers?

**MATHEMATICAL PRACTICE(S)**
- 8.MP.5. Use appropriate tools strategically.

**DOK Range Target for Instruction & Assessment**

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<td>Students should be able to:</td>
<td>Perform operations using numbers expressed in scientific notations. Use scientific notation to express very large and very small quantities.</td>
<td>Interpret scientific notation that has been generated by technology. Choose appropriate units of measure when using scientific notation.</td>
<td>Tasks assessing concepts, skills, and procedures. Tasks assessing expressing mathematical reasoning. Tasks assessing modeling/applications.</td>
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</tbody>
</table>

**EXPLANATIONS AND EXAMPLES**

Students can convert decimal forms to scientific notation and apply rules of exponents to simplify expressions. In working with calculators or spreadsheets, it is important that students recognize scientific notation. Students should recognize that the output of 2.45E+23 is 2.45 x 1023 and 3.5E-4 is 3.5 x 10-4. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.
### Cluster

2. Understand the connections between proportional relationships, lines, and linear equations.

They will master the following concepts and skills:

- Graph a proportional relationship given a table, equation, or contextual situation.
- Recognize unit rate as slope and interpret the meaning of the slope in context.
- Recognize that proportional relationships include the point (0,0).
- Compare different representations of two proportional relationships represented as contextual situations, graphs, or equations.

### Big Idea

- The graph of a line is always represented by the equation $ax + by = c$.

### Academic Vocabulary

- Unit rate
- Proportional relationships
- Slope
- Vertical
- Horizontal
- Similar triangles
- Y-intercept

### Standard and Deconstruction

8.EE.5 | Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

#### Description

Students build on their work with unit rates from 6th grade and proportional relationships in 7th grade to compare graphs, tables, and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables, and equations to compare two proportional relationships represented in different ways.

**Example 1:** Compare the scenarios to determine which represents a greater speed. Explain your choice including a written description of each scenario. Be sure to include the unit rates in your explanation.

**Scenario 1:**

$$y = 55x$$

- $x$ is the time in hours.
- $y$ is the distance in miles.

**Scenario 2:**

- $y = 60x$
- $x$ is the time in hours.
- $y$ is the distance in miles.

**Solution:**

Scenario 1 has the greater speed since the unit rate is 60 miles per hour. The graph shows this rate since 60 is the distance traveled in one hour. Scenario 2 has a unit rate of 55 miles per hour shown as the coefficient in the equation.

Given an equation of a proportional relationship, students draw a graph of the relationship. Students recognize that the unit rate is the coefficient of $x$ and that this value is also the slope of the line.
ESSENTIAL QUESTION(S)

Why are there different strategies for solving a linear equation?

MATHEMATICAL PRACTICE(S)

- 8.MP.1. Make sense of problems and persevere in solving them.
- 8.MP.3. Construct viable arguments and critique the reasoning of others.
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.
- 8.MP.8. Look for and express regularity in repeated reasoning.

DOK Range Target for Instruction & Assessment

| 1 | 2 | 3 | 4 |

Instructional Targets

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<tr>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Graph proportional relationships.</td>
<td>Compare two different proportional relationships represented in different ways. Interpret the unit rate of proportional relationships as the slope of the graph.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

EXPLANATIONS AND EXAMPLES

Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationships. Students are expected to both sketch and interpret graphs.

Example:

Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation.

**Scenario 1:**

<table>
<thead>
<tr>
<th>Traveling Time</th>
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<tbody>
<tr>
<td><img src="image1" alt="Graph Image" /></td>
</tr>
</tbody>
</table>

**Scenario 2:**

\[ y = 50x \]

- \( x \) is the time in hours.
- \( y \) is the distance in miles.
### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>8.EE.6</th>
<th>Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.</th>
</tr>
</thead>
</table>

#### DESCRIPTION

Triangles are similar when there is a constant rate of proportionality between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line.

**Example 1:**
The triangle between A and B has a vertical height of 2 and a horizontal length of 3.
The triangle between B and C has a vertical height of 4 and a horizontal length of 6.
The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of 2/3 for the line, indicating that the triangles are similar.

Given an equation in slope-intercept form, students graph the line represented.

Students write equations in the form $y = mx$ for lines going through the origin, recognizing that $m$ represents the slope of the line.

**Example 2:**
Write an equation to represent the graph to the right.

**Solution:** $y = -3/2x$

Students write equations in the form $y = mx + b$ for lines not passing through the origin, recognizing that $m$ represents the slope and $b$ represents the y-intercept.

**Solution:** $y = 2/3x - 2$
### ESSENTIAL QUESTION(S)

Why is the equation $y = mx + b$ an efficient way of representing a non-vertical line?

### MATHEMATICAL PRACTICE(S)

- 8.MP.3. Construct viable arguments and critique the reasoning of others.
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.
- 8.MP.8. Look for and express regularity in repeated reasoning.

### DOK Range Target for Instruction & Assessment

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<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Identify characteristics of similar triangles.</td>
<td>Analyze patterns for points on a line that pass through the origin.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Find the slope of a line.</td>
<td>Derive an equation of the form $y = mx$ for a line through the origin.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Determine the y-intercept of a line.</td>
<td>Analyze patterns for points on a line that do not pass through or include the origin.</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Derive an equation of the form $y = mx + b$ for a line intercepting the vertical axis at $b$ (the y-intercept).</td>
<td></td>
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<td></td>
<td></td>
<td>Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane.</td>
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</table>

### EXPLANATIONS AND EXAMPLES

**Example:**

Explain why $\triangle ABC$ is similar to $\triangle DFE$, and deduce that $\overline{AB}$ has the same slope as $\overline{BE}$. Express each line as an equation.
### Cluster 3. Analyze and solve linear equations and pairs of simultaneous linear equations.

They will identify and provide examples of systems of equations that have one solution, infinitely many solutions or no solutions; solve a system of equations algebraically; estimate solutions by graphing systems of equations; and create and utilize systems of linear equation to model real-world situations.

### Big Idea

- A linear graph uses points connected by lines which represent successive changes in the value of a variable quantity or quantities.

### Academic Vocabulary

elimination, intersection, intersecting, parallel lines, coefficient, distributive property, like terms, substitution, solution, solve, system of linear equations

### Standard and Deconstruction

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<td>8.EE.7</td>
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<tr>
<td>Solve linear equations in one variable.</td>
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</table>

**Description**

Students solve one-variable equations including those with the variables being on both sides of the equal sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. Equations shall include rational numbers, distributive property, and combining like terms.

**Example 1:**

Equations have one solution when the variables do not cancel out. For example, $10x - 23 = 29 - 3x$ can be solved to $x = 4$. This means that when the value of $x$ is 4, both sides will be equal. If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be $(4, 17)$.

\[
10 \cdot 4 - 23 = 29 - 3 \cdot 4 \\
40 - 23 = 29 - 12 \\
17 = 17
\]

**Example 2:**

Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for $x$ that will make the sides equal.

\[
-x + 7 - 6x = 19 - 7x
\]

Combine like terms.

\[
-7x + 7 = 19 - 7x
\]

Add $7x$ to each side.

\[
7 \neq 19
\]

This solution means that no matter what value is substituted for $x$ the final result will never be equal to each other.

If each side of the equation were treated as a linear equation and graphed, the lines would be parallel.
Example 3:
An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation. For example the following equation, when simplified will give the same values on both sides.

\[-\frac{1}{2}(36a-6)=\frac{2}{3}(4-24a)\]

\[-18a + 3 = 3 - 18a\]

If each side of the equation were treated as a linear equation and graphed, the graph would be the same line. Students write equations from verbal descriptions and solve.

Example 4:
Two more than a certain number is 15 less than twice the number. Find the number.

**Solution:**

\[n + 2 = 2n - 15\]

\[17 = n\]

### ESSENTIAL QUESTION(S)

Why find the solution of a linear equation?

### MATHEMATICAL PRACTICE(S)

- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.

### SUBSTANDARD DECONSTRUCTED

8.EE.7a. **Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.** Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form \(x = a, a = a,\text{ or } a = b\) results (where \(a\) and \(b\) are different numbers).

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<td><strong>Assessment Types</strong></td>
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<tr>
<td><strong>Students should be able to:</strong></td>
<td>Identify examples of linear equations in one variable with one solution.</td>
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<tr>
<td></td>
<td>Identify examples of linear equations in one variable with infinitely many solutions.</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Identify examples of linear equations in one variable with no solution.</td>
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<td></td>
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<tr>
<td></td>
<td>Show how to transform given equations into simpler forms, until the result is an equivalent equation of the form (x=a, a=a,\text{ or } a=b).</td>
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</table>
8.EE.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

**Instructional Targets**

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**Students should be able to:**

- Solve linear equations with rational number coefficients.
- Solve equations whose solutions require expanding expressions using the distributive property and/or collecting like terms.

**EXPLANATIONS AND EXAMPLES**

As students transform linear equations in one variable into simpler forms, they discover the equations can have one solution, infinitely many solutions, or no solutions.

When the equation has one solution, the variable has one value that makes the equation true as in $12 - 4y = 16$. The only value for $y$ that makes this equation true is $-1$.

When the equation has infinitely many solutions, the equation is true for all real numbers as in $7x + 14 = 7(x+2)$. As this equation is simplified, the variable terms cancel leaving $14 = 14$ or $0 = 0$. Since the expressions are equivalent, the value for the two sides of the equation will be the same regardless which real number is used for the substitution.

When an equation has no solutions it is also called an inconsistent equation. This is the case when the two expressions are not equivalent as in $5x - 2 = 5(x+1)$. When simplifying this equation, students will find that the solution appears to be two numbers that are not equal or $-2 = 1$. In this case, regardless which real number is used for the substitution, the equation is not true and therefore has no solution.

**Examples:**

**Solve for $x$:**

- $-3(x + 7) = 4$
- $3x - 8 = 4x - 8$
- $3(x + 1) - 5 = 3x - 2$

**Solve:**

- $7(m - 3) = 7$
- $\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y$
8.EE.8 Analyze and solve pairs of simultaneous linear equations.

Systems of linear equations can also have one solution, infinitely many solutions, or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically.

Students graph a system of two linear equations, recognizing that the ordered pair for the point of intersection is the x-value that will generate the given y-value for both equations. Students recognize that graphed lines with one point of intersection (different slopes) will have one solution, parallel lines (same slope, different y-intercepts) have no solutions, and lines that are the same (same slope, same y-intercept) will have infinitely many solutions.

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole numbers and/or decimals/fractions. Students define variables and create a system of linear equations in two variables.

Example 1:
1. Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same.

Solution:
Let \( W \) = number of weeks
Let \( H \) = height of the plant after \( W \) weeks

<table>
<thead>
<tr>
<th>( W )</th>
<th>( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

2. Based on the coordinates from the table, graph lines to represent each plant.

Solution:

<table>
<thead>
<tr>
<th>( W )</th>
<th>( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>

3. Write an equation that represents the growth rate of Plant A and Plant B.

Solution:

Plant A \( H = 2W + 4 \)
Plant B \( H = 4W + 2 \)
4. At which week will the plants have the same height?

Solution:

\[2W + 4 = 4W + 2\]  
Set height of Plant A equal to height of Plant B

\[2W - 2W + 4 = 4W - 2W + 2\]  
Solve for \(W\)

\[4 = 2W + 2\]  
\[2W + 2 = 2W\]  
\[2 = 2W\]  
\[1W = 2\]

After one week, the height of Plant A and Plant B are both 6 inches.

Check:

\[2(1)+4=4(1)+2\]
\[2+4=4+2\]
\[6=6\]

Given two equations in slope-intercept form (Example 1) or one equation in standard form and one equation in slope-intercept form, students use substitution to solve the system.

Example 2:

Solve: Victor is half as old as Maria. The sum of their ages is 54. How old is Victor?

Solution:

Let \(v = \) Victor's age  
Let \(m = \) Maria's age

\[v + m = 54\]

\[v = 1/2m\]

\[1/2m + m = 54\]  
Substitute \(1/2m\) for \(v\) in the first equation.

\[1 1/2m = 54\]

\[m=36\]

If Maria is 36, then substitute 36 into \(v + m = 54\) to find Victor's age of 18.

Note: Students are not expected to change linear equations written in standard form to slope-intercept form or solve systems using elimination.

For many real-world contexts, equations may be written in standard form. Students are not expected to change the standard form to slope-intercept form. However, students may generate ordered pairs recognizing that the values of the ordered pairs would be solutions for the equation. For example, in the equation above, students could make a list of the possible ages of Victor and Maria that would add to 54. The graph of these ordered pairs would be a line with all the possible ages for Victor and Maria.
**Essential Question(s)**
Why do simultaneous linear equations have different types of solutions?

**Mathematical Practice(s)**
- 8.MP.1. Make sense of problems and persevere in solving them.
- 8.MP.3. Construct viable arguments and critique the reasoning of others.
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.
- 8.MP.8. Look for and express regularity in repeated reasoning.

**DOK Range Target for Instruction & Assessment**
- X: 1
- X: 2
- X: 3
- □: 4

### Substandard Deconstructed

**8.EE.8a.** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assessment Types</strong></td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td><strong>Students should be able to:</strong></td>
<td>Identify the solution(s) to a system of two linear equations in two variables as the point(s) of intersection of their graphs.</td>
<td>Describe the point(s) of intersection between two lines as points that satisfy both equations simultaneously.</td>
<td></td>
</tr>
</tbody>
</table>
### 8.EE.8b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be $5$ and $6$.

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
<th>Think: Tasks assessing expressing mathematical reasoning</th>
<th>Do: Tasks assessing modeling/applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Identify cases in which a system of two equations with two unknowns has no solution.</td>
<td>Estimate the point(s) of intersection for a system of two equations in two unknowns by graphing the equations.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identify cases in which a system of two equations with two unknowns has an infinite number of solutions.</td>
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<td></td>
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<tr>
<td></td>
<td>Solve a system of two equations (linear) with two unknowns algebraically.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solve simple cases of systems of two linear equations with two variables by inspection.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Estimate the point(s) of intersection for a system of two equations with two unknowns by graphing the equations.</td>
<td></td>
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<tr>
<td></td>
<td>Solve a system of two equations (linear) with two unknowns algebraically.</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Solve simple cases of systems of two linear equations with two variables by inspection.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 8.EE.8c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
<th>Think: Tasks assessing expressing mathematical reasoning</th>
<th>Do: Tasks assessing modeling/applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Solve real-world problems leading to two linear equations in two variables.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Given each set of coordinates, graph their corresponding lines.

**Solution:**

Write an equation that represent the growth rate of Plant A and Plant B.

**Solution:**

Plant A: \( H = 2W + 4 \)

Plant B: \( H = 4W + 2 \)

At which week will the plants have the same height?

**Solution:**

The plants have the same height after one week.

Plant A: \( H = 2W + 4 \)  
Plant B: \( H = 4W + 2 \)  
Plant A: \( H = 2(1) + 4 \)  
Plant B: \( H = 4(1) + 2 \)  
Plant A: \( H = 6 \)  
Plant B: \( H = 6 \)

After one week, the height of Plant A and Plant B is both 6 inches.
DOMAIN:

FUNCTIONS (F)

EIGHTH GRADE
MATHEMATICS
### Functions (F)

#### Domains

<table>
<thead>
<tr>
<th>Domain</th>
<th>Clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Functions (F)</td>
<td>1. Define, evaluate, and compare functions. 2. Use functions to model relationships between quantities.</td>
</tr>
</tbody>
</table>

### Functions (F)

#### Early Equations and Expressions

<table>
<thead>
<tr>
<th>Seventh</th>
<th>Eighth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linear Functions</strong></td>
<td><strong>Linear Functions</strong></td>
</tr>
<tr>
<td>8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</td>
<td></td>
</tr>
<tr>
<td>8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</td>
<td></td>
</tr>
<tr>
<td>8.F.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.</td>
<td></td>
</tr>
<tr>
<td>8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
<td></td>
</tr>
<tr>
<td>8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</td>
<td></td>
</tr>
<tr>
<td><strong>Systems of Two Linear Equations</strong></td>
<td><strong>Systems of Two Linear Equations</strong></td>
</tr>
<tr>
<td>8.EE.8.a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</td>
<td></td>
</tr>
<tr>
<td>8.EE.8.b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.</td>
<td></td>
</tr>
<tr>
<td>8.EE.8.c Solve real-world and mathematical problems leading to two linear equations in two variables.</td>
<td></td>
</tr>
</tbody>
</table>

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**Common Core State Standards Deconstructed for Classroom Impact**

**Lexile Grade Level Bands: 1010L to 1185L**

**Eighth Grade**

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**Domain:** Functions (F)

**Clusters:**

1. Define, evaluate, and compare functions.
2. Use functions to model relationships between quantities.
1. **Define, evaluate, and compare functions.**

   Understand that functions describe relationships where one variable determines a unique value of the other. Recognize a graph of a function as the set of ordered pairs consisting of an input and corresponding output.\(^1\)

**BIG IDEA**

- Relations and functions are relationships that can be represented as verbal rules, equations, tables, and graphs.

**ACADEMIC VOCABULARY**

functions, y-value, x-value, vertical line test, input, output, rate of change, linear function, nonlinear function

**STANDARD AND DECONSTRUCTION**

<table>
<thead>
<tr>
<th>STANDARD</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.F.1</td>
<td>Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</td>
</tr>
</tbody>
</table>

**DESCRIPTON**

Students understand rules that take x as input and gives y as output is a function. Functions occur when there is exactly one y-value is associated with any x-value. Using y to represent the output we can represent this function with the equations \(y = x^2 + 5x + 4\). Students are not expected to use the function notation \(f(x)\) at this level.

Students identify functions from equations, graphs, and tables/ordered pairs.

**Graphs**

Students recognize graphs such as the one below as a function using the vertical line test, showing that each x-value has only one y-value;

\[
\text{Graph Image}
\]

whereas, graphs such as the following are not functions since there are 2 y-values for multiple x-values.

\[
\text{Graph Image}
\]
### ESSENTIAL QUESTION(S)
What is a function?

### MATHEMATICAL PRACTICE(S)
8.MP.2. Reason abstractly and quantitatively.
8.MP.6. Attend to precision.

### DOK Range Target for Instruction & Assessment

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Instructional Targets

<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

**Students should be able to:**
- Identify cases in which a system of two equations with two unknowns has no solution.
- Identify cases in which a system of two equations with two unknowns has an infinite number of solutions.
- Solve a system of two equations (linear) with two unknowns algebraically.
- Solve simple cases of systems of two linear equations with two variables by inspection.
- Estimate the point(s) of intersection for a system of two equations with two unknowns by graphing the equations.

### EXPLANATIONS AND EXAMPLES
For example, the rule that takes x as input and gives \(x^2+5x+4\) as output is a function. Using y to stand for the output we can represent this function with the equation \(y = x^2+5x+4\), and the graph of the equation is the graph of the function. Students are not yet expected use function notation such as \(f(x) = x^2+5x+4\).
### 8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

**DESCRIPTION**

Students compare two functions from different representations.

**Example 1:**

Compare the following functions to determine which has the greater rate of change.

Function 1: \( y = 2x + 4 \)

Function 2:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**Solution:**

The rate of change for function 1 is 2; the rate of change for function 2 is 3. Function 2 has the greater rate of change.

**Example 2:**

Compare the two linear functions listed below and determine which has a negative slope.

Function 1: Gift Card

Samantha starts with $20 on a gift card for the bookstore. She spends $3.50 per week to buy a magazine. Let \( y \) be the amount remaining as a function of the number of weeks, \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>16.50</td>
</tr>
<tr>
<td>2</td>
<td>13.00</td>
</tr>
<tr>
<td>3</td>
<td>9.50</td>
</tr>
</tbody>
</table>

Function 2: Calculator rental

The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost \( c \) of renting a calculator as a function of the number of months \( m \).

\[ c = 10 + 5m \]

**Solution:**

Function 1 is an example of a function whose graph has a negative slope. Both functions have a positive starting amount; however, in function 1, the amount decreases 3.50 each week, while in function 2, the amount increases 5.00 each month.

**NOTE:** Functions could be expressed in standard form. However, the intent is not to change from standard form to slope-intercept form but to use the standard form to generate ordered pairs. Substituting a zero \((0)\) for \( x \) and \( y \) will generate two ordered pairs. From these ordered pairs, the slope could be determined.
## Example 3:

2x + 3y = 6

Let x = 0:

\[ 2(0) + 3y = 6 \]

\[ 3y = 6 \]

\[ y = 2 \]

Ordered pair: (0, 2)

Let y = 0:

\[ 2x + 3(0) = 6 \]

\[ 2x = 6 \]

\[ x = 3 \]

Ordered pair: (3, 0)

Using (0, 2) and (3, 0) students could find the slope and make comparisons with another function.

## Essential Question(s)

What are efficient strategies for comparing functions?

## Mathematical Practice(s)

8.MP.1. Make sense of problems and persevere in solving them.
8.MP.2. Reason abstractly and quantitatively.
8.MP.3. Construct viable arguments and critique the reasoning of others.
8.MP.5. Use appropriate tools strategically.
8.MP.6. Attend to precision.
8.MP.7. Look for and make use of structure.
8.MP.8. Look for and express regularity in repeated reasoning.

## DOK Range Target for Instruction & Assessment

| 1 | 2 | 3 | 4 |

## Instructional Targets

<table>
<thead>
<tr>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

## Students should be able to:

- Identify functions algebraically including slope and y-intercept.
- Identify functions using graphs, tables, and verbal descriptions.
- Compare and contrast two functions with different representations.
- Draw conclusions based on different representations of functions.
**Examples:**

Compare the two linear functions listed below and determine which equation represents a greater rate of change.

**Function 1:**

The function whose input $x$ and output $y$ are related by $y = 3x + 7$.

**Function 2:**

Compare the two linear functions listed below and determine which has a negative slope.

**Function 1: Gift Card**

Samantha starts with $20 on a gift card for the book store. She spends $3.50 per week to buy a magazine. Let $y$ be the amount remaining as a function of the number of weeks, $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>16.50</td>
</tr>
<tr>
<td>2</td>
<td>13.00</td>
</tr>
<tr>
<td>3</td>
<td>9.50</td>
</tr>
<tr>
<td>4</td>
<td>6.00</td>
</tr>
</tbody>
</table>

**Function 2:**

The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost ($c$) of renting a calculator as a function of the number of months ($m$).

**Solution:**

Function 1 is an example of a function whose graph has negative slope. Samantha starts with $20 and spends money each week. The amount of money left on the gift card decreases each week. The graph has a negative slope of -3.5, which is the amount the gift card balance decreases with Samantha’s weekly magazine purchase.

Function 2 is an example of a function whose graph has positive slope. Students pay a yearly non-refundable fee for renting the calculator and pay $5 for each month they rent the calculator. This function has a positive slope of 5 which is the amount of the monthly rental fee. An equation for Example 2 could be $c = 5m + 10$. 
### STANDARD AND DECONSTRUCTION

| 8.F.3 | Interpret the equation \( y = mx + b \) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function \( A = s^2 \) giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4), and (3, 9), which are not on a straight line. |

### DESCRIPTION

Students understand that linear functions have a constant rate of change between any two points. Students use equations, graphs and tables to categorize functions as linear or nonlinear.

**Example 1:**

Determine if the functions listed below are linear or nonlinear. Explain your reasoning.

1. \( y = -2x^2 + 3 \)
2. \( y = 0.25 + 0.5(x - 2) \)
3. \( A = \pi r^2 \)
4. \( y \) | \( x \) | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>3</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. \( y \) vs \( x \) graph with points indicating a non-linear relationship.

**Solution:**

1. Nonlinear
2. Linear
3. Nonlinear
4. Nonlinear; there is not a constant rate of change.
5. Nonlinear; the graph curves indicating the rate of change is not constant.

### ESSENTIAL QUESTION(S)

What are efficient strategies for determining if an equation is a linear function?

### MATHEMATICAL PRACTICE(S)

- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.
### DOK Range Target for Instruction & Assessment

<table>
<thead>
<tr>
<th>DOK Level</th>
<th>Instructional Targets</th>
<th>Assessment Types</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Know: Concepts/Skills</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
</tr>
<tr>
<td></td>
<td>Think</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
</tr>
<tr>
<td>3</td>
<td>Do</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

### Instructional Targets

#### Know: Concepts/Skills
- Recognize that a linear function is graphed as a straight line.
- Recognize the equation $y = mx + b$ is the equation of a function whose graph is a straight line where $m$ is the slope and $b$ is the y-intercept.
- Provide examples of nonlinear functions using multiple representations.

### Assessment Types

- Tasks assessing expressing mathematical reasoning.
- Tasks assessing modeling/applications.

### Students should be able to:
- Compare the characteristics of linear and nonlinear functions using various representations.

### EXPLANATIONS AND EXAMPLES

**Example:** Determine which of the functions listed below are linear and which are not linear and explain your reasoning.

- $y = -2x^2 + 3$ nonlinear
- $y = 2x$ linear
- $A = \pi r^2$ nonlinear
- $y = 0.25 + 0.5(x - 2)$ linear
### 8.F.4 Construct a function to model a linear relationship between two quantities.

Determine and interpret the rate of change and initial value of the function from a description of a relationship or from two \((x,y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of a situation it models, and in terms of its graph or a table of values.

**DESCRIPTION**

Students identify the rate of change (slope) and initial value \((y\text{-intercept})\) from tables, graphs, equations or verbal descriptions to write a function (linear equation). Students understand that the equation represents the relationship between the 
\(x\)-value and the \(y\)-value; what math operations are performed with the 
\(x\)-value to give the \(y\)-value. Slopes could be undefined slopes or zero slopes.

**Tables:**

Students recognize that in a table the \(y\)-intercept is the \(y\)-value when \(x\) is equal to 0. The slope can be determined by finding the ratio \(y/x\) between the change in two \(y\)-values and the change between the two corresponding \(x\)-values.

**Example 1:**

Write an equation that models the linear relationship in the table below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Solution:**

The \(y\)-intercept in the table above would be \((0, 2)\). The distance between 8 and -1 is 9 in a negative direction \(\rightarrow -9\); the distance between -2 and 1 is 3 in a positive direction. The slope is the ratio of rise to run or \(y/x\) or \(-9/3 = -3\). The equation would be \(y = -3x + 2\).
Graphs:
Using graphs, students identify the y-intercept as the point where the line crosses the y-axis and the slope as the rise/run.

Example 2:
Write an equation that models the linear relationship in the graph below.

Solution: The y-intercept is 4. The slope is \( \frac{1}{4} \), found by moving up 1 and right 4 going from (0,4) to (4,5). The linear equation would be \( y = \frac{1}{4}x + 4 \).

Equations:
In a linear equation the coefficient of \( x \) is the slope and the constant is the y-intercept. Students need to be given the equations in formats other than \( y = mx + b \), such as \( y = ax + b \) (format from graphing calculator), \( y = b + mx \) (often the format from contextual situations), etc.

Point and Slope:
Students write equations to model lines that pass through a given point with the given slope.

Example 3:
A line has a zero slope and passes through the point (-5, 4). What is the equation of the line?

Solution: \( y = 4 \)

Example 4:
Write an equation for the line that has a slope of \( \frac{1}{2} \) and passes through the point (-2, 5)

Solution: \( y = \frac{1}{2}x + 6 \)

Students could multiply the slope \( \frac{1}{2} \) by the x-coordinate -2 to get -1. Six (6) would need to be added to get to 5, which gives the linear equation.

Students also write equations given two ordered pairs. Note that point-slope form is not an expectation at this level. Students use the slope and y-intercepts to write a linear function in the form \( y = mx + b \).

Contextual Situations:
In contextual situations, the y-intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem. Rates of change can often occur over years. In these situations it is helpful for the years to be “converted” to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

Example 5:
The company charges $45 a day for the car as well as charging a one-time $25 fee for the car’s navigation system (GPS). Write an expression for the cost in dollars, \( c \), as a function of the number of days, \( d \), the car was rented.

Solution: \( C = 45d + 25 \)

Students interpret the rate of change and the y-intercept in the context of the problem. In Example 5, the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one-time fees vs. recurrent fees will help students model contextual situations.
### EIGHTH GRADE

**LEXILE GRADE LEVEL BANDS: 1010L TO 1185L**

**ESSENTIAL QUESTION(S)**

What quantitative information can be extracted from a linear function?

**MATHEMATICAL PRACTICE(S)**

8.MP.1. Make sense of problems and persevere in solving them.
8.MP.2. Reason abstractly and quantitatively.
8.MP.3. Construct viable arguments and critique the reasoning of others.
8.MP.5. Use appropriate tools strategically.
8.MP.6. Attend to precision.
8.MP.7. Look for and make use of structure.
8.MP.8. Look for and express regularity in repeated reasoning.

**DOK Range Target for Instruction & Assessment**

<table>
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<tr>
<th>1</th>
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<th>4</th>
</tr>
</thead>
</table>

**Instructional Targets**

<table>
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<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should be able to:</td>
<td>Recognize that slope is determined by the constant rate of change.</td>
<td>Construct a function to model a linear relationship between two quantities.</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
</tr>
<tr>
<td></td>
<td>Recognize that the y-intercept is the initial value where $x = 0$.</td>
<td>Relate the rate of change and initial value to real-world quantities in a linear function in terms of the situation modeled and in terms of its graph or a table of values.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
</tr>
<tr>
<td></td>
<td>Determine the rate of change from two $(x,y)$ values, a verbal description, values in a table, or graph.</td>
<td></td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td></td>
<td>Determine the initial value from two $(x,y)$ values, a verbal description, values in a table, or graph.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 1:
The table below shows the cost of renting a car. The company charges $45 a day for the car as well as charging a one-time $25 fee for the car’s navigation system (GPS). Write an expression for the cost in dollars, c, as a function of the number of days, d.

Students might write the equation $c = 45d + 25$ using the verbal description or by first making a table.

<table>
<thead>
<tr>
<th>Days (d)</th>
<th>Cost (c) in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>115</td>
</tr>
<tr>
<td>3</td>
<td>160</td>
</tr>
<tr>
<td>4</td>
<td>205</td>
</tr>
</tbody>
</table>

Students should recognize that the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one time fees vs. recurrent fees will help students model contextual situations.

Example 2:
When scuba divers come back to the surface of the water, they need to be careful not to ascend too quickly. Divers should not come to the surface more quickly than a rate of 0.75 ft per second. If the divers start at a depth of 100 feet, the equation $d = 0.75t - 100$ shows the relationship between the time of the ascent in seconds (t) and the distance from the surface in feet (d).

Will they be at the surface in 5 minutes? How long will it take the divers to surface from their dive?
Make a table of values showing several times and the corresponding distance of the divers from the surface. Explain what your table shows. How do the values in the table relate to your equation?
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

**DESCRIPTION**

Given a verbal description of a situation, students sketch a graph to model that situation. Given a graph of a situation, students provide a verbal description of the situation.

**Example 1:**
The graph below shows John's trip to school. He walks to Sam's house and, together, they ride a bus to school. The bus stops once before arriving at school.

Describe how each part A – E of the graph relates to the story.

**Solution:**

- A. John is walking to Sam's house at a constant rate.
- B. John gets to Sam's house and is waiting for the bus.
- C. John and Sam are riding the bus to school. The bus is moving at a constant rate, faster than John's walking rate.
- D. The bus stops.
- E. The bus resumes at the same rate as in part C.

**Example 2:**
Describe the graph of the function between $x = 2$ and $x = 5$.

**Solution:**

The graph is nonlinear and decreasing.
**ESSENTIAL QUESTION(S)**

What is the relationship between a linear equation and a linear graph?

**MATHEMATICAL PRACTICE(S)**

- 8.MP.3. Construct viable arguments and critique the reasoning of others.
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

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<tbody>
<tr>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
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</tr>
</tbody>
</table>

**Students should be able to:**

- Analyze a graph and describe the functional relationship between two quantities using the qualities of the graph.
- Sketch a graph, given a verbal description of its qualitative features.
- Interpret the relationship between x and y values by analyzing a graph.

**EXPLANATIONS AND EXAMPLES**

**Example:**

The graph below shows a student’s trip to school. This student walks to his friend’s house and, together, they ride a bus to school. The bus stops once before arriving at school.

Describe how each part A-E of the graph relates to the story.

![Graph showing a student’s trip to school](image)
DOMAIN:

GEOMETRY (G)

EIGHTH GRADE
MATHEMATICS
### Geometry (G)

1. Understand congruence and similarity using physical models, transparencies, or geometry software.
2. Understand and apply the Pythagorean Theorem.
3. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

### SEVENTH GRADE

<table>
<thead>
<tr>
<th>Similarity and Congruence, including Constructions and Transformations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.G.1</strong> Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</td>
</tr>
<tr>
<td><strong>8.G.2</strong> Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</td>
</tr>
</tbody>
</table>

### EIGHTH GRADE

<table>
<thead>
<tr>
<th>Similarity and Congruence, including Constructions and Transformations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.G.1</strong> Verify experimentally the properties of rotations, reflections, and translations.</td>
</tr>
<tr>
<td><strong>8.G.4</strong> Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</td>
</tr>
<tr>
<td><strong>8.G.5</strong> Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</td>
</tr>
<tr>
<td><strong>8.G.7</strong> Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</td>
</tr>
<tr>
<td><strong>8.G.6</strong> Explain a proof of the Pythagorean Theorem and its converse.</td>
</tr>
</tbody>
</table>

### Pythagorean Theorem

**8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form \( x^2 = p \) and \( x^3 = p \), where \( p \) is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

**8.G.6** Explain a proof of the Pythagorean Theorem and its converse.
## Geometry (G)

<table>
<thead>
<tr>
<th>Seventh</th>
<th>Eighth</th>
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<tbody>
<tr>
<td><strong>Length, Area, and Volume</strong></td>
<td><strong>Area and Volume of Geometrical Shapes and Solids</strong></td>
</tr>
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</tr>
<tr>
<td>8.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td>
<td>8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</td>
</tr>
<tr>
<td>8.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</td>
<td></td>
</tr>
<tr>
<td>8.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</td>
<td></td>
</tr>
</tbody>
</table>
**EIGHTH GRADE**

**LEXILE GRADE LEVEL BANDS: 1010L TO 1185L**

### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>8.G.1</th>
<th>Verify experimentally the properties of rotations, reflections, and translations.</th>
</tr>
</thead>
</table>

**DESCRIPTION**

Students use compasses, protractors and rulers, or technology to explore figures created from translations, reflections, and rotations. Characteristics of figures, such as lengths of line segments, angle measures, and parallel lines, are explored before the transformation (pre-image) and after the transformation (image). Students understand that these transformations produce images of exactly the same size and shape as the pre-image and are known as rigid transformations.

**ESSENTIAL QUESTION(S)**

How are geometric attributes of a shape affected when movement occurs?

**MATHEMATICAL PRACTICE(S)**

- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.
- 8.MP.8. Look for and express regularity in repeated reasoning.

### SUBSTANDARD DECONSTRUCTED

**8.G.1a. Lines are taken to lines, and line segments to line segments of the same length.**

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</tbody>
</table>

**Instructional Targets**

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<th>Assessment Types</th>
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<th>Do</th>
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<tbody>
<tr>
<td></td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

**Students should be able to:**

- Definitions and properties of lines and angles.
- Measure line segments.
- Define and identify rotations, reflections, and translations.
- Identify corresponding sides and corresponding angles.
- Identify center of rotation.
- Identify direction and degree of rotation.

Verify that congruence of angles are maintained through rotation, reflection, and translation.
### 8.G.1b. Angles are taken to angles of the same measure.

<table>
<thead>
<tr>
<th>Instructional Targets</th>
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<tbody>
<tr>
<td><strong>Assessment Types</strong></td>
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<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td><strong>Students should be able to:</strong></td>
<td>Definitions and properties of lines and angles. Measure line segments. Define and identify rotations, reflections, and translations. Identify corresponding sides and corresponding angles. Identify center of rotation. Identify direction and degree of rotation.</td>
<td></td>
<td>Verify that congruence of angles are maintained through rotation, reflection, and translation.</td>
</tr>
</tbody>
</table>

### 8.G.1c. Parallel lines are taken to parallel lines.

<table>
<thead>
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</tr>
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<tbody>
<tr>
<td><strong>Assessment Types</strong></td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td><strong>Students should be able to:</strong></td>
<td>Definitions and properties of segments, lines, and parallel lines. Define and identify rotations, reflections, and translations. Understand prime notation to describe an image after a translation, reflection, or rotation.</td>
<td>Verify that when parallel lines are rotated, reflected, or translated, each in the same way, they remain parallel lines.</td>
<td>Use physical models, transparencies, or geometry software to verify the properties of rotations, reflections, and translations.</td>
</tr>
</tbody>
</table>

### EXPLANATIONS AND EXAMPLES

Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same angle after they have been rotated, reflected, and/or translated.

Students are not expected to work formally with properties of dilations until high school.
8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

This standard is the students’ introduction to congruency. Congruent figures have the same shape and size. Translations, reflections, and rotations are examples of rigid transformations. A rigid transformation is one in which the pre-image and the image both have exactly the same size and shape since the measures of the corresponding angles and corresponding line segments remain equal (are congruent).

Students examine two figures to determine congruency by identifying the rigid transformation(s) that produced the figures. Students recognize the symbol for congruency (≅) and write statements of congruency.

Example 1:
Is Figure A congruent to Figure A'? Explain how you know.

Solution:
These figures are congruent since A' was produced by translating each vertex of Figure A 3 to the right and 1 down.

Example 2:
Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.

Solution:
Figure A' was produced by a 90° clockwise rotation around the origin.
**ESSENTIAL QUESTION(S)**

How are geometric attributes of a shape affected when movement occurs?

**MATHEMATICAL PRACTICE(S)**

- 8.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

| 1 | 2 | 3 | 4 |

**Instructional Targets**

<table>
<thead>
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<td>Tasks assessing modeling/applications.</td>
<td></td>
</tr>
</tbody>
</table>

**Students should be able to:**

- Define congruency.
- Identify symbols for congruency.
- Describe the sequence of rotations, reflections, and translations needed to exhibit the congruence between 2-D figures.
- Apply the concept of congruency to write congruent statements.
- Reason that a 2-D figure is congruent to another if the second can be obtained by a sequence of rotations, reflections, translations.

**EXPLANATIONS AND EXAMPLES**

**Examples:**

Is Figure A congruent to Figure A'? Explain how you know.

Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.

![Diagram of Figure A and Figure A']

![Diagram of Figure A and Figure A']
**STANDARD AND DECONSTRUCTION**

<table>
<thead>
<tr>
<th>8.G.3</th>
<th>Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</th>
</tr>
</thead>
</table>

**DESCRIPTION**

Students identify resulting coordinates from translations, reflections, and rotations (90°, 180° and 270° both clockwise and counterclockwise), recognizing the relationship between the coordinates and the transformation.

**Translations**

Translations move the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is congruent to its pre-image. Triangle ABC has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from x = 1 to x = 8) and 3 units up (from y = 5 to y = 8). Points B and C also move in the same direction (7 units to the right and 3 units up), resulting in the same changes to each coordinate.

**Reflections**

A reflection is the “flipping” of an object over a line, known as the “line of reflection”. In the 8th grade, the line of reflection will be the x-axis and the y-axis. Students recognize that when an object is reflected across the y-axis, the reflected x-coordinate is the opposite of the pre-image x-coordinate (see figure below).

Likewise, a reflection across the x-axis would change a pre-image coordinate (3, -8) to the image coordinate of (3, 8) -- note that the reflected y-coordinate is opposite of the pre-image y-coordinate.

**Rotations**

A rotation is a transformation performed by “spinning” the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise up to 360° (at 8th grade, rotations will be around the origin and a multiple of 90°). In a rotation, the rotated object is congruent to its pre-image.

Consider when triangle DEF is 180° clockwise about the origin. The coordinate of triangle DEF are D(2,5), E(2,1), and F(8,1). When rotated 180° about the origin, the new coordinates are D'(-2,-5), E'(-2,-1) and F'(-8,-1). In this case, each coordinate is the opposite of its pre-image (see figure below).
Dilations
A dilation is a non-rigid transformation that moves each point along a ray which starts from a fixed center, and multiplies distances from this center by a common scale factor. Dilations enlarge (scale factors greater than one) or reduce (scale factors less than one) the size of a figure by the scale factor. In 8th grade, dilations will be from the origin. The dilated figure is similar to its pre-image.

Students recognize the relationship between the coordinates of the pre-image, the image, and the scale factor for a dilation from the origin. Using the coordinates, students are able to identify the scale factor (image/pre-image).

Students identify the transformation based on given coordinates. For example, the pre-image coordinates of a triangle are A(4, 5), B(3, 7), and C(5, 7). The image coordinates are A(-4, 5), B(-3, 7), and C(-5, 7). What transformation occurred?

ESSENTIAL QUESTION(S)
How are geometric attributes of a shape affected when movement occurs?

MATHEMATICAL PRACTICE(S)
8.MP.3. Construct viable arguments and critique the reasoning of others.
8.MP.5. Use appropriate tools strategically.
8.MP.6. Attend to precision.
8.MP.7. Look for and make use of structure.

DOK Range Target for Instruction & Assessment

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<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Define dilations as a reduction or enlargement of a figure.</td>
<td>Identify scale factor of the dilation.</td>
<td>Describe the effects of dilations, translations, rotations, and reflections on 2-D figures using coordinates.</td>
</tr>
</tbody>
</table>
**EXPLANATIONS AND EXAMPLES**

**Dilation:** A dilation is a transformation that moves each point along a ray emanating from a fixed center, and multiplies distances from the center by a common scale factor. In dilated figures, the dilated figure is similar to its pre-image.

**Translation:** A translation is a transformation of an object that moves the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is congruent to its pre-image. Triangle ABC has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from x = 1 to x = 8) and 3 units up (from y = 5 to y = 8). Points B and C also move in the same direction (7 units to the right and 3 units up).

![Translation Diagram]

**Reflection:** A reflection is a transformation that flips an object across a line of reflection (in a coordinate grid the line of reflection may be the x or y axis). In a rotation, the rotated object is congruent to its pre-image.

![Reflection Diagram]

When an object is reflected across the y axis, the reflected x coordinate is the opposite of the pre-image x coordinate.

![Reflection Across Y Axis Diagram]

**Rotation:** A rotated figure is a figure that has been turned around a fixed point. This is called the center of rotation. A figure can be rotated up to 360°. Rotated figures are congruent to their pre-image figures.

Consider \( \triangle DEF \) when it is rotated 180° clockwise about the origin. The coordinates of \( \triangle DEF \) are D(2,5), E(2,1), and F(8,1). When rotated 180°, \( \triangle DEF \) has new coordinates D'(-2,-5), E'(-2,-1) and F'(-8,-1). Each coordinate is the opposite of its pre-image.

![Rotation Diagram]
<table>
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<th>STANDARD AND DECONSTRUCTION</th>
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<tbody>
<tr>
<td><strong>8.G.4</strong>  <strong>Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DESCRIPTION</th>
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</table>
| Similar figures and similarity are first introduced in the 8th grade. Students understand similar figures have congruent angles and sides that are proportional. Similar figures are produced from dilations. Students describe the sequence that would produce similar figures, including the scale factors. Students understand that a scale factor greater than one will produce an enlargement in the figure, while a scale factor less than one will produce a reduction in size.  
Example 1:  
Is Figure A similar to Figure A'? Explain how you know.  

![Figure A and A'](image)

Solution:  
Dilated with a scale factor of 1/2 then reflected across the x-axis, making Figures A and A' similar.  
Students need to be able to identify that triangles are similar or congruent based on given information.  
Example 2:  
Describe the sequence of transformations that results in the transformation of Figure A to Figure A'. |
**ESSENTIAL QUESTION(S)**
What geometric attributes are considered when comparing two shapes?

**MATHEMATICAL PRACTICE(S)**
8.MP.2. Reason abstractly and quantitatively.
8.MP.5. Use appropriate tools strategically.
8.MP.6. Attend to precision.
8.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

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<td>Tasks assessing modeling/applications.</td>
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<tr>
<td>Students should be able to:</td>
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<tr>
<td>Define similar figures as corresponding angles are congruent and corresponding sides are proportional.</td>
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<tr>
<td>Recognize symbol for similar.</td>
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<tr>
<td>Apply the concept of similarity to write similarity statements.</td>
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<tr>
<td>Reason that a 2-D figure is similar to another if the second can be obtained by a sequence of rotations, reflections, translations, or dilation.</td>
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</tr>
<tr>
<td>Describe the sequence of rotations, reflections, translations, or dilations that exhibits the similarity between 2-D figures using words and/or symbols.</td>
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</tr>
</tbody>
</table>

**EXPLANATIONS AND EXAMPLES**

**Examples:**
Is Figure A similar to Figure A'? Explain how you know.

Describe the sequence of transformations that results in the transformation of Figure A to Figure A':
### STANDARD AND DECONSTRUCTION

| 8.G.5 | Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. |

### DESCRIPTION

Students use exploration and deductive reasoning to determine relationships that exist between the following: a) angle sums and exterior angle sums of triangles, b) angles created when parallel lines are cut by a transversal, and c) the angle-angle criterion for similarity of triangle.

Students construct various triangles and find the measures of the interior and exterior angles. Students make conjectures about the relationship between the measure of an exterior angle and the other two angles of a triangle. (the measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles) and the sum of the exterior angles (360°). Using these relationships, students use deductive reasoning to find the measure of missing angles.

Students construct parallel lines and a transversal to examine the relationships between the created angles. Students recognize vertical angles, adjacent angles, and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles. Using these relationships, students use deductive reasoning to find the measure of missing angles.

Example 1: You are building a bench for a picnic table. The top of the bench will be parallel to the ground. If \( m\angle 1 = 148^\circ \), find \( m\angle 2 \) and \( m\angle 3 \). Explain your answer.

**Solution:**

Angle 1 and angle 2 are alternate interior angles, giving angle 2 a measure of 148°. Angle 2 and angle 3 are supplementary. Angle 3 will have a measure of 32° so the \( m\angle 2 + m\angle 3 = 180^\circ \).

Example 2:

Show that \( m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ \) if line \( l \) and \( m \) are parallel lines and \( t_1 \) and \( t_2 \) are transversals.

**Solution:**

\[ \angle 1 + \angle 2 + \angle 3 = 180^\circ \]

\[ \angle 5 \equiv \angle 1 \text{ Corresponding angles are congruent therefore } \angle 1 \text{ can be substituted for } \angle 5. \]

\[ \angle 4 \equiv \angle 2 \text{ Alternate interior angles are congruent therefore } \angle 4 \text{ can be substituted for } \angle 2. \]

Therefore \( \angle 3 + \angle 4 + \angle 5 = 180^\circ. \)

Students can informally conclude that the sum of the angles in a triangle is 180° (the angle-sum theorem) by applying their understanding of lines and alternate interior angles.
Example 3:
In the figure below Line X is parallel to Line YZ. Prove that the sum of the angles of a triangle is 180°.

Solution:
Angle $a$ is 35° because it alternates with the angle inside the triangle that measures 35°. Angle $c$ is 80° because it alternates with the angle inside the triangle that measures 80°. Because lines have a measure of 180°, and angles $a + b + c$ form a straight line, then angle $b$ must be $65° \rightarrow 180 - (35 + 80) = 65$. Therefore, the sum of the angles of the triangle is $35° + 65° + 80°$.

Example 4:
What is the measure of angle 5 if the measure of angle 2 is 45° and the measure of angle 3 is 60°?

Solution:
Angles 2 and 4 are alternate interior angles, therefore the measure of angle 4 is also 45°. The measure of angles 3, 4, and 5 must add up to 180°. If angles 3 and 4 add up to 105° the angle 5 must be equal to 75°.

Students construct various triangles having line segments of different lengths but with two corresponding congruent angles. Comparing ratios of sides will produce a constant scale factor, meaning the triangles are similar. Students solve problems with similar triangles.

**ESSENTIAL QUESTION(S)**
What geometric attributes are considered when examining angles, lines, and triangles?

**MATHEMATICAL PRACTICE(S)**
8.MP.3. Construct viable arguments and critique the reasoning of others.
8.MP.5. Use appropriate tools strategically.
8.MP.6. Attend to precision.
8.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
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<tr>
<td><strong>Assessment Types</strong></td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td><strong>Students should be able to:</strong></td>
<td>Define similar triangles. Define and identify transversals. Identify angles created when parallel line is cut by transversal (alternate interior, alternate exterior, corresponding, vertical, adjacent, etc.)</td>
<td>Justify that the sum of interior angles equals 180. Justify that the exterior angle of a triangle is equal to the sum of the two remote interior angles. Use Angle-Angle Criterion to prove similarity among triangles; give an argument in terms of transversals, why this is so.</td>
<td></td>
</tr>
</tbody>
</table>
Examples:

Students can informally prove relationships with transversals.

- Show that $m \angle 3 + m \angle 4 + m \angle 5 = 180^\circ$ if $l$ and $m$ are parallel lines and $t_1$ & $t_2$ are transversals.

  $\angle 1 + \angle 2 + \angle 3 = 180^\circ$. Angle 1 and Angle 5 are congruent because they are corresponding angles ($\angle 5 \cong \angle 1$). $\angle 1$ can be substituted for $\angle 5$.

- $\angle 4 \cong \angle 2$ because alternate interior angles are congruent. $\angle 4$ can be substituted for $\angle 2$.

Therefore $m \angle 3 + m \angle 4 + m \angle 5 = 180^\circ$.

Students can informally conclude that the sum of a triangle is $180^\circ$ (the angle-sum theorem) by applying their understanding of lines and alternate interior angles. In the figure below, line $x$ is parallel to line $yz$:

Angle $a$ is $35^\circ$ because it alternates with the angle inside the triangle that measures $35^\circ$. Angle $c$ is $80^\circ$ because it alternates with the angle inside the triangle that measures $80^\circ$. Because lines have a measure of $180^\circ$, and angles $a + b + c$ form a straight line, then angle $b$ must be $65^\circ$ ($180 - 35 + 80 = 65$). Therefore, the sum of the angles of the triangle are $35^\circ + 65^\circ + 80^\circ$. 
2. Understand and apply the Pythagorean Theorem.

**BIG IDEA**

- 2-D and 3-D objects have unique attributes that can be used when developing mathematical arguments about geometric relationships.

**ACADEMIC VOCABULARY**

- right triangle, hypotenuse, legs, Pythagorean Theorem, Pythagorean triple

<table>
<thead>
<tr>
<th>STANDARD AND DECONSTRUCTION</th>
<th>8.G.6</th>
<th>Explain a proof of the Pythagorean Theorem and its converse.</th>
</tr>
</thead>
</table>
| **DESCRIPTION** | Using models, students explain the Pythagorean Theorem, understanding that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students also understand that given three side lengths with this relationship forms a right triangle. **Example 1:** The distance from Jonestown to Maryville is 180 miles, the distance from Maryville to Elm City is 300 miles, and the distance from Elm City to Jonestown is 240 miles. Do the three towns form a right triangle? Why or why not? **Solution:** If these three towns form a right triangle, then 300 would be the hypotenuse since it is the greatest distance.  

\[ 180^2 + 240^2 = 300^2 \]  

\[ 32400 + 57600 = 90000 \]  

\[ 90000 = 90000\sqrt{} \]  

These three towns form a right triangle.
### ESSENTIAL QUESTION(S)

What is a strategy to determine with accuracy if a given triangle is a right triangle?

### MATHEMATICAL PRACTICE(S)

- 8.MP.3. Construct viable arguments and critique the reasoning of others.
- 8.MP.7. Look for and make use of structure.

### DOK Range Target for Instruction & Assessment

<table>
<thead>
<tr>
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<th>1</th>
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</table>

### Instructional Targets

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<th>Assessment Types</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

**Students should be able to:**

- Define key vocabulary: square root, Pythagorean Theorem, right triangle, legs a & b, hypotenuse, sides, right angle, converse, base, height, proof.
- Identify the legs and hypotenuse of a right triangle.
- Explain a proof of the Pythagorean Theorem.
- Explain a proof of the converse of the Pythagorean Theorem.
### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>8.G.7</th>
<th>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</th>
</tr>
</thead>
</table>

**DESCRIPTION**

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

**Example 1:** The Irrational Club wants to build a tree house. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the tree house be off the ground?

**Solution:**

\[
a^2 + 5^2 = 9^2
\]

\[
a^2 + 25 = 81
\]

\[
a^2 = 56
\]

\[
\sqrt{a^2} = \sqrt{56}
\]

\[
a = 56 \text{ or } \approx 7.5
\]

**Example 2:**

Find the length of \(d\) in the figure to the right if \(a = 8\) in., \(b = 3\) in. and \(c = 4\) in.

**Solution:**

First find the distance of the hypotenuse of the triangle formed with legs \(a\) and \(b\).

\[
8^2 + 3^2 = c^2
\]

\[
64 + 9 = c^2
\]

\[
73 = c^2
\]

\[
\sqrt{73} = \sqrt{c^2}
\]

\[
73 \text{ in.} = c
\]

The \(\sqrt{73}\) is the length of the base of a triangle with \(c\) as the other leg and \(d\) is the hypotenuse. To find the length of \(d\):

\[
73^2 + 4^2 = d^2
\]

\[
73 + 16 = d^2
\]

\[
89 = d^2
\]

\[
\sqrt{89} = \sqrt{d^2}
\]

\[
89 \text{ in.} = d
\]

Based on this work, students could then find the volume or surface area.
### ESSENTIAL QUESTION(S)
What is a strategy to determine with accuracy the length of a side in a right triangle?

### MATHEMATICAL PRACTICE(S)
8.MP.1. Make sense of problems and persevere in solving them.
8.MP.2. Reason abstractly and quantitatively.
8.MP.5. Use appropriate tools strategically.
8.MP.6. Attend to precision.
8.MP.7. Look for and make use of structure.

### DOK Range Target for Instruction & Assessment
2

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
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<th>Do</th>
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<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Recall the Pythagorean Theorem and its converse.</td>
<td>Apply Pythagorean Theorem in solving real-world problems dealing with two- and three-dimensional shapes. Solve basic mathematical Pythagorean Theorem problems and its converse to find missing lengths of sides of triangles in two- and three-dimensions.</td>
<td></td>
</tr>
</tbody>
</table>

### EXPLANATIONS AND EXAMPLES
Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean triplets.
**EIGHTH GRADE**  
LEXILE GRADE LEVEL BANDS: 1010L TO 1185L

## STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>8.G.8</th>
<th>Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</th>
</tr>
</thead>
</table>

**DESCRIPTION**

One application of the Pythagorean Theorem is finding the distance between two points on the coordinate plane. Students build on work from 6th grade (finding vertical and horizontal distances on the coordinate plane) to determine the lengths of the legs of the right triangle drawn connecting the points. Students understand that the line segment between the two points is the length of the hypotenuse.

NOTE: The use of the distance formula is not an expectation.

**Example 1:**

Find the length of AB.

1. Form a right triangle so that the given line segment is the hypotenuse.
2. Use Pythagorean Theorem to find the distance (length) between the two points.

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<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>36</td>
<td>49</td>
</tr>
<tr>
<td>85</td>
<td></td>
</tr>
</tbody>
</table>
```

**Example 2:**

Find the distance between (-2, 4) and (-5, -6).

**Solution:**

The distance between -2 and -5 is the horizontal length; the distance between 4 and -6 is the vertical distance.

Horizontal length: 3
Vertical length: 10

\[
10^2 + 3^2 = c^2 \\
100 + 9 = c^2 \\
109 = c^2 \\
\sqrt{109} = \sqrt{c^2} \\
\sqrt{109} = c
\]

Students find area and perimeter of two-dimensional figures on the coordinate plane, finding the distance between each segment of the figure. (Limit one diagonal line, such as a right trapezoid or parallelogram.)
**ESSENTIAL QUESTION(S)**

What is a strategy to calculate with accuracy the distance between any two points?

**MATHEMATICAL PRACTICE(S)**

8.MP.1. Make sense of problems and persevere in solving them.
8.MP.2. Reason abstractly and quantitatively.
8.MP.5. Use appropriate tools strategically.
8.MP.6. Attend to precision.
8.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

<table>
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**Instructional Targets**

<table>
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<td></td>
</tr>
</tbody>
</table>

**Students should be able to:**

- Recall the Pythagorean Theorem and its converse.
- Determine how to create a right triangle from two points on a coordinate graph.
- Use the Pythagorean Theorem to solve for the distance between the two points.

**EXPLANATIONS AND EXAMPLES**

**Example:**

Students will create a right triangle from the two points given (as shown in the diagram below) and then use the Pythagorean Theorem to find the distance between the two given points.

```
(-2, 4)

(-3, -6)
```

[Diagram showing a right triangle with points (-2, 4) and (-3, -6)]
### EIGHTH GRADE

**LEXILE GRADE LEVEL BANDS: 1010L TO 1185L**

<table>
<thead>
<tr>
<th>CLUSTER</th>
<th>3. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIG IDEA</td>
<td>• Objects can be measured using a simple multiple of the unit of measure defined within a standard measurement system.</td>
</tr>
<tr>
<td>ACADEMIC VOCABULARY</td>
<td>cones, cylinders, spheres, radius, volume, height, Pi</td>
</tr>
</tbody>
</table>

#### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>8.G.9</th>
<th>Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESCRIPTION</td>
<td>Students build on understandings of circles and volume from 7th grade to find the volume of cylinders, finding the area of the base ( \pi r^2 ) and multiplying by the number of layers (the height).</td>
</tr>
</tbody>
</table>

\[ V = \pi r^2 h \]

Find the area of the base and multiply by the number of layers.

Students understand that the volume of a cylinder is 3 times the volume of a cone having the same base area and height or that the volume of a cone is 1/3 the volume of a cylinder having the same base area and height.

\[ V = \frac{1}{3} \pi r^2 h \quad \text{or} \quad V = \pi r^2 h \]

A sphere can be enclosed with a cylinder, which has the same radius and height of the sphere (Note: the height of the cylinder is twice the radius of the sphere). If the sphere is flattened, it will fill 2/3 of the cylinder. Based on this model, students understand that the volume of a sphere is 2/3 the volume of a cylinder with the same radius and height. The height of the cylinder is the same as the diameter of the sphere or 2r. Using this information, the formula for the volume of the sphere can be derived in the following way:

\[ V = \pi r^2 \]

\[ V = \frac{2}{3} \pi r^2 h \]

multiply by \( \frac{2}{3} \) since the volume of a sphere is \( \frac{2}{3} \) the cylinder’s volume

\[ V = \frac{2}{3} \pi r^2 2r \]

substitute 2r for height since 2r is the height of the sphere

\[ V = \frac{4}{3} \pi r^3 \]

simplify
Students find the volume of cylinders, cones, and spheres to solve real world and mathematical problems. Answers could also be given in terms of Pi.

**Example 2:**
How much yogurt is needed to fill the cone to the right? Express your answers in terms of Pi.

**Solution:**

\[
V = \frac{1}{3} \pi r^2 h \\
V = \frac{1}{3} \pi (3^2)(5) \\
V = \frac{1}{3} \pi (45) \\
V = 15 \pi \text{ cm}^3
\]

**Example 3:**
Approximately how much air would be needed to fill a soccer ball with a radius of 14 cm?

**Solution:**

\[
V = \frac{4}{3} \pi r^3 \\
V = \frac{4}{3} (3.14)(14^3) \\
V = 11.5 \text{ cm}^3
\]

“Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of why the formula works and how the formula relates to the measure (volume) and the figure. This understanding should be for all students.

**Note:** At this level composite shapes will not be used and only volume will be calculated.
**ESSENTIAL QUESTION(S)**
What are efficient strategies for calculating the volumes of cones, cylinders, and spheres?

**MATHEMATICAL PRACTICE(S)**
- 8.MP.1. Make sense of problems and persevere in solving them.
- 8.MP.3. Construct viable arguments and critique the reasoning of others.
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.
- 8.MP.8. Look for and express regularity in repeated reasoning.

**DOK Range Target for Instruction & Assessment**

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<tr>
<td><strong>Students should be able to:</strong></td>
<td>Identify and define vocabulary: cone, cylinder, sphere, radius, diameter, circumference, area, volume, π, base, height. Know formulas for volume of cones, cylinders, and spheres.</td>
<td>Compare the volumes of cones, cylinders, and spheres. Determine and apply appropriate volume formulas in order to solve mathematical and real-world problems for the given shape. Given the volume of a cone, cylinder, or sphere, find the radii, height, or approximate using π.</td>
<td></td>
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</table>

**EXPLANATIONS AND EXAMPLES**

**Example:**
James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter’s volume.
EIGHTH GRADE

MATHEMATICS

DOMAIN:

STATISTICS AND PROBABILITY (SP)
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<tbody>
<tr>
<td><strong>DOMAIN</strong></td>
<td>Statistics and Probability (SP)</td>
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<tr>
<td><strong>CLUSTERS</strong></td>
<td>1. Investigate patterns of associations in bivariate data.</td>
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### VARIATION, DISTRIBUTION, AND MODELING

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<th><strong>SEVENTH</strong></th>
<th><strong>EIGHTH</strong></th>
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<tr>
<td><strong>Bivariate Data, Scatter Plots, and Basic Linear Regression</strong></td>
<td><strong>Bivariate Data, Scatter Plots, and Basic Linear Regression</strong></td>
</tr>
<tr>
<td>8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</td>
<td></td>
</tr>
<tr>
<td>8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.</td>
<td></td>
</tr>
<tr>
<td>8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</td>
<td></td>
</tr>
<tr>
<td>8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.</td>
<td></td>
</tr>
</tbody>
</table>
1. Investigate patterns of associations in bivariate data.

Big Idea:
- Tables, charts, and graphs are methods for visually representing statistical data.
- General conclusions can be made about a set of data from an appropriate visual representation.

Academic Vocabulary:
- bivariate data, scatter plot, linear model, clustering, linear association, nonlinear association, outliers, positive association, negative association, categorical data, two-way table, relative frequency

Standard and Deconstruction:

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Description:
Bivariate data refers to two-variable data, one to be graphed on the x-axis and the other on the y-axis. Students represent numerical data on a scatter plot, to examine relationships between variables. They analyze scatter plots to determine if the relationship is linear (positive, negative association, or no association) or nonlinear. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets. (http://nces.ed.gov/nceskids/createagraph/default.aspx)

Data can be expressed in years. In these situations it is helpful for the years to be “converted” to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

Example 1:
Data for 10 students’ Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
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<tbody>
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<td>50</td>
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<td>27</td>
<td>74</td>
<td>63</td>
<td>40</td>
<td>96</td>
</tr>
</tbody>
</table>

Solution:
This data has a positive association.

Example 2:
Data for 10 students’ Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td>64</td>
<td>50</td>
<td>85</td>
<td>34</td>
<td>56</td>
<td>24</td>
<td>72</td>
<td>63</td>
<td>42</td>
<td>93</td>
</tr>
<tr>
<td>Distance from School (miles)</td>
<td>0.5</td>
<td>1.8</td>
<td>1</td>
<td>2.3</td>
<td>3.4</td>
<td>0.2</td>
<td>2.5</td>
<td>1.6</td>
<td>0.8</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Solution:
There is no association between the math score and the distance a student lives from school.
Example 3:
Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order in the table below. Describe the association between the number of staff and the average time for filling an order.

<table>
<thead>
<tr>
<th>Number of Staff</th>
<th>Average time to fill order (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>72</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
</tr>
<tr>
<td>7</td>
<td>42</td>
</tr>
<tr>
<td>8</td>
<td>93</td>
</tr>
</tbody>
</table>

Solution:
There is a positive association.

Example 4:
The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Expectancy (in years)</td>
<td>70.8</td>
<td>72.6</td>
<td>73.7</td>
<td>74.7</td>
<td>75.4</td>
<td>75.8</td>
<td>76.8</td>
<td>77.4</td>
</tr>
</tbody>
</table>

Solution:
There is a positive association.

Students recognize that points may be away from the other points (outliers) and have an effect on the linear model.

NOTE: Use of the formula to identify outliers is not expected at this level.

Students recognize that not all data will have a linear association. Some associations will be nonlinear as in the example below:
**Essential Question(s)**

How can a scatter plot be useful when analyzing data?

**Mathematical Practice(s)**

- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
</table>

**Instructional Targets**

<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
<td></td>
</tr>
</tbody>
</table>

**Students should be able to:**

- Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- Construct scatter plots for bivariate measurement data.
- Interpret scatter plots for bivariate (two different variables such as distance and time) measurement data to investigate patterns of association between two quantities.

**Explanations and Examples**

Students build on their previous knowledge of scatter plots examine relationships between variables. They analyze scatter plots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets. ([http://nces.ed.gov/nceskids/createagraph/default.aspx](http://nces.ed.gov/nceskids/createagraph/default.aspx))

**Examples:**

Data for 10 students' Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td>64</td>
<td>50</td>
<td>85</td>
<td>34</td>
<td>56</td>
<td>24</td>
<td>72</td>
<td>63</td>
<td>42</td>
<td>93</td>
</tr>
<tr>
<td>Science</td>
<td>68</td>
<td>70</td>
<td>83</td>
<td>33</td>
<td>60</td>
<td>27</td>
<td>74</td>
<td>63</td>
<td>40</td>
<td>96</td>
</tr>
</tbody>
</table>

Data for 10 students' Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math score</td>
<td>64</td>
<td>50</td>
<td>85</td>
<td>34</td>
<td>56</td>
<td>24</td>
<td>72</td>
<td>63</td>
<td>42</td>
<td>93</td>
</tr>
<tr>
<td>Dist from school (miles)</td>
<td>0.5</td>
<td>1.8</td>
<td>1</td>
<td>2.3</td>
<td>3.4</td>
<td>0.2</td>
<td>2.5</td>
<td>1.6</td>
<td>0.8</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order in the table below. Describe the association between the number of staff and the average time for filling an order.

<table>
<thead>
<tr>
<th>Number of staff</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average time to fill order (seconds)</td>
<td>180</td>
<td>138</td>
<td>120</td>
<td>108</td>
<td>96</td>
<td>84</td>
</tr>
</tbody>
</table>

The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Expectancy (in years)</td>
<td>70.8</td>
<td>72.6</td>
<td>73.7</td>
<td>74.7</td>
<td>75.4</td>
<td>75.8</td>
<td>76.8</td>
<td>77.4</td>
</tr>
</tbody>
</table>
### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>STANDARD</th>
<th>KNOWLEDGE AND DECONSTRUCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.SP.2</td>
<td>Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</td>
</tr>
</tbody>
</table>

### DESCRIPTION
Students understand that a straight line can represent a scatter plot with linear association. The most appropriate linear model is the line that comes closest to most data points. The use of linear regression is not expected. If there is a linear relationship, students draw a linear model. Given a linear model, students write an equation.

### ESSENTIAL QUESTION(S)
How can the understanding of linear functions and scatter plots be useful when analyzing data?

### MATHEMATICAL PRACTICE(S)
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.

### DOK Range Target for Instruction & Assessment
- **1**
- **2**
- **3**
- **4**

### Instructional Targets

<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Tasks assessing concepts, skills, and procedures.</th>
<th>Tasks assessing expressing mathematical reasoning.</th>
<th>Tasks assessing modeling/applications.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should be able to:</td>
<td>Know straight lines are used to model relationships between two quantitative variables.</td>
<td>Informally fit a straight line in scatter plot data.</td>
<td>Informally assess the model fit by judging the closeness of the data points to the line.</td>
</tr>
</tbody>
</table>

### EXPLANATIONS AND EXAMPLES

**Examples:**
The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

<table>
<thead>
<tr>
<th>Miles Traveled</th>
<th>0</th>
<th>75</th>
<th>120</th>
<th>160</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallons Used</td>
<td>0</td>
<td>2.3</td>
<td>4.5</td>
<td>5.7</td>
<td>9.7</td>
<td>10.7</td>
</tr>
</tbody>
</table>
**STANDARD AND DECONSTRUCTION**

| 8.SP.3 | Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. |

**DESCRIPTION**

Linear models can be represented with a linear equation. Students interpret the slope and y-intercept of the line in the context of the problem.

**Example 1:**

1. Given data from students’ math scores and absences, make a scatterplot.

   ![Scatterplot](image1.png)

<table>
<thead>
<tr>
<th>Absences</th>
<th>Math Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>95</td>
</tr>
<tr>
<td>2</td>
<td>85</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

2. Draw a linear model paying attention to the closeness of the data points on either side of the line.

![Linear Model](image2.png)

3. From the linear model, determine an approximate linear equation that models the given data (about $y = -25/3x + 95$)

4. Students should recognize that 95 represents the y-intercept and $-25/3$ represents the slope of the line. In the context of the problem, the y-intercept represents the math score a student with 0 absences could expect. The slope indicates that the math scores decreased 25 points for every 3 absences.

5. Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4 absences should expect to receive a math score of about 62. They can then compare this value to their line.
### ESSENTIAL QUESTION(S)
How can the understanding of linear functions and scatter plots be useful when analyzing data?

### MATHEMATICAL PRACTICE(S)
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.

### DOK Range Target for Instruction & Assessment
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
</table>

### Instructional Targets

<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

**Students should be able to:**
- Find the slope and intercept of a linear equation.
- Interpret the meaning of the slope and intercept of a linear equation in terms of the situation.
- Solve problems using the equation of a linear model.
Examples:
Given data from students’ math scores and absences, make a scatterplot.

<table>
<thead>
<tr>
<th>Absences</th>
<th>Math Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>65</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>95</td>
</tr>
<tr>
<td>1</td>
<td>85</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>45</td>
</tr>
<tr>
<td>8</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>71</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>95</td>
</tr>
<tr>
<td>6</td>
<td>85</td>
</tr>
<tr>
<td>6</td>
<td>42</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>0</td>
<td>92</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>80</td>
</tr>
</tbody>
</table>

Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.

From the line of best fit, determine an approximate linear equation that models the given data (about $y = \frac{-25}{3}x + 95$).

Students should recognize that 95 represents the *y* intercept and $\frac{-25}{3}$ represents the slope of the line.

Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4 absences should expect to receive a math score of about 62. They can then compare this value to their line.
### Standard and Deconstruction

**8.SP.4**

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

### Description

Students understand that a two-way table provides a way to organize data between two categorical variables. Data for both categories needs to be collected from each subject. Students calculate the relative frequencies to describe associations.

**Example 1:**

Twenty-five students were surveyed and asked if they received an allowance and if they did chores. The table below summarizes their responses.

<table>
<thead>
<tr>
<th></th>
<th>Receive Allowance</th>
<th>No Allowance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do Chores</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Do Not Do Chores</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Of the students who do chores, what percent do not receive an allowance?

**Solution:**

5 of the 20 students who do chores do not receive an allowance, which is 25%.
### Essential Question(s)

How can the understanding of tables and patterns be useful when analyzing data?

### Mathematical Practice(s)

- 8.MP.3. Construct viable arguments and critique the reasoning of others.
- 8.MP.5. Use appropriate tools strategically.
- 8.MP.7. Look for and make use of structure.

### DOK Range Target for Instruction & Assessment

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
</table>

### Instructional Targets

<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should be able to:</td>
<td>Recognize patterns shown in comparison of two sets of data. Know how to construct a two-way table.</td>
<td>Interpret the data in the two-way table to recognize patterns. Use relative frequencies of the data to describe relationships (positive, negative, or no correlation).</td>
<td>Tasks assessing conceptual knowledge. Tasks assessing expressing mathematical reasoning. Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

### Explanations and Examples

#### Example:

The table illustrates the results when 100 students were asked the survey questions: Do you have a curfew? and Do you have assigned chores? Is there evidence that those who have a curfew also tend to have chores?

<table>
<thead>
<tr>
<th>Curfew</th>
<th>Chores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>40</td>
</tr>
<tr>
<td>No</td>
<td>10</td>
</tr>
<tr>
<td>Yes</td>
<td>10</td>
</tr>
<tr>
<td>No</td>
<td>40</td>
</tr>
</tbody>
</table>

#### Solution:

Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores.