COMMON CORE State Standards

DECONSTRUCTED for CLASSROOM IMPACT

5TH GRADE
MATHEMATICS

The
COMMON CORE
Institute

MATHEMATICS

Introduction

The Common Core Institute is pleased to offer this grade-level tool for educators who are teaching with the Common Core State Standards.

The Common Core Standards Deconstructed for Classroom Impact is designed for educators by educators as a two-pronged resource and tool 1) to help educators increase their depth of understanding of the Common Core Standards and 2) to enable teachers to plan College & Career Ready curriculum and classroom instruction that promotes inquiry and higher levels of cognitive demand.

What we have done is not all new. This work is a purposeful and thoughtful compilation of preexisting materials in the public domain, state department of education websites, and original work by the Center for College & Career Readiness. Among the works that have been compiled and/or referenced are the following: Common Core State Standards for Mathematics and the Appendix from the Common Core State Standards Initiative; Learning Progressions from The University of Arizona's Institute for Mathematics and Education, chaired by Dr. William McCallum; the Arizona Academic Content Standards; the North Carolina Instructional Support Tools; and numerous math practitioners currently in the classroom.

We hope you will find the concentrated and consolidated resource of value in your own planning. We also hope you will use this resource to facilitate discussion with your colleagues and, perhaps, as a lever to help assess targeted professional learning opportunities.

Understanding the Organization

The Overview acts as a quick-reference table of contents as it shows you each of the domains and related clusters covered in this specific grade-level booklet. This can help serve as a reminder of what clusters are part of which domains and can reinforce the specific domains for each grade level.

Key Changes identifies what has been moved to and what has been moved from this particular grade level, as appropriate. This section also includes **Critical Areas of Focus**, which is designed to help you begin to approach how to examine your curriculum, resources, and instructional practices. A review of the **Critical Areas of Focus** might enable you to target specific areas of professional learning to refresh, as needed.

Math Fluency Standards	
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 Add/subtract within 100 (pencil & paper)
3	Multiply/divide within 100 Add/subtract within 1000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations
7	Solve $px + q = r$, $p(x + q) = r$
8	Solve simple 2 x 2 systems by inspection

For each domain is the domain itself and the associated clusters. Within each domain are sections for each of the associated clusters. The cluster-specific content can take you to a deeper level of understanding. Perhaps most importantly, we include here the **Learning Progressions.** The **Learning Progressions** provide context for the current domain and its related standards. For any grade except Kindergarten, you will see the domain-specific standards for the current

grade in the center column. To the left are the domain-specific standards for the preceding grade and to the right are the domain-specific standards for the following grade. Combined with the **Critical Areas of Focus,** these Learning Progressions can assist you in focusing your planning.

For each cluster, we have included four key sections: Description, Big Idea, Academic Vocabulary, and Deconstructed Standard.

The cluster **Description** offers clarifying information, but also points to the **Big Idea** that can help you focus on that which is most important for this cluster within this domain. The **Academic Vocabulary** is derived from the cluster description and serves to remind you of potential challenges or barriers for your students.

Each standard specific to that cluster has been deconstructed. There **Deconstructed Standard** for each standard specific to that cluster and each **Deconstructed Standard** has its own subsections, which can provide you with additional guidance and insight as you plan. Note the deconstruction drills down to the sub-standards when appropriate. These subsections are:

- Standard Statement
- Standard Description
- Essential Question(s)
- Mathematical Practice(s)
- DOK Range Target for Learning and Assessment
- Learning Expectations
- Explanations and Examples

As noted, first are the **Standard Statement** and **Standard Description**, which are followed by the **Essential Question(s)** and the associated **Mathematical Practices**. The **Essential Question(s)** amplify the **Big Idea**, with the intent of taking you to a deeper level of understanding; they may also provide additional context for the **Academic Vocabulary**.

The **DOK Range Target for Learning and Assessment** remind you of the targeted level of cognitive demand. The **Learning Expectations** correlate to the DOK and express the student learning targets for student proficiency for KNOW, THINK, and DO, as appropriate. In some instances, there may be no learning targets for student proficiency for one or more of KNOW, THINK or DO. The learning targets are expressions of the deconstruction of the Standard as well as the alignment of the DOK with appropriate consideration of the Essential Questions.

The last subsection of the **Deconstructed Standard** includes **Explanations and Examples**. This subsection might be quite lengthy as it can include additional context for the standard itself as well as examples of what student work and student learning could look like. **Explanations and Examples** may offers ideas for instructional practice and lesson plans.

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Standards for Mathematical Practice in Fifth Grade

Each of the explanations below articulates some of the knowledge and skills expected of students to demonstrate grade-level mathematical proficiency.

PRACTICE	EXPLANATION
Make sense and persevere in problem solving.	Students solve problems by applying their understanding of the appropriate mathematical concepts. Students determine the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
Reason abstractly and quantitatively.	Students recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
Construct viable arguments and critique the reasoning of others.	Students may construct arguments using concrete referents, such as objects and graphical representations. They explain calculations based upon models and properties of operations and rules that generate patterns. They refine their mathematical communication skills as they participate in discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), and graphical representations. Students connect the different representations and explain the connections. They also evaluate the utility of models to determine which models are most useful and efficient to solve a problem.
Use appropriate tools strategically.	Students consider the available tools (including estimation) when solving a problem and decide when certain tools might be helpful.
Attend to precision.	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology; they are careful about specifying units of measure and state the meaning of the symbols they choose.
Look for and make use of structure.	Students look closely to discover a pattern or structure. They can examine numerical patterns and relate them to a rule or a graphical representation.
Look for and express regularity in repeated reasoning.	Students use repeated reasoning to understand operations and algorithms and make generalizations about patterns.

Operations and Algebraic Thinking (OA)
- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten (NBT)
- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions (NF)
- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data (MD)
- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry (G)
- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices (MP)
MP 1. Make sense of problems and persevere in solving them.

MP 2. Reason abstractly and quantitatively.

MP 3. Construct viable arguments and critique the reasoning of others.

MP 4. Model with mathematics.

MP 5. Use appropriate tools strategically.

MP 6. Attend to precision.

MP 7. Look for and make use of structure.

MP 8. Look for and express regularity in repeated reasoning.

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KEY CHANGES

NEW TO 5TH GRADE	- Patterns in zeros when multiplying (5.NBT.2) - Extend understandings of multiplication and division of fractions (5.NF.3, 5.NF.45.NF.5, 5.NF.7) - Conversions of measurements within the same system (5.MD.1) - Volume (5.MD.3, 5.MD.4, 5.MD.5) - Coordinate System (5.G.1, 5.02) - Two-dimensional figures – hierarchy (5.G.3, 5.G.4) - Line plot to display measurements (5.MD.2)
MOVED FROM FIFTH GRADE	- Estimate measure of objects from on system to another system (2.01) - Measure of angles (2.01) - Describe triangles and quadrilaterals (3.01) - Angles, diagonals, parallelism and perpendicularity (3.02, 3.04) - Symmetry - line and rotational (3.03) - Data -stem-and-leaf plots, different representations, median, range and mode (4.01, 4.02, 4.03) - Constant and carrying rates of change (5.03)

KEY CHANGES

CRITICAL AREAS OF FOCUS

1. **Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).**

   - Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them.

   - Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.) .

2. **Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.**

   - Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
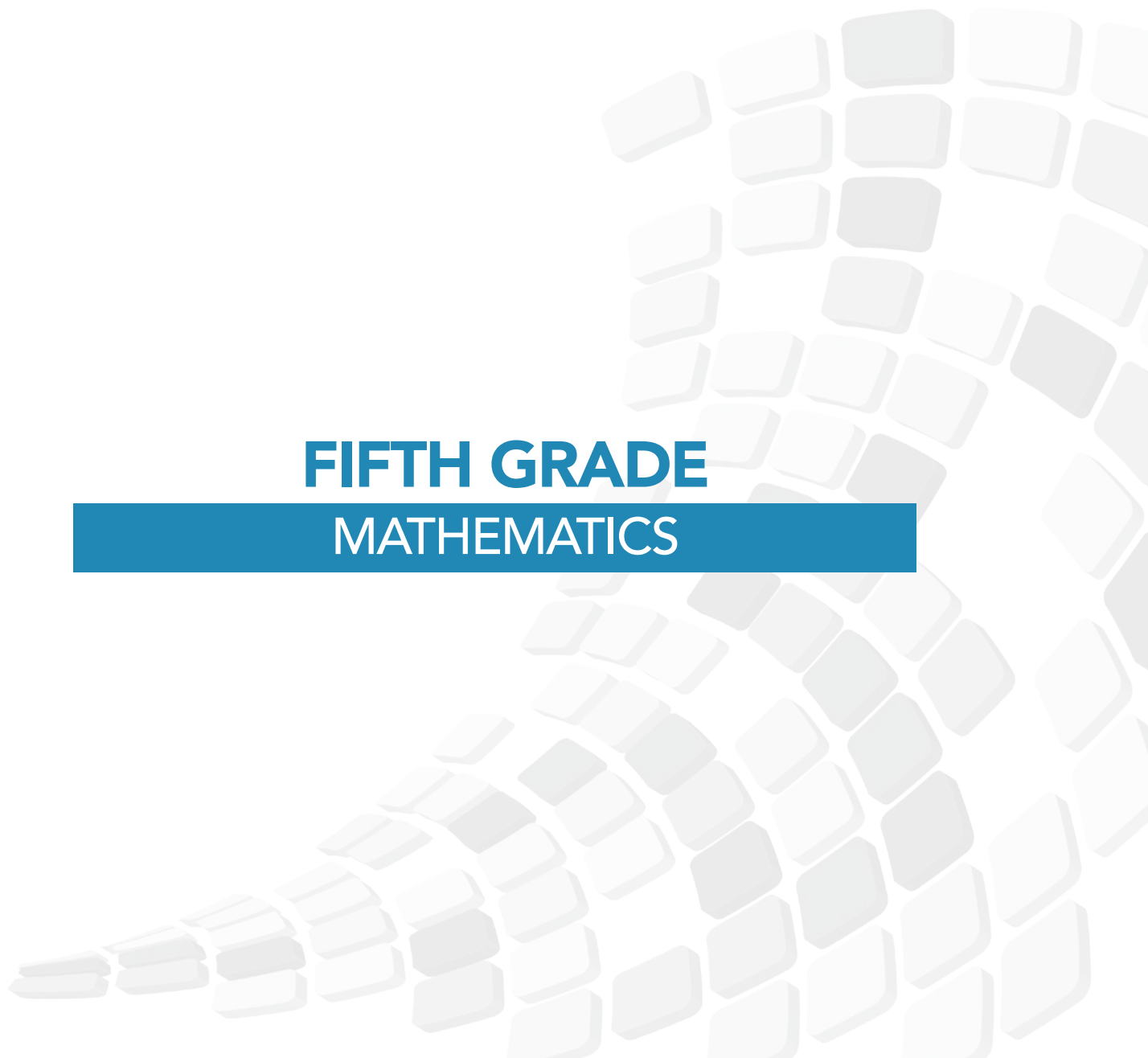
2. **Developing understanding of volume. Students recognize volume as an attribute of three-dimensional space.**

   - They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

DOMAIN:

OPERATIONS AND ALGEBRAIC THINKING (OA)

FIFTH GRADE

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DOMAIN	Operations and Algebraic Thinking (OA)
CLUSTERS	1. Write and interpret numerical expressions. 2. Analyze patterns and relationships.

OPERATIONS AND ALGEBRAIC THINKING (OA)

FOURTH	FIFTH	SIXTH
EARLY EQUATIONS AND EXPRESSIONS		
Exploring arithmetic and geometric patterns/sequences	*Exploring arithmetic and geometric patterns/sequences*	*Exploring arithmetic and geometric patterns/sequences*
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	
Working with Expressions	*Working with Expressions*	*Working with Expressions*
	5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	
	5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	
		6.EE.2.a Write expressions that record operations with numbers and with letters standing for numbers.
		6.EE.2.b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
		6.EE.2.c Evaluate expressions at specific values of their variables. Include expressions embedded in formulas or equations from real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
		6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
		6.EE.3 Apply the properties of operations to generate equivalent expressions.
		6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Source: turnonccmath.net, NC State University College of Education

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CLUSTER:	1. Write and interpret numerical expressions.
	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
BIG IDEA:	Numerical expressions bring order and precision to calculations.
ACADEMIC VOCABULARY:	Parentheses, brackets, braces, numerical expressions.

STANDARD AND DECONSTRUCTION

5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
DESCRIPTION	The order of operations is introduced in third grade and is continued in fourth. This standard calls for students to evaluate expressions with parentheses ( ), brackets [ ] and braces { }. In upper levels of mathematics, evaluate means to substitute for a variable and simplify the expression. However at this level students are to only simplify the expressions because there are no variables.

Example:

Evaluate the expression 2{ 5[12 + 5(500 - 100) + 399]}

Students should have experiences working with the order of first evaluating terms in parentheses, then brackets, and then braces.

The first step would be to subtract 500 – 100 = 400. Then multiply 400 by 5 = 2,000.

Inside the bracket, there is now [12 + 2,000 + 399]. That equals 2,411.

Next multiply by the 5 outside of the bracket. 2,411 x 5 = 12,055.

Next multiply by the 2 outside of the braces. 12,055 x 2= 24,110.

Mathematically, there cannot be brackets or braces in a problem that does not have parentheses. Likewise, there cannot be braces in a problem that does not have both parentheses and brackets.

In fifth grade, students work with exponents only dealing with powers of ten (5.NBT.2). Students are expected to evaluate an expression that has a power of ten in it.

Example:

3 {2 + 5 [5 + 2 x 10^4]+ 3}

In fifth grade students begin working more formally with expressions. They write expressions to express a calculation, e.g., writing 2 x (8 + 7) to express the calculation "add 8 and 7, then multiply by 2." They also evaluate and interpret expressions, e.g., using their conceptual understanding of multiplication to interpret 3 x (18932 x 921) as being three times as large as 18932 + 921, without having to calculate the indicated sum or product. Thus, students in Grade 5 begin to think about numerical expressions in ways that prefigure their later work with variable expressions (e.g., three times an unknown length is 3 . L). In Grade 5, this work should be viewed as exploratory rather than for attaining mastery; for example, expressions should not contain nested grouping symbols, and they should be no more complex than the expressions one finds in an application of the associative or distributive property, e.g., (8 + 27) + 2 or (6 x 30) (6 x 7). Note however that the numbers in expressions need not always be whole numbers. (Progressions for the CCSSM, Operations and Algebraic Thinking, CCSS Writing Team, April 2011, page 32)

ESSENTIAL QUESTION(S)	What do the symbols (parentheses, brackets, braces) represent when evaluating an expression?
MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them. 5.MP.5. Use appropriate tools strategically. 5.MP.8. Look for and express regularity in repeated reasoning.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use order of operations including parenthesis, brackets, or braces.	Evaluate expressions using the order of operations (including using parenthesis, brackets, or braces).	

EXPLANATIONS AND EXAMPLES	This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions.

Examples:

- $(26 + 18) + 4$ Answer: 11
- $\{[2 \times (3+5)] - 9\} + [5 \times (23-18)]$ Answer: 32
- $12 - (0.4 \times 2)$ Answer: 11.2
- $(2 + 3) \times (1.5 - 0.5)$ Answer: 5

-              Answer: 5⅙

- $\{ 80 \div [2 \times (3½ + 1½)] \} + 100$ Answer: 108

STANDARD AND DECONSTRUCTION

5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8+7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum of product.

DESCRIPTION	This standard refers to expressions. Expressions are a series of numbers and symbols (+, -, x, ÷) without an equals sign. Equations result when two expressions are set equal to each other (2 + 3 = 4 + 1). Example: 4(5 + 3) is an expression. When we compute 4(5 + 3) we are evaluating the expression. The expression equals 32. 4(5 + 3) = 32 is an equation. This standard calls for students to verbally describe the relationship between expressions without actually calculating them. This standard calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations. Example: Write an expression for the steps "double five and then add 26." Student (2 x 5) + 26 Describe how the expression 5(10 x 10) relates to 10 x 10. Student The expression 5(10 x 10) is 5 times larger than the expression 10 x 10 since I know that I that 5(10 x 10) means that I have 5 groups of (10 x 10).
ESSENTIAL QUESTION(S)	What do the symbols (parentheses, brackets, braces) represent when evaluating an expression?
MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them. 5.MP.2. Reason abstractly and quantitatively. 5.MP.7. Look for and make use of structure. 5.MP.8. Look for and express regularity in repeated reasoning.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Write numerical expressions for given numbers with operation words. Write operation words to describe a given numerical expression.	Interpret numerical expressions without evaluating them.	Solve addition and subtraction word problems within 10.

EXPLANATIONS AND EXAMPLES	Students use their understanding of operations and grouping symbols to write expressions and interpret the meaning of a numerical expression. Examples: • Students write an expression for calculations given in words such as "divide 144 by 12, and then subtract ⅞." They write (144 ÷ 12) – ⅞. • Students recognize that 0.5 x (300 ÷ 15) is ½ of (300 ÷ 15) without calculating the quotient.

CLUSTER:	**2. Analyze patterns and relationships.**
	Make sense of problems and persevere in solving them.
BIG IDEA:	Numerical expressions bring order and precision to calculations.
ACADEMIC VOCABULARY:	Numerical patterns, rules, ordered pairs, coordinate plane.

STANDARD AND DECONSTRUCTION

5.OA.3	**Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and the given rule "Add 6" and the starting number 0, generate the terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.**
DESCRIPTION	This standard extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function which is why we get the straight lines. The Days are the independent variable, Fish are the dependent variables, and the constant rate is what the rule identifies in the table.

Make a chart (table) to represent the number of fish that Sam and Terri catch.

Days	Sam's Total Number of Fish	Terri's Total Number of Fish
0	0	0
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

Example:

Describe the pattern:

Since Terri catches 4 fish each day, and Sam catches 2 fish, the amount of Terri's fish is always greater. Terri's fish is also always twice as much as Sam's fish. Today, both Sam and Terri have no fish. They both go fishing each day. Sam catches 2 fish each day. Terri catches 4 fish each day. How many fish do they have after each of the five days? Make a graph of the number of fish.

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DESCRIPTION
(continued)

Plot the points on a coordinate plane and make a line graph, and then interpret the graph.

Student:

My graph shows that Terri always has more fish than Sam. Terri's fish increases at a higher rate since she catches

4 fish every day. Sam only catches 2 fish every day, so his number of fish increases at a smaller rate than Terri.

Important to note as well that the lines become increasingly further apart. Identify apparent relationships between corresponding terms. Additional relationships: The two lines will never intersect; there will not be a day in which boys have the same total of fish, explain the relationship between the number of days that has passed and the number of fish a boy has (2n or 4n, n being the number of days).

Catching Fish

ESSENTIAL QUESTION(S)	How can I compare two numerical patterns?		
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Generate two numerical patterns using two given rules. Form ordered pairs consisting of corresponding terms for the two patterns. Graph generated ordered pairs on a coordinate plane.	Analyze and explain the relationships between corresponding terms in the two numerical patterns.	

EXPLANATIONS AND EXAMPLES

Example:

Use the rule "add 3" to write a sequence of numbers. Starting with a 0, students write 0, 3, 6, 9, 12, . . .

Use the rule "add 6" to write a sequence of numbers. Starting with 0, students write 0, 6, 12, 18, 24, . . .

After comparing these two sequences, the students notice that each term in the second sequence is twice the corresponding terms of the first sequence. One way they justify this is by describing the patterns of the terms. Their justification may include some mathematical notation (See example below). A student may explain that both sequences start with zero and to generate each term of the second sequence he/she added 6, which is twice as much as was added to produce the terms in the first sequence. Students may also use the distributive property to describe the relationship between the two numerical patterns by reasoning that $6 + 6 + 6 = 2 (3 + 3 + 3)$.

$$0, \ ^{+3} 3, \ ^{+3} 6, \ ^{+3} 9, \ ^{+3}12, . . .$$

$$0, \ ^{+6} 6, \ ^{+6} 12, \ ^{+6}18, \ ^{+6} 24, . . .$$

Once students can describe that the second sequence of numbers is twice the corresponding terms of the first sequence, the terms can be written in ordered pairs and then graphed on a coordinate grid. They should recognize that each point on the graph represents two quantities in which the second quantity is twice the first quantity.

Ordered pairs

(0, 0)

(3, 6)

(6, 12)

(9, 18)

(12, 24)

DOMAIN:

NUNBER AND OPERATIONS IN BASE TEN (NBT)

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DOMAIN	Number in Operations Base Ten (NBT)
CLUSTERS	1. Understand the place value system. 2. Perform operations with multi-digit whole numbers and with decimals to hundredths.

NUMBERS AND OPERATIONS IN BASE TEN (NBT)		
FOURTH	**FIFTH**	**SIXTH**
PLACE VALUE AND DECIMALS		
Decimal Numbers, Integer Exponents, and Scientific Notation	*Decimal Numbers, Integer Exponents, and Scientific Notation*	*Decimal Numbers, Integer Exponents, and Scientific Notation*
4.NF.6 Use decimal notation for fractions with denominators 10 or 100.	5.NBT.3.a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$.	6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithms.
4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	5.NBT.3.b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.
	5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.	
	5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	
	5.NBT.4 Use place value understanding to round decimals to any place.	
	5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, and explain.	

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NUMBERS AND OPERATIONS IN BASE TEN (NBT)		
FOURTH	**FIFTH**	**SIXTH**
MULTIPLICATION AND DIVISION		
Factors and Multiples	*Factors and Multiples*	*Factors and Multiples*
4.OA.4 Find all factor pairs for a whole number in the range 1 - 100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1 - 100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1 - 100 is prime or composite.	5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.	6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
	5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	6.NS.2 Fluently divide multi-digit numbers using the standard algorithm
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		
Factors and Multiples	*Factors and Multiples*	*Factors and Multiples*
4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.		
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.		

Source: turnonccmath.net, NC State University College of Education

CLUSTER:	**1. Understand the place value system.**
	Understand the place value system.
BIG IDEA:	Deep understanding of place value is needed to understand and use multi-digit numbers (whole numbers, fractions, decimals).
ACADEMIC VOCABULARY:	Place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round.

STANDARD AND DECONSTRUCTION

5.NBT.1	**Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and ⅟₁₀ of what it represents in the place to its left.**

DESCRIPTION	Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths. This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that he tens place is ten times as much as the ones place, and the ones place is ⅟₁₀th the size of the tens place.
	Example:
	The 2 in the number 542 is different from the value of the 2 in 324. The 2 in 542 represents 2 ones or 2, while the 2 in 324 represents 2 tens or 20. Since the 2 in 324 is one place to the left of the 2 in 542 the value of the 2 is 10 times greater. Meanwhile, the 4 in 542 represents 4 tens or 40 and the 4 in 324 represents 4 ones or 4. Since the 4 in 324 is one place to the right of the 4 in 542 the value of the 4 in the number 324 is ⅟₁₀th of its value in the number 542.
	Base on the base-10 number system digits to the left are times as great as digits to the right; likewise, digits to the right are ⅟₁₀th of digits to the left. For example, the 8 in 845 has a value of 800 which is ten times as much as the 8 in the number 782. In the same spirit, the 8 in 782 is ⅟₁₀th the value of the 8 in 845.
ESSENTIAL QUESTION(S)	How does a digit's position affect its value?
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and ⅟₁₀ of what it represents in the place to its left.		

MATHEMATICS

EXPLANATIONS AND EXAMPLES

In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons.

Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and ¹⁄₁₀ of what it represents in the place to its left.

A student thinks, "I know that in the number 5555, the 5 in the tens place (5555) represents 50 and the 5 in the hundreds place (5555) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is ¹⁄₁₀ of the value of a 5 in the hundreds place.

To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe ¹⁄₁₀ of that model using fractional language ("This is 1 out of 10 equal parts. So it is ¹⁄₁₀". I can write this using ¹⁄₁₀ or 0.1"). They repeat the process by finding ¹⁄₁₀ of a ¹⁄₁₀ (e.g., dividing ¹⁄₁₀ into 10 equal parts to arrive at ¹⁄₁₀₀ or 0.01) and can explain their reasoning, "0.01 is ¹⁄₁₀ of ¹⁄₁₀ thus is ¹⁄₁₀₀ of the whole unit."

In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.

The 5 that the arrow points to is ¹⁄₁₀ of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is ¹⁄₁₀ of 50 and 10 times five tenths.

The 5 that the arrow points to is ¹⁄₁₀ of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.

STANDARD AND DECONSTRUCTION

5.NBT.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

DESCRIPTION

Multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left.

Example:

Multiplying by 104 is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit 4 places to the left.

Patterns in the number of 0s in products of a whole numbers and a power of 10 and the location of the decimal point in products of decimals with powers of 10 can be explained in terms of place value. Because students have developed their understandings of and computations with decimals in terms of multiples rather than powers, connecting the terminology of multiples with that of powers affords connections between understanding of multiplication and exponentiation. (Progressions for the CCSSM, Number and Operation in Base Ten, CCSS Writing Team, April 2011, page 16)

This standard includes multiplying by multiples of 10 and powers of 10, including 102 which is 10 x 10=100, and 103 which is 10 x 10 x 10=1,000. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10.

Example:

2.5 x 103 = 2.5 x (10 x 10 x 10) = 2.5 x 1,000 = 2,500 Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.

350 ÷ 103 = 350 ÷ 1,000 = 0.350 = 0.35$^{350}/_{10}$ = 35, $^{35}/_{10}$ = 3.5$^{3.5}/_{10}$ =.0.35, or 350 x $^1/_{10}$, 35 x $^1/_{10}$, 3.5 x $^1/_{10}$ this will relate well to subsequent work with operating with fractions. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10 , the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left.

ESSENTIAL QUESTION(S)

How does a digit's position affect its value?

MATHEMATICAL PRACTICE(S)

5.MP.2. Reason abstractly and quantitatively.

5.MP.6. Attend to precision.

5.MP.7. Look for and make use of structure.

DOK Range Target for Instruction & Assessment

☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Represent powers of 10 using whole number exponents. Translate between powers of 10 written as 10 raised to a whole number exponent, the expanded form, and standard notation.	Explain the patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain the relationship of the placement of the decimal point when a decimal is multiplied or divided by a power of 10.	

EXPLANATIONS AND EXAMPLES

Examples:

Students might write:

- $36 \times 10 = 36 \times 10^1 = 360$

- $36 \times 10 \times 10 = 36 \times 10^2 = 3600$

- $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36,000$

- $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360,000$

Students might think and/or say:

- I noticed that every time, I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left.

- When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones).

Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense.

- $523 \times 10^3 = 523,000$ The place value of 523 is increased by 3 places.

- $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places.

- $52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by one place.

NUMBERS & OPERATIONS IN BASE TEN

STANDARD AND DECONSTRUCTION

5.NBT.3	Read, write, and compare decimals to thousandths.
DESCRIPTION	This standard references expanded form of decimals with fractions included and comparing decimals builds on work from fourth grade.
ESSENTIAL QUESTION(S)	How does a digit's position affect its value?
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Read and write decimal to thousandths using base-ten numerals, number names, and expanded form. Use >, =, and < symbols to record the results of comparisons between decimals.	Compare two decimals to the thousandths, based on the place value of each digit.	

SUBSTANDARD DECONSTRUCTED	**5.NBT.3a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).**
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Read and write decimal to thousandths using base-ten numerals, number names, and expanded form.		

MATHEMATICS

SUBSTANDARD DECONSTRUCTED	5.NBT.3b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use >, =, and < symbols to record the results of comparisons between decimals. Compare two decimals to the thousandths, based on the place value of each digit.		

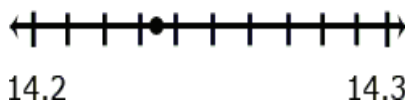| ESSENTIAL QUESTION(S)	How does a digit's position affect its value?		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Read and write decimal to thousandths using base-ten numerals, number names, and expanded form. Use >, =, and < symbols to record the results of comparisons between decimals.	Compare two decimals to the thousandths, based on the place value of each digit.	

EXPLANATIONS AND EXAMPLES

Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation as show in the standard 3a. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800).

Example:

Some equivalent forms of 0.72 are:

$\frac{72}{100}$ $\qquad\qquad\qquad\qquad\qquad$ $\frac{70}{100} + \frac{2}{100}$

$\frac{7}{10} + \frac{2}{100}$ $\qquad\qquad\qquad\qquad\quad$ 0.720

$7 \times (\frac{1}{10}) + 2 \times (\frac{1}{100})$ $\qquad\qquad$ $7 \times (\frac{1}{10}) + 2 \times (\frac{1}{100}) + 0 \times (\frac{1}{1000})$

$0.70 + 0.02$ $\qquad\qquad\qquad\qquad$ $\frac{720}{1000}$

Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.

Example:

Comparing 0.25 and 0.17, a student might think, "25 hundredths is more than 17 hundredths". They may also think that it is 8 hundredths more. They may write this comparison as 0.25 > 0.17 and recognize that 0.17 < 0.25 is another way to express this comparison.

Comparing 0.207 to 0.26, a student might think, "Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, "I know that 0.207 is 207 thousandths (and may write $\frac{207}{1000}$). 0.26 is 26 hundredths (and may write $\frac{26}{100}$) but I can also think of it as 260 thousandths ($\frac{260}{1000}$). So, 260 thousandths is more than 207 thousandths.

MATHEMATICS

STANDARD AND DECONSTRUCTION

5.NBT.3	Read, write, and compare decimals to thousandths:

DESCRIPTION

This standard refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding.

Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0., 0.5, 1, 1.5 are examples of benchmark numbers.

Example:

Which benchmark number is the best estimate of the shaded amount in the model below? Explain your thinking.

ESSENTIAL QUESTION(S)

How does a digit's position affect its value?

MATHEMATICAL PRACTICE(S)

5.MP.2. Reason abstractly and quantitatively.

5.MP.6. Attend to precision.

5.MP.7. Look for and make use of structure.

DOK Range Target for Instruction & Assessment

☒ 1   ☐ 2   ☐ 3   ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use knowledge of base ten and place value to round decimals to any place.		

EXPLANATIONS AND EXAMPLES

When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers.

Example:
* Students recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).

14.2   14.3

CLUSTER:	2. Perform operations with multi-digit whole numbers and with decimals to hundredths.
	Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
BIG IDEA:	Precision is required to apply the four operations to solving multi-digit whole number and decimal problems.
ACADEMIC VOCABULARY:	Multiplication/multiply, division/divide, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning .

STANDARD AND DECONSTRUCTION

5.NBT.5	Fluently multiply multi-digit whole numbers using the standard algorithm.

DESCRIPTION

In fifth grade, students fluently compute products of whole numbers using the standard algorithm. Underlying this algorithm are the properties of operations and the base-ten system. Division strategies in fifth grade involve breaking the dividend apart into like base-ten units and applying the distributive property to find the quotient place by place, starting from the highest place. (Division can also be viewed as finding an unknown factor: the dividend is the product, the divisor is the known factor, and the quotient is the unknown factor.) Students continue their fourth grade work on division, extending it to computation of whole number quotients with dividends of up to four digits and two-digit divisors. Estimation becomes relevant when extending to two-digit divisors. Even if students round appropriately, the resulting estimate may need to be adjusted.

Recording division after an underestimate

$1655 \div 27$

Rounding 27 to 30 produces the underestimate 50 at the first step but this method allows the division process to be continued

(Progressions for the CCSSM, Number and Operation in Base Ten, CCSS Writing Team, April 2011, page 16)

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

This standard refers to fluency which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or breaking numbers apart also using strategies according to the numbers in the problem, 26 x 4 may lend itself to (25 x 4) + 4 where as another problem might lend itself to making an equivalent problem 32 x 4 = 64 x 2)). This standard builds upon students' work with multiplying numbers in third and fourth grade. In fourth grade, students developed understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop conceptual understanding. The size of the numbers should NOT exceed a three-digit factor by a two-digit factor.

STANDARD AND DECONSTRUCTION

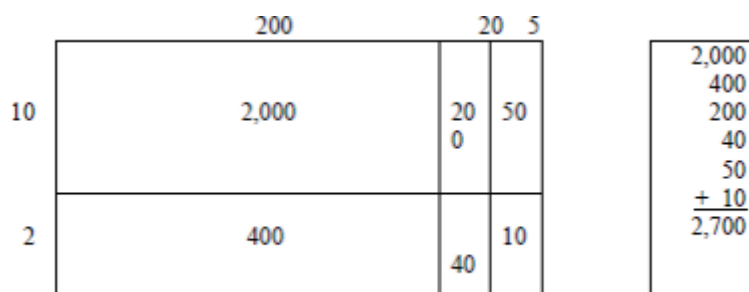|	
DESCRIPTION (continued)	Examples of alternative strategies: There are 225 dozen cookies in the bakery. How many cookies are there? **Student 1** 225 x 12 I broke 12 up into 10 and 2. 225 x 10 = 2,250 225 x 2 = 450 2,250 + 450 = 2,700 **Student 2** 225x12 I broke up 225 into 200 and 25. 200 x 12 = 2,400 I broke 25 up into 5 x 5, so I had 5 x 5 x12 or 5 x 12 x 5. 5 x12= 60. 60 x 5 = 300 I then added 2,400 and 300 2,400 + 300 = 2,700. **Student 3** I doubled 225 and cut 12 in half to get 450 x 6. I then doubled 450 again and cut 6 in half to get 900 x 3. 900 x 3 = 2,700. Draw a array model for 225 x 12…. 200 x 10, 200 x 2, 20 x 10, 20 x 2, 5 x 10, 5 x 2 225 x 12 $$\begin{array}{c|c|c|c} & 200 & 20 & 5 \\ \hline 10 & 2,000 & 200 & 50 \\ \hline 2 & 400 & 40 & 10 \end{array}$$ 2,000 400 200 40 50 + 10 2,700

ESSENTIAL QUESTION(S)	Why is the standard algorithm an efficient method for multiplication?
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure. 5.MP.8. Look for and express regularity in repeated reasoning.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Fluently multiply multi-digit whole numbers using the standard algorithm.		

EXPLANATIONS AND EXAMPLES	In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value. Example: • 123 x 34. When students apply the standard algorithm, they, decompose 34 into 30 + 4. Then they multiply 123 by 4, the value of the number in the ones place, and multiply 123 by 30, the value of the 3 in the tens place, and add the two products.

NUMBERS & OPERATIONS IN BASE TEN

STANDARD AND DECONSTRUCTION

5.NBT.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

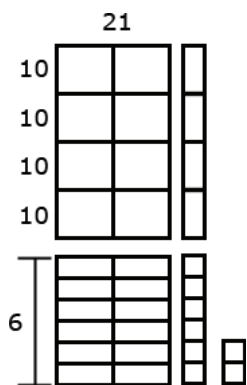| DESCRIPTION	This standard references various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is critical. Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups. In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Example:

There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them?

Student 1
1,716 divided by 16
There are 100 16's in 1,716.
1,716 – 1,600 = 116
I know there areat least 6 16's.
116 – 96 = 20
I can take out at least 1 more 16.
20 – 16 = 4
There are 107 teams with 4 students left over. If we put the extra students on different team, 4 teams will have 17 students.

Student 2
1,716 divided by 16
There are 100 16's in 1,716.
Ten groups of 16 is 160. That's too big.
Half of that is 80, which is 5 groups.
I know that 2 groups of 16's is 32.
I have 4 students left over.

1716 -1600	100
116 -80	5
36 -32	2
4	

Student 3
1,716 ÷ 16
I want to get to 1,716.
I know that 100 16's equals 1,600
I know that 5 16's equals 80
1,600 + 80 = 1,680
Two more groups of 16's equals 32, which gets us to 1,716
So we had 100 + 6 + 1 = 107 teams
Those other 4 students can just hang out

Student 4
How many 16's are there in 1,716?
We have an area of 1,716. I know that one side of my array is 16 units long. I used 16 as the height. I am trying to answer the question what is the width of my rectangle if the area is 1,716 and the height is 16. 100 + 7 = 107 R 4

	100	7
16	100 x 16 = 1,600	7 x 16 = 112
	1,716 – 1,600 = 116	116 - 112 = 4

MATHEMATICS

ESSENTIAL QUESTION(S)	What is an efficient strategy for dividing numbers?		
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.3. Construct viable arguments and critique the reasoning of others. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.	Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division to solve division problems. Illustrate and explain division calculations by using equations, rectangular arrays, and/or area models.	

NUMBERS & OPERATIONS IN BASE TEN

EXPLANATIONS AND EXAMPLES

In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Example:

- Using expanded notation ~ 2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25

- Using his or her understanding of the relationship between 100 and 25, a student might think ~

  - I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
  - 600 divided by 25 has to be 24.
  - Since 3 x 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5. (Note that a student might divide into 82 and not 80)
  - I can't divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
  - 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7.

Using an equation that relates division to multiplication, 25 x n = 2682, a student might estimate the answer to be slightly larger than 100 because s/he recognizes that 25 x 100 = 2500.

Example: 968 ÷ 21

- Using base ten models, a student can represent 962 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.

Example: 968 ÷ 21

- Using base ten models, a student can represent 962 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.

STANDARD AND DECONSTRUCTION

5.NBT.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

DESCRIPTION (continued)	Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers. Like base-ten units must be added and subtracted, so students need to attend to aligning the corresponding places correctly (this also aligns the decimal points). It can help to put 0s in places so that all numbers show the same number of places to the right of the decimal point. Although whole numbers are not usually written with a decimal point, but that a decimal point with 0s on its right can be inserted (e.g., 16 can also be written as 16.0 or 16.00). The process of composing and decomposing a base-ten unit is the same for decimals as for whole numbers and the same methods of recording numerical work can be used with decimals as with whole numbers.

For example, students can write digits representing new units below on the addition or subtraction line, and they can decompose units wherever needed before subtracting.

General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. Before students consider decimal multiplication more generally, they can study the effect of multiplying by 0.1 and by 0.01 to explain why the product is ten or a hundred times as small as the multiplicand (moves one or two places to the right). They can then extend their reasoning to multipliers that are single-digit multiples of 0.1 and 0.01 (e.g., 0.2 and 0.02, etc.).

There are several lines of reasoning that students can use to explain the placement of the decimal point in other products of decimals. Students can think about the product of the smallest base-ten units of each factor. For example, a tenth times a tenth is a hundredth, so 3.2 x 7.1 will have an entry in the hundredth place. Note, however, that students might place the decimal point correctly for 3.2 x 8.5 unless they take into account the 0 in the ones place of 32 x 85. (Or they can think of 0.2 x 0.5 as 10 hundredths.)

Students can also think of the decimals as fractions or as whole numbers divided by 10 or 100.5.NF.3. When they place the decimal point in the product, they have to divide by a 10 from each factor or 100 from one factor. For example, to see that 0.6 x 0.8 = 0.48, students can use fractions: $\frac{6}{10}$ x $\frac{8}{10}$ = $\frac{48}{100}$.5.NF.4 Students can also reason that when they carry out the multiplication without the decimal point, they have multiplied each decimal factor by 10 or 100, so they will need to divide by those numbers in the end to get the correct answer. Also, students can use reasoning about the sizes of numbers to determine the placement of the decimal point. For example, 3.2 x 8.5 should be close to 3 x 9, so 27.2 is a more reasonable product for 3.2 x 8.5 than 2.72 or 272. This estimation-based method is not reliable in all cases, however, especially in cases students will encounter in later grades. For example, it is not easy to decide where to place the decimal point in 0.023 x 0.0045 based on estimation. Students can summarize the results of their reasoning such as those above as specific numerical patterns and then as one general overall pattern such as "the number of decimal places in the product is the sum of the number of decimal places in each factor." General methods used for computing quotients of whole numbers extend to decimals with the additional issue of placing the decimal point in the quotient. As with decimal multiplication, students can first examine the cases of dividing by 0.1 and 0.01 to see that the quotient becomes 10 times or 100 times as large as the dividend. For example, students can view 7 ÷ 0.1 = as asking how many tenths are in 7.5.NF.7b Because it takes 10 tenths make 1, it takes 7 times as many tenths to make 7, so 7 ÷ 0.1 = 7 x 10 = 70. Or students could note that 7 is 70 tenths, so asking how many tenths are in 7 is the same as asking how many tenths are in 70 tenths, which is 70. In other words, 7 ÷ 0.1 is the same as 70 ÷1. So dividing by 0.1 moves the number 7 one place to the left, the quotient is ten times as big as the dividend. As with decimal multiplication, students can then proceed to more general cases. For example, to calculate 7 ÷ 0.2, students can reason that 0.2 is 2 tenths and 7 is 70 tenths, so asking how many 2 tenths are in 7 is the same as asking how many 2 tenths are in 70 tenths.

In other words, 7 ÷ 0.2 is the same as 70 ÷ 2; multiplying both the 7 and the 0.2 by 10 results in the same quotient. Or students could calculate 7 ÷ 0.2 by viewing 0.2 as 2 x 0.1, so they can first divide 7 by 2, which is 3.5, and then divide that result by 0.1, which makes 3.5 ten times as large, namely 35. Dividing by a decimal less than 1 results in a quotient larger than the dividend5.NF.5 and moves the digits of the dividend one place to the left. Students can summarize the results of their reasoning as specific numerical patterns then as one general overall pattern such as "when the decimal point in the divisor is moved to make a whole number, the decimal point in the dividend should be moved the same number of places."(Progressions for the CCSSM, Number and Operation in Base Ten, CCSS Writing Team, April 2011, page 17-18)

STANDARD AND DECONSTRUCTION

DESCRIPTION
(continued)

This standard builds on the work from fourth grade where students are introduced to decimals and compare them. In fifth grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations (2.25 x 3= 6.75), but this work should not be done without models or pictures. This standard includes students' reasoning and explanations of how they use models, pictures, and strategies.

Example:

A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

Student 2

I saw that the 0.25 in 1.25 and the 0.75 for water would combine to equal 1 whole.

I then added the 2 wholes and the 0.40 to get 2.40.

Example of Multiplication:

A gumball costs $0.22. How much do 5 gumballs cost? Estimate the total, and then calculate. Was your estimate close?

STANDARD AND DECONSTRUCTION

DESCRIPTION
(continued)

I estimate that the total cost will be a little more than a dollar. I know that 5 20's equal 100 and we have 5 22's.

I have 10 whole columns shaded and 10 individual boxes shaded. The 10 columns equal 1 whole. The 10 individual boxes equal 10 hundredths or tenth. My answer is $1.10.

My estimate was a little more than a dollar, and my answer was $1.10. I was really close.

Example of Division:

A relay race lasts 4.65 miles. The relay team has 3 runners. If each runner goes the same distance, how far does each team member run? Make an estimate, find your actual answer, and then compare them.

My estimate is that each runner runs between 1 and 2 miles. If each runner went 2 miles, that would be a total of 6 miles which is too high. If each runner ran 1 mile, that would be 3 miles, which is too low.

I used the 5 grids above to represent the 4.65 miles. I am going to use all of the first 4 grids and 65 of the squares in the 5th grid. I have to divide the 4 whole grids and the 65 squares into 3 equal groups. I labeled each of the first 3 grids for each runner, so I know that each team member ran at least 1 mile. I then have 1 whole grid and 65 squares to divide up. Each column represents one-tenth. If I give 5 columns to each runner, that means that each runner has run 1 whole mile and 5 tenths of a mile. Now, I have 15 squares left to divide up. Each runner gets 5 of those squares. So each runner ran 1 mile, 5 tenths and 5 hundredths of a mile. I can write that as 1.55 miles.

My answer is 1.55 and my estimate was between 1 and 2 miles. I was pretty close.

NUMBERS & OPERATIONS IN BASE TEN

ESSENTIAL QUESTION(S)	What is an efficient strategy for adding, subtracting, multiplying and dividing decimals?		
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.3. Construct viable arguments and critique the reasoning of others. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Relate the strategy to a written method and explain the reasoning used to solve decimal operation calculations.	

EXPLANATIONS AND EXAMPLES	This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.
	
	 Examples:
	 • 3.6 + 1.7
	  ◆ A student might estimate the sum to be larger than 5 because 3.6 is more than 3½ and 1.7 is more than 1½.
	 • 5.4 – 0.8
	  ◆ A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.
	 • 6 x 2.4
	  ◆ A student might estimate an answer between 12 and 18 since 6 x 2 is 12 and 6 x 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 x 2½ and think of 2½ groups of 6 as 12 (2 groups of 6) + 3 (½ of a group of 6).
	
	 This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.
	
	 Example: 4 - 0.3
	 • 3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.

EXPLANATIONS AND EXAMPLES

The answer is 3 and $\frac{7}{10}$ or 3.7.

Example: An area model can be useful for illustrating products

$$\begin{array}{r} 2.4 \\ \times\ 1.3 \\ \hline .12 \\ .60 \\ .40 \\ +\ 2.00 \\ \hline 3.12 \end{array}$$

Students should be able to describe the partial products displayed by the area model. For example,

"$\frac{3}{10}$ times $\frac{4}{10}$ is $\frac{12}{100}$.

$\frac{3}{10}$ times 2 is $\frac{6}{10}$ or $\frac{60}{100}$.

1 group of $\frac{4}{10}$ is $\frac{4}{10}$ or $\frac{40}{100}$.

1 group of 2 is 2."

Example of division: finding the number in each group or share

- Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as 2.4÷4=0.6

0.6 0.6 0.6 0.6

Example of division: find the number of groups

- To divide to find the number of groups, a student might

  ◆ draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.

  ◆ count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $\frac{10}{10}$, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . .16 tenths, a student can count 8 groups of 2 tenths.

  ◆ Use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of $\frac{2}{10}$ is $\frac{16}{10}$ or $1\frac{6}{10}$."

Technology Connections: Create models using Interactive Whiteboard software (such as SMART Notebook.

DOMAIN:

NUMBER AND OPERATIONS - FRACTIONS (NF)

FIFTH GRADE
MATHEMATICS

DOMAIN	Number and Operations - Fractions (NF)
CLUSTERS	1. Use equivalent fractions as a strategy to add and subtract fractions. 2. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

NUMBERS AND OPERATIONS - FRACTIONS (NF)		
FOURTH	**FIFTH**	**SIXTH**
EQUAPARTITIONING		
Equipartitioning Multiple Wholes	*Equipartitioning Multiple Wholes*	*Equipartitioning Multiple Wholes*
	5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	
FRACTIONS		
Operations with Fractions	*Operations with Fractions*	*Operations with Fractions*
4.NF.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.	5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	
4.NF.3.a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	
4.NF.3.c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.		
4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.		
4.NF.3.d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.		

MATHEMATICS

NUMBERS AND OPERATIONS IN BASE TEN (NBT)		
FOURTH	**FIFTH**	**SIXTH**
MULTIPLICATION AND DIVISION		
Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)	*Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)*	*Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)*
4.NF.4.a Understand a fraction a/b as a multiple of 1/b.	5.NF.4.a Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b.	6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4.b Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.	5.NF.4.b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	
4.NF.4.c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.	5.NF.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence	
	5.NF.5.a Interpret multiplication as scaling (resizing), by Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	
	5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	
	5.NF.7.a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.	
	5.NF.7.b Interpret division of a whole number by a unit fraction, and compute such quotients.	
	5.NF.7.c Solve real world problems involving division of unit fractions using visual models and equations.	

Source: turnonccmath.net, NC State University College of Education

NUMBERS & OPERATIONS -FRACTIONS

CLUSTER:	1. Use equivalent fractions as a strategy to add and subtract fractions.
	Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them.
BIG IDEA:	Adding and Subtracting Fractions provide a process to share and split things equally.
ACADEMIC VOCABULARY:	Fraction, equivalent, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, mixed numbers.

STANDARD AND DECONSTRUCTION

5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, a/b + c/d = (ad + bc)/bd).
DESCRIPTION	5.NF.1 builds on the work in fourth grade where students add fractions with like denominators. In fifth grade, the example provided in the standard $\frac{2}{3} + \frac{3}{4}$ has students find a common denominator by finding the product of both denominators. This process should come after students have used visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm describes in the standard The use of these visual fraction models allows students to use reasonableness to find a common denominator prior to using the algorithm. For example, when adding $\frac{1}{3} + \frac{1}{6}$, Grade 5 students should apply their understanding of equivalent fractions and their ability to rewrite fractions in an equivalent form to find common denominators.

1/3   is the same as   2/6

I drew a rectangle and shaded ⅓. I knew that if I cut every third in half then I would have sixths. Based on my picture, ⅓ equals ⅔. Then I shaded in another ⅙ with stripes. I ended up with an answer of ⅔, which is equal to ½.

On the contrary, based on the algorithm that is in the example of the Standard, when solving $\frac{1}{3} + \frac{1}{6}$, multiplying 3 and 6 gives a common denominator of 18. Students would make equivalent fractions $\frac{6}{18} + \frac{3}{18} = \frac{9}{18}$ which is also equal to one-half. Please note that while multiplying the denominators will always give a common denominator, this may not result in the smallest denominator.

Fifth grade students will need to express both fractions in terms of a new denominator with adding unlike denominators. For example, in calculating $\frac{2}{3} + \frac{5}{4}$ they reason that if each third in $\frac{2}{3}$ is subdivided into fourths and each fourth in $\frac{5}{4}$ is subdivided into thirds, then each fraction will be a sum of unit fractions with denominator 3 x 4 = 4 x 3 + 12:

$$\frac{2}{3} + \frac{5}{4} = \frac{2 \times 4}{3 \times 4} + \frac{5 \times 3}{4 \times 3} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}.$$

DESCRIPTION (Continued)	It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding adding fractions. (Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 10) Example: Present students with the problem ⅓ + ⅙. Encourage students to use the clock face as a model for solving the problem. Have students share their approaches with the class and demonstrate their thinking using the clock model.
ESSENTIAL QUESTION(S)	How do I add or subtract fractions with unlike denominators?
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.4. Model with mathematics. 5.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Generate equivalent fractions to find the like denominator.	Solve addition and subtraction problems involving fractions (including mixed numbers) with like and unlike denominators using an equivalent fraction strategy.	

EXPLANATIONS AND EXAMPLES	Students should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator. Examples: $$\frac{2}{5} + \frac{7}{8} = \frac{16}{40} + \frac{35}{40} = \frac{51}{40}$$ $$3\frac{1}{4} - \frac{1}{6} = 3\frac{3}{12} - \frac{2}{12} = 3\frac{1}{12}$$

NUMBERS & OPERATIONS -FRACTIONS

STANDARD AND DECONSTRUCTION

5.NF.2	**Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g. by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result ⅖ + ½ = ³⁄₇, by observing that ³⁄₇ < ½.**

DESCRIPTION	This standard refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as ⅞ is greater than ¾ because ⅞ is missing only ⅛ and ¾ is missing ¼ so ⅞ is closer to a whole Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. Example here such as ⅝ is greater than ⁶⁄₁₀ because ⅝ is ⅛ larger than ½ (⁴⁄₈) and ⁶⁄₁₀ is only ¹⁄₁₀ larger than ½ (⁵⁄₁₀).

Example:

Your teacher gave you ½ of the bag of candy. She also gave your friend ⅓ of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you have? Estimate your answer and then calculate. How reasonable was your estimate?

> Student 1
> 1/7 is really close to 0. 1/3 is larger than 1/7, but still less than 1/2. If we put them together we might get close to 1/2.
> 1/7 + 1/3= 3/21 + 7/21 = 10/21. The fraction does not simplify. I know that 10 is half of 20, so 10/21 is a little less than ½.
> Another example: 1/7 is close to 1/6 but less than 1/6, and 1/3 is equivalent to 2/6, so I have a little less than 3/6 or ½.

ESSENTIAL QUESTION(S)	What is an efficient strategy for adding and subtracting, fractions?

MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them. 5.MP.2. Reason abstractly and quantitatively. 5.MP.3. Construct viable arguments and critique the reasoning of others. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure. 5.MP.8. Look for and express regularity in repeated reasoning.

DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

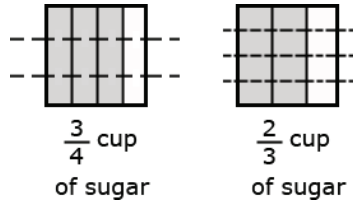| **Learning Expectations**	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Generate equivalent fractions to find like denominators.	Evaluate the reasonableness of an answer, using fractional number sense, by comparing it to a benchmark fraction. Solve word problems involving addition and subtraction of fractions with unlike denominators referring to the same whole.	

COMMON CORE STATE STANDARDS DECONSTRUCTED FOR CLASSROOM IMPACT

EXPLANATIONS AND EXAMPLES

Jerry was making two different types of cookies. One recipe needed ¾ cup of sugar and the other needed ⅔ cup of sugar. How much sugar did he need to make both recipes?
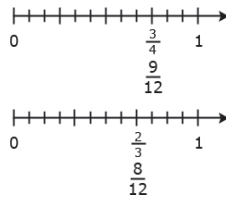
- Mental estimation:

  - A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to ½ and state that both are larger than ½ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2.

- Area model

$$\frac{3}{4}\text{ cup} \qquad \frac{2}{3}\text{ cup}$$
$$\text{of sugar} \qquad \text{of sugar}$$

$$\frac{3}{4}=\frac{9}{12} \quad \frac{2}{3}=\frac{8}{12} \quad \frac{3}{4}+\frac{2}{3}=\frac{17}{12}=\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$$

- Linear model

Solution:

Example: Using a bar diagram

- Sonia had 2⅓ candy bars. She promised her brother that she would give him ½ of a candy bar. How much will she have left after she gives her brother the amount she promised?

- If Mary ran 3 miles every week for 4 weeks, she would reach her goal for the month. The first day of the first week she ran 1¾ miles. How many miles does she still need to run the first week?

  - Using addition to find the answer: 1¾ + n = 3

  - A student might add 1¼ to 1¾ to get to 3 miles. Then he or she would add ⅙ more. Thus 1¼ miles + ⅙ of a mile is what Mary needs to run during that week.

Example: Using an area model to subtract

- This model shows 1¾ subtracted from 3⅙ leaving 1 + ¼ + ⅙ which a student can then change to 1 + ³⁄₁₂ + ²⁄₁₂ = 1⁵⁄₁₂.

EXPLANATIONS AND EXAMPLES

- This diagram models a way to show how 3 13 $\frac{1}{16}$ and 1¾ can be expressed with a denominator of 12. Once this is done a student can complete the problem, 2¹⁴⁄₁₂ − 1⁹⁄₁₂ = 1⁵⁄₁₂.

- This diagram models a way to show how 3 13 $\frac{1}{16}$ and 1¾ can be expressed with a denominator of 12. Once this is accomplished, a student can complete the problem, 2¹⁴⁄₁₂ − 1⁹⁄₁₂ = 1⁵⁄₁₂.

$$2 \qquad \frac{6}{6} = \frac{12}{12} \quad \frac{1}{6} = \frac{2}{12}$$

$$1 \qquad \frac{9}{12}$$

Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students' work with whole number operations and can be supported through the use of physical models.

Example:

- Elli drank ⅗ quart of milk and Javier drank ¹⁄₁₀ of a quart less than Ellie. How much milk did they drink all together?

Estimation skills

$$\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10} \qquad \text{This is how much milk Javier drank}$$

$$\frac{3}{5} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10} \qquad \text{Together they drank } 1\frac{1}{10} \text{ quarts of milk}$$

This solution is reasonable because Ellie drank more than ½ quart and Javier drank ½ quart so together they drank slightly more than one quart.

MATHEMATICS

CLUSTER:	**2. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**
	Cluster Description: Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
BIG IDEA:	Fractions are numbers where the operations on whole numbers can be applied.
ACADEMIC VOCABULARY:	Fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing.

STANDARD AND DECONSTRUCTION

5.NF.3	**Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ¾. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?**
DESCRIPTION	Fifth grade student should connect fractions with division, understanding that $5 \div 3 = \frac{5}{3}$

Students should explain this by working with their understanding of division as equal sharing.

How to share 5 objects equally among 3 shares:
$$5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$$

If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is $\frac{1}{3}$ of an object, and so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object.

(Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 11)

Students should also create story contexts to represent problems involving division of whole numbers.

STANDARD AND DECONSTRUCTION

5.NBT.3	Read, write, and compare decimals to thousandths:

DESCRIPTION (continued)	Example: If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? This can be solved in two ways. First, they might partition each pound among the 9 people, so that each person gets 50 x ⅑ = ⁵⁰⁄₉ pounds. Second, they might use the equation 9 x 5= 45 to see that each person can be given 5 pounds, with 5 pounds remaining. Partitioning the remainder gives 5⅑ pounds for each person. *(Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 11)* This standard calls for students to extend their work of partitioning a number line from third and fourth grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities. Example: Your teacher gives 7 packs of paper to your group of 4 students. If you share the paper equally, how much paper does each student get?
ESSENTIAL QUESTION(S)	What is an efficient strategy for dividing whole numbers and fractions to obtain an answer in the form of a mixed number?
MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them. 5.MP.2. Reason abstractly and quantitatively. 5.MP.3. Construct viable arguments and critique the reasoning of others. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Interpret a fraction as division of the numerator by the denominator.	Interpret the remainder as a fractional part of the problem. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.	

EXPLANATIONS AND EXAMPLES

Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read ⅗ as "three fifths" and after many experiences with sharing problems, learn that ⅗ can also be interpreted as "3 divided by 5."

Examples:

- Ten team members are sharing 3 boxes of cookies. How much of a box will each student get?

    ◆ When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, 10 x n = 3 (10 groups of some amount is 3 boxes) which can also be written as n = 3 ÷ 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting ³⁄₁₀ of a box.

- Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend?

- The six fifth grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive?

Students may recognize this as a whole number division problem but should also express this equal sharing problem as ²⁷⁄₆. They explain that each classroom gets ²⁷⁄₆ boxes of pencils and can further determine that each classroom get 4³⁄₆ or 4½ boxes of pencils.

STANDARD AND DECONSTRUCTION

5.NF.4	**Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.**

DESCRIPTION

Each student receives 1 whole pack of paper and ¼ of the each of the 3 packs of paper. So each student gets 1¾ packs of paper.

Students need to develop a fundamental understanding that the multiplication of a fraction by a whole number could be represented as repeated addition of a unit fraction (e.g., 2 x (¼) = ¼ + ¼

This standard extends student's work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction such as ⅗ actually could be represented as 3 pieces that are each one-fifth (3 x (⅕)). This standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.

Example:

Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?

This question is asking what ⅔ of ¾ is, or what is ⅔ x ¾. What is ⅔ x ¾, in this case you have ⅔ groups of size ¾ (a way to think about it in terms of the language for whole numbers is 4 x 5 you have 4 groups of size 5.

The array model is very transferable from whole number work and then to binomials.
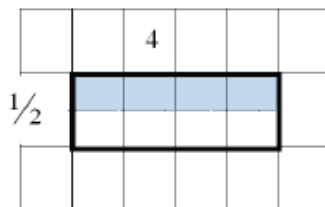
STANDARD AND DECONSTRUCTION

DESCRIPTION
(continued)

This standard extends students' work with area. In third grade students determine the area of rectangles and composite rectangles. In fourth grade students continue this work. The fifth grade standard calls students to continue the process of covering (with tiles). Grids (see picture) below can be used to support this work.
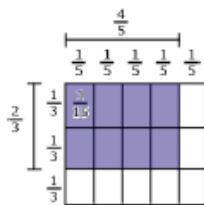
Example:

The home builder needs to cover a small storage room floor with carpet. The storage room is 4 meters long and half of a meter wide. How much carpet do you need to cover the floor of the storage room? Use a grid to show your work and explain your answer. In the grid below I shaded the top half of 4 boxes. When I added them together, I added ½ four times, which equals 2. I could also think about this with multiplication ½ x 4 is equal to ⁴⁄₂ which is equal to 2.

Example:

In solving the problem ⅔ x ⅘, students use an area model to visualize it as a 2 by 4 array of small reactangles each of which has side lengths ⅓ and ⅕. They reason that ⅓ x ⅕ = 1/(3 x 5) by counting squares in the entire rectangle, so the area of the shaded area is (2 x 4) x (3 x 5) = $\frac{2 \times 4}{3 \times 5}$. Thet can explain that the product is less than ⅘ because they are finding ⅔ of ⅘. They can further estimate that the answer must be between ⅖ and ⅘ because ⅔ of ⅘ is more than ½ of ⅘ and less than one group of ⅘.

The area model and the line segments show that the area is the same quantity as the product of the side lengths.

NUMBERS & OPERATIONS -FRACTIONS

ESSENTIAL QUESTION(S)	What is an efficient strategy to find the area of a rectangle with fractional side lengths?
MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them. 5.MP.2. Reason abstractly and quantitatively. 5.MP.3. Construct viable arguments and critique the reasoning of others. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure. 5.MP.8. Look for and express regularity in repeated reasoning.

SUBSTANDARD DECONSTRUCTED	**5.NF.4a Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)**

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Multiply fractions by whole numbers. Multiply fractions by fractions.	Interpret the product of a fraction times a whole number as total number of parts of the whole. Determine the sequence of operations that result in the total number of parts of the whole. Interpret the product of a fraction times a fraction as the total number of parts of the whole.	

SUBSTANDARD DECONSTRUCTED	**5.NF.4b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.**

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Find area of a rectangle with fractional side lengths using different strategies.	Represent fraction products as rectangular areas. Justify multiplying fractional side lengths to find the area is the same as tiling a rectangle with unit squares of the appropriate unit fraction side lengths.	

EXPLANATIONS AND EXAMPLES

Students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.

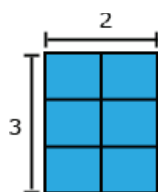As they multiply fractions such as ⅗ x 6, they can think of the operation in more than one way.

- ◆ 3 x (6 ÷ 5) or (3 x ⁶⁄₅)
- ◆ (3 x 6) ÷ 5 or 18 ÷ 5 (¹⁸⁄₅)

Students create a story problem for ⅗ x 6 such as,

- Isabel had 6 feet of wrapping paper. She used ⅗ of the paper to wrap some presents. How much does she have left?

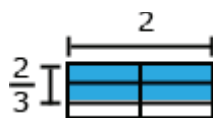- Every day Tim ran ⅗ of mile. How far did he run after 6 days? (Interpreting this as 6 x ⅗)

Examples: Building on previous understandings of multiplication

- Rectangle with dimensions of 2 and 3 showing that 2 x 3 = 6.

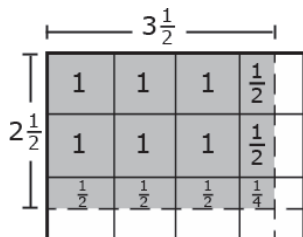Rectangle with dimensions of 2 and ⅔ showing that 2 x ⅔ = ⁴⁄₃

- 2 ½ groups of 3 ½

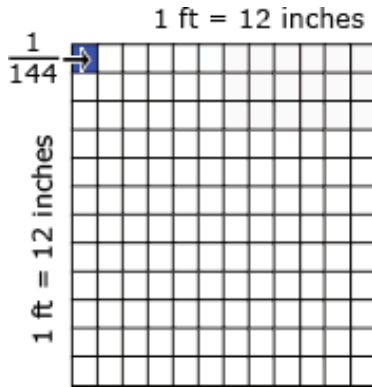- In solving the problem ⅔ x ⅘, students use an area model to visualize it as a 2 by 4 array of small rectangles each of which has side lengths ⅓ and ⅕. They reason that ⅓ x ⅕ = 1/(3 x 5) by counting squares in the entire rectangle, so the area of the shaded area is (2 x 4) x 1/(3 x 5) = $\frac{2 \times 4}{3 \times 5}$. They can explain that the product is less than ⅘ because they are finding ⅔ of ⅘. They can further estimate that the answer must be between ⅖ and ⅘ because ⅔ of ⅘ is more than ½ of ⅘ and less than one group of ⅘.

- Larry knows that ___ x ___ is ___ . To prove this he makes the following array.

Technology Connections:

- Create story problems for peers to solve using digital tools.
  Use a tool such as Jing to digitally communicate story problems.

STANDARD AND DECONSTRUCTION	
5.NF.5	**Interpret multiplication as scaling (resizing), by:**

DESCRIPTION	This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation. Example: Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and ⅚ meters wide. The second flower bed is 5 meters long and ⅝ meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer. Example: Example: $2\frac{2}{3} \times 8$ must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So the answer must be close to, but less than 24. $\frac{3}{4} = \frac{5 \times 3}{5 \times 4}$ because multiplying $\frac{3}{4}$ by $\frac{5}{5}$ is the same as multiplying by 1.
ESSENTIAL QUESTION(S)	Why does the product of whole number multiplication differ from the multiplication of a whole number and fraction?
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.4. Model with mathematics. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure.

SUBSTANDARD DECONSTRUCTED	**5.NF.5a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.**

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Know that scaling (resizing) involves multiplication.	Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	

SUBSTANDARD DECONSTRUCTED	**5.NF.5b** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.

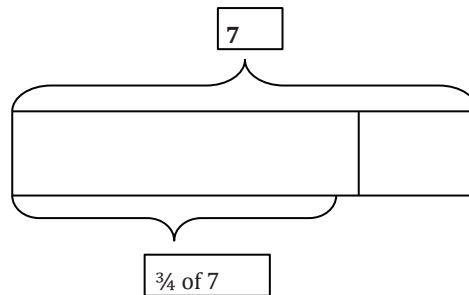| DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Know that multiplying whole numbers and fractions result in products greater than or less than one depending upon the factors.	Draw a conclusion that multiplying a fraction greater than one will result in a product greater than the given number. Draw a conclusion that when you multiply a fraction by one the resulting fraction is equivalent. Draw a conclusion that when you multiply a fraction by a fraction, the product will be smaller than the given number.	

EXPLANATIONS AND EXAMPLES	Examples:

- $\frac{3}{4} \times 7$ is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.

7

¾ of 7

- 2 ⅔ x 8 must be more than 8 because 2 groups of 8 is 16 and 2 ⅔ is almost 3 groups of 8. So the answer must be close to, but less than 24.

- $\frac{3}{4} = \frac{5 \times 3}{5 \times 4}$ because multiplying $\frac{3}{4}$ by $\frac{5}{5}$ is the same as multiplying by 1.

STANDARD AND DECONSTRUCTION

5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

DESCRIPTION	This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.

Example:

There are 2, bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. ⅖ of the students on each bus are girls. How many busses would it take to carry only the girls?

Student 1
I drew 3 grids and 1 grid represents 1 bus. I cut the third grid in half and I marked out the right half of the third grid, leaving 2 ½ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls. When I added up the shaded pieces, 2/5 of the 1st and 2nd bus were both shaded, and 1/5 of the last bus was shaded.

2/5   +   2/5   +   1/5   = 5/5 = 1 whole bus.

Student 2
2 ½ x 2/5=
I split the 2 ½ into 2 and ½
2 x 2/5 = 4/5
½ x 2/5 = 2/10
I then added 4/5 and 2/10. That equals 1 whole bus load.

ESSENTIAL QUESTION(S)	What models or equations can be used to efficiently solve problems using fractions and mixed numbers?

MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them. 5.MP.2. Reason abstractly and quantitatively. 5.MP.3. Construct viable arguments and critique the reasoning of others. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure. 5.MP.8. Look for and express regularity in repeated reasoning.

DOK Range Target for Instruction & Assessment	☒ 1　　☒ 2　　☐ 3　　☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Represent word problems involving multiplication of fractions and mixed numbers.	Solve real world problems involving multiplication of fractions and mixed numbers.	

NUMBERS & OPERATIONS -FRACTIONS

EXPLANATIONS AND EXAMPLES

Examples:
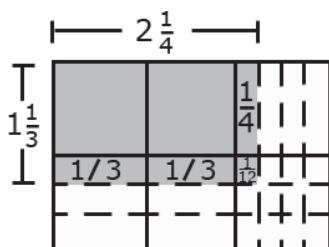
- Evan bought 6 roses for his mother.   of them were red. How many red roses were there?

- Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.

- A student can use an equation to solve.

$$\frac{2}{3} \times 6 = \frac{12}{3} = 4$$

- Mary and Joe determined that the dimensions of their school flag needed to be   ft. by 2   ft. What will be the area of the school flag?

- A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. Thinking ahead a student may decide to multiply by   instead of 2

The explanation may include the following:
- First, I am going to multiply 2   by 1 and then by   .
- When I multiply 2   by 1, it equals 2   .
- Now I have to multiply 2   by   .
-      times 2 is   .
-      times   is   .
- So the answer is 2   +   +   or   +   +   = 2   = 3

MATHEMATICS

STANDARD AND DECONSTRUCTION

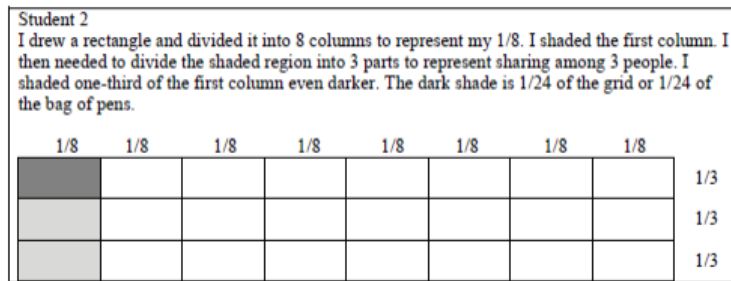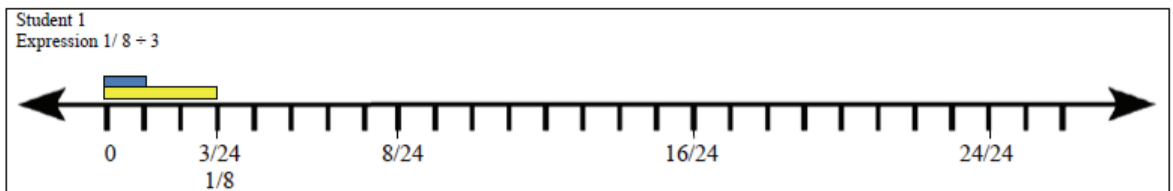| 5.NF.7	**Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.** *Recognize any solid figure packed without gaps or overlaps and filled with n unit cubes indicates the total cubic units or volume.*

DESCRIPTION

5.NF.7 is the first time that students are dividing with fractions. In fourth grade students divided whole numbers, and multiplied a whole number by a fraction. The concept unit fraction is a fraction that has a one in the denominator. For example, the fraction ⅗ is 3 copies of the unit fraction ⅕. ⅕ + ⅕ + ⅕ = ⅗ = ⅕ x 3 or 3 x ⅕.

5.NF.7a This standard asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.

Example:

You have ⅛ of a bag of pens and you need to share them among 3 people. How much of the bag does each person

Student 1
Expression 1/ 8 ÷ 3

0   3/24   8/24   16/24   24/24
     1/8

Student 2
I drew a rectangle and divided it into 8 columns to represent my 1/8. I shaded the first column. I then needed to divide the shaded region into 3 parts to represent sharing among 3 people. I shaded one-third of the first column even darker. The dark shade is 1/24 of the grid or 1/24 of the bag of pens.

Student 3
1/8 of a bag of pens divided by 3 people. I know that my answer will be less than 1/8 since I'm sharing 1/8 into 3 groups. I multiplied 8 by 3 and got 24, so my answer is 1/24 of the bag of pens. I know that my answer is correct because (1/24) x 3 = 3/24 which equals 1/8.

5.NF.7b This standard calls for students to create story contexts and visual fraction models for division situations where a whole number is being divided by a unit fraction.

Example:

Create a story context for 5 ÷ ⅙. Find your answer and then draw a picture to prove your answer and use multiplication to reason about whether your answer makes sense. How many ⅙ are there in 5?

Student
The bowl holds 5 Liters of water. If we use a scoop that holds 1/6 of a Liter, how many scoops will we need in order to fill the entire bowl?

I created 5 boxes. Each box represents 1 Liter of water. I then divided each box into sixths to represent the size of the scoop. My answer is the number of small boxes, which is 30. That makes sense since 6 x 5 = 30.

1 = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 a whole has 6/6 so five wholes would be 6/6 + 6/6 + 6/6 + 6/6 + 6/6 =30/6

NUMBERS & OPERATIONS -FRACTIONS

STANDARD AND DECONSTRUCTION

DESCRIPTION (continued)	5.NF.7c extends students' work from other standards in 5.NF.7. Student should continue to use visual fraction models and reasoning to solve these real-world problems.
	Example:
	How many ⅓-cup servings are in 2 cups of raisins?
	Student I know that there are three 1/3 cup servings in 1 cup of raisins. Therefore, there are 6 servings in 2 cups of raisins. I can also show this since 2 divided by 1/3 = 2 x 3 = 6 servings of raisins.
ESSENTIAL QUESTION(S)	What models or equations can be used to efficiently solve problems using fractions and mixed numbers?
MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them.
	5.MP.2. Reason abstractly and quantitatively.
	5.MP.3. Construct viable arguments and critique the reasoning of others.
	5.MP.4. Model with mathematics.
	5.MP.5. Use appropriate tools strategically.
	5.MP.6. Attend to precision.
	5.MP.7. Look for and make use of structure.
	5.MP.8. Look for and express regularity in repeated reasoning.

SUBSTANDARD DECONSTRUCTED	**5.NF.7a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient.**
	Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Know the relationship between multiplication and division.	Interpret division of a unit fraction by a whole number and justify your answer using the relationship between multiplication and division, by creating story problems, using visual models, and relationship to multiplication, etc.	

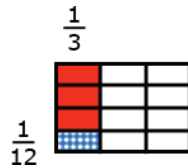| **SUBSTANDARD DECONSTRUCTED**	**5.NF.7b** Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Know the relationship between multiplication and division.	Interpret division of a whole number by a unit fraction and justify your answer using the relationship between multiplication and division, and by representing the quotient with a visual fraction model.	

SUBSTANDARD DECONSTRUCTED	**5.NF.7c** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Know the relationship between multiplication and division.	Solve real world problems involving division of unit fractions by whole numbers other than 0 and division of whole numbers by unit fractions using strategies such as visual fraction models and equations.	

NUMBERS & OPERATIONS -FRACTIONS

EXPLANATIONS AND EXAMPLES

In fifth grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) or with unit fraction divisors and whole number dividends. Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each group/share. In sixth grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions.

Division Example: Knowing the number of groups/shares and finding how many/much in each group/share:

- Four students sitting at a table were given ⅓ of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?

  - The diagram shows the ⅓ pan divided into 4 equal shares with each share equaling ¹⁄₁₂ of the pan.

$$\frac{1}{3}$$

$$\frac{1}{12}$$

Examples:

Knowing how many in each group/share and finding how many groups/shares

- Angelo has 4 lbs of peanuts. He wants to give each of his friends ⅕ lb. How many friends can receive ⅕ lb of peanuts?

A diagram for 4 ÷ ⅕ is shown below. Students explain that since there are five fifths in one whole, there must be 20 fifths in 4 lbs.

1 lb. of peanuts

$$\frac{1}{5} \text{ lb.}$$

- How much rice will each person get if 3 people share ½ lb of rice equally?

$$\frac{1}{2} \div 3 = \frac{3}{6} \div 3 = \frac{1}{6}$$

- A student may think or draw ½ and cut it into 3 equal groups then determine that each of those part is ⅙.
- A student may think of ½ as equivalent to ³⁄₆. ³⁄₆ divided by 3 is ⅙.

DOMAIN:

MEASUREMENT AND DATA (MD)

FIFTH GRADE
MATHEMATICS

DOMAIN	Measurement and Data (MD)
CLUSTERS	1. Convert like measurement units within a given measurement system. 2. Represent and interpret data. 3. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

MEASUREMENT AND DATA

LENGTH, AREA, AND VOLUME

FOURTH	FIFTH	SIXTH
Area and Perimeter	*Area and Perimeter*	*Area and Perimeter*
4.MD.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.	5.MD.3.b A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	
	5.MD.3.a A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.	
	5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	
	5.MD.5.a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	
	5.MD.5.b Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.	
	5.MD.5.c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.	
Conversion	*Conversion*	*Conversion*
4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.	
4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.		

MEASUREMENT AND DATA		
FOURTH	**FIFTH**	**SIXTH**
LENGTH, AREA, AND VOLUME		
Area and Volume of Geometrical Shapes and Solids	*Area and Volume of Geometrical Shapes and Solids*	*Area and Volume of Geometrical Shapes and Solids*
		6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
		6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\,w\,h$ and $V = b\,h$ to find volumes of right
		6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Source: turnonccmath.net, NC State University College of Education

CLUSTER:	1. Convert like measurement units within a given measurement system
	Convert like measurement units within a given measurement system.
BIG IDEA:	Converting and using measurement quantify and represent daily tasks (distance, time, volume, mass, money).
ACADEMIC VOCABULARY:	Conversion/convert, metric and customary measurement From previous grades: relative size, liquid volume, mass, length, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), hour, minute, second .

STANDARD AND DECONSTRUCTION

5.MD.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

DESCRIPTION

5.MD.1 calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume.

Students should explore how the base-ten system supports conversions within the metric system.
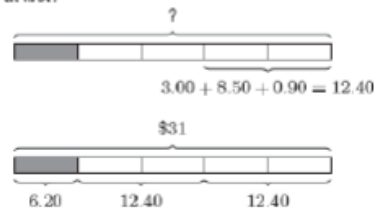
Example: 100 cm = 1 meter.

In Grade 5, students extend their abilities from Grade 4 to express measurements in larger or smaller units within a measurement system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and connection between fractions and decimals (e.g., 2 . meters can be expressed as 2.5 meters or 250 centimeters). For example, building on the table from Grade 4, Grade 5 students might complete a table of equivalent measurements in feet and inches. Grade 5 students also learn and use such conversions in solving multi-step, real world problems (see example below).

Feet	Inches
0	0
	1
	2
	3

In Grade 6, this table can be discussed in terms of ratios and proportional relationships (see the Ratio and Proportion Progression). In Grade 5, however, the main focus is on arriving at the measurements that generate the table.

Multi-step problem with unit conversion

Kumi spent a fifth of her money on lunch. She then spent half of what remained. She bought a card game for $3, a book for $8.50, and candy for 90 cents. How much money did she have at first?

$3.00 + 8.50 + 0.90 = 12.40$

$31

6.20   12.40   12.40

Students can use tape diagrams to represent problems that involve conversion of units, drawing diagrams of important features and relationships (MP1).

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, August 2011, page 26)

MATHEMATICS

ESSENTIAL QUESTION(S)	What is the relationship of units for the SI and metric systems?
MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them. 5.MP.2. Reason abstractly and quantitatively. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision.
DOK Range Target for Instruction & Assessment	☒ 1　　☒ 2　　☐ 3　　☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize units of measurement within the same system. Divide and multiply to change units.	Convert units of measurement within the same system. Solve multi-step, real world problems that involve converting units.	

EXPLANATIONS AND EXAMPLES	In fifth grade, students build on their prior knowledge of related measurement units to determine equivalent measurements. Prior to making actual conversions, they examine the units to be converted, determine if the converted amount will be more or less units than the original unit, and explain their reasoning. They use several strategies to convert measurements. When converting metric measurement, students apply their understanding of place value and decimals.

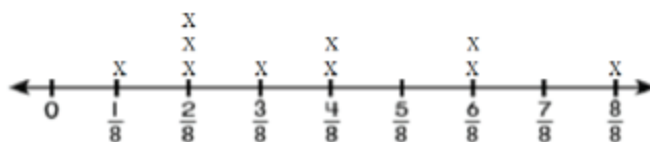| CLUSTER:	Represent and interpret data.
	Represent and interpret data.
BIG IDEA:	onverting and using measurement quantify and represent daily tasks (distance, time, volume, mass, money).
ACADEMIC VOCABULARY:	Line plot, length, mass, liquid volume.

STANDARD AND DECONSTRUCTION

5.MD.2	Make a line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

DESCRIPTION	5.MD.2 This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Example:
Students measured objects in their desk to the nearest ½, ¼, or 1/8 of an inch then displayed data collected on a line plot. How many object measured ¼? ½? If you put all the objects together end to end what would be the total length of **all** the objects?

ESSENTIAL QUESTION(S)	Why is it important to draw or select an accurate line plot to interpret data?

MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them.
	5.MP.2. Reason abstractly and quantitatively.
	5.MP.4. Model with mathematics.
	5.MP.5. Use appropriate tools strategically.
	5.MP.6. Attend to precision.
	5.MP.7. Look for and make use of structure.

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

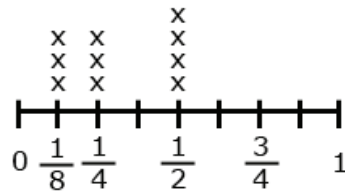| Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Identify benchmark fractions. Make a line plot to display a data set of measurements in fractions of a unit.	Solve problems involving information presented in line plots which use fractions of a unit by adding, subtracting, multiplying, and dividing fractions.	

EXPLANATIONS AND EXAMPLES

Ten beakers, measured in liters, are filled with a liquid.

Liquid in Beakers

```
            x
x  x        x
x  x        x
x  x        x
├──┼──┼──┼──┼──┼──┼──┼──┤
0  1/8 1/4  1/2   3/4   1
```

Amount of Liquid (in Liters)

The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

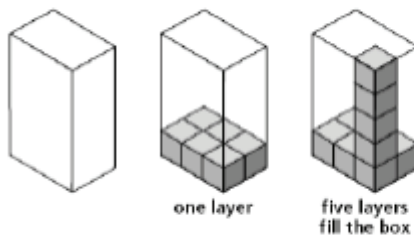| CLUSTER:	3. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
	Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.
BIG IDEA:	Volume is an attribute of solid figures (that can describe patterns and reason in the physical world) that can be measured.
ACADEMIC VOCABULARY:	Measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in., cubic ft., nonstandard cubic units), multiplication, addition, edge lengths, height, area of base

STANDARD AND DECONSTRUCTION

5.MD.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement
DESCRIPTION	5. MD.3, 5.MD.4, and 5. MD.5 These standards represent the first time that students begin exploring the concept of volume. In third grade, students begin working with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of cube) with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer (see picture below). Students should have ample experiences with concrete manipulatives before moving to pictorial representations. Students' prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in3, m3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc are helpful in developing an image of a cubic unit. Students' estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.

(3 x 2) represented by first layer
(3 x 2) x 5 represented by number of 3 x 2 layers
(3 x 2) + (3 x 2) + (3 x 2) + (3 x 2) + (3 x 2) = 6 + 6 + 6 + 6 + 6 = 30
6 representing the size/area of one layer

one layer   five layers fill the box

The major emphasis for measurement in Grade 5 is volume. Volume not only introduces a third dimension and thus a significant challenge to students' spatial structuring, but also complexity in the nature of the materials measured. That is, solid units are "packed," such as cubes in a three-dimensional array, whereas a liquid "fills" three-dimensional space, taking the shape of the container. The unit structure for liquid measurement may be psychologically one dimensional for some students.

MATHEMATICS

ESSENTIAL QUESTION(S)	How do I calculate volume?
MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure.

SUBSTANDARD DECONSTRUCTED	**5.MD.3a A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize that volume is the measurement of the space inside a solid three-dimensional figure. Recognize a unit cube has 1 cubic unit of volume and is used to measure volume of three-dimensional shapes.		

SUBSTANDARD DECONSTRUCTED	**5.MD.3b A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize any solid figure packed without gaps or overlaps and filled with n unit cubes indicates the total cubic units or volume.		

EXPLANATIONS AND EXAMPLES	Students' prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in3, m3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc are helpful in developing an image of a cubic unit. Student's estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.

STANDARD AND DECONSTRUCTION

5.MD.4	**Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.**

DESCRIPTION

"Packing" volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube.5.MD.3 They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build.5.MD.4 They can use the results to compare the volume of right rectangular prisms that have different dimensions. Such experiences enable students to extend their spatial structuring from two to three dimensions. That is, they learn to both mentally decompose and recompose a right rectangular prism built from cubes into layers, each of which is composed of rows and columns. That is, given the prism, they have to be able to decompose it, understanding that it can be partitioned into layers, and each layer partitioned into rows, and each row into cubes. They also have to be able to compose such as structure, multiplicatively, back into higher units. That is, they eventually learn to conceptualize a layer as a unit that itself is composed of units of units—rows, each row composed of individual cubes—and they iterate that structure. Thus, they might predict the number of cubes that will be needed to fill a box given the net of the box.

Another complexity of volume is the connection between "packing" and "filling." Often, for example, students will respond that a box can be filled with 24 centimeter cubes, or build a structure of 24 cubes, and still think of the 24 as individual, often discrete, not necessarily units of volume. They may, for example, not respond confidently and correctly when asked to fill a graduated cylinder marked in cubic centimeters with the amount of liquid that would fill the box. That is, they have not yet connected their ideas about filling volume with those concerning packing volume. Students learn to move between these conceptions, e.g., using the same container, both filling (from a graduated cylinder marked in ml or cc) and packing (with cubes that are each 1 cm3). Comparing and discussing the volume-units and what they represent can help students learn a general, complete, and interconnected conceptualization of volume as filling three-dimensional space.

Students then learn to determine the volumes of several right rectangular prisms, using cubic centimeters, cubic inches, and cubic feet. With guidance, they learn to increasingly apply multiplicative reasoning to determine volumes, looking for and making use of structure. That is, they understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes.5.MD.5a They also learn that the height of the prism tells how many layers would fit in the prism. That is, they understand that volume is a derived attribute that, once a length unit is specified, can be computed as the product of three length measurements or as the product of one area and one length measurement.

ESSENTIAL QUESTION(S)	How can I accurately find the volume of a shape?

MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision.

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Measure volume by counting unit cubes, cubic cm, cubic in, cubic ft, and improvised units.		

EXPLANATIONS AND EXAMPLES

Students understand that same sized cubic units are used to measure volume. They select appropriate units to measure volume. For example, they make a distinction between which units are more appropriate for measuring the volume of a gym and the volume of a box of books. They can also improvise a cubic unit using any unit as a length (e.g., the length of their pencil). Students can apply these ideas by filling containers with cubic units (wooden cubes) to find the volume. They may also use drawings or interactive computer software to simulate the same filling process.

Technology Connections:

http://illuminations.nctm.org/ActivityDetail.aspx?ID=6
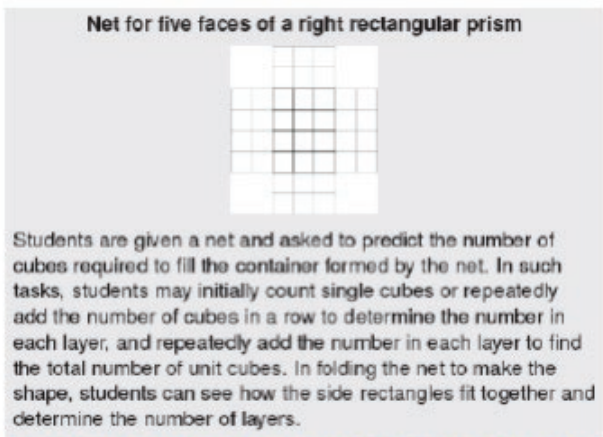

STANDARD AND DECONSTRUCTION

5.MD.5	**Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume**

DESCRIPTION

Then, students can learn the formulas V =l x w x h and V = B x h for right rectangular prisms as efficient methods for computing volume, maintaining the connection between these methods and their previous work with computing the number of unit cubes that pack a right rectangular prism.5.MD.5b They use these competencies to find the volumes of right rectangular prisms with edges whose lengths are whole numbers and solve real-world and mathematical problems involving such prisms.

Students also recognize that volume is additive and they find the total volume of solid figures composed of two right rectangular prisms.5.MD.5c For example, students might design a science station for the ocean floor that is composed of several rooms that are right rectangular prisms and that meet a set criterion specifying the total volume of the station. They draw their station and justify how their design meets the criterion.
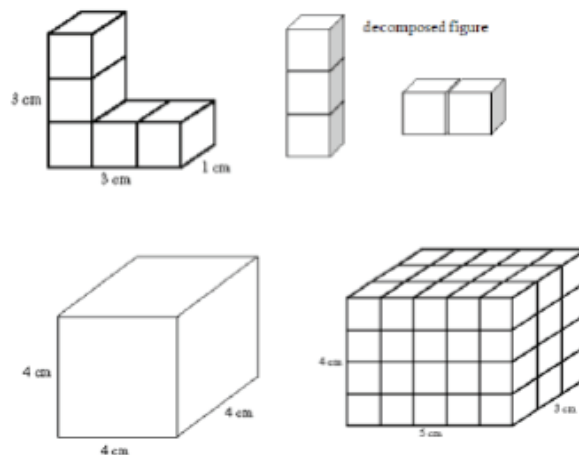

Net for five faces of a right rectangular prism

Students are given a net and asked to predict the number of cubes required to fill the container formed by the net. In such tasks, students may initially count single cubes or repeatedly add the number of cubes in a row to determine the number in each layer, and repeatedly add the number in each layer to find the total number of unit cubes. In folding the net to make the shape, students can see how the side rectangles fit together and determine the number of layers.

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, August 2011, page 26)

5. MD.5a & b These standards involve finding the volume of right rectangular prisms (see picture above).Students should have experiences to describe and reason about why the formula is true. Specifically, that they are covering the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length x width x height) is an extension of the formula for the area of a rectangle.

5.MD.5c This standard calls for students to extend their work with the area of composite figures into the context of volume. Students should be given concrete experiences of breaking apart (decomposing) 3-dimensional figures into right rectangular prisms in order to find the volume of the entire 3-dimensional figure.

Examples:

MATHEMATICS

ESSENTIAL QUESTION(S)	How do I solve real world problems using volume efficiently?
MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them. 5.MP.2. Reason abstractly and quantitatively. 5.MP.3. Construct viable arguments and critique the reasoning of others. 5.MP.4. Model with mathematics. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure. 5.MP.8. Look for and express regularity in repeated reasoning.

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Identify a right rectangular prism. Multiply the three dimensions in any order to calculate volume (Commutative and Associative properties). Know that "B" is the area of the base. Recognize volume as additive.	Develop volume formula for a rectangle prism by comparing volume when filled with cubes to volume by multiplying the height by the area of the base, or when multiplying the edge lengths (L x W x H). Apply the following formulas to right rectangular prisms having whole number edge lengths in the context of real world mathematical problems: Volume = length x width x height Volume = area of base x height. Solve real world problems by decomposing a solid figure into two non-overlapping right rectangular prisms and adding their volumes.	Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes.

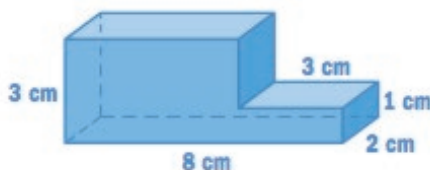| **SUBSTANDARD DECONSTRUCTED**	**5.MD.5c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize volume as additive.	Solve real world problems using volume of solid figures with non-overlapping parts.	

EXPLANATIONS AND EXAMPLES

Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.
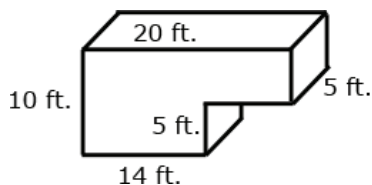
Examples:

- When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

- Students determine the volume of concrete needed to build the steps in the diagram below.
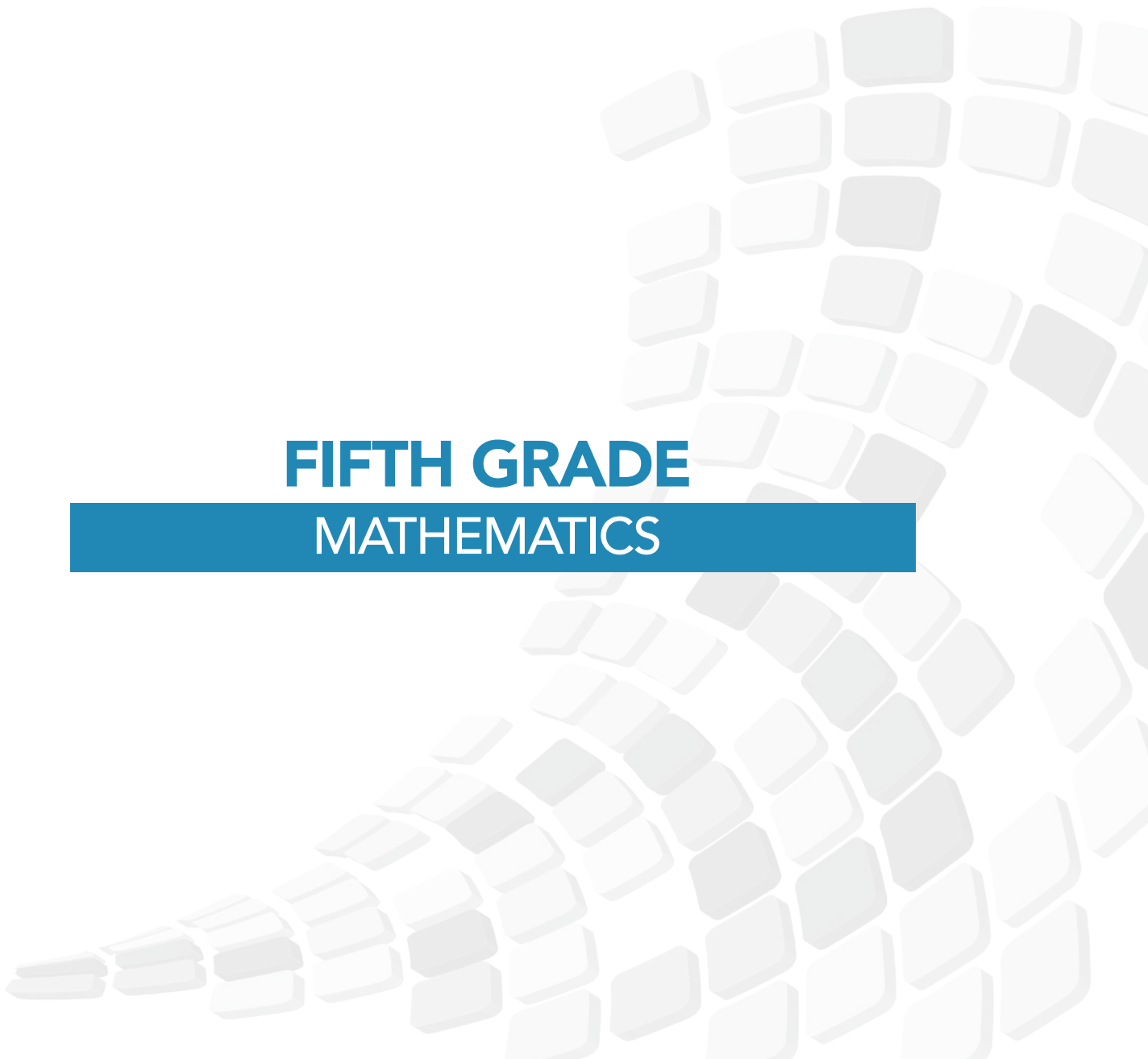

- A homeowner is building a swimming pool and needs to calculate the volume of water needed to fill the pool. The design of the pool is shown in the illustration below.

DOMAIN:

GEOMETRY (G)

FIFTH GRADE
MATHEMATICS

DOMAIN	Geometry
CLUSTERS	1. **Graph points on the coordinate plane to solve real-world and mathematical problems.** 2. **Classify two-dimensional figures into categories based on their properties.**

GEOMETRY		
FOURTH	**FIFTH**	**SIXTH**
INTEGERS, NUMBER LINES, AND COORDINATE PLANES		
Integers on the Number Line	*Integers on the Number Line*	*Integers on the Number Line*
	5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	6.NS.7.a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
		6.NS.6.a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., −(−3) = 3, and that 0 is its own opposite.
		6.NS.7.c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
		6.NS.7.d Distinguish comparisons of absolute value from statements about order.
		6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
		6.NS.6.b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
		6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

MATHEMATICS

GEOMETRY		
FOURTH	**FIFTH**	**SIXTH**
SHAPES AND ANGLES		
Shapes and Properties	*Shapes and Properties*	*Shapes and Properties*
4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	5.G.4 Classify two-dimensional figures in a hierarchy based on properties.	
	4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 5.G.4 Classify two-dimensional figures in a hierarchy based on properties.	

Source: turnonccmath.net, NC State University College of Education

CLUSTER:	1. Graph points on the coordinate plane to solve real-world and mathematical problems.
	Graph points on the coordinate plane to solve real-world and mathematical problems.
BIG IDEA:	The Coordinate Plane describes distance and position to solve problems.
ACADEMIC VOCABULARY:	Coordinate system, coordinate plane, first quadrant, points, lines, axis/axes, x-axis, y-axis, horizontal, vertical, intersection of lines, origin, ordered pairs, coordinates, x-coordinate, y-coordinate.

STANDARD AND DECONSTRUCTION

5.G.1	**Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate)**
DESCRIPTION	5.G.1 and 5.G.2 These standards deal with only the first quadrant (positive numbers) in the coordinate plane Although students can often "locate a point," these understandings are beyond simple skills. For example, initially, students often fail to distinguish between two different ways of viewing the point (2, 3), say, as instructions: "right 2, up 3"; and as the point defined by being a distance 2 from the y-axis and a distance 3 from the x-axis. In these two descriptions the 2 is first associated with the x-axis, then with the y-axis.

Example:
Connect these points in order on the coordinate grid below:
(2, 2) (2, 4) (2, 6) (2, 8) (4, 5) (6, 8) (6, 6) (6, 4) and (6, 2).

What letter is formed on the grid?

Solution: "M" is formed.

Example:
Plot these points on a coordinate grid.
Point A: (2,6)
Point B: (4,6)
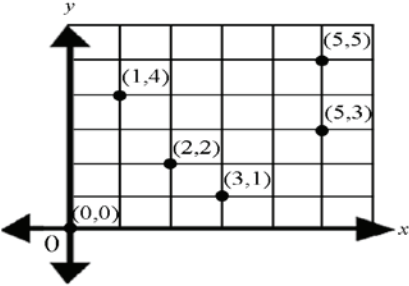Point C: (6,3)
Point D: (2,3)
Connect the points in order. Make sure to connect Point D back to Point A.
1. What geometric figure is formed? What attributes did you use to identify it?
2. What line segments in this figure are parallel?
3. What line segments in this figure are perpendicular?
solutions: trapezoid, line segments AB and DC are parallel, segments AD and DC are perpendicular

Example:
Emanuel draws a line segment from (1, 3) to (8, 10). He then draws a line segment from (0, 2) to (7, 9). If he wants to draw another line segment that is parallel to those two segments what points will he use?

ESSENTIAL QUESTION(S)	What are the parts of the coordinate system?		
MATHEMATICAL PRACTICE(S)	5.MP.4. Model with mathematics. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define the coordinate system. Identify the x- and y-axes. Locate the origin on the coordinate system. Identify coordinates of a point on a coordinate system. Recognize and describe the connection between the ordered pair and the x- and y-axes from the origin.		

EXPLANATIONS AND EXAMPLES

Examples:

- Students can use a classroom size coordinate system to physically locate the coordinate point (5, 3) by starting at the origin point (0,0), walking 5 units along the x axis to find the first number in the pair (5), and then walking up 3 units for the second number in the pair (3). The ordered pair names a point in the plane.

- Graph and label the points below in a coordinate system.
  - A (0, 0)
  - B (5, 1)
  - C (0, 6)
  - D (2.5, 6)
  - E (6, 2)
  - F (4, 1)
  - G (3, 0)

GEOMETRY

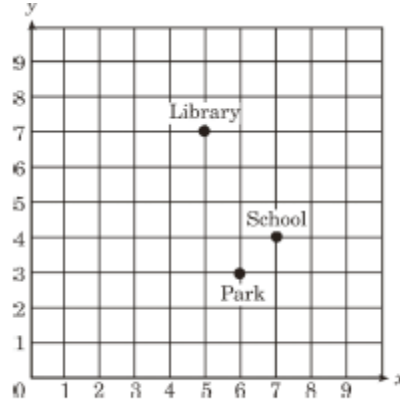## STANDARD AND DECONSTRUCTION

5.G.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

DESCRIPTION	This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms.

Example:
Using the coordinate grid, which ordered pair represents the location of the School?
Explain a possible path from the school to the library.

ESSENTIAL QUESTION(S)	How can you use the coordinate graph to solve problems?

MATHEMATICAL PRACTICE(S)	5.MP.1. Make sense of problems and persevere in solving them.
	5.MP.2. Reason abstractly and quantitatively.
	5.MP.4. Model with mathematics.
	5.MP.5. Use appropriate tools strategically.
	5.MP.6. Attend to precision.
	5.MP.7. Look for and make use of structure.

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

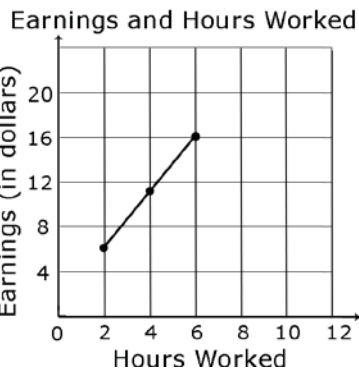| Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Graph points in the first quadrant.	Interpret coordinate values of points in real world context and mathematical problems. Represent real world and mathematical problems by graphing points in the first quadrant.	

EXPLANATIONS AND EXAMPLES

Examples:

- Sara has saved $20. She earns $8 for each hour she works.

  ⬧ If Sara saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours?

  ⬧ Create a graph that shows the relationship between the hours Sara worked and the amount of money she has saved.

  ⬧ What other information do you know from analyzing the graph?

- Use the graph below to determine how much money Jack makes after working exactly 9 hours.

Earnings and Hours Worked

STANDARD AND DECONSTRUCTION

5.G.3	**Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.**

| **DESCRIPTION** | This standard calls for students to reason about the attributes (properties) of shapes. Student should have experiences discussing the property of shapes and reasoning.

Example:

Examine whether all quadrilaterals have right angles. Give examples and non-examples.

The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do not appear until middle school.

TEACHER NOTE: In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (Progressions for the CCSSM: Geometry, The Common Core Standards Writing Team, June 2012.) |
|---|---|
| **ESSENTIAL QUESTION(S)** | How can the attributes of two-dimensional figures be organized? |
| **MATHEMATICAL PRACTICE(S)** | 5.MP.2. Reason abstractly and quantitatively.

5.MP.6. Attend to precision.

5.MP.7. Look for and make use of structure. |
| **DOK Range Target for Instruction & Assessment** | ☒ 1 ☒ 2 ☐ 3 ☐ 4 |

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize that some two-dimensional shapes can be classified into more than one category based on their attributes.

Recognize if a two-dimensional shape is classified into a category, that it belongs to all subcategories of that category. | | |

EXPLANATIONS AND EXAMPLES

Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line).

Example:

- If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms

A sample of questions that might be posed to students include:

- A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms?
- Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons.
- All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False?
- A trapezoid has 2 sides parallel so it must be a parallelogram. True or False?

Technology Connections:

http://illuminations.nctm.org/ActivityDetail.aspx?ID=70

STANDARD AND DECONSTRUCTION

5.G.4	Classify two-dimensional figures in a hierarchy based on properties.

DESCRIPTION

This standard builds on what was done in 4th grade.

Figures from previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, kite
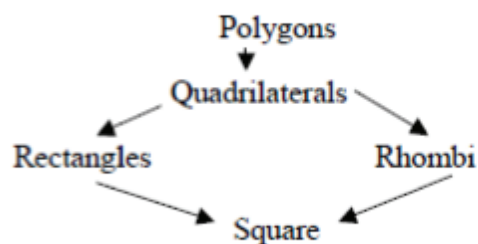
A kite is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (adjacent to) each other.

Example:

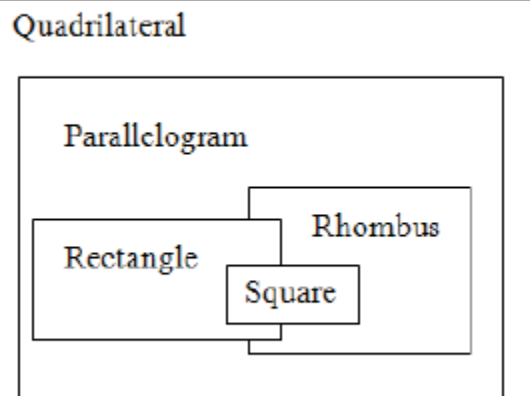Create a Hierarchy Diagram using the following terms:

polygons – a closed plane figure formed from line segments that meet only at their endpoints.
quadrilaterals - a four-sided polygon.
rectangles - a quadrilateral with two pairs of congruent parallel sides and four right angles.
rhombi – a parallelogram with all four sides equal in length.
square – a parallelogram with four congruent sides and four right angles.

Possible student solution:

quadrilateral – a four-sided polygon.
parallelogram – a quadrilateral with two pairs of parallel and congruent sides.
rectangle – a quadrilateral with two pairs of congruent, parallel sides and four right angles.
rhombus – a parallelogram with all four sides equal in length.
square – a parallelogram with four congruent sides and four right angles.

Possible student solution:

Student should be able to reason about the attributes of shapes by examining: What are ways to classify triangles? Why can't trapezoids and kites be classified as parallelograms? Which quadrilaterals have opposite angles congruent and why is this true of certain quadrilaterals?, and How many lines of symmetry does a regular polygon have?

TEACHER NOTE: In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (Progressions for the CCSSM: Geometry, The Common Core Standards Writing Team, June 2012.)

MATHEMATICS

ESSENTIAL QUESTION(S)	How can the attributes of two-dimensional figures be organized?

MATHEMATICAL PRACTICE(S)	5.MP.2. Reason abstractly and quantitatively. 5.MP.3. Construct viable arguments and critique the reasoning of others. 5.MP.5. Use appropriate tools strategically. 5.MP.6. Attend to precision. 5.MP.7. Look for and make use of structure.

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize the hierarchy of two-dimensional shapes based on their attributes.	Analyze properties of two-dimensional figures in order to place into a hierarchy. Classify two-dimensional figures into categories and/or sub-categories based on their attributes.	

EXPLANATIONS AND EXAMPLES	Properties of figure may include:

Properties of figure may include:
- Properties of sides—parallel, perpendicular, congruent, number of sides
- Properties of angles—types of angles, congruent

Examples:
- A right triangle can be both scalene and isosceles, but not equilateral.
- A scalene triangle can be right, acute and obtuse.

Triangles can be classified by:
- Angles
  - Right: The triangle has one angle that measures 90º.
  - Acute: The triangle has exactly three angles that measure between 0º and 90º.
  - Obtuse: The triangle has exactly one angle that measures greater than 90º and less than 180º.
- Sides
  - Equilateral: All sides of the triangle are the same length.
  - Isosceles: At least two sides of the triangle are the same length.
  - Scalene: No sides of the triangle are the same length.

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