COMMON CORE State Standards

DECONSTRUCTED for CLASSROOM IMPACT

HIGH SCHOOL ALGEBRA

MATHEMATICS

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Supporting Today's Educators, Shaping Tomorrow's Leaders
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Introduction

The Common Core Institute is pleased to offer this grade-level tool for educators who are teaching with the Common Core State Standards.

The Common Core Standards Deconstructed for Classroom Impact is designed for educators by educators as a two-pronged resource and tool 1) to help educators increase their depth of understanding of the Common Core Standards and 2) to enable teachers to plan College & Career Ready curriculum and classroom instruction that promotes inquiry and higher levels of cognitive demand.

What we have done is not all new. This work is a purposeful and thoughtful compilation of preexisting materials in the public domain, state department of education websites, and original work by the Center for College & Career Readiness. Among the works that have been compiled and/or referenced are the following: Common Core State Standards for Mathematics and the Appendix from the Common Core State Standards Initiative; Learning Progressions from The University of Arizona's Institute for Mathematics and Education, chaired by Dr. William McCallum; the Arizona Academic Content Standards; the North Carolina Instructional Support Tools; and numerous math practitioners currently in the classroom.

We hope you will find the concentrated and consolidated resource of value in your own planning. We also hope you will use this resource to facilitate discussion with your colleagues and, perhaps, as a lever to help assess targeted professional learning opportunities.

Understanding the Organization

The Overview acts as a quick-reference table of contents as it shows you each of the domains and related clusters covered in this specific grade-level booklet. This can help serve as a reminder of what clusters are part of which domains and can reinforce the specific domains for each grade level.

For each cluster, we have included four key sections: Description, Big Idea, Academic Vocabulary, and Deconstructed Standard.

The cluster **Description** offers clarifying information, but also points to the **Big Idea** that can help you focus on that which is most important for this cluster within this domain. The **Academic Vocabulary** is derived from the cluster description and serves to remind you of potential challenges or barriers for your students.

Math Fluency Standards	
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 Add/subtract within 100 (pencil & paper)
3	Multiply/divide within 100 Add/subtract within 1000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations
7	Solve $px + q = r$, $p(x + q) = r$
8	Solve simple 2 x 2 systems by inspection

Each standard specific to that cluster has been deconstructed. There **Deconstructed Standard** for each standard specific to that cluster and each **Deconstructed Standard** has its own subsections, which can provide you with additional guidance and insight as you plan. Note the deconstruction drills down to the sub-standards when appropriate. These subsections are:

- Standard Statement
- Standard Description
- Essential Question(s)
- Mathematical Practice(s)
- DOK Range Target for Learning and Assessment
- Learning Expectations
- Explanations and Examples

As noted, first are the **Standard Statement** and **Standard Description**, which are followed by the **Essential Question(s)** and the associated **Mathematical Practices**. The **Essential Question(s)** amplify the **Big Idea**, with the intent of taking you to a deeper level of understanding; they may also provide additional context for the **Academic Vocabulary**.

The **DOK Range Target for Learning and Assessment** remind you of the targeted level of cognitive demand. The **Learning Expectations** correlate to the DOK and express the student learning targets for student proficiency for KNOW, THINK, and DO, as appropriate. In some instances, there may be no learning targets for student proficiency for one or more of KNOW, THINK or DO. The learning targets are expressions of the deconstruction of the Standard as well as the alignment of the DOK with appropriate consideration of the Essential Questions.

The last subsection of the **Deconstructed Standard** includes **Explanations and Examples**. This subsection might be quite lengthy as it can include additional context for the standard itself as well as examples of what student work and student learning could look like. **Explanations and Examples** may offer ideas for instructional practice and lesson plans.

A wonderful resource for explanations and examples, which we often referred to and cited as a source in this tool, is www.shmooop.com.

Standards for Mathematical Practice in High School Mathematics Courses

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

PRACTICE	EXPLANATION
MP.1 Make sense and persevere in problem solving.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
MP.2 Reason abstractly and quantitatively.	Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

PRACTICE	EXPLANATION
MP.3 Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
MP.4 Model with mathematics.	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, 2-by-2 tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
MP.5 Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students interpret graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

PRACTICE	EXPLANATION
MP.6 Attend to precision.	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.
MP.7 Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as composed of several objects. For example, they can see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
MP.8 Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11, that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)(x – 1) = 3$.

Expressions

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and Inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x2 – 2 = 0 are real numbers, not rational numbers; and the solutions of x2 + 2 = 0 are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, A = ((b1+b2)/2)h, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Seeing Structure in Expressions (A-SSE)

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

Arithmetic with Polynomials and Rational Expressions (A-APR)

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

Creating Equations (A-CED)

- Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities (A-REI)

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Mathematical Practices (MP)

MP 1. Make sense of problems and persevere in solving them.
MP 2. Reason abstractly and quantitatively.
MP 3. Construct viable arguments and critique the reasoning of others.
MP 4. Model with mathematics.
MP 5. Use appropriate tools strategically.
MP 6. Attend to precision.
MP 7. Look for and make use of structure.
MP 8. Look for and express regularity in repeated reasoning.

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

High School Pathways for Traditional and Integrated Courses

	Domain	HS Algebra I	Mathematics I	Geometry
Number & Quantity	The Real Number System (RN)	RN. 1, 2, 3		
	Quantities (Q)	Q. 1, 2, 3	Q. 1, 2, 3	
	The Complex Number System (CN)			
	Vector Quantities and Matrices (VM)			
Algebra	Seeing Structure in Expressions (SSEE)	SSE. 1a, 1b, 2, 3a, 3b, 3c	SSE. 1a, 1b	
	Arithmetic with Polynomials and Rational Expressions (APR)	APR. 1		
	Creating Equations (CED)	CED. 1, 2, 3, 4	CED. 1, 2, 3, 4	
	Reasoning with Equations and Inequalities (REI)	REI. 1, 3, 4a, 4b, 5, 6, 7, 10, 11, 12	REI. 1, 3, 5, 6, 10, 11, 12	
Functions	Interpreting Functions (IF)	IF. 1, 2, 3, 4, 5, 6, 7a, 7b, 7c, 8a, 8b, 9	IF. 1, 2, 3, 4, 5, 6, 7a, 7c, 9	
	Building Functions (BF)	BF. 1a, 1b, 2, 3, 4a	BF. 1a, 1b, 2, 3	
	Linear, Quadratic, and Exponential Models (LE)	LE. 1a, 1b, 1c, 2, 3, 5	LE. 1a, 1b, 1c, 2, 3, 5	
	Trigonometric Functions (TF)			
Geometry	Congruence (CO)		CO. 1, 2, 3, 4, 5, 6, 7, 8, 12, 13	CO. 1-13
	Similarity, Right Triangles, and Trigonometry (SRT)			SRT. 1-11
	Circle (C)			C. 1-5
	Expressing Geometric Properties with Equations (GPE)		GPE. 4, 5, 7	GPE. 1, 2, 4-7
	Geometric Measurement and Dimension (GMD)			GMD. 1, 3, 4
	Modeling with Geometry (MG)			MG. 1, 2, 3
Statistic and Probability	Interpreting Categorical and Quantitative Data (ID)	ID. 1-3, 5-9	ID. 1-3, 5-9	
	Making Interference and Justifying Conclusions (IC)			
	Conditional Probability and the Rules of Probability (CP)			CP. 1-9
	Using Probability to Make Decisions (MD)			MD. 6-7

High School Pathways for Traditional and Integrated Courses

		Mathematics II	HS Algebra II	Mathematics III	4th Courses (T)	4th Courses (I)
Number & Quantity		RN. 1, 2, 3				
		CN. 1, 2, 7, 8, 9	CN. 1, 2, 7, 8, 9	CN. 8, 9	CN. 3, 4, 5, 6	CN. 3, 4, 5, 6
					VM. 1, 2, 3, 4 (a-c), 5(a-b), 6, 7, 8, 9, 10, 11, 12	VM. 1, 2, 3, 4 (a-c), 5(a-b), 6, 7, 8, 9, 10, 11, 12
Algebra		SSE.1a, 1b, 2, 3a, 3b, 3c	SSE. 1, 1b, 2, 4	SSE. 1, 1b, 2, 4		
		APR. 1	APR. 1-7	APR. 1-7		
		CED. 1, 2, 4	CED. 1, 2, 3, 4	CED. 1, 2, 4		
		REI. 4a, 4b, 7	REI. 2, 11	REI. 2, 11	REI. 8, 9	REI. 8, 9
Functions		IF. 4, 5, 6, 7a, 7b, 8a, 8b, 9	IF. 4, 5, 6, 7b, 7c, 7e, 8, 9	IF. 4, 5, 6, 7b, 7c, 7e, 8, 9	IF. 7d	IF. 7d
		BF. 1a, 1b, 3, 4a	BF. 1b, 3, 4a	BF. 1b, 3, 4a	BF. 1c, 4b, 4c, 4d, 5	BF. 1c, 4b, 4c, 4d, 5
		LE. 3	LE. 4	LE. 4		
		TF. 8	TF. 1, 2, 5, 8	TF. 1, 2, 5	TF. 3, 4, 6, 7, 9	TF. 3, 4, 6, 7, 9
Geometry		CO. 9, 10, 11				
		SRT. 1a, 1b, 2, 3, 4, 5, 6, 7, 8		SRT. 9, 10, 11		
		C. 1-5				
		GPE. 1, 2, 4			GPE. 3	GPE. 3
		GMD. 1, 3		GMD. 4	GMD. 2	GMD. 2
				MG. 1, 2, 3		
Statistic and Probability			ID. 4	ID. 4		
			IC. 1-6	IC. 1-6		
		CP. 1-9				
		MD. 6-7	MD. 6-7	MD. 6-7	MD. 1-5	MD. 1-5

Learning Progressions for Integrated Courses

Algebra			
Math 1	**Math 2**	**Math 3**	**Math 4**
Seeing Structure in Expressions (SSE)			
A.SSE.1(a-b)	A.SSE.1(a-b)	A.SSE.1(a-b)	
	A.SSE.2	A.SSE.2	
	A.SSE.3		
		A.SSE.4	
Arithmetic with Polynomials and Rational Expressions (APR)			
	A.APR.1	A.APR.1	
		A.APR.2	
		A.APR.3	
		A.APR.4	
		A.APR.5	
		A.APR.6	
		A.APR.7	
Creating Equations (CED)			
A.CED.1	A.CED.1	A.CED.1	
A.CED.2	A.CED.2	A.CED.2	
A.CED.3	A.CED.3	A.CED.3	
A.CED.4	A.CED.4	A.CED.4	
Reasoning with Equations and Inequalities (REI)			
A.REI.1			
		A.REI.2	
A.REI.3			
	A.REI.4(a-b)		
A.REI.5			
A.REI.6			
	A.REI.7		
			A.REI.8
A.REI.10			A.REI.9
A.REI.11		A.REI.11	
A.REI.12			

Learning Progressions for Traditional Courses

Algebra			
Algebra 1 (Grade 9)	**Geometry (Grade 10)**	**Algebra 2 (Grade 11)**	**PreCalculus (Grade 12)**
Seeing Structure in Expressions (SSE)			
A.SSE.1(a-b)		A.SSE.1(a-b)	
A.SSE.2		A.SSE.2	
A.SSE.3(a-c)			
		A.SSE.4	
Arithmetic with Polynomials and Rational Expressions (APR)			
A.APR.1		A.APR.1	
		A.APR.2	
		A.APR.3	
		A.APR.4	
		A.APR.5	
		A.APR.6	
		A.APR.7	
Creating Equations (CED)			
A.CED.1		A.CED.1	
A.CED.2		A.CED.2	
A.CED.3		A.CED.3	
A.CED.4		A.CED.4	
Reasoning with Equations and Inequalities (REI)			
A.REI.1			
		A.REI.2	
A.REI.3			
A.REI.4(a-b)			
A.REI.5			
A.REI.6			
A.REI.7			
			A.REI.8
			A.REI.9
A.REI.10			
A.REI.11		A.REI.11	
A.REI.12			

DOMAIN:

SEEING STRUCTURE IN EXPRESSIONS (A-SSE)

HIGH SCHOOL
ALGEBRA

MATHEMATICS

DOMAIN	Seeing Structure in Expressions (A-SSE)
CLUSTERS	1. **Interpret the structure of expressions.** 2. **Write expressions in equivalent forms to solve problems.**
ACADEMIC VOCABULARY	Algebra I absolute value, complement of an event, compound, conjunction, direct and inverse variation, disjunction, domain & range, exponential growth (and decay), interest (simple and compound), irrational numbers, joint and conditional probability, law of large numbers, mathematical model, measure of spread (range, interquartile range), midpoint formula, outlier, parent function, Pascal's triangle, polynomial (binomial, trinomial), quadratic formula (including discriminant), quantitative and qualitative data, radicand, rational expression, real number properties, real roots (zeros, solutions, x-intercepts), relative frequency, sequences (arithmetic, geometric, Fibonacci), simulations, subsets of real numbers Algebra II amplitude, asymptote, binomial theorem, combination, common ratio (geometric sequence), complete the square, complex conjugate, complex number, composition (of functions), conic sections (circles, parabola, ellipse, hyperbola), empirical rule, factorial, focus (pl. foci), independent and dependent events, inverse of a relation, logarithm, normal distribution, period, permutation, piece-wise function, radian measure, rational function, regression equation, series (arithmetic, geometric, finite, infinite, etc.), sigma, standard deviation, step function, synthetic division, transcendental function, trigonometric function, trigonometric identity, unit circle, variance

CLUSTER	1. **Interpret the structure of expressions.**
BIG IDEA	• **Expressions are used to show a mathematical relationship and to designate value.** • **Variables, expressions, and equations are algebraic representations of mathematical situations that dictate the unknown to be solved in real-world problems.** • **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.SSE.1	Interpret expressions that represent a quantity in terms of its context. (★)
DESCRIPTION	A.SSE.1a Identify the different parts of the expression and explain their meaning within the context of a problem. A.SSE.1b Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.
ESSENTIAL QUESTION(S)	How can the understanding of the parts of an expression lead to effective problem solving? How are terms, factors and coefficients related?
MATHEMATICAL PRACTICE(S)	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

SUBSTANDARD DECONSTRUCTED	A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define and recognize parts of an expression, such as terms, factors, and coefficients.	Interpret parts of an expression, such as terms, factors, and coefficients in terms of the context.	

SUBSTANDARD DECONSTRUCTED	A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:		Interpret complicated expressions, in terms of the context, by viewing one or more of their parts as a single entity.	

EXPLANATIONS AND EXAMPLES

Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context.

The only way that could be more general is if it said, "Do things to things in terms of other things." Luckily, we have just a smidgeon more to work with.

At its core, this standard wants students to start thinking of math as a language, not a pile of numbers. Just like any other language, math can help us communicate thoughts and ideas with each other, but students need to know the basics before they can really understand it.

Your students probably already have some idea of what an expression is in a general sense. Start from this point. At its simplest, an expression is a thought or idea communicated by language. In the same way, a mathematical expression can be considered a mathematical thought or idea communicated by the language of mathematics.

Emphasize that mathematics is a *language*, just as English, French, German, and Pig Latin are languages. Students should use the ocabulary-vay of athematics-may correctly to become fluent in it. After all, the best way to learn a new language is to immerse yourself in it.

EXPLANATIONS AND EXAMPLES

(a.) Example.

Just like English has nouns, verbs, and adjectives, mathematics has terms, factors, and coefficients. Well, sort of.

Students should know that **terms** are the pieces of the expression that are separated by plus or minus signs, except when those signs are within grouping symbols like parentheses, brackets, curly braces, or absolute value bars. Every mathematical expression has at least one term. For instance, the expression $3x + 2$ has two terms: $3x$ and 2. A term that has no variables is often called a constant because it never changes.

Within each term, there can be two or more **factors**, the numbers and/or variables multiplied together. The term $3x$ has two factors: 3 and x. There are always *at least* two factors, though one of them may be the number 1, which isn't usually written. But that 1 is always there...watching us.

Finally, a **coefficient** is a factor (usually numeric) that is multiplying a variable. Using the example, the 3 in the first term is the coefficient of the variable x.

The **order** or **degree** of a mathematical expression is the largest sum of the exponents of the variables when the expression is written as a sum of terms. For the example $3x + 2$, the order is 1, since the variable x in the first term has an exponent of 1 and there are no other terms with variables.

The expression $5x^2 – 3x + 2$ has order 2, whereas the expression $3xy + 5x^2y^3 – 7x + 32y^4$ has order 5, because the exponents of x and y in the second term are 2 and 3, respectively, and $2 + 3 = 5$. No other term has a higher exponent sum.

Now that we have our words, we can start putting them together and make expressions. A good way to see if students really understand an expression like $3x + 2$ is to have them translate mathematical expressions into English and vice versa. For instance, the expression $3x + 2$ could also be written as, "the sum of 3 times a number and 2," or, "2 more than three times a number."

Clearly, it's much easier to write the mathematical expression than to write it in English (not to mention Pig Latin). The two are directly related to each other, however, and students should be able to translate back and forth. At first, students might want to make use of a "dictionary" like the table below to help them go from one language to the other.

English	Mathematical	Example
a number, some number	a variable	$x, y, z, \rho, \delta, \lambda, \ldots$
sum of, plus, more than	$+$	the sum of a number and 1 : $b+1$ 10 more than a number : $w+10$
difference of, minus, less than	-	the difference of 5 and a number : $5-z$ 3 less than a number : $d-3$
product of, times	$\times, \cdot$	the product of 2 and a number : $2 \cdot g$ or $2g$ 20 times a number : $20 \times k$ or $20k$
quotient of, divided by	$\div$	the quotient of a number and 4 : $a \div 4$ or $\frac{a}{4}$ 1 divided by a number : $1 \div q$ or $\frac{1}{q}$
power of, raised to	exponentiation	the fifth power of a number : t^5 a number raised to the nth power : f^n
root of	$\sqrt[\cdots]{\ldots}$ (radical)	the third root of a number : $\sqrt[3]{m}$
reciprocal	$\frac{1}{\ldots}$	the reciprocal of a number : $\frac{1}{q}$
absolute value of	$\lvert\ldots\rvert$	the absolute value of a number : $\lvert x\rvert$
the quantity	$(\ldots)$	3 times the quantity "the sum of a number and 1" : $3(h+1)$

b). **Example.**

Let's consider a more complex expression: $5x – (2 – 4y)$. In English, this could be stated as "the difference between 5 times a number and the quantity 4 times another number less than 2."' That's a mouthful, and this expression isn't even that complex! It should be obvious why we do math in symbol notation now: it's much easier to write.

Notice how the English expression mentioned "a number" and "another number." This is a clue that two different variables must be used in the mathematical expression. These two variables might represent two different physical quantities in some situation, and the expression shows how each quantity contributes to the overall behavior.

How many terms are in that expression? Your students will probably say three, but there are only two the way the expression is written. The two terms inside the parentheses are treated as a single thing, so the first term is $5x$ and the second term is $-(2 – 4y)$. Since there is no number immediately following the minus sign in the second term, we assume the number is actually 1. So the second term could be written as $-1(2 – 4y)$. This should be interpreted as a -1 multiplying everything inside the parentheses.

Of course, we can have expressions, which have even more variables, if there are more changing or unknown quantities involved. For example, the compound interest expression $P(1 + r)^n$ has three variables: P, r, and n, each representing a different physical quantity. As written, this expression has only one term, consisting of two factors, P and $(1 + r)^n$. The first factor depends only on P, while the second depends on r and n.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.SSE.2	**Use the structure of an expression to identify ways to rewrite it.** ***For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$***		
DESCRIPTION	A.SSE.2 Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms. A.SSE.2 Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely. A.SSE.2 Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.		
ESSENTIAL QUESTION(S)	What strategies can be applied to create equivalent expressions?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Identify ways to rewrite expressions, such as difference of squares, factoring out a common monomial, and regrouping. Identify various structures of expressions.	Use the structure of an expression to identify ways to rewrite it. Classify expressions by structure and develop strategies to assist in classification.	

EXPLANATIONS AND EXAMPLES

Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further.

In English, we have many different ways of saying the same thing. “Stop that!” and “Stop eating that!” and “Stop eating my sandwich!” all mean the same thing, but it’s important to be able to know which one to use. Well, if your sandwich is being eaten, then any of those will probably work.

Math works much the same way. Writing mathematical expressions in different ways is incredibly important, especially in algebra. It’s not about redundancy; it’s about simplicity. And about keeping a firm grip on that sandwich of yours.

Students should be able to convert mathematical expressions to alternative but equivalent forms by factoring. This is important when students want to explain certain properties of an expression or the quantity which the expression represents, solve equations involving mathematical expressions, or simplify complex expressions. They might not want to at first, but if you tell them it’s on the chapter test, they’ll most likely be very interested.

Factoring is the conversion of a mathematical expression into an equivalent form that consists of a single term composed of nothing but terms multiplied together, called factors. Basically, it’s the opposite of the distributive property. Students should recall that the distributive property is:

$a(b + c) = ab + ac$

Note that the a has been *distributed* to the two terms inside the parentheses. Also note that we have taken an expression with one term and transformed it into an equivalent expression with more than one term. We’ve *expanded* the expression.

Factoring is the *reverse* process, so it can be stated as:

$ab + ac = a(b + c)$

We have converted from a two-term expression to a one-term expression which consists of nothing but factors, namely a and $(b + c)$. We repeat the process on any factors which can be factored further. Once we have factored everything which can be factored, the expression is said to be **fully factored.** Fancy that.

EXPLANATIONS AND EXAMPLES (continued)

The factor we pull outside the parentheses (a, in our previous example) is called the **greatest common factor** or **GCF** if it is the *largest* factor that is common to all of the terms. The GCF can be a number, a variable, a multi-term expression, or any combination of these.

The GCF technique is the very first thing that should be checked when confronted with any factoring problem. Doing so will often simplify subsequent factoring steps. Strong multiplication and division skills are critical to being efficient at GCF determination, so it may be helpful to go over divisibility rules with your students. (All even numbers are divisible by 2, if the sum of a number's digits is divisible by 3 then the number itself is divisible by 3, and so on.)

Factoring by grouping is a variant of the GCF technique. In this case, we have to group certain terms together, then treat each grouping as a GCF problem. We continue that process until we just can't go any further.

For instance, we can factor the expression $8x^2 - 4x - 40x + 20$ by a common numerical factor of 4. There is no common variable factor, though. We can factor this as $4(2x^2 - x - 10x + 5)$. What about that mess inside the parentheses?

If we group the first two terms together, we see they have a common factor of x, so they can be rewritten as x(2x – 1). The last two terms have a common factor of -5. They can then be written as $-5(2x - 1)$. Putting this all together we get the following:

$4[x(2x - 1) - 5(2x - 1)]$

Our four-term expression is now a two-term expression. These two terms have a common factor of (2x – 1), however, so we can factor further:

$4(2x - 1)(x - 5)$

This is the fully factored form.

As important as GCF factorizations are, students should know how to factor expressions of a higher order. Yes, we're talking about exponents.

Students should know that expressions of order two are called quadratic expressions, and that they can be written in the form ax2 + bx + c, where a, b, and c are numbers and a ≠ 0. (Ask the students to explain why that restriction is necessary.)

Factoring quadratics is a little more challenging, so it helps to examine the factoring process in reverse. Consider the expression $(2x - 5)(3x + 2)$. Clearly, this is a factored form of a quadratic expression. If we use the distributive property to expand the expression we would get

$(2x - 5)(3x + 2)$

$2x(3x + 2) - 5(3x + 2)$

$6x^2 + 4x - 15x - 10$

$6x^2 - 11x - 10$

The last expression is the quadratic expression written in standard form. We can identify $a = 6$, $b = -11$, and $c = -10$.

Notice that if we run this distributive process in reverse, it's like taking the standard form quadratic and splitting up the middle term into two pieces and then using the grouping technique. How would we know to break up the middle term in that way, though? There are many ways to write -11 as the sum of two numbers, but only one will give us what we need.

How would we know to pick +4 and -15? Notice that the product of +4 and -15 is -60. This is also the product of the first and last coefficients in the standard form, a and c: (6)(-10) = -60. So, one might assume we multiplied a and c together, then found two numbers which multiply together to get that number but add together to give b. If you study several examples, you'll see this is correct. This is always the case. So the steps in the general technique for factoring quadratics which can be factored are:

1. Write the quadratic in standard form.
2. Multiply a and c together.
3. Find two numbers which multiply to produce the number in step 2, but add to produce b. Be careful with signs!
4. Rewrite the quadratic, but replace the middle term with two terms using the two numbers from step 3.
5. Use the grouping technique to factor the expression.

Students should know that this general technique works for all factorable quadratics, but that there are some special cases to consider. For instance, the difference of two squares $x^2 - y^2$ can be factored as $(x - y)(x + y)$. Also, the perfect trinomial $x^2 + 2^{ax} + a^2$ can be factored to equal $(x + a)^2$. These two work even when x, y, and a are quantities made of multiple terms. That's pretty nifty.

EXPLANATIONS AND EXAMPLES (continued)

In general, students should follow these rules:

1. Determine if there is a greatest common factor in the expression. You typically want to factor that out first, as it leaves a simpler expression to concentrate on in subsequent steps.
2. Determine if it is possible to use the grouping technique.
3. If the expression is a quadratic, put it in standard form first.
4. Examine the form of the expression and determine if one of the special forms—the difference of two squares or the perfect square trinomial—applies. If so, apply the rules given previously for writing the factored form.
5. If the quadratic is not a special form, use the general method for factoring quadratics.

Factoring takes a lot of practice, but it is probably the single most important skill students will develop in math. It is critical to just about every topic in higher levels of math, so it is worthwhile to spend a fair amount of time developing this skill. Once students start getting the hang of it, many of them will be able to just "see" the factors. Encourage them to always look for patterns, to pay attention to the individual pieces of the expressions they get and how they appear to relate to the pieces of the original expression.

(Source: www.shmoop.com)

Example:

Factor $x^3 - 2x^2 - 35x$

In the traditional pathway, linear, quadratic, and exponential expressions are the focus in Algebra I, and integer exponents are extended to rational exponents (only those with square or cubed roots). In Algebra II, the expectation is to extend to polynomial and rational expressions.

In the international pathway, CCSS Mathematics I focuses on linear expressions and exponential expressions with integer exponents. CCSS Mathematics II extends exponential expressions from the first course and includes quadratic expressions. Course II also extends integer exponents to rational exponents, focusing on those with square or cubed roots. In CCSS Mathematics III the expectation is polynomial and rational expressions.

CLUSTER	**2. Write expressions in equivalent forms to solve problems.**
BIG IDEA	**• Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.SSE.3	**Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (★)**
DESCRIPTION	A.SSE.3a Write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros. Given a quadratic function explain the meaning of the zeros of the function. That is if $f(x) = (x – c) (x – a)$ then $f(a) = 0$ and $f(c) = 0$. Given a quadratic expression, explain the meaning of the zeros graphically. That is for an expression (x – a) (x – c), a and c correspond to the x-intercepts (if a and c are real). A.SSE.3b Write expressions in equivalent forms by completing the square to convey the vertex form, to find the maximum or minimum value of a quadratic function, and to explain the meaning of the vertex. A.SSE.3c Use properties of exponents (such as power of a power, product of powers, power of a product, and rational exponents, etc.) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay.

ESSENTIAL QUESTION(S)	What strategies can be applied to create equivalent expressions that lead to accuracy and efficiency? What strategies can be used to factor a quadratic equation? What does completing the square in a quadratic expression reveal? What is the most efficient way to transform expressions for exponential functions?
MATHEMATICAL PRACTICE(S)	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

SUBSTANDARD DECONSTRUCTED	**A.SSE.3a. Factor a quadratic expression to reveal the zeros of the function it defines.**		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Explain the connection between the factored form of a quadratic expression and the zeros of the function it defines. Explain the properties of the quantity represented by the quadratic expression. Factor a quadratic expression to produce an equivalent form of the original expression.	Choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the original expression.	

SUBSTANDARD DECONSTRUCTED	**A.SSE.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.**		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Explain the connection between the completed square form of a quadratic expression and the maximum or minimum value of the function it defines. Complete the square on a quadratic expression to produce an equivalent form of an expression.	Explain the properties of the quantity or quantities represented by the transformed exponential expression. Choose and produce an equivalent form of an exponential expression to reveal and explain properties of the quantity represented by the original expression.	

SUBSTANDARD DECONSTRUCTED	**A.SSE.3c. Use the properties of exponents to transform expressions for exponential functions.** ***For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.***		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use the properties of exponents to transform simple expressions for exponential functions.	Choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the original expression.	

EXPLANATIONS AND EXAMPLES

Students will use the properties of operations to create equivalent expressions.

Your students should be gaining fluency in mathematics, and be able to write and rewrite expressions. The spells of nausea, hyperventilation, and paranoia surrounding the beautiful language of mathematics should have subsided by now. If they haven't, you may want to consult the school nurse.

Rather than rewriting expressions for the fun of it (and it is fun, isn't it?), students should understand what these different expressions can tell us about the quantities they represent. These mathematical expressions can tell us the zeros (or roots or x-intercepts) and the maximum and minimum values of a function, and have plenty of other applications in the real world. Most of them involving money.

(a.) Example.

Students should already be comfortable with finding the zeros of functions. In other words, finding the x values that make the mathematical expression equal to 0. For linear expressions like $5x - 10$, it's relatively easy to find the zeros. All we have to do is set the expression to equal 0 and solve for x. In this case, x = 2. Duh.

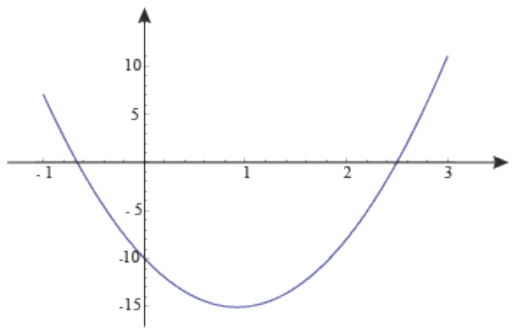
But what about quadratic expressions that aren't that simple? The simplest way to find roots for higher order expressions is by factoring. We can actually demonstrate that using our linear expression. Recall that we set our linear expression equal to 0 in order to find the root. Mathematically, we had $5x - 10 = 0$. Your students most likely added 10 to both sides and then divided by 5, but let's give factoring a shot.

We can factor that expression by pulling out 5 (the coefficient of x) as the GCF. That would yield the factored form $5(x - 2) = 0$. We've turned our two-term expression into a one-term expression which has two factors, 5 and $(x - 2)$. The product of those factors is zero, however. By the zero product rule, at least one of the factors must be zero. Well, obviously, 5 can never be 0. Therefore, $x - 2$ must be equal to zero. If we solve the equation $x - 2 = 0$, we get x = 2, which is the answer we got before.

So how does this work for a quadratic? When finding the zeros of a quadratic equation, the best thing to do is try and factor. We can take an expression like $6x^2 - 11x - 10$ and turn it into $(2x - 5)(3x + 2)$. To find the zeros of the expression, we set this factored form equal to zero, then apply the zero product rule. In this case, both factors contain variable expressions, so we must consider that either of the factors may be equal to zero, or both may be equal to zero simultaneously. A common mistake students make is to set the variable equal to zero, but that's not right! We must set each factor equal to zero.

EXPLANATIONS AND EXAMPLES (continued)

So we have to solve two first order equations: $2x - 5 = 0$ and $3x + 2 = 0$. The first one gives. The first one gives x=5/2. The second one gives x=2/3. There should be two roots, since this is a second order expression, and these are the two roots we are looking for. The equation $y = 6x^2 - 11x - 10$ has two x-intercepts! The graph of that equation looks like this:



It crosses the x-axis two times, and at the locations we calculated. This curve is called a parabola. All quadratic expressions of this type have the same basic shape.

Sometimes, quadratics have only one x-intercept and other times they don't have any real roots at all. It just means the function doesn't cross the *x-axis*, or that it crosses the *x-axis* once (at the minimum or maximum). Make sure your students don't have a panic attack when that happens.

(b) Example.

Before you talk about completing the square, remind your students about the perfect square trinomial. In general form, a perfect square trinomial is a quadratic of the form $m^2 \pm 2mn + n^2$, which can be factored into the form $(m \pm n)(m \pm n) = (m \pm n)^2$.

Notice that half of the middle term is equal to the square root of the first term times the square root of the last term. This is a requirement for a quadratic to be a perfect square trinomial. Not all quadratics are in this form. Some algebraic manipulation can be used to put any quadratic equation into a form where one side is a perfect square trinomial. That's called completing the square.

What about the equation $y = x^2 - 4x + 3$? This quadratic is factorable as $(x - 1)(x - 3)$. Clearly not a perfect square trinomial, but no one's perfect, right?

First, subtract 3 from both sides to get $y - 3 = x^2 - 4x$. Now, we can add whatever we want to the right-hand side, as long as we do exactly the same thing to the left-hand side. We want a 4 where the 3 used to be (because 4 is a perfect square of 2), so we get $y - 3 + 4 = x^2 - 4x + 4$. The right-hand side is now in the proper perfect square trinomial form, and can be factored as $(x - 2)^2$. Our equation is then $y + 1 = (x - 2)^2$.

For every x-value we plug into the right hand side of the equation, there is a corresponding y-value on the left-hand side. When $x = -2$, y must be equal to 15, for example. Notice that the right hand side of the equation is never negative, since we are squaring a number, and that always produces a positive number.

The smallest thing the right-hand side can possibly be is 0 (when $x = 2$). This is going to translate to the smallest possible value that y can be ($y + 1 = 0$, which means $y = -1$). In mathematical terms, that point represents the minimum of the function. In this case, the point (2, -1) represents the minimum point of our parabola, which is called the vertex.

Every quadratic equation has either a minimum or a maximum, depending on whether it points up or down (whether it looks like a happy face or a sad face). Notice that we could not have easily found the coordinates of the vertex from $y = (x - 1)(x - 3)$. We had to put the quadratic in a different form that highlighted the coordinates of the vertex in order to extract this information.

(c) Example.

At this point, the students should have a firm understanding of the concept of an exponent—a power to which some other quantity is raised. Up until this point, however, exponents probably have always been numbers. Students may not be aware that exponents can be variables, too.

Consider an expression like r^n. The r is identified as the base of the expression, while the n is identified as the exponent. Since both r and n are variables, this is a more general type of expression involving exponents than something like r^2 or x^{10}, where the base is the variable and the exponent is a number. These general expressions are known as (surprise, surprise) exponential expressions, while the others are usually referred to as power law expressions.

EXPLANATIONS AND EXAMPLES (continued)

Students should know that the rules for manipulating power law expressions apply for exponential expressions too. The following table summarizes those rules:

Rule	*Action*	*Example*
product	add the exponents	$r^m \cdot r^n = r^{m+n}$
quotient	subtract the exponents	$\frac{r^m}{r^n} = r^{m-n}$
power	multiply the exponents	$(r^m)^n = r^{mn}$
root	divide the exponent by the root index	$\sqrt[n]{r^m} = (\sqrt[n]{r})^m = r^{\frac{m}{n}}$

In multiplying and dividing exponential expressions, just as in power law expressions, the bases have to be the same. We can use the rules for exponential expressions to transform them into alternate forms, allowing us to interpret the expressions in different ways.

Everyone loves earning money, so let's find out ways to do that. No, we don't mean playing the lotto, because that's all chance and probability. We want exponential growth, so we can earn some dinero the good old-fashioned way: investing.

The compound interest expression allows us to calculate the new value of an investment after a certain period of time has elapsed if interest is earned at a certain rate for each period of time. It sounds really complicated, but it looks pretty simple: $P(1 + r)^n$. In this expression, P is the initial amount or present value, r is the interest rate for each period expressed in decimal form, and n is the number of periods over which the new value is to be calculated.

Let's focus on the second factor of that expression: $(1 + r)^n$. As an example, let's say we open a savings account which offers a rate of 15% per year. In other words, our money grows by 15% each year. This is represented by the mathematical expression $(1 + 0.15)^n = (1.15)^n$, where n is the number of years over which we allow this account to grow.

What if we were thinking about switching banks, and they offered a savings account, but expressed their rates on a per month basis rather than a per year basis? Obviously, we want to get the highest rate we can, since that's how we make more money. How could we compare the two?

We use the laws of exponentials. If we want our periods to be in months, we have to transform our exponential expression to be on a per month basis. There are 12 months in a year. So, if the account accrues interest for *n* years, it accrues interest for 12*n* months. The exponent of our transformed expression must be 12*n*.

We can't just change the exponent without also changing the part inside the parentheses, though, because we would be creating a completely different expression. To balance multiplying the exponent by 12, we have to divide any exponent inside the parentheses by 12. Even though it isn't written, there is an exponent there, since $1.15^1 = 1.15$. So, the new exponent inside the parentheses must become $\frac{1}{12}$, and the transformed exponential expression is $(1.15^{\frac{1}{12}})^{12n}$.

Using the rule that when raising an exponential expression to a power you multiply the exponents, the students can verify that we get the original expression back. We use a calculator to evaluate $1.15^{\frac{1}{12}} \approx 1.012$. The final expression, in terms of monthly periods is $(1.012)^{12n} = (1 + 0.012)^{12n}$. If the annual rate is 15%, the equivalent monthly rate is approximately 1.2%, and we can now compare this to what the other bank offers.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.SSE.4	**Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* (★)**		
DESCRIPTION	A.SSE.4 Develop the formula for the sum of a finite geometric series when the ratio is not 1. A.SSE.4 Use the formula to solve real world problems such as calculating the height of a tree after n years given the initial height of the tree and the rate the tree grows each year. Calculate mortgage payments.		
ESSENTIAL QUESTION(S)	Why can rewriting exponential expressions in another form lead to efficiency when solving a problem?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Find the first term in a geometric sequence given at least two other terms. Define a geometric series as a series with a constant ratio between successive terms. Use the formula S + a (1-rn)/(1-r) to solve problems.	Derive a formula [i.e. equivalent to the formula S + a (1-rn)/(1-r)] for the sum of a finite geometric series (when the common ratio is not 1).	

EXPLANATIONS AND EXAMPLES

Although students may not know it, they already know a lot about sequences and series. Not only are sequences and series on TV, in music, and all over the Internet, they were also taught between naptime and making those dried macaroni picture frames. You could have a sequence as easy as one, two, three.

In mathematics, a **sequence** is a bunch of numbers listed one after the other. In many cases, it is possible to determine a particular member of a sequence simply from its location, just like when counting natural numbers. That sequence starts with the number 1 and every other member of the sequence can be found by taking the previous member of the sequence and adding 1 to it.

Students should know the difference between an **arithmetic sequence** and a **geometric sequence**. In arithmetic sequences, successive members have a *common difference*. That is, the difference between each member and the one before it is some constant value. In geometric sequences, all numbers have a *common ratio*, meaning that the quotient of each member and the one before it is some constant value.

It's best to give several examples of both **arithmetic** and **geometric sequences** and series, and then finally give a formula for each. For instance, if a is the first term in an arithmetic series and the common difference is d, then the series would be the sum of $a, a + d, a + 2d, a + 3d$, and so on. For a geometric sequence with common ratio r, we'll have $a, ar, ar^2, ar^3, ar^4, ar^5$, and so on.

Notice that the first member of the sequence has no factors of r. The second member has 1 factor of r, the third has 2 factors of r, the fourth has 3 factors of r, and so on. The number of factors of r is always one less than the number of that particular term in the sequence. If n represents the number of the nth member of the sequence, then that number has a value of ar^{n-1}.

Students should know that a series is the sum of the members of a sequence. We can form sums of both arithmetic and geometric sequences. For an n-term geometric sequence, we can write the series as $sn = a + ar + ar^2 + ar^3 + ar^4 + ar5 + ... + ar^{n-1}$.

EXPLANATIONS AND EXAMPLES (continued)

This is all fine if we have 6 or 7 terms, but imagine trying to use this to calculate a series with 100 terms. Or 200 terms. Or an infinite number of terms. That would be an awful, not to mention time-consuming calculation to do by hand. Luckily, we don't have to, because we can perform a little magic to greatly simplify this expression. (It's actually not magic, but don't tell your students that. Some of them are still waiting for their invitation to Hogwarts, and this is the closest they'll ever get.)

Let's take the expression for the series and multiply it by the common ratio, r.

$rs_n = ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6 + ... + ar^n$

Notice that the number of factors of r in each term in the sum increased by 1. Now, let's take our original expression for s and subtract this new expression from it. There is nothing that says we can't do that, right?

$s_n - rs_n = (a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ... + ar^{n-1}) - (ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6 + ... + ar^n)$

Notice that except for the very first term in the expression for s and the very last term in the expression for rs, there are matching terms in the two sets of parentheses. That means all of those terms cancel each other out, and we're left with $s_n - rs_n = a - ar^n$.

We can then factor the left-hand side of that equation as $sn(1 - r)$. Dividing both sides by $1 - r$ and factoring out the a in the numerator on the right side gives us a much simpler way to find the sum of a geometric series.

$$s_n = \frac{a - ar^n}{1 - r} = a\frac{1 - r^n}{1 - r}$$

Your students should be applauding right about now. We know it's tough to compete with Hogwarts, but come on. This math is clearly not meant for mere muggles.

We do have to be careful about one thing, though. Notice that we divide by $1 - r$. If $r = 1$, we have a serious problem since that's dividing by 0. So we have to place a restriction on this formula, since using it when $r = 1$ might bring back He-Who-Must-Not-Be-Named...again.

Geometric series appear in many places, but one case which touches just about everyone is in the area of finance and economics. Calculations involving loan payments (for a mortgage, for instance) often make use of geometric series.

Suppose you took out a $200,000 loan for a dragon. It sounds expensive, but it's worth it if you're a frequent flyer. Plus, *it's a dragon*.

The beastly $200,000 loan has a 30-year fixed interest rate of 3.6% per year paid monthly. By convention, this means that each month, an interest rate of $0.036 \div 12 = 0.003$, or 0.3% will be applied to the unpaid principal to calculate the interest payment for that month. This will continue for each month until the term of 30 years has been reached, which is $(12)(30) = 360$ months. If m is the monthly amount, P is the principal amount on the loan, r is the rate, and n is the number of months, we can use the following formula to find the monthly amount.

$$m = \frac{Pr(1 + r)^n}{(1 + r)^n - 1}$$

There are calculators online, which will do this sort of calculation for you, but it is nice to understand how the calculation is done so your students can verify it for themselves. As far as dragons go, nobody wants to pay more than they have to. On second thought, that probably applies to just about anything.

(Source: www.shmoop.com)

DOMAIN:

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS (A-APR)

HIGH SCHOOL
ALGEBRA

MATHEMATICS

DOMAIN	Arithmetic with Polynomials and Rational Expressions (A-APR)
CLUSTERS	1. **Perform arithmetic operations on polynomials.** 2. **Understand the relationship between zeros and factors of polynomials.** 3. **Use polynomial identities to solve problems.** 4. **Rewrite rational expressions.**
ACADEMIC VOCABULARY	Algebra I absolute value, complement of an event, compound, conjunction, direct and inverse variation, disjunction, domain & range, exponential growth (and decay), interest (simple and compound), irrational numbers, joint and conditional probability, law of large numbers, mathematical model, measure of spread (range, interquartile range), midpoint formula, outlier, parent function, Pascal's triangle, polynomial (binomial, trinomial), quadratic formula (including discriminant), quantitative and qualitative data, radicand, rational expression, real number properties, real roots (zeros, solutions, x-intercepts), relative frequency, sequences (arithmetic, geometric, Fibonacci), simulations, subsets of real numbers Algebra II amplitude, asymptote, binomial theorem, combination, common ratio (geometric sequence), complete the square, complex conjugate, complex number, composition (of functions), conic sections (circles, parabola, ellipse, hyperbola), empirical rule, factorial, focus (pl. foci), independent and dependent events, inverse of a relation, logarithm, normal distribution, period, permutation, piece-wise function, radian measure, rational function, regression equation, series (arithmetic, geometric, finite, infinite, etc.), sigma, standard deviation, step function, synthetic division, transcendental function, trigonometric function, trigonometric identity, unit circle, variance

CLUSTER	1. **Perform arithmetic operations on polynomials**
BIG IDEA	• **The operations of addition, subtraction and multiplication can be applied to polynomials.** • **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.APR.1	**Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.**
DESCRIPTION	A.APR.1 Understand the definition of a polynomial. A.APR.1 Understand the concepts of combining like terms and closure. A.APR.1 Add, subtract, and multiply polynomials and understand how closure applies under these operations. Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x. In the traditional pathway, linear and quadratic polynomial expressions are the expectation in Algebra I and beyond quadratic polynomial expressions is the expectation for Algebra II. In the international pathway, CCSS Mathematics II places emphasis on polynomials that simplify to quadratics and CCSS Mathematics III extends beyond quadratics.

ESSENTIAL QUESTION(S)	What strategies can be used to perform arithmetic operations on polynomials?		
MATHEMATICAL PRACTICE(S)	HS.MP.8. Look for regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define closure. Identify that the sum, difference, or product of two polynomials will always be a polynomial, which means that polynomials are closed under the operations of addition, subtraction, and multiplication. Apply arithmetic operations of addition, subtraction, and multiplication to polynomials.		

EXPLANATIONS AND EXAMPLES

Students should understand that polynomials, like integers, are "closed" when it comes to addition, subtraction, and multiplication. Basically, this just means they're kind of cliquey as far as these operations are concerned.

An integer plus an integer is an integer, an integer minus an integer is an integer, and an integer times an integer is an integer. Similarly, a polynomial plus a polynomial is a polynomial, a polynomial minus a polynomial is a polynomial, and a polynomial times a polynomial is a polynomial. If that isn't cliquey, we don't know what is.

Students should know that a **polynomial** is any expression that is a combination of more than one term via addition or subtraction. Each individual term is called a **monomial**. Monomials can be constant (like single numbers) or include variables to different degrees (like x6). As long as it's in one lump with no plus or minus signs, it's a monomial.

Some examples of polynomials include:

- $x + 4$
- $x^2 + 2$
- $2x^2 - 3x + 5$
- $x^6 + 4x^5 - 3x^2 + x$

POLYNOMIALS & RATIONAL EXPRESSIONS

EXPLANATIONS AND EXAMPLES (continued)

Polynomials are really nice to work with because they're continuous and defined for all values. In other words, we can replace x with any real number, and we'll get a real number as our result. Just don't rub it in Pinocchio's ever-growing nose. He's always wanted to be real.

For example, take the polynomial $x^3 + 2^x – 5$. Input any value of x, like 6, and we'll get a real number, like $(6)^3 + 2(6) – 5 = 223$. Students should know that adding, subtracting, and multiplying two or more polynomials together will give them a polynomial. A different polynomial, but still a polynomial.

However, polynomials are not closed (so they're…open?) under division because sometimes the quotient won't be another polynomial. Take this quotient of polynomials, for example:

$$\frac{x^3 + 4x^2 + 2x - 5}{x^2 + 3x + 1}$$

This is a rational expression, not a polynomial. Somehow, polynomials seem a lot more rational.

Definitions are crucial for students to understand before learning how to actually perform operations on polynomials.

Students should know that when adding and subtracting polynomials, we can only combine like terms with like terms. Constants can only be added to constants, x terms can only be added to x terms, x^2 terms can only be added to x^2 terms, and so on.

If addition and subtraction are like OCD, meticulously pairing terms that go together, then multiplication is like ADHD, combining any and all terms together in one big dog pile regardless of what they are.

Students should know that multiplying a polynomial by a monomial means distribution, and that multiplying two polynomials together means a lot of distribution. More specifically, we have to make sure to multiply every term in one polynomial by every term in the other polynomial.

For instance, students performing the operation $(x + 2) \times (x^3 + x – 7)$ should know to first distribute the x to get $x^4 + x^2 – 7x$, and then the 2 to get $2x^3 + 2x – 14$, and then to add the two together so that the final answer is $x^4 + 2x^3 + x^2 – 5x – 14$.

When two binomials are multiplied together, like $(x + 1)(x + 3)$, most students prefer to remember the acronym FOIL, which stands for multiplying the First, Outer, Inner, and Last numbers together.

The best way to get students to feel comfortable is to have them practice, but have them stick to operations like addition, subtraction, and multiplication at first. They'll get to more complicated operations such as lobotomies and appendectomies later…like, after med school..

(Source: www.shmoop.com)

CLUSTER	**2. Understand the relationship between zeros and factors of polynomials.**
BIG IDEA	• **The operations of addition, subtraction and multiplication can be applied to polynomials.** • **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.APR.2	**Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by *x – a* is *p(a)*, so *p(a)* = 0 if and only if *(x – a)* is a factor of p(x).**
DESCRIPTION	A.APR.2 Understand and apply the Remainder Theorem. A.APR.2 Understand how this standard relates to A.SSE.3a. A.APR.2 Understand that a is a root of a polynomial function if and only if x-a is a factor of the function.

ESSENTIAL QUESTION(S)	How are zeros and factors of a polynomial related?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define the remainder theorem for polynomial division and divide polynomials. Given a polynomial p(x) and a number a, divide p(x) by (x – a) to find p(a) then apply the remainder theorem and conclude that p(x) is divisible by x – a, if and only if p(a) = 0.		

EXPLANATIONS AND EXAMPLES

Before introducing remainders of polynomials to your students, be sure they remember what a remainder is. No, not a reindeer. A remainder.

For example, tell them to divide 13 by 4. If they've gotten this far in math, this really shouldn't be a problem for them. Most of them will give you the answer 3.25. Instead, backtrack a few years of schooling and tell them to write their answer as "3 remainder 1." It's the same thing, only for third graders.

If we divide two integers, sometimes they make another integer ($6 \div 2 = 3$), and other times they have remainders ($13 \div 4 = 3$ remainder 1). A remainder of 0 means that the second number is a factor of first number. For instance, 2 is a factor of 6 because we can multiply 2 by an integer to get 6.

Once they've gotten over the intense déjà-vu, slowly and gently explain to them that polynomials are the same way. If dividing polynomial p(x) by x – a has a remainder of 0, we'll know that $x - a$ is a factor of $p(x)$. In other words, $p(x) = q(x) \times (x - a)$ where $q(x)$ is a polynomial or an integer.

A remainder of 0 also means that if we substitute a for x, we'll end up with $p(a) = 0$ regardless of what $q(a)$ is. That's because $p(a) = q(a) \times (a - a) = q(a) \times 0$. Hopefully, by now your students know that anything multiplied by zero is zero.

If $p(x)$ divided by $x - a$ has a remainder, we can write the equation as $p(x) = q(x) \times (x - a) + r(x)$, where $r(x)$ is the remainder. Sometimes $r(x)$ is a number and other times it's another polynomial. If we set $x = a$ in this situation, we'll end up with $p(a) = r(a)$. Would you look at that?

Essentially, any polynomial $p(x)$ can be written as a product of $(x - a)$ and some quotient $q(x)$, plus the remainder $p(a)$.

$p(x) = q(x) \times (x - a) + p(a)$

When $p(a) = 0$, $(x - a)$ is a factor of $p(x)$.

EXPLANATIONS AND EXAMPLES (continued)	
	Students should know how to perform all these calculations and rearrangements. For their answers to be at least somewhat reasonable, they should be comfortable with factoring polynomials, finding remainders, and of course, division. You'd think that last part goes without saying, but you never know. If students find these tasks particularly confusing, we suggest practicing with integers. That way, students can perform these same calculations within their comfort zones. Plus, they can check their answers, too. For instance, $8/6$ = 1 remainder 2 Converting this into the formula, we get 8 = 1(6) + 2 because 8 is like *p(x)*, 1 is like *q(x)*, and 2 is like *p(a)*. Make sense? Now get them to try some polynomials instead. *(Source: www.shmoop.com)*

STANDARD AND DECONSTRUCTION

A.APR.3	**Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.**		
DESCRIPTION	A.APR.3 Find the zeros of a polynomial when the polynomial is factored. A.APR.3 Use the zeros of a function to sketch a graph of the function.		
ESSENTIAL QUESTION(S)	How can a graph of a function be estimated based on the zeros and factors of a polynomial?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Factor polynomials using any available method. Create a sign chart for a polynomial f(x) using the polynomial's x-intercepts and testing the domain intervals for which f(x) greater than and less than zero. Use the x-intercepts of a polynomial function and the sign chart to construct a rough graph of the function.		

EXPLANATIONS AND EXAMPLES

Graphing calculators or programs can be used to generate graphs of polynomial functions.

In plain English, this standard is all about factoring polynomials and finding their zeros. It's about time too, because it can be frustrating dealing with students who claim that $x3 - 10x2 - 2x + 24$ has 1 zero because of 10x2. So frustrating that you may or may not want to quit this teaching gig and get a job at the local circus. At least they'd take you more seriously there.

First, your students should know that finding the zeros of a polynomial doesn't mean counting how many zeros they can find. That's not algebra. That's counting.

The zeros of a polynomial are the x values when we set the polynomial itself to equal zero. In other words, when we plug in any of the zeros of a polynomial in for x, our answer should be 0. So the zeros of $x^3 - 10x^2 - 2x + 24$ are the x values that make the equation $x^3 - 10x^2 - 2x + 24 = 0$ true.

Why are these zero values important? On the coordinate plane, they're the places where the function crosses the *x-axis*. The zeros of the polynomial (also called the solutions or "roots") are the *x-intercepts* of the graph.

To find the zeros, students should factor polynomials into linear factors (where x is only to the first power). Then, setting each linear factor equal to 0 and solving for the variable will give us the *x-intercepts*, which can be used to graph the function.

Students should also have a rough idea of how different polynomials should look when graphed. Linear functions make lines, quadratic functions make parabolas, and so on. They don't have to know exactly which term affects which aspect of the graph, but their answers shouldn't look like a game of connect-the-dots.

For example, we're told to graph the function defined by the polynomial $x^2 + 5x + 6$. Students should feel comfortable factoring this into a product of two linear factors: $(x + 2)(x + 3)$. Solving the equation $x^2 + 5x + 6 = 0$ would be difficult, but $(x + 2)(x + 3) = 0$ is much easier.

The entire polynomial equals 0 when each individual factor equals 0. That gives us $x + 2 = 0$ and $x + 3 = 0$, or $x = -2$ and $x = -3$. So our *x-intercepts* are at $x = -2$ and $x = -3$. Knowing these intercepts makes it easier to graph the function.

(Source: www.shmoop.com)

CLUSTER	**3. Use polynomial identities to solve problems.**
BIG IDEA	• **The operations of addition, subtraction and multiplication can be applied to polynomials.** • **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.APR.4	**Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.***		
DESCRIPTION	A.APR.4 Understand that polynomial identities include but are not limited to the product of the sum and difference of two terms, the difference of two squares, the sum and difference of two cubes, the square of a binomial, etc . A.APR.4 Prove polynomial identities by showing steps and providing reasons. A.APR.4 Illustrate how polynomial identities are used to determine numerical relationships such as $25^2=(20 + 5)^2 = 20^2 + 2 \cdot 5 + 5^2$.		
ESSENTIAL QUESTION(S)	How can proving polynomial identities help solve problems efficiently?		
MATHEMATICAL PRACTICE(S)	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Explain that an identity shows a relationship between two quantities or expressions, which is true for all values of the variables, over a specified set.	Prove polynomial identities. Use polynomial identities to describe numerical relationships.	

EXPLANATIONS AND EXAMPLES

With the increase in technology and this huge new thing called the Internet, identity theft has become a worldwide problem. For this reason, it is paramount to keep important information such as addresses and telephone numbers as private as possible when online. If not, you might have thieves coming to your home and stealing your polynomial identities.

Sure, *your* identity should be kept safe too, but we were talking more about polynomial identities. A **polynomial identity** is just a true equation, often generalized so that it can apply to more than one situation. For instance, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ is true for any x and y. But how can we be sure that's true unless we prove it?

Students should be able to prove identities by showing that one side of an equation is equal to the other. That just takes the same skills they use to organize equations and expressions. They can leave their room-cleaning skills at home, since we all know how organized *those* are.

To prove polynomial identities, students can either work from one side of the equation to try to derive the other side, or can work from both sides to get the same thing. Both are just fine as long as both sides end up being exactly the same. The goal is to make both sides identical.

Students should also understand the use of certain identities. Sometimes, memorizing an identity is quicker than working out the algebra longhand, and other times the identity applies to a particular context.

We can prove the identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ by manipulating the equation step by step, showing our work as we go. We'll change only one side of the equation and see if we can change it into the other side.

$(x^2 + y^2)^2$

$= (x^2 + y^2)(x2 + y^2)$

$= x^4 + x^2y^2 + y^2x^2 + y^4$

$= x^4 + 2x^2y^2 + y4$

$= x^4 - 2x^2y^2 + 4x^2y^2 + y^4$

$= (x^4 - 2x^2y^2 + y^4) + 4x^2y^2$

$= (x^2 - y^2)^2 + (2xy)^2$

There. Since both sides are equal, we can call this equation an identity. The identity is true for all values of x and y. For x = 4 and $y = 2$? Yep. How about for $x = -1$ and $y = 398{,}128$? You betcha.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.APR.5	**(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.**
DESCRIPTION	A.APR.5 For small values of n, use Pascal's Triangle to determine the coefficients of the binomial expansion. A.APR.5 Use the Binomial Theorem to find the nth term in the expansion of a binomial to a positive power.
ESSENTIAL QUESTION(S)	How can proving polynomial identities help solve problems efficiently?
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Define the Binomial Theorem and compute combinations. Apply the Binomial theorem to expand (x+y)n, when n is a positive integer and x and y are any number, rather than expanding by multiplying.	Explain the connection between Pascal's Triangle and the determination of the coefficients in the expansion of (x+y)n, when n is a positive integer and x and y are any number.	

EXPLANATIONS AND EXAMPLES

The Binomial Theorem and Pascal's Triangle are very important concepts for two reasons. First, they're capitalized, and when words are capitalized, you know they're a big deal. Second, both the Binomial Theorem and Pascal's Triangle make expanding polynomials way easier.

Students should be able to expand $(x + y)n$ using Pascal's Triangle, where x and y can be anything. Pascal's Triangle provides the coefficients for the expansion, and the Binomial Theorem explains Pascal's Triangle. They go together like ramma lamma lamma ke ding a de dinga dong…whatever that means.

The Binomial Theorem can be explained using combinations, usually because they're the easiest to visualize and explain. For example, flipping a coin is a classic example since there are two sides.

How many combinations are possible with 1 heads and 2 tails? We could have HTT, THT, and TTH. There are three possible combinations. We can assign values to our total number of trials $(n = 3)$ and our total number of desired outcomes (heads, for example, would be $k = 1$).

The notation ${}_nC_k$ is a way of saying, "The number of different combinations we can have if, out of n trials, our desired outcome occurs k times." For example, we can use ${}_nC_k$ if we flip a coin n times and we want to know how many different ways we can have k heads (or k tails, whichever we want). We can calculate ${}_nC_k$ using the formula:

$${}_nC_k = \frac{n!}{(n-k)!k!}$$

Those exclamation points aren't just because we're excited about n and k. They're mathspeak for "factorial," which translates to $n \times (n - 1) \times (n - 2) \times \ldots \times 2 \times 1$. But you can be excited about n and k too.

In our example of three coin flips ($n = 3$) and 1 heads ($k = 1$), this equals:

$$\frac{3!}{(3-1)!1!} = \frac{3 \cdot 2 \cdot 1}{(2 \cdot 1) \cdot 1} = \frac{6}{2} = 3$$

There are 3 possible ways to get 1 heads out of 3 coin flips.

(Source: www.shmoop.com)

EXPLANATIONS AND EXAMPLES (continued)

We can use this same logic when expanding $(x + y)$n. How many ways can we get 3 *x*'s and 2 *y*'s, for example? (Note that there are the same number of ways to get 2 *x*'s and 3 *y*'s.) This can be written as $_5C_3$, which equals

$$\frac{5!}{(5-3)!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)} = 10$$

Let's now think about all ways of getting *k x*'s and $(n - k)$ *y*'s out of *n* outcomes.

When $n = 0$, we can have $_0C_0$, which equals 1. Our values for $n = 2$ are $_1C_0$ and $_1C_1$, both of which equal 1 as well. For $n = 2$, $_2C_0$ and $_2C_2$ equal 1, but $_2C_1$ equals 2. Students should know that continuing with these calculations would get us Pascal's Triangle, in which each number is the sum of the two numbers above it.

$n = 0$ 1

$n = 1$ 1 1

$n = 2$ 1 2 1

$n = 3$ 1 3 3 1

$n = 4$ 1 4 6 4 1

$n = 5$ 1 5 10 10 5 1

Let's use Pascal's Triangle to expand (x + y)5.

$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

The coefficients of each term correspond to the row of Pascal's Triangle in which $n = 5$. The exponents of the x's start from *n* and decrease to 0, while the orders of the *y*'s start at 0 and increase until we get to *n*. Pretty awesome, right?

(Source: www.shmoop.com)

CLUSTER	**4. Rewrite rational expressions.**
BIG IDEA	• **The operations of addition, subtraction and multiplication can be applied to polynomials.** • **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.APR.6	**Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.**
DESCRIPTION	AA.APR.6 Rewrite rational expressions $\frac{a(x)}{b(x)}$, in the form $q(x)+\frac{r(x)}{b(x)}$ by using factoring, long division, or synthetic division. Use a computer algebra system for complicated examples to assist with building a broader conceptual understanding.

ESSENTIAL QUESTION(S)	What strategy can be used to rewrite rational expressions in different forms?
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Use inspection to rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x). Use long division to rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x). Use a computer algebra system to rewrite complicated rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x).		

EXPLANATIONS AND EXAMPLES

The polynomial *q(x)* is called the quotient and the polynomial *r(x)* is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes.

This standard is like a Rube Goldberg machine. It looks far more complicated than it really is.

It all boils down to dividing polynomials and expressing the answer properly. Students should have already touched division and fractions at least a little, having been through the third grade and all. In a way, it's just a continuation of the Remainder Theorem, so we recommend covering that first.

Students should already know how to divide polynomials by factoring or long division. As with many divisions, they won't all be perfect and a remainder will be left over. Instead of just writing what the remainder is, we now expect students to actually do something with it.

Let's say we're dividing *a(x)* by *b(x)*, and our answer is *q(x)* with remainder *r(x)*. Just like the Remainder Theorem, if *r(x)* = 0, then *b(x)* is a factor of *a(x)*. We know that already.

But what if *b(x)* doesn't divide *a(x)* with remainder 0? Well, just like simplifying 13⁄4 to 3 with a remainder of 1, or 3¼, we can write $^{a(x)}/_{b(x)}$ as *q(x)* with remainder *r(x)*, or $^{r(x)}/_{b(x)}$. Just like a remainder of 1 divided by 4 means ¼, a remainder of *r(x)* divided by *b(x)* will give us $^{r(x)}/_{b(x)}$.

EXPLANATIONS AND EXAMPLES (continued)	All the talk about the degree of *r(x)* being less than the degree of *b(x)* just means that *r(x)* should be "smaller" than *b(x)*. It wouldn't make sense to split $\frac{13}{4}$ into 2 $\frac{5}{4}$ because we can still divide 5 by 4. It's the same idea, only polynomial-style. We suggest relating these polynomial quotients to fractions of integers so that students don't feel overwhelmed. It's understandable for them to be confused when we throw seven different functions at them, but they'll be a lot more receptive when they're working with concepts they already know. Students should also not be afraid of the big bad long division monster. Often, factoring is near impossible to figure out when remainders are involved and there are times when synthetic division just won't cut it. Students should give in and embrace long division and their lives will be better for it. *(Source: www.shmoop.com)*

STANDARD AND DECONSTRUCTION

A.APR.7	**(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.**		
DESCRIPTION	A.APR.7 Simplify rational expressions by adding, subtracting, multiplying, or dividing. A.APR.7 Understand that rational expressions are closed under addition, subtraction, multiplication, and division (by a nonzero expression).		
ESSENTIAL QUESTION(S)	How is adding, subtracting, multiplying and dividing of rational expressions similar to that of rational numbers?		
MATHEMATICAL PRACTICE(S)	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Add, subtract, multiply, and divide rational expressions.	Informally verify that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.	

EXPLANATIONS AND EXAMPLES

A rational expression is any polynomial divided by any polynomial except for zero. If your students don't already know, dividing by zero is catastrophic. In fact, many esteemed mathematicians believe the apocalypse will result from dividing by zero.

Students should understand that rational expressions are closed under addition, subtraction, multiplication, and division, meaning that:

- A rational expression plus a rational expression is a rational expression.
- A rational expression minus a rational expression is a rational expression.
- A rational expression times a rational expression is a rational expression.
- A rational expression divided by a rational expression is a rational expression.

It's only rational.

Students should also know how to add, subtract, multiply, and divide rational expressions. They're really not that different from adding, subtracting, and multiplying polynomials. We're just throwing division in there, too.

Let's denote a rational expression as

$$\frac{p(x)}{q(x)}$$

We could worry about remainders here, but keeping rational expressions as fractions will help us out in the long run. In fact, rational expressions and fractions are practically identical when it comes to adding, subtracting, multiplying, and dividing.

If we want to add or subtract $\frac{1}{4}$ and $\frac{2}{5}$, we first have to find a common denominator. Only after they both have a denominator of $5 \times 4 = 20$, can we add and subtract them. We don't need common denominators to multiply $\frac{1}{4}$ and $\frac{2}{5}$, though. Just multiply the numerators together and multiply the denominators together. When division is involved, it's easier to just flip the second fraction and then multiply, so $\frac{1}{4} \div \frac{2}{5}$ becomes $\frac{1}{4} \times \frac{5}{2}$.

EXPLANATIONS AND EXAMPLES (continued)

Well guess what. Rational expressions are exactly the same, only the integers are replaced with polynomials. Basically, the rules simplify to these:

$$\frac{p(x)}{q(x)} + \frac{m(x)}{n(x)} = \frac{p(x) \cdot n(x) + q(x) \cdot m(x)}{q(x) \cdot n(x)}$$

$$\frac{(x)}{(x)} - \frac{m(x)}{n(x)} = \frac{p(x) \cdot n(x) - q(x) \cdot m(x)}{q(x) \cdot n(x)}$$

$$\frac{p(x)}{q(x)} \cdot \frac{m(x)}{n(x)} = \frac{p(x) \cdot m(x)}{q(x) \cdot n(x)}$$

$$\frac{p(x)}{q(x)} \div \frac{m(x)}{n(x)} = \frac{p(x)}{q(x)} \cdot \frac{n(x)}{m(x)} = \frac{p(x) \cdot n(x)}{q(x) \cdot m(x)}$$

In each case, we end up with a polynomial divided by a polynomial, which is a rational expression. Since polynomials can be defined anywhere, we can't have any x under the radical. As long as students know how to multiply and factor polynomials, they should be fine with rational expressions.

$$\frac{p(x)}{q(x)} + \frac{m(x)}{n(x)} = \frac{p(x) \cdot n(x) + q(x) \cdot m(x)}{q(x) \cdot n(x)}$$

$$\frac{p(x)}{q(x)} \cdot \frac{m(x)}{n(x)} = \frac{p(x) \cdot m(x)}{q(x) \cdot n(x)}$$

$$\frac{p(x)}{q(x)} \div \frac{m(x)}{n(x)} = \frac{p(x)}{q(x)} \cdot \frac{n(x)}{m(x)} = \frac{p(x) \cdot n(x)}{q(x) \cdot m(x)}$$

(Source: www.shmoop.com)

DOMAIN:

CREATING EQUATIONS (A-CED)

HIGH SCHOOL
ALGEBRA

MATHEMATICS

DOMAIN	Domain: Creating Equations (A-CED) (★)
CLUSTERS	**1. Create equations that describe numbers or relationships.**
ACADEMIC VOCABULARY	Algebra I absolute value, complement of an event, compound, conjunction, direct and inverse variation, disjunction, domain & range, exponential growth (and decay), interest (simple and compound), irrational numbers, joint and conditional probability, law of large numbers, mathematical model, measure of spread (range, interquartile range), midpoint formula, outlier, parent function, Pascal's triangle, polynomial (binomial, trinomial), quadratic formula (including discriminant), quantitative and qualitative data, radicand, rational expression, real number properties, real roots (zeros, solutions, x-intercepts), relative frequency, sequences (arithmetic, geometric, Fibonacci), simulations, subsets of real numbers Algebra II amplitude, asymptote, binomial theorem, combination, common ratio (geometric sequence), complete the square, complex conjugate, complex number, composition (of functions), conic sections (circles, parabola, ellipse, hyperbola), empirical rule, factorial, focus (pl. foci), independent and dependent events, inverse of a relation, logarithm, normal distribution, period, permutation, piece-wise function, radian measure, rational function, regression equation, series (arithmetic, geometric, finite, infinite, etc.), sigma, standard deviation, step function, synthetic division, transcendental function, trigonometric function, trigonometric identity, unit circle, variance

CLUSTER	**1. Create equations that describe numbers or relationships.**
BIG IDEA	• **Real-world problems are described by creating and solving equations and inequalities.** • **Equations describe numbers and relationships.** • **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.CED.1	**Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.***
DESCRIPTION	A.CED.1 Create linear, quadratic, rational and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.

ESSENTIAL QUESTION(S)	How are equations used to describe numbers or relationships and solve problems?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Solve linear and exponential equations in one variable. Solve inequalities in one variable. Solve all available types of equations and inequalities, including root equations and inequalities, in one variable. Describe the relationships between the quantities in the problem (for example, how the quantities are changing or growing with respect to each other); express these relationships using mathematical operations to create an appropriate equation or inequality to solve.	Use all available types of functions to create such equations, including root functions, but constrain to simple cases. Create equations and inequalities in one variable to model real-world situations. Create equations (linear and exponential) and inequalities in one variable and use them to solve problems.	

EXPLANATIONS AND EXAMPLES

Equations can represent real-world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.

Students should be able to interpret word problems and form equations and inequalities in order to solve the problem. That means translating a word problem to an algebraic equation.

Let's be real, here. Math is another language, just like Spanish, Japanese, or Icelandic. When you start learning a language, you don't start by translating words like "absquatulate" or "loquacious" or "pneumonoultramicroscopicsilicovolcanoconiosis" (and yes, that is a real word).

It's better to start easier, with words like "cat" and "girl" and slowly work your way up. Just the same, if you use simple linear equations that are familiar to students, they can focus on the translation process and it'll all go a lot smoother.

Translation is a useful analogy in and of itself because it emphasizes that the algebraic equation is the same as the word problem, just presented in a different way. In addition to helping students to understand the process, the translation analogy can also help reassure struggling learners and encourage practice.

After they've gotten a hang of the basics, students can start learning quadratic, rational, and exponential functions to address all aspects of this standard. Once students are familiar with these operations individually, they should be asked to distinguish them from each other.

As students gain experience, there are additional strategies that should be introduced. One experienced problem solver strategy is to read the question twice before beginning. It's a useful piece of advice in general, actually.

EXPLANATIONS AND EXAMPLES (continued)

Writing a list of what is known and a list of what needs to be calculated is also an excellent strategy. Such lists are especially useful when sorting out unnecessary information, identifying an appropriate formula to utilize, or constructing a proof. These strategies should be suggested and shown to students after they are proficient with the basic translation process.

To start off, the chart below may be presented as a dictionary to support word to symbol translation. Students can also add to the chart as they find other key words or phrases.

(Source: www.shmoop.com)

Algebra Symbol	Key Words
= equals	1. all 2. equals 3. gives 4. is, are, was, were, will be 5. results 6. same 7. yields
< is less than	1. below 2. less than
≤ is less than or equal to	1. maximum of 2. not more than
> is greater than	1. greater than 2. more than 3. over
≥ is greater than or equal to	1. at least 2. minimum of 3. not less than
+ addition	1. add 2. and 3. combine 4. increase 5. more 6. plus 7. raise 8. sum 9. together 10. total
– subtraction	1. decrease 2. difference 3. fewer 4. less 5. lose 6. minus 7. reduce
× multiplication	1. directly proportional 2. double(× 2), triple(× 3), etc. 3. group of 4. linear 5. multiplied 6. product 7. times

Algebra Symbol	Key Words
/ division	1. average 2. cut 3. divided by/into 4. each 5. inversely proportional 6. out of 7. per 8. pieces 9. quotient 10. ratio 12. share 13. split
x^n power	1. power 2. square (n = 2), cube (n = 3), etc.
n^x exponential	1. decays 2. doubles (n = 2), triples (n = 3), quadruples (n = 4), etc. 3. grows 4. rate of n per x

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.CED.2	**Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.**
DESCRIPTION	A.CED.2 Create equations in two or more variables to represent relationships between quantities. A.CED.2 Graph equations in two variables on a coordinate plane and label the axes and scales.

ESSENTIAL QUESTION(S)	How are equations used to describe numbers or relationships and solve problems?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Identify the quantities in a mathematical problem or real-world situation that should be represented by distinct variables and describe what quantities the variables represent. Graph one or more created equations on coordinate axes with appropriate labels and scales.	Create at least two equations in two or more variables to represent relationships between quantities. Justify which quantities in a mathematical problem or real-world situation are dependent and independent of one another and which operations represent those relationships. Determine appropriate units for the labels and scale of a graph depicting the relationship between equations created in two or more variables.	

EXPLANATIONS AND EXAMPLES

This standard has two significant components. The first is translating word problems into equations with two or more variables. The more the merrier. Well, maybe not in this case. Translating word problems to create simple equations with two or more variables is not that different conceptually from creating equations with one variable.

The main difference is that more complicated mathematical relationships such as systems of equations, functions, and proportions may develop (along with nausea, headaches, and spontaneous yodeling). In any case, this aspect of this standard should be taught with the previous one.

The second component is creating graphs of equations on coordinate axes, which incorporates multiple skills such as visual perception, interpreting data, and synthesizing information. Such graphs relate to equations with multiple equations by relating one variable to another.

Take lines, for example. In the form $y = mx + b$, we can look at either x or y and any defined value for x will give us a defined value for y, and vice versa. Graphs can help visualize these relationships between variables and facilitate the connection of equations to the graphs that represent them. Yearnin' for more graphin'? Don't worry. There'll be more down the line.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.CED.3	**Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.** ***For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.***
DESCRIPTION	A.CED.3 Write and use a system of equations and/or inequalities to solve a real world problem. Recognize that the equations and inequalities represent the constraints of the problem. Use the Objective Equation and the Corner Principle to determine the solution to the problem. (Linear Programming)

ESSENTIAL QUESTION(S)	How are equations used to describe numbers or relationships and solve problems?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Recognize when a modeling context involves constraints.	Interpret solutions as viable or nonviable options in a modeling context. Determine when a problem should be represented by equations, inequalities, systems of equations and/ or inequalities. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities.	

EXPLANATIONS AND EXAMPLES

Students have already translated words into algebraic equations (sometimes with more than one variable!) and have actually taken the time to solve the problem. We hate to make you the villain here, but you have to tell them their work isn't over. Students now have to interpret the results. This standard is about one thing: analysis. Well… actually it's about three things:

1. Creating equations/inequalities or systems of equations/inequalities.
2. Solving these equations/inequalities or system of equations.
3. Interpreting the answer properly.

To analyze problems in which multiple relationships affect multiple variables, students must be able to create systems of equations, solve them, and interpret the results appropriately.

Creating Systems of Equations

In order to create systems of equations from word problems or other contexts, students need to be able to differentiate the relations and create equations for each. To support this, should already be able to create equations from word problems.

Creating equations from a word problem or similar context is a three-step translation process:

1. Translate the equality or inequality. (=, <, >, ≤, or ≥)
2. Translate the operations. (+, −, ×, ÷, x^n, n^x)
3. Translate the numbers and variables.

Systems of equations are identified during step one of this process. Students need to be able to read a problem and identify how many equality and inequality relations are described. Then, they should write each down separately.

CREATING EQUATIONS

EXPLANATIONS AND EXAMPLES (continued)

Once these are written down, students perform steps two and three (translating the operations, numbers, and variables) independently for each equation. They should also simplify each equation individually before working to solve the system of equations.

Solving Systems of Equations

Solving systems of equations can be done through substitution or adding the two equations together to cancel out one of the variables. The goal is to eliminate one variable so that we can find the solution for the other and then substitute that answer back in to find the value of the second variable.

Hopefully, students already know how to solve systems of equations. After all, it's necessary in order to interpret results from a set of equations.

Interpreting Results from Systems of Equations

In many situations, students struggle to understand what an algebraic result means in the context of a word problem. This is especially true when systems of equations are involved and when they arrive at solutions that are correct algebraically, but incorrect in context.

For instance, a question about how many tops hats a giraffe can wear might produce the number 6.25 as the answer. This might make sense algebraically, but in the context of giraffes wearing top hats—how can a giraffe wear one fourth of a hat? The logical answer would then have to be 6 (although logic might not be our biggest concern if we're talking about giraffes in top hats).

Such algebraic solutions also present a challenge when multiple roots are encountered or when cancelling rational expressions. Describing what values of a variable are allowed when recording the variable information is one strategy for dealing with this problem. (For instance, we could write that giraffes only wear top hats in whole numbers.)

A common error is to report an answer based on a different variable in the problem. Recording the variable information helps to prevent such errors. If this is a common issue, students should try highlighting the quantity of interest in the problem, the matching variable name, and the eventual result. They can then check all highlighted items to ensure that they match.

Multiple variables present even more of a challenge, as students often need to find information for more than one variable in a problem. Highlighting the different information requested in different colors is one strategy. Another possibility is to have students solve for all variables before interpreting a solution; however, this becomes troublesome in more complex problems.

Interpreting results should be performed throughout the algebra curriculum. With enough practice, students will perform whatever strategies work best for them naturally.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.CED.4	**Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.** ***For example, rearrange Ohm's law V = IR to highlight resistance R.***
DESCRIPTION	A.CED.4 Solve multi-variable formulas or literal equations, for a specific variable.
ESSENTIAL QUESTION(S)	How are equations used to describe numbers or relationships and solve problems?
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Define a quantity of interest to mean any numerical or algebraic quantity (e.g. , 2(a/b)=d in which 2 is the quantity of interest showing that d must be even; ($\pi r^2h/3$)=V and πr2h=V showing that V =3* V)	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g. πr^2 can be re-written as (π r)r which makes the form of this expression resemble Bh. The quantity of interest could also be = + +…+.	

EXPLANATIONS AND EXAMPLES

Students can rearrange formulas until their pencils whittle down to toothpicks, but it won't help them one bit if the formulas they use won't enable them actually solve the problem. That means students should be able to match commonly encountered formulas to context in word problems as well as rearranging them to solve for whatever value they want.

Matching Formulas to Create Equations

In addition to being able to translate word problems into equations, students also need to be able to identify when a common formula is needed for the given context. These are most commonly geometric formulas (like perimeter, area, or volume of various shapes) or physical formulas (such as *F = ma, p = mv, V = IR, v = d⁄t, KE = ½mv2, or GPE = mgh*).

It is important to note that this assumes that students are already familiar with the relevant formulas from previous learning. Students who are not already familiar with the formulas need to be supported in understanding them before they will be able to match them to contexts. Matching formulas may be treated as a variation from the creating equations process. When attempting to write down the equality or inequality, students will notice that there isn't enough information. A general relationship might be implied, but no specific equality or inequality is described. What ever will they do?

Well, students need to identify the formula that describes that relationship. They can look for clues such as appropriate units (volume, for example, is always in cubic units). Once the correct formula is matched to the contextual relationship, the students can continue solving the problem as usual.

Rearranging Formulas

Once a formula is written algebraically, students can manipulate it however they want (within mathematical reason). The process for rearranging it is identical to the process of rearranging any equation or system of equations. It involves simplifying expressions and solving equations, which students should already know how to do.

The best way to rearrange questions when looking for a particular value is to isolate that value. Students should rearrange the equations so that the desired value is on one side of the equal sign, and there's a whole big mess of stuff on the other. That way, they'll be able to plug in the values they know and end up with whatever they want equals some number.

(Source: www.shmoop.com)

DOMAIN:

REASONING WITH EQUATIONS AND INEQUALITIES (A-REI)

HIGH SCHOOL
ALGEBRA

MATHEMATICS

DOMAIN	Reasoning with Equations and Inequalities (A-REI)
CLUSTERS	**1. Understand solving equations as a process of reasoning and explain the reasoning.** **2. Solve equations and inequalities in one variable.** **3. Solve systems of equations.** **4. Represent and solve equations and inequalities graphically.**
ACADEMIC VOCABULARY	Algebra I absolute value, complement of an event, compound, conjunction, direct and inverse variation, disjunction, domain & range, exponential growth (and decay), interest (simple and compound), irrational numbers, joint and conditional probability, law of large numbers, mathematical model, measure of spread (range, interquartile range), midpoint formula, outlier, parent function, Pascal's triangle, polynomial (binomial, trinomial), quadratic formula (including discriminant), quantitative and qualitative data, radicand, rational expression, real number properties, real roots (zeros, solutions, x-intercepts), relative frequency, sequences (arithmetic, geometric, Fibonacci), simulations, subsets of real numbers Algebra II amplitude, asymptote, binomial theorem, combination, common ratio (geometric sequence), complete the square, complex conjugate, complex number, composition (of functions), conic sections (circles, parabola, ellipse, hyperbola), empirical rule, factorial, focus (pl. foci), independent and dependent events, inverse of a relation, logarithm, normal distribution, period, permutation, piece-wise function, radian measure, rational function, regression equation, series (arithmetic, geometric, finite, infinite, etc.), sigma, standard deviation, step function, synthetic division, transcendental function, trigonometric function, trigonometric identity, unit circle, variance

CLUSTER	**1. Understand solving equations as a process of reasoning and explain the reasoning.**
BIG IDEA	• **Reasoned explanations are required to solve equations and inequalities.** • **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.** • **Relationships between two sets of numbers can be described by mathematical rules, where a function is a unique rule that has a one-to-one correspondence.**

STANDARD AND DECONSTRUCTION

A.REI.1	**Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.**		
DESCRIPTION	A.REI.1 Assuming an equation has a solution, construct a convincing argument that justifies each step in the solution process. Justifications may include the associative, commutative, and division properties, combining like terms, multiplication by 1, etc.		
ESSENTIAL QUESTION(S)	What are the steps and strategies to justify a solution to a problem?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Know that solving an equation means that the equation remains balanced during each step. Recall the properties of equality. Explain why, when solving equations, it is assumed that the original equation is equal.	Determine if an equation has a solution. Choose an appropriate method for solving the equation. Justify solution(s) to equations by explaining each step in solving a simple equation using the properties of equality, beginning with the assumption that the original equation is equal. Construct a mathematically viable argument justifying a given, or self-generated, solution method.	

EXPLANATIONS AND EXAMPLES

Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions.

Who was it that said, "A journey of a thousand miles begins with a single step"? Was it Socrates? Confucius? Bugs Bunny?

Whoever it was, they probably weren't thinking about algebra when they coined that gem. To many students, solving an algebraic equation may feel like a journey of a thousand miles. Before students begin with their single step, they should probably get a compass or something.

Students should be able to figure out the logical next step in solving an equation using the previous step. Sounds simple, but it takes more than just putting one foot in front of the other. Knowing that whatever is done to one side of the equation must be done to the other is a good start, but that doesn't tell them what to do.

Ideally, students should not be memorizing a set of rules and procedures and using them to solve equations. They should understand how the next step in solving an equation can be logically derived from the previous step, but there is a general format for how to get started on their algebraic journey.

When we solve an equation, it's usually a good idea to get the variable we want on one side of the equal sign. Then, we do what we can to simplify it as much as we can.

EXPLANATIONS AND EXAMPLES (continued)

For example, let's solve the equation $x^2 - 3x - 7 = 2x + 17$. Subtracting $2x + 17$ from both sides gets all our x's on one side of the equation. That's a good place to start.

$x^2 - 3x - 7 - (2x + 17) = 2x + 17 - (2x + 17)$

$x^2 - 5x - 24 = 0$

We also took the opportunity to simplify by combining like terms. Since the equation we have is a quadratic equation, we can factor it into the product of two linear terms.

$(x - 8)(x + 3) = 0$

This product will equal 0 when either $x - 8$ or $x + 3$ is equal to 0, so we can solve it by setting each of them to 0. As a result, $x = 8$ and -3.

If students are really struggling, we suggest starting simple. Give them easier linear equations and slowly work your way to more complex ones. Point out patterns in equations, make sure they know the quadratic formula, and remind them of helpful factoring tricks.

Most of all, get them to practice. It's difficult to take even the first step of a journey if you can't walk. After enough exercises, your students should be able to tackle any thousand-mile journey faster than the Road Runner.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.REI.2	**Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.**
DESCRIPTION	A.REI.2 Solve simple rational and radical equations in one variable and provide examples of how extraneous solutions arise.

ESSENTIAL QUESTION(S)	What are the steps and strategies to justify a solution to a problem?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Determine the domain of a rational function. Determine the domain of a radical function. Solve radical equations in one variable. Solve rational equations in one variable.	Give examples showing how extraneous solutions may arise when solving rational and radical equations.	

EXPLANATIONS AND EXAMPLES

Rational equations might sometimes be a bit more radical than students would like. Those square root signs can instill terror in the heart of any student unprepared for them. If students overcame their fear of monsters under the bed (and hopefully they did), they'll get over their fear of radicals too.

Rational equations mean that fractions are involved. Radical equations mean that square roots are involved. Students should know how to deal with both separately and together.

A radical equation is one in which the variable is under the radical sign. When solving radical equations, it's usually best to leave the radicals for last unless there's a quick and easy way to get rid of all of them. If only there were a quick and easy way to get rid of monsters.

For example, $\sqrt{x} - 4 = 12$ is a radical equation. To solve, students should add 4 to both sides and get $\sqrt{x} - 4 + 4 = 12 + 4$, or $\sqrt{x} = 16$. All that's left is squaring both sides to get x = 256. Not too scary, right?

Of course, solving radical equations means students have to understand how to combine radicals. For example, $\sqrt{x} + 3\sqrt{x} = 4\sqrt{x}$ since both terms have the same thing under the radical. We cannot combine $\sqrt{x}$ and $\sqrt{2x}$ this same way. We could, however, rewrite $\sqrt{2x}$ as $\sqrt{2} \cdot \sqrt{x}$ in which case $\sqrt{x} + \sqrt{2} \cdot \sqrt{x} = (\sqrt{x})(1 + \sqrt{2})$. Not as pretty, but nonetheless doable.

Students should know how to combine, manipulate, and rewrite radical expressions. This usually takes practice and repetition. When all else fails, tell students to treat radicals like they'd treat variables. This is the one time the Golden Rule doesn't apply.

Students should already know how to solve rational equations. They should be able to find x faster than a pirate with a treasure map. And if he has an eye patch then it shouldn't even be a contest.

A rational equation might look like

$$\frac{3}{(x-4)(x+2)} = \frac{1}{x-4}$$

In this case, let's say it does. First, cross-multiply to get 3(x – 4) = (x – 4)(x + 2). Since we have (x – 4) on both sides, we can reduce the equation to 3 = x + 2. Our final answer is x = 1.

EXPLANATIONS AND EXAMPLES (continued)	Students should know that sometimes, algebraic manipulation produces extraneous solutions. For example, multiplying the fairly simple equation x + 5 = 0 by x will give x2 + 5x = 0. Now, both x = 0 and x = -5 will satisfy that quadratic. However, looking at x + 5 = 0, we can see that x = 0 won't work for the original equation (because 0 + 5 ≠ 0). That means x = 0 is an extraneous solution. To check for extraneous solutions, students should plug in their final answers back into the original equation. If the equation produces an incorrect statement (like 5 = 0), then they'll know that solution didn't really exist. Much like the monsters under the bed…we hope. *(Source: www.shmoop.com)*

CLUSTER	**2. Solve equations and inequalities in one variable.**
BIG IDEA	• **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.REI.3	**Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.**		
DESCRIPTION	A.REI.3 Solve linear equations in one variable, including coefficients represented by letters. A.REI.3 Solve linear inequalities in one variable, including coefficients represented by letters.		

ESSENTIAL QUESTION(S)	What are the steps and strategies to justify a solution to a problem?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Recall properties of equality. Solve multi-step equations in one variable. Solve multi-step inequalities in one variable.	Determine the effect that rational coefficients have on the inequality symbol and use this to find the solution set. Solve equations and inequalities with coefficients represented by letters.	

EXPLANATIONS AND EXAMPLES

With linear equations, students should be able to find the solution or solutions that make the equation true. We usually get one or several specific answers with equations, but inequalities sing a slightly different tune. As different as Under Pressure and Ice Ice Baby.

Students should first understand the difference between an equation and inequality. An equation uses the = sign while an equality may use <, >, ≤, or ≥. If we find that x ≤ 2, we know that x can be 2 or anything less than 2. If we know that $x < 2$, x cannot be 2, but it can be anything less than 2.

With inequalities, students should find the *set of numbers* that make the inequality true. Inequalities won't tell us exactly which number x will equal. Instead, it'll give us a range of possible x values, all of which will work for the inequality.

Students should also know how to work with inequalities. Algebraically, they aren't that different from an equal sign. Still, multiplying and dividing by negative numbers switches the direction of the sign (1 > -2 but multiplying both sides by -1 gives us -1 < 2).

If students are unsure, it might be helpful for them to visualize inequalities on a number line. Sometimes, letters may represent constants and coefficients in equations. Students should know how to treat these as numbers. For instance, the answer to the equation $x + 4m = 2x + m$ would be written as $x = 3m$. It's okay for our solution to be in terms of *m* because *m* is treated as a constant. Isn't that nice, nice, baby?

We apologize in advance if that baseline is stuck in your head for the rest of the day.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.REI.4	**Solve quadratic equations in one variable.**
DESCRIPTION	A.REI.4a Transform a quadratic equation written in standard form to an equation in vertex form - by completing the square. $(x - p)^2 = q$ A.REI.4a Derive the quadratic formula by completing the square on the standard form of a quadratic equation. A.REI.4b Solve quadratic equations in one variable by simple inspection, taking the square root, factoring, and completing the square. A.REI.4b Understand why taking the square root of both sides of an equation yields two solutions. A.REI.4b Use the quadratic formula to solve any quadratic equation, recognizing the formula produces all complex solutions. Write the solutions in the form, where a and b are real numbers. $a \pm bi$ A.REI.4b Explain how complex solutions affect the graph of a quadratic equation.

ESSENTIAL QUESTION(S)	What are the steps and strategies to justify a solution to a problem? How can the method of completing the square be used to derive the quadratic formula? How do I determine the most appropriate and efficient strategy to use to solve quadratic equations?
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4

SUBSTANDARD DECONSTRUCTED	**A.REI.4a Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.**		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x-p)2 = q that has the same solutions. Solve quadratic equations in one variable.	Derive the quadratic formula by completing the square on the standard form of a quadratic equation.	

SUBSTANDARD DECONSTRUCTED

A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Solve quadratic equations by inspection taking square roots, completing the square, the quadratic formula and factoring. Express complex solutions as a ± bi for real numbers solutions as a and b.	Recognize when the quadratic formula gives complex solutions. Determine appropriate strategies to solve problems involving quadratic equations, as appropriate to the initial form of the equation.	

EXPLANATIONS AND EXAMPLES

Students should know how to solve equations involving terms with one variable to the second degree. These equations may be written in any form, the most common being the standard form of a quadratic equation, $ax^2 + bx + c = 0$. Students should know and apply the three main ways to solve a quadratic equation: stop, drop, and roll.

Oh, wait. Scratch that. We meant factor, complete the square, and the quadratic formula.

Your students should already know what factoring is, and that it's possible with simple quadratic equations like $x^2 + x - 12 = 0$. When factoring, students should look for two numbers that add to the coefficient b (in this case, 1) and multiply to get the constant c (in this case, -12). Easier said than done, unless you've got a mouthful of peanut butter.

For the equation $x^2 + x - 12$, the two numbers that work are -3 and 4. They add to get 1 and multiply to get -12, so we can factor $x^2 + x - 12$ into $(x - 3)(x + 4)$. Now it's way easier to solve when we set the equation to equal 0. Since $x - 3 = 0$ and $x + 4 = 0$, our answers are $x = 3$ and $x = -4$. Note that this method only works when a is 1.

(a.) Example.

This is a sneaky and fun way to solve unruly quadratic equations. It involves creating and factoring a "perfect square" (meaning, a quadratic expression whose two linear factors are the same). Let's solve the same equation as we did before: $x^2 + x - 12 = 0$.

First, add 12 to both sides: $x^2 + x = 12$. Next, complete the square by making the left side a perfect square. To do that, we take b, divide by 2, and square the result. In this case, our $b = 1$, which ends up being ¼. Add this to both sides of the equation.

$$x^2 + x + \frac{1}{4} = 12 + \frac{1}{4}$$

The left side is a perfect square because its two linear factors are the same: $x + ½$. Therefore, we can re-write the left side of the equation and simplify the right-hand side.

$$(x + \frac{1}{2})^2 = \frac{49}{4}$$

From there, we can square root both sides and subtract both sides by ½.

$$x + \frac{1}{2} = \pm\frac{7}{2}$$

$$x = \frac{7}{2} - \frac{1}{2}, -\frac{7}{2} - \frac{1}{2}$$

$x = 3, -4$

(b) Example.

The quadratic equation is essentially a shortcut to completing the square. It may not seem like a shortcut to your students, but it beats spending an hour trying to factor an equation that just won't budge.

Students should know the quadratic formula and how to use it. When given a quadratic equation $ax^2 + bx + c = 0$, the quadratic formula is this ravishing thing:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXPLANATIONS AND EXAMPLES (continued)	Of course, if all that business under the radical turns out to be negative, we've got a real problem. We can't square root negative numbers, right? As long as students are familiar with complex numbers and i, this real problem becomes an imaginary one very quickly. Then we can simplify the rest so that it follows the form a ± bi. While students may cringe at this monstrous behemoth of a formula, they'll soon appreciate its sophisticated and simple beauty. In fact, many of them will memorize it without even realizing it. Armed with those three ways, students should be able to attack just about any quadratic formula using only a pencil and their bright and shiny noggins (and maybe a calculator, if you're feeling generous). *(Source: www.shmoop.com)*

CLUSTER	**3. Solve systems of equations.**
BIG IDEA	• **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

A.REI.5	**Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.**		
DESCRIPTION	A.REI.5 Solve systems of equations using the elimination method (sometimes called linear combinations). A.REI.5 Solve a system of equations by substitution (solving for one variable in the first equation and substituting it into the second equation).		
ESSENTIAL QUESTION(S)	What strategy is used to solve systems of two equations?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Recognize and use properties of equality to maintain equivalent systems of equations.	Justify that replacing one equation in a two-equation system with the sum of that equation and a multiple of the other will yield the same solutions as the original system.	

EXPLANATIONS AND EXAMPLES

Students should be able to understand how manipulating systems of equations will lead to those equations' solutions. Let's say we have two functions that we're trying to solve simultaneously. We'll call our functions f(x, y)and g(x, y), because they're funky and groovy. Yeah, we took our functions straight out of the 70s.

For instance, let's say that *f(x, y)* is $2x + 4 = 3y$ and *g(x, y)* is $-x + 8y = 2$. We can rewrite these as $2x - 3y + 4 = 0$ and $x - 8y + 2 = 0$, in which case $f(x, y) = 2x - 3y + 4$ and $g(x, y) = x - 8y + 2$. Now, we have our awesome functions (or…funk-tions?).

If one solution to both $f(x, y) = 0$ and $g(x, y) = 0$ is when $x = a$ and $y = b$, then $f(a, b) = 0$ and $g(a, b) = 0$. That means *a* and *b* must also be the *x* and y values for the system of equations $f(x, y) + g(x, y) = 0$ and $nf(x, y) = 0$, where n is a constant. After all, $f(a, b) + g(a, b) = 0 + 0 = 0$, and $nf(a, b) = n(0) = 0$.

Basically, this means that system of equations $f(x, y) = 0$ and $g(x, y) = 0$ will have the same solutions as $f(x, y) + g(x, y) = 0$ and $nf(x, y) = 0$.

Students should know that this is useful when solving a system of equations (as in, 2 or more of 'em) because it means we can multiply equations by constants and add them together all we want. That'll help us find the x and y values that work for all equations in the system.

We strongly urge you to relate this information back to graphing linear and quadratic equations. Explain that the solution to the system of equations is really the point (or points) where the functions intersect. For example, functions *f(x, y)* and *g(x, y)* intersect at *(a, b)* since those are the *x* and *y* values that make the two equations equal to each other. It's all connected, just like we're all connected. Pretty groovy, right?

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.REI.6	**Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.**		
DESCRIPTION	A.REI.6 Solve systems of equations using graphs.		

ESSENTIAL QUESTION(S)	How and when are systems of two linear equations solved precisely and graphically?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Solve systems of linear equations by any method.	Justify the method used to solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables.	

EXPLANATIONS AND EXAMPLES

The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations.

By now, students should know that a linear equation given by $y = mx + b$ (or a variation of it) and makes a line on a graph. They should also know that a system of linear equations means that we have more than one $y = mx + b$ equation in the mix. So far, so good.

Solving a system of linear equations means finding the point at which the two (or more?) lines intersect. This happens when the same set of x and y values satisfy all the linear equations in the system.

What about lines that never intersect? Well, they're parallel, for starters. But that just means that the system of parallel lines will have no solution. (It may be helpful to tell your students that "No solution" is a legitimate answer, but only after they've done the work to prove it. Otherwise you'll get "No solution" as the answer to every homework problem.)

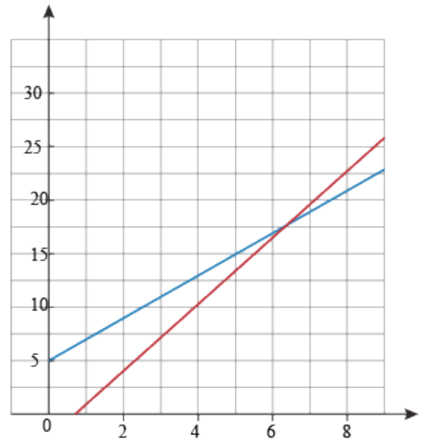
Students should be shown that a system of linear equations can be solved either through graphs or straight algebra, but that these two methods arrive at the same answer because they mean the same thing. The goal is to find the point at which the two lines intersect.

Graphs

Graphically, solving a system of two linear equations is easy. Using the slopes and y-intercepts of both lines, students can graph each line individually and find the intersection point. It's pretty and simple (and pretty simple), but not always accurate visually. For instance, what's the solution to the following system of equations?

EXPLANATIONS AND EXAMPLES (continued)

Is it at point (6.2, 17)? Or maybe (6.3, 17.5)? It's difficult to tell exactly. Very helpful visually, but a bit less useful in terms of coming up with numerical answers.



Algebra

It'd be helpful to know that the two lines above have equations of $y = 2x + 5$ and $y = 3x - 1.25$. With the linear equations, students should be able to graph the lines on a coordinate plane, as well as, solve them exactly using substitution. When the lines intersect, the x and y values in both linear equations are the same. That means we can set the two equations equal to each other.

For instance, we can say that $3x - 1.25 = 2x + 5$, because both the y coordinates have to be equal. (By the way, that gives $x = 6.25$ as our answer. If we substitute 6.25 for x back into either equation, we should get 17.5.)

If students are struggling with these concepts, emphasizing the connection between the visual graphs and the algebraic calculations should help cement their understanding..

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.REI.7	**Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.***
DESCRIPTION	A.REI.7 Solve a system containing a linear equation and a quadratic equation in two variables (conic sections possible) graphically and symbolically.

ESSENTIAL QUESTION(S)	What is the most efficient method to solve systems of 2 equations that include a quadratic and linear equation, algebraically and graphically?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Transform a simple system consisting of a linear equation and a quadratic equation in two variables so that a solution can be found algebraically and graphically.	Explain the correspondence between the algebraic and graphical solutions to a simple system consisting of a linear equation and a quadratic equation in two variables.	

EXPLANATIONS AND EXAMPLES

Students should know that this is a slight expansion on the previous standard. Our only change is that instead of having two lines, we have a linear equation (a straight line) and a quadratic equation (a not so straight line). No big deal.

In the simplest terms, the quadratic equation is just a linear equation with a square sign over a variable. It can have more terms, but as long as the largest exponent over a variable is 2, it's a quadratic equation. Simple enough, right?

Students must know the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

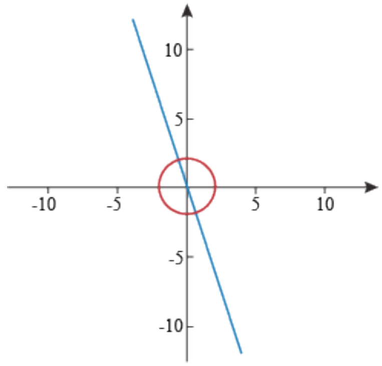
Actually, they should have it tattooed on their faces. It'll be a great icebreaker at parties.

The variables in the quadratic equation (lovingly named *a*, *b*, and *c*) are derived from our standard quadratic equation $ax^2 + bx + c = 0$.

Let's use the actual problem in the standard itself as an example. We have $y = -3x$ as our linear equation, and $x^2 + y^2 = 3$ as our quadratic. (Technically, the second equation isn't a "true" quadratic, but there's no shame in torturing your students a little bit to teach them some more. We're only kidding about the torturing part.)

If we graph these bad boys, we can already see that we have two points of intersection.

EXPLANATIONS AND EXAMPLES (continued)



We can prove why this is the case algebraically and calculate the exact points of intersection. First, we can solve for our quadratic as we did with linear equations: by substituting one equation into another.

That turns $x^2 + y^2 = 3$ into $x^2 + (-3x)^2 = 3$. We can eventually get it into the $ax^2 + bx + c = 0$ format (which ends up being $10x^2 - 3 = 0$) so that we can use the quadratic formula. In our case, $a = 10$, $b = 0$, and $c = -3$.

If we plug in those values into the quadratic formula, we get

$$x = \frac{-0 \pm \sqrt{0^2 - 4(10)(-3)}}{2(10)}$$

$$x = \frac{\pm\sqrt{120}}{20}$$

$x \approx \pm 0.55$

Since we know what x is now, we can simply plug those values into our $y = -3x$ equation to get our answers of ± 1.64. That means our points of intersection are (-0.55, 1.64) and (0.55, -1.64). It matches up with the graph, so we're done.

Students should also make use of the discriminant in order to figure out whether the line and quadratic intersect once, twice, or not at all. The discriminant is the part of the quadratic formula under the radical: $b^2 - 4ac$.

If the value of the discriminant is less than 0, there are no points of intersection. If the answer is equal to 0, there is one point of intersection (a tangent line). If the answer is greater than 0, there are two points of intersection.

The value of our discriminant is $(0)^2 - (4 \times 10 \times -3) = 120$. That means there are two points of intersection. Using the graph, the quadratic formula, and this little tidbit, we've triple checked our answer. Talk about thorough..

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.REI.8	**(+) Represent a system of linear equations as a single matrix equation in a vector variable.**
DESCRIPTION	A.REI.8 Write a system of linear equations as a single matrix equation.
ESSENTIAL QUESTION(S)	How does a matric equation provide a viable representation of a system of linear equations?
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Write a system of linear equations in vector variable form. Write a system of linear equations as a single matric equation.	Solve systems of equations.	

EXPLANATIONS AND EXAMPLES

Students should understand that a system of equations can be expressed as a matrix of coefficients multiplied by a vector matrix of variables. Sometimes, it's even better to do it that way.

Your students will probably curse you for stressing matrices time and time again, but just reassure them that it's for their own good. In order to work with matrices, your students should already understand what they are and how to perform various functions with them. The same goes for linear equations and systems of linear equations.

Students should know that a matrix equation takes the form $AX = B$, where A represents the coefficient of our variables, X represents our variables, and B represents the output to our equations.

In order to turn linear equations into matrices, we have to rearrange them all into the same format. Among the preferred formats are $ax + by + cz = d$. The linear equations $y = 2x + 5$ and $y = 3x - 2$ become $-2x + y = 5$ and $-3x + y = -2$. Painless, no?

Then we take each term in the equations and split them up into their proper matrix. Taking the equations from above, we'll have the A matrix (the one with all the coefficients) equaling $\begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix}$

Since our only variables are x and y (and we put the coefficients into the A matrix in that order), our X matrix becomes:

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

The final matrix, B, is what's on the other side of our equal signs. That means it turns into:

$$\begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

In matrix form, it looks like this:

$$\begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

This standard only indicates that students should know how to represent systems of linear equations in matrix equation form, but not why doing so is useful (aside from organizational purposes). That is covered in the next standard.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

A.REI.9	**(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).**
DESCRIPTION	A.REI.9 Find the inverse of the coefficient matrix in the equation, if it exists. Use the inverse of the coefficient matrix to solve the system. Use technology for matrices with dimensions 3 by 3 or greater. A.REI.9 Find the dimension of matrices. A.REI.9 Understand when matrices can be multiplied. A.REI.9 Understand that matrix multiplication is not commutative. A.REI.9 Understand the concept of an identity matrix. A.REI.9 Understand why multiplication by the inverse of the coefficient matrix yields a solution to the system (if it exists).
ESSENTIAL QUESTION(S)	How does a matric equation provide a viable representation of a system of linear equations?
MATHEMATICAL PRACTICE(S)	HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Solve a system of linear equations using inverse matrices. Solve a system of linear equations with three or more variables using technology.	Find the inverse of a matrix.	

EXPLANATIONS AND EXAMPLES

Students will perform multiplication, addition, subtraction, and scalar multiplication of matrices. They will use the inverse of a matrix to solve a matrix equation. Students may use graphing calculators, programs, or applets to model and find solutions for systems of equations.

This is where actual calculations take place. We've finally gotten to the fun stuff. Your students may think otherwise, but we know the truth.

Students should already know how to form a matrix equation of $AX = B$ from a system of linear equations and be familiar with the concept of inverse matrices. They will use the inverse of a matrix in order to solve for the variables of the matrix that is formed from given equations.

Students should know that if the determinant of a square matrix is zero (ad – bc = 0), there is no inverse to the matrix. A matrix that takes the form.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

has an inverse of

$$\frac{1}{ad-bc} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Assuming we've already translated our equations -2x + y = 5 and -3x + y = -2 into matrix equation form, we can find the inverse matrix of the A matrix (the one with all the coefficients).

$$\begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix}$$

EXPLANATIONS AND EXAMPLES (continued)

First, we take our matrix and find its determinant. That would be -2 × 1 – 1 × -3 = -2 + 3 = 1. The inverse matrix exists. Now, let's give our creation life! After switching around the numbers, we have to multiply by the inverse of the determinant, which is 1⁄1 or just 1. So our inverse matrix is fine as is.

$$\begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$$

So your students have found the inverse matrix. Now what? Well, the whole point of this process is to find the values of x and y. (They should write that down. That's important.) To do that, we can multiply the inverse matrix we found by the B matrix (the one with all the solutions). That should give us the values for *x* and *y*.

$$\begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

That gives us 1 × 5 + (-1 × -2) = 5 + 2 = 7 = x, and 3 × 5 + (-2 × -2) = 15 + 4 = 19 = y. If we plug those values back into the original linear equations, they should all hold up.

While this method might seem a bit cumbersome, students should appreciate that it's especially useful for systems of equations with many, many variables. On the other hand, the inverses for matrices larger than 2 × 2 should be calculated using technology. The whole point, though, is that they can help us solve for the values of the variables.

(Source: www.shmoop.com)

CLUSTER	**4. Represent and solve equations and inequalities graphically.**
BIG IDEA	**• Relationships between two sets of numbers can be described by mathematical rules, where a function is a unique rule that has a one-to-one correspondence.**

STANDARD AND DECONSTRUCTION

A.REI.10	**Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).**		
DESCRIPTION	A.REI.10 Understand that all solutions to an equation in two variables are contained on the graph of that equation.		
ESSENTIAL QUESTION(S)	How does a graphed solution of equations and inequalities in two variables indicate the set of all its solutions?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Recognize that the graphical representation of an equation in two variables is a curve, which may be a straight line. Explain why each point on a curve is a solution to its equation.		

EXPLANATIONS AND EXAMPLES

Students should understand that equations with two variables can be represented graphically. The shape that results on the coordinate plane is a visual representation of all the solutions to that equation.

What does that mean? It means we're not just pulling rabbits out of hats! The equations actually mean something visually.

An equation with two variables can be anything from $y = x$ to $x^2 + y^2 = 4$ to $19x^{13} = y$. Some are simpler than others, of course, but they all have an *x* and a *y*.

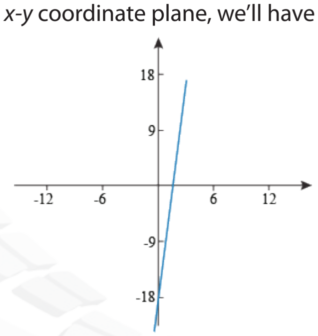
That means instead of having an equation with one variable (and therefore one solution), we can have many different solutions. Graphically, we can represent these solutions by drawing a curve or line through all the pairs of solutions (one for *x* and one for *y*) that work for that particular equation.

Let's take the equation $7x - 18 = y$ and see how we can represent this graphically.

How do we prove to a student that indeed, a line of a two-variable equation, when graphed, shows all of the solutions? Let's show them how to pull that rabbit out of the hat themselves.

Since any two points define a line, all we need to do is input two values for x and see what the output y values are. We'll pick three points just to be sure our graph is a line and not some weird curve.

Let's pick the numbers -1, 0, and 3 for *x*. Plugging in the numbers for x into our equation $7x - 18 = y$ gives us -25, -18, and 3 for the *y* values. So the points in our graph become (-1, -25), (0, -18), and (3, 3). If we graph these on the *x*-*y* coordinate plane, we'll have this:



EXPLANATIONS AND EXAMPLES (continued)	
EXPLANATIONS AND EXAMPLES (continued)	If your students don't believe you, prove to them that the equation and graph correspond to one another. Take a point on the line that is easily identifiable, say (2, -4), and plug the values into the equation. If we do that, we'll have -4 = 7(2) – 18, which simplifies to -4 = -4. That way, students will be sure that points on the line or curve are valid solutions to the equation, and vice versa. But don't stop there. It's also important to prove the opposite. For example, the coordinate (4, 1), which is not on the line, is also not a solution to our equation. If we plug in the coordinates, we can confirm this: 1 = 7(4) – 18 is false because 1 ≠ 10. This means (4, 1) isn't a solution to our equation and not a point on the line. Now you can pat yourself on the back and prove to students that teachers aren't just up to some magic tricks. Everything in math pretty much works as it's supposed to. This method can be applied to two-variable equations of higher orders. The generic shapes of these equations (such as quadratics making a parabola) should be known and associated with each other already. Otherwise, students will need to graph several points before verifying the graph that corresponds with the particular equation. *(Source: www.shmoop.com)*

STANDARD AND DECONSTRUCTION

A.REI.11	**Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f(x)* and *y* = *g(x)* intersect are the solutions of the equation *f(x)* = *g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (★)**		
DESCRIPTION	A.REI.11 Explain why the intersection of *y* = *f(x)* and *y* = *g(x)* is the solution of *f(x)* = *g(x)* for any combination of linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Find the solution(s) by: • Using technology to graph the equations and determine their point of intersection. • Using tables of values. • Using successive approximations that become closer and closer to the actual value.		
ESSENTIAL QUESTION(S)	How does a graphed solution of equations and inequalities in two variables indicate the set of all its solutions?		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/ applications.
Students should be able to:	Recognize that if (x_1, y_1) and (x_2, y_2) share the same location in the coordinate plane that $x_1 = x_2$ and $y_1 = y_2$. Recognize that f(x) = g(x) means that there may be particular inputs of f and g for which the outputs of f and g are equal. Recognize and use function notation to represent linear, polynomial, rational, absolute value, exponential, and logarithmic equations.	Explain why the x-coordinates of the points where the graph of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x). Approximate/find the solution(s) using an appropriate method. For example, using technology to graph the functions, make tables of values or find successive approximations.	
EXPLANATIONS AND EXAMPLES	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions. Students should understand that an equation and its graph are just two different representations of the same thing. The graph of the line or curve of a two-variable equation shows in visual form all of the solutions (infinite as they may be) to our equation in written form. When two equations are set to equal one another, their solution is the point at which graphically they intersect one another. Depending on the equations (and the alignment of the planets), there might be one solution, or more, or none at all. Students can arrive at the correct answer(s) through graphing the functions and plotting their intersection points, creating a table of x and f(x) values, and solving for x algebraically when f(x) = g(x). These strategies should be provided to students and practiced with students so that the connection between graphs and equations are solidified. (We wouldn't want them to be liquefied, now would we?) *(Source: www.shmoop.com)*		

STANDARD AND DECONSTRUCTION

A.REI.12	**Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.**
DESCRIPTION	A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary for non-inclusive inequalities. A.REI.12 Graph the solution set to a system of linear inequalities in two variables as the intersection of their corresponding half-planes.

ESSENTIAL QUESTION(S)	How does a graphed solution of equations and inequalities in two variables indicate the set of all its solutions?		
MATHEMATICAL PRACTICE(S)	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Identify characteristics of a linear inequality and a system of linear inequalities, such as: boundary line, shading, and determine the appropriate points to test and derive a solution set from.	Explain the meaning of the intersection of the shaded regions in a system of linear inequalities.	Graph a line, or boundary line, and shade the appropriate region for a two variable linear inequality. Graph a system of linear inequalities and shade the appropriate overlapping region for a system of linear inequalities.

EXPLANATIONS AND EXAMPLES

Students may use graphing calculators, programs, or applets to model and find solutions for inequalities or systems of inequalities.

All this is asking us to do is what we already know from the previous standards, plus one simple step. In fact, this step is fun (as long as you color inside the lines). Students should know how to graph a linear inequality, complete with all the nuts and bolts.

A **linear inequality** is the same as a linear equation, but instead of an equal sign, we'll have to use the inequality signs (like ≤, ≥, <, and >).

What's all this "half-plane" business? Just mathematical mumbo-jumbo. It means that because we're graphing an inequality and our linear equation is with a different sign now, it'll be shaded above or below the line as part of our solution. That's it.

When dealing with inequalities, your students should ask themselves two questions:

1. Which part of the graph do I shade in?
2. Do I draw a dotted or a solid line?

If the inequality is greater than, or greater than or equal to (using either > or ≥), then we shade the upper half of the graph. If the inequality if less than, or less than or equal to (using either < or ≤), then we shade the lower half of the graph.

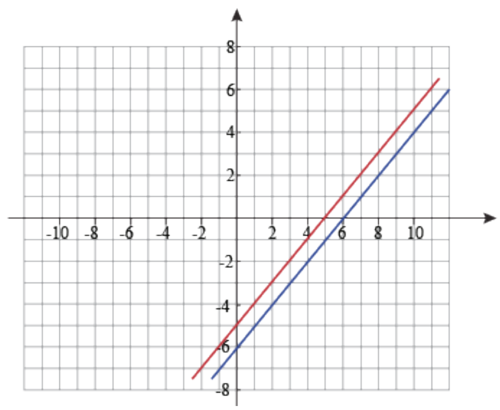
If students are struggling with which half to shade, the simplest way to remove all doubt is to plug in the coordinates of a point that's very obviously on one side of the boundary. If the inequality is true for that point, then we know to shade the "half-plane" containing that point. If it's false, we'll shade in the other half. Make sure to bring your colored pencils.

EXPLANATIONS AND EXAMPLES (continued)

The line that graphs our linear equation is dashed or dotted if we use greater than or less than (using > or <) in our inequality. That's so we know the line is a boundary, but all the points on it don't satisfy the inequality. The line we'll use is solid if the inequality has a greater than or equal to, or less than or equal to (using ≥ or ≤), symbol because the boundary includes possible solutions to our inequality.

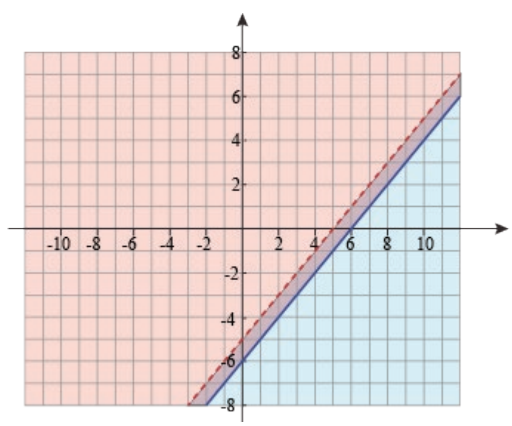
Students should understand how to graph not one, but two inequalities. It's just like graphing one inequality, and then graphing another right on top of it. Using the same graph saves trees.

Given a pair of inequalities (such as $y < x - 5$ and $y \geq x - 6$, for instance), we draw them as though they were equations first. We can do this through a computer, a graphing calculator, or by creating a table of values to calculate enough points to get us a straight line.



Time to bust out those colored pencils. Since our first inequality is "less than," this means we must shade below the line. We'll color it red. Then comes the ultimate question: solid or dotted? Well, there's no "equal to" component, so our set of solutions to the inequality does not include the boundary line itself. That means it must be drawn as a dotted line.

For the second inequality, we know that it must be "greater than or equal to," meaning we shade above the line. We'll pick blue. Because of its "equal to" part, we must include the line. It must remain solid.



Red and blue make purple. The overlapping purple area is the solution to our system of inequalities. That means that only within the overlapping area will the values of x and y work for both the inequalities we listed.

Students should know how to graph inequalities, shade in the half-planes, and find the set of solutions for a system of inequalities. If students are struggling, have them plug in coordinates that are on the boundary or very clearly to one side. This will help connect the graph and the inequality, as well as make sense of what's going algebraically and graphically.

Also, make sure they pick colors that go together.

(Source: www.shmoop.com)

COMMON CORE State Standards

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