COMMON CORE State Standards

DECONSTRUCTED for CLASSROOM IMPACT

HS

HIGH SCHOOL GEOMETRY

MATHEMATICS

The COMMON CORE Institute

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Introduction

The Common Core Institute is pleased to offer this grade-level tool for educators who are teaching with the Common Core State Standards.

The Common Core Standards Deconstructed for Classroom Impact is designed for educators by educators as a two-pronged resource and tool 1) to help educators increase their depth of understanding of the Common Core Standards and 2) to enable teachers to plan College & Career Ready curriculum and classroom instruction that promotes inquiry and higher levels of cognitive demand.

What we have done is not all new. This work is a purposeful and thoughtful compilation of preexisting materials in the public domain, state department of education websites, and original work by the Center for College & Career Readiness. Among the works that have been compiled and/or referenced are the following: Common Core State Standards for Mathematics and the Appendix from the Common Core State Standards Initiative; Learning Progressions from The University of Arizona's Institute for Mathematics and Education, chaired by Dr. William McCallum; the Arizona Academic Content Standards; the North Carolina Instructional Support Tools; and numerous math practitioners currently in the classroom.

We hope you will find the concentrated and consolidated resource of value in your own planning. We also hope you will use this resource to facilitate discussion with your colleagues and, perhaps, as a lever to help assess targeted professional learning opportunities.

Understanding the Organization

Math Fluency Standards	
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 Add/subtract within 100 (pencil & paper)
3	Multiply/divide within 100 Add/subtract within 1000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations
7	Solve $px + q = r$, $p(x + q) = r$
8	Solve simple 2 x 2 systems by inspection

The Overview acts as a quick-reference table of contents as it shows you each of the domains and related clusters covered in this specific grade-level booklet. This can help serve as a reminder of what clusters are part of which domains and can reinforce the specific domains for each grade level.

Critical Areas of Focus is designed to help you begin to approach how to examine your curriculum, resources, and instructional practices. A review of the **Critical Areas of Focus** might enable you to target specific areas of professional learning to refresh, as needed.

For each domain is the domain itself and the associated clusters. Within each domain are sections for each of the associated clusters. The cluster-specific content can take you to a deeper level of understanding. Perhaps most importantly, we include here the **Learning Progressions.** The **Learning Progressions** provide context for the current domain and its related standards. For any grade except Kindergarten, you will see the domain-specific standards for the current grade in the center column. To the left are the domain-specific standards for the preceding grade and to the right are the domain-specific standards for the following grade. Combined with the **Critical Areas of Focus,** these Learning Progressions can assist you in focusing your planning.

For each cluster, we have included four key sections: Description, Big Idea, Academic Vocabulary, and Deconstructed Standard.

The cluster **Description** offers clarifying information, but also points to the **Big Idea** that can help you focus on that which is most important for this cluster within this domain. The **Academic Vocabulary** is derived from the cluster description and serves to remind you of potential challenges or barriers for your students.

Each standard specific to that cluster has been deconstructed. There **Deconstructed Standard** for each standard specific to that cluster and each **Deconstructed Standard** has its own subsections, which can provide you with additional guidance and insight as you plan. Note the deconstruction drills down to the sub-standards when appropriate. These subsections are:

- Standard Statement
- Standard Description
- Essential Question(s)
- Mathematical Practice(s)
- DOK Range Target for Learning and Assessment
- Learning Expectations
- Explanations and Examples

As noted, first are the **Standard Statement** and **Standard Description**, which are followed by the **Essential Question(s)** and the associated **Mathematical Practices**. The **Essential Question(s)** amplify the **Big Idea**, with the intent of taking you to a deeper level of understanding; they may also provide additional context for the **Academic Vocabulary**.

The **DOK Range Target for Learning and Assessment** remind you of the targeted level of cognitive demand. The **Learning Expectations** correlate to the DOK and express the student learning targets for student proficiency for KNOW, THINK, and DO, as appropriate. In some instances, there may be no learning targets for student proficiency for one or more of KNOW, THINK or DO. The learning targets are expressions of the deconstruction of the Standard as well as the alignment of the DOK with appropriate consideration of the Essential Questions.

The last subsection of the **Deconstructed Standard** includes **Explanations and Examples**. This subsection might be quite lengthy as it can include additional context for the standard itself as well as examples of what student work and student learning could look like. **Explanations and Examples** may offers ideas for instructional practice and lesson plans. A wonderful resource for explanations and examples, which we often referred to and cited as a source in this tool, is www.shmooop.com.

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

Standards for Mathematical Practice in High School Mathematics Courses

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

PRACTICE	EXPLANATION
MP.1 Make sense and persevere in problem solving.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
MP.2 Reason abstractly and quantitatively.	Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

PRACTICE	EXPLANATION
MP.3 Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
MP.4 Model with mathematics.	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, 2-by-2 tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
MP.5 Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students interpret graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

PRACTICE	EXPLANATION
MP.6 Attend to precision.	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
MP.7 Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as composed of several objects. For example, they can see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
MP.8 Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

OVERVIEW

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

OVERVIEW

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Congruence (G-CO)

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry (G-SRT)

- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Apply trigonometry to general triangles.

Circles (G-C)

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.

Expressing Geometric Properties with Equations (G-GPE)

- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems algebraically.

Geometric Measurement and Dimension (G-GMD)

- Explain volume and formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry (G-MG)

- Apply geometric concepts in modeling situations.

Mathematical Practices (MP)

MP 1. Make sense of problems and persevere in solving them.
MP 2. Reason abstractly and quantitatively.
MP 3. Construct viable arguments and critique the reasoning of others.
MP 4. Model with mathematics.
MP 5. Use appropriate tools strategically.
MP 6. Attend to precision.
MP 7. Look for and make use of structure.
MP 8. Look for and express regularity in repeated reasoning.

High School Pathways for Traditional and Integrated Courses

	Domain	HS Algebra I	Mathematics I	Geometry
Number & Quantity	The Real Number System (RN)	RN, 1, 2, 3		
	Quantities (Q)	Q.1, 2, 3	Q.1, 2, 3	
	The Complex Number System (CN)			
	Vector Quantities and Matrices (VM)			
Algebra	Seeing Structure in Expressions (SSEE)	SSE.1a, 1b, 2, 3a, 3b, 3c	SSE.1a, 1b	
	Arithmetic with Polynomials and Rational Expressions (APR)	APR .1		
	Creating Equations (CED)	CED. 1, 2, 3, 4	CED. 1, 2, 3, 4	
	Reasoning with Equations and Inequalities (REI)	REI. 1, 3, 4a, 4b, 5, 6, 7, 10, 11, 12	REI. 1, 3, 5, 6, 10, 11, 12	
Functions	Interpreting Functions (IF)	IF, 1, 2, 3, 4, 5, 6, 7a, 7b, 7c, 8a, 8b, 9	IF. 1, 2, 3, 4, 5, 6, 7a, 7c, 9	
	Building Functions (BF)	BF. 1a, 1b, 2, 3, 4a	BF. 1a, 1b, 2, 3	
	Linear, Quadratic, and Exponential Models (LE)	LE. 1a, 1b, 1c, 2, 3, 5	LE. 1a, 1b, 1c, 2, 3, 5	
	Trigonometric Functions (TF)			
Geometry	Congruence (CO)		CO. 1, 2, 3, 4, 5, 6, 7, 8, 12, 13	CO. 1-13
	Similarity, Right Triangles, and Trigonometry (SRT)			SRT. 1-11
	Circle (C)			C. 1-5
	Expressing Geometric Properties with Equations (GPE)		GEP. 4, 5, 7	GPE. 1, 2, 4-7
	Geometric Measurement and Dimension (GMD)			GMD. 1, 3, 4
	Modeling with Geometry (MG)			MG. 1, 2, 3
Statistic and Probability	Interpreting Categorical and Quantitative Data (D)	ID. 1-3, 5-9	ID. 1-3, 5-9	
	Making Interference and Justifying Conclusions (IC)			
	Conditional Probability and the Rules of Probability (CP)			CP. 1-9
	Using Probability to Make Decisions (MD)			MD. 6-7

High School Pathways for Traditional and Integrated Courses

	Mathematics II	HS Algebra II	Mathematics III	4th Courses (T)	4th Courses (I)
Number & Quantity	RN, 1, 2, 3				
	CN.1, 2, 7, 8, 9	CN.1, 2, 7, 8, 9	CN. 8, 9	CN. 3, 4, 5, 6	CN. 3, 4, 5, 6
				VM. 1, 2, 3, 4 (a-c), 5(a-b), 6, 7, 8, 9, 10, 11, 12	VM. 1, 2, 3, 4 (a-c), 5(a-b), 6, 7, 8, 9, 10, 11, 12
Algebra	SSE.1a, 1b, 2, 3a, 3b, 3c	SSE. 1, 1b, 2, 4	SSE. 1, 1b, 2, 4		
	APR .1	APR .1-7	APR .1-7		
	CED. 1, 2, 4	CED. 1, 2, 3, 4	CED. 1, 2, 4		
	REI. 4a, 4b, 7	REI. 2, 11	REI. 2, 11	REI. 8, 9	REI. 8, 9
Functions	IF. 4, 5, 6, 7a, 7b, 8a, 8b, 9	IF. 4, 5, 6, 7b, 7c, 7e, 8, 9	IF. 4, 5, 6, 7b, 7c, 7e, 8, 9	IF. 7d	IF. 7d
	BF. 1a, 1b, 3, 4a	BF. 1b, 3, 4a	BF. 1b, 3, 4a	BF. 1c, 4b, 4c, 4d, 5	BF. 1c, 4b, 4c, 4d, 5
	LE. 3	LE. 4	LE. 4		
	TF. 8	TF. 1, 2, 5, 8	TF. 1, 2, 5	TF. 3, 4, 6, 7, 9	TF. 3, 4, 6, 7, 9
Geometry	CO. 9, 10, 11				
	SRT. 1a, 1b, 2, 3, 4, 5, 6, 7, 8		SRT. 9, 10, 11		
	C. 1-5				
	GPE. 1, 2, 4			GPE. 3	GPE. 3
	GMD. 1, 3		GMD. 4	GMD. 2	GMD. 2
			MG. 1, 2, 3		
Statistic and Probability		ID. 4	ID. 4		
		IC. 1-6	IC. 1-6		
	CP. 1-9				
	MD. 6-7	MD. 6, 7	MD. 6-7	MD. 1-5	MD. 1-5

Learning Progressions for Integrated Courses

GEOMETRY			
Math 1	**Math 2**	**Math 3**	**Math 4**
CONGRUENCE (CO)			
G.CO.1			
G.CO.2			
G.CO.3			
G.CO.4			
G.CO.5			
G.CO.6			
G.CO.7			
G.CO.8			
	G.CO.9		
	G.CO.10		
	G.CO.11		
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY (SRT)			
	G.SRT.1		
	G.SRT.2		
	G.SRT.3		
	G.SRT.4		
	G.SRT.5		
	G.SRT.6		
	G.SRT.7		
	G.SRT.8		
		G.SRT.9	
		G.SRT.10	
		G.SRT.11	
CIRCLES (C)			
	G.C.1		
	G.C.2		
	G.C.3		
	G.C.4		
	G.C.5		
EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS (GPE)			
	G.GPE.1		
	G.GPE.2		
			G.GPE.3
G.GPE.4	G.GPE.4		
G.GPE.5			
G.GPE.6			
G.GPE.7			
GEOMETRIC MEASUREMENT AND DIMENSION (GMD)			
	G.GMD.1		
			G.GMD.2
	G.GMD.3		
	G.GMD.4	G.GMD.4	
MODELING WITH GEOMETRY (MG)			
		G.GM.1	
		G.GM.2	
		G.GM.3	

Learning Progressions for Traditional Courses

GEOMETRY			
Math 1	**Math 2**	**Math 3**	**Math 4**
CONGRUENCE (CO)			
	G.CO.1		
	G.CO.2		
	G.CO.3		
	G.CO.4		
	G.CO.5		
	G.CO.6		
	G.CO.7		
	G.CO.8		
	G.CO.9		
	G.CO.10		
	G.CO.11		
	G.CO.12		
	G.CO.13		
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY (SRT)			
	G.SRT.1		
	G.SRT.2		
	G.SRT.3		
	G.SRT.4		
	G.SRT.5		
	G.SRT.6		
	G.SRT.7		
	G.SRT.8		
	G.SRT.9		
	G.SRT.10		
	G.SRT.11		
CIRCLES (C)			
	G.C.1		
	G.C.2		
	G.C.3		
	G.C.4		
	G.C.5		
EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS (GPE)			
	G.GPE.1		
	G.GPE.2		
			G.GPE.3
	G.GPE.4		
	G.GPE.5		
	G.GPE.6		
	G.GPE.7		
GEOMETRIC MEASUREMENT AND DIMENSION (GMD)			
	G.GMD.1		
			G.GMD.2
	G.GMD.3		
	G.GMD.4		
MODELING WITH GEOMETRY (MG)			
	G.GM.1		
	G.GM.2		
	G.GM.3		

DOMAIN:

CONGRUENCE (GCO)

HIGH SCHOOL
GEOMETRY

MATHEMATICS

DOMAIN	Congruence
CLUSTERS	1. Experiment with transformations in the plane 2. Understand congruence in terms of rigid motions. 3. Prove geometric theorems. 4. Make geometric constructions.
ACADEMIC VOCABULARY	angle, circle, perpendicular lines, parallel lines, line segments, point, line, arc, rigid motion, congruent, angle-side-angle(asa), side-angle-side (sas), side-side-side (sss), inscribed, scale factor, dilation, aa similarity, theorem, law of sines and law of cosines

CLUSTER	1. Experiment with transformation in the plane.
BIG IDEA	• Congruence and similarity can be used to prove geometric relationships and to solve problems.

STANDARD AND DECONSTRUCTION

GCO.1	**Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.**
DESCRIPTION	Understand and use definitions of angles, circles, perpendicular lines, parallel lines, and line segments based on the undefined term of a point, a line, the distance along a line, and the length of an arc.

ESSENTIAL QUESTION(S)	What types of transformations can be done in plane?		
MATHEMATICAL PRACTICE(S)	HS.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Describe the terms point, line, and distance along a line in a plane. Define perpendicular lines, parallel lines, line segments, and angles. Define circle and the distance around a circular arc.		

EXPLANATIONS AND EXAMPLES

Congruence is the overarching theme of the next dozen standards in geometry, so let's tackle that one before running down the rest of the line. Pun intended.

In its most basic sense, "congruency" means being in a state of agreement. No, not with your partner, children, or even your students. In the world of geometry, two figures are called congruent if one can be "carried onto" another.

What does that mean? In brief, it means that all corresponding pairs of sides and angles are equal. They're kind of like "identical twins" without the hassles of sibling rivalry or having to decide how long they'll be dressing alike.

More than just congruence, students should know the precise definitions of an angle, circle, perpendicular line, parallel line, and line segment. After all, they can't work with something if they don't know what it is. That's like trying to kick a field goal without knowing what those big yellow posts are at each end of the field.

Speaking of which—those huge yellow posts at either end? Yeah, the goal posts. Each clearly has two massive poles standing straight up in the air. They are *parallel*. The one at the bottom of each, which is parallel to the *ground* but connects them? It's *perpendicular* to the first two and to the one in the center that reaches into the grass below and holds up the whole thing.

There are several angles here, too. For example, one at each end of the bottom of that square "U" formed by the two parallel posts and their perpendicular bar.

Okay, so what about those *undefined notions*? Don't worry. It's a lot simpler than it sounds. It all boils down to the fact that we have to start somewhere.

In football, for example, the kicker has to trust that when he sends the ball to the other team, it's going to move in a certain direction based on where and with how much force his foot hits it. He may not realize it, but he trusts the laws of physics.

EXPLANATIONS AND EXAMPLES (Continued)

In geometry, when your students set out to prove or construct something, they have to accept a few concepts that seem perfectly understandable, but are not actually formally defined. They can think of them as the laws of geometry.

In this case, these "notions" are a point, a line, the distance along a line, and the distance around a circular arc. A **point** is simply a location, represented by a dot. A point has no size, no length, no nothing. Only a location.

A **line** is simply is straight, infinite length. It's named simply by any two points on the line. It can also be named by a single letter, normally shown in the lower case. The **distance along a line** is a decided or arbitrary length along the line.

The **distance around a circular arc** is like the distance along a line, but it's not straight. It's a length equidistant from a point rather than equidistant from a line.

You may also see these called "fundamental" or "primitive" notions. They're the same thing, but try to be consistent. Your students might get confused if you refer to the same thing in three different ways.

Using these undefined notions, we can define an **angle** as a two lines or line segments or rays that share a common endpoint and a circle as a set of points equidistant from a given point (better known as "the center").

What about all this business with lines? Well, since lines in the real world can't extend forever, we have **line segments**, which are pieces of the line that run between two endpoints, but don't extend beyond them.

Parallel lines are lines that never intersect. Ever. No matter how far you extend them. Perpendicular lines not only intersect, they meet each other head-on, at a 90° angle.

Students should be able to not only define but also recognize and *draw* a proper angle, circle, perpendicular line, parallel line, and line segment, and more. These are concepts that most students shouldn't have a problem with. If some of your students are struggling, give lots of definitions and examples so that they master these concepts before moving on.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GCO.2	**Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).**
DESCRIPTION	Use various technologies such as transparencies, geometry software, interactive whiteboards, and digital visual presenters to represent and compare rigid and size transformations of figures in a coordinate plane. Comparing transformations that preserve distance and angle to those that do not. Describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, to include translations and horizontal and vertical stretching.

ESSENTIAL QUESTION(S)	What types of transformations can be done in plane?		
MATHEMATICAL PRACTICE(S)	HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Describe the different types of transformations including translations, reflections, rotations, and dilations. Describe transformations as functions that take points in the coordinate plane as inputs and give other points as outputs.	Represent transformations in the plane. Write functions to represent transformations. Compare transformations that preserve distance and angle to those that do not (e g , translation vs. horizontal stretch).	

EXPLANATIONS AND EXAMPLES

Students may use geometry software and/or manipulatives to model and compare transformations.

In today's world, it seems more and more people are trying to transform themselves in some way. Whether they want to transform their *inner* selves through self-help books and meditation or their outer selves with exercise and more cosmetic surgery than Joan Collins can handle, we shouldn't forget that shapes sometimes need a change as well. These changes are called transformations.

No, not Transformers. They're cool and all, but they're not the same.

A **transformation** means somehow altering a shape on the coordinate plane. Whether we move, flip, stretch, shrink, or turn the shape, we're performing a transformation. Unfortunately, an Extreme Makeover doesn't count when it comes to shapes.

The three main types of transformations—translations, reflections, and rotations—are called "rigid" because they preserve the distance and angles of their shapes.

Though it means something much different in your students' foreign language classes, **translation** means moving a shape in one direction. They can think of it as "sliding" that shape. No turning, no flipping, no complex movements. Just a straight slide. The shape stays exactly the same, but in a new place.

Just as it sounds, **rotation** is a circular movement. The shape moves around a central point. It's like turning a dial in one direction or the other, causing the object to stay where it is, but spin clockwise or counterclockwise. After 360°, though, you end up right where you started.

EXPLANATIONS AND EXAMPLES (Continued)

Students should think of **reflection** as looking at a shape in a mirror. Everything about the shape is the same, except one is the mirror image of the other. If imagining a mirror is strange or difficult, your right and left hand are reflections of each other, too.

Students should know that when any of these transformations are done in the coordinate plane, they can be described as functions that take points in the plane as inputs and give other points as outputs. The best way to see this is through coordinates of a simple shape, like a triangle. Moving up one unit means we add 1 to every y coordinate. Our input is y and our output is $y + 1$. Easy peasy.

We've been talking about transformations that move shapes without altering the *shape itself*—only the coordinates on which it sits. In other words, these transformations "preserve distance and angle," but there are some transformations that *change* them. This happens when multiplication is used, as opposed to addition or subtraction.

When you multiply, the distances between lines of the shape on the coordinate plane are lengthened. Multiply the x coordinates by 2, and the shape grows twice as wide; multiply the y coordinates by 2, and the shape grows twice as tall. This is called a "stretch" and tell your students to be careful with them. Otherwise, we may end up with gigantic shapes that'll want to take over the world.

On the other hand, if you multiply the coordinates by a number less than 1, the transformation is called compression. If compression were made into a Hollywood movie, it'd be called, "Honey, I Shrunk the Shapes."

Students should know how to move a given figure around the coordinate plane according to the different types of transformations. They should also represent a translation as a function using coordinates and understand the difference between transformations that preserve distance and angle and those that do not.

In no time, they'll be transforming themselves into the best geometry students you could hope for! Awww.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GCO.3	**Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.**
DESCRIPTION	Describe the rotations and reflections of a rectangle, parallelogram, trapezoid, or regular polygon that maps each figure onto itself.

ESSENTIAL QUESTION(S)	What types of transformations can be done in plane?		
MATHEMATICAL PRACTICE(S)	HS.MP.3 Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Describe rotations and reflections that carry a rectangle, parallelogram, trapezoid, or regular polygon onto it.		

EXPLANATIONS AND EXAMPLES

Students may use geometry software and/or manipulatives to model transformations.

By this point, students should know that transformations are movements of geometric figures on the coordinate plane. Now, it's simply a matter of building on this concept. It's just an extension of the last two standards, so hopefully it'll be kind of a no-brainer.

Students should be able to work with some geometric shapes, including parallelograms, trapezoids, and more. They should not only describe, but also identify and use *points* of symmetry.

In plain English, students have to illustrate how a figure is *mapped* onto itself by simply using a transformation without changing the shape or size of the figure. To do that, they'll need to calculate how much symmetry exists between the images, both among their lines (through reflection) and degrees (through rotational movement).

One important thing to keep in mind, though: properties such as side lengths and angle measures of the objects are key. You only have to rotate a regular hexagon 60° for it to carry onto itself, but a square needs 90°.

Both the concepts of symmetry and "carrying a geometric shape or figure onto itself" can be found in things like wallpapers, clothing, computer screensavers, and much more. If you ever need examples for symmetry, nature's got your back. And if nature's raining on your parade, then computer-generated images get pretty close to the real deal nowadays..

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GCO.4	**Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.**		
DESCRIPTION	Using previous comparisons and descriptions of transformations, develop and understand the meaning of rotations, reflections, and translations based on angles, circles, perpendicular lines, parallel lines, and line segments.		
ESSENTIAL QUESTION(S)	What types of transformations can be done in plane?		
MATHEMATICAL PRACTICE(S)	HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recall definitions of angles, circles, perpendicular and parallel lines, and line segments. Define rotations, reflections, and translations.	Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments.	

EXPLANATIONS AND EXAMPLES

Students may use geometry software and/or manipulatives to model transformations. Students may observe patterns and develop definitions of rotations, reflections, and translations.

Students should be able to recognize and visualize transformations of geometric shapes and not only reflect, rotate, and translate shapes, but also *combine* these three different transformations. By defining and describing these transformations in reference to angles, circles, lines and line segments, students should gain a deeper understanding of these transformations and when these operations apply to real world situations.

Show how rotations, reflections, and translations are drawn, but also to use polygons in order to perform a few of these transformations. Then, explain how each step meshes with the definition of that transformation. The more examples, the merrier!

Also, remind students that when a geometric shape undergoes a *rigid* transformation, its angles cannot change. Rigid transformations do not affect length, area, or angle measure of a geometric shape. They do, however, affect your flexibility in yoga class.

Students should use the three transformations listed to explore and prove geometric properties. The best way to analyze these transformations in terms of angles, circles, line segments, and perpendicular and parallel lines is to consider each individually.

For example, your students are driving (and definitely *not* texting) and approaching a four-way intersection. The two streets will be perpendicular to one another. If we picture a line down the middle of one of the streets, either horizontally or vertically, and we imagine folding the streets over each other, we will have a reflection transformation.

The angles and the angles are all mirrored, but due to the 90° angles all around, they are congruent either way. Of course, actually folding streets over would be some crazy Inception-style infrastructure, but you get the point.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GCO.5	**Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.**
DESCRIPTION	Transform a geometric figure given a rotation, reflection, or translation using graph paper, tracing paper, or geometric software. Create sequences of transformations that map a geometric figure on to itself and another geometric figure.

ESSENTIAL QUESTION(S)	What types of transformations can be done in plane?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.		Draw a transformed figure and specify the sequence of transformations that were used to carry the given figure onto the other.

EXPLANATIONS AND EXAMPLES

Students may use geometry software and/or manipulatives to model transformations and demonstrate a sequence of transformations that will carry a given figure onto another.

Rotations, reflections, and translations might sound like moves in a *Cirque du Soleil* show. Rotating about the trapeze, reflected in a shiny disco ball, and translated from French to English. But really, rotations, reflections, and translations are all about staying rigid rather than flexible. Sorry, contortionists.

Students should already be well aware of rigid transformations, only now, they'll actually be using them instead of theorizing about them. Students should come up with sets of transformations (meaning more than one) that map figures onto themselves and other figures.

So, while not death-defying like some circus acts, what's exciting about this standard for students is that they are taking definitions and rules they've learned and actually bringing them into practice on paper.

Following this lesson, students should be able to use visuals to improve their sense and knowledge of how a transformation maps one shape onto another. They should also understand why moving and manipulating figures is a useful tool in real-world situations.

Make sure to keep your students motivated. This standard is the last of those dealing with working on transformations in the plane. Since they've already learned so much about congruence and transformations (or they should have, anyway), it's not as if they have to jump through hoops to master this concept.

All they have to do is put pencil to paper and start drawing. Well, transforming drawings, actually.

(Source: www.shmoop.com)

CLUSTER	**2. Understand congruence in terms of rigid motions**
BIG IDEA	• **Congruence and similarity can be used to prove geometric relationships and to solve problems.**

STANDARD AND DECONSTRUCTION

GCO.6	**Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.**
DESCRIPTION	Use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane. Knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent.

ESSENTIAL QUESTION(S)	How can I use rigid motions to understand congruence?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use geometric descriptions of rigid motions to transform figures. Use geometric descriptions of rigid motions to predict the effect of a given motion on a given figure.	Determine if two figures are congruent using the definition of congruence in terms of rigid motions.	

EXPLANATIONS AND EXAMPLES

A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.

Students may use geometric software to explore the effects of rigid motion on a figure(s).

The words "rigid" and "motion" sound like complete opposites, don't they? Well, not in geometry. A rigid motion is a motion—or *transformation*, in geometric lingo—that preserves distance and lengths; in other words, every segment of the image is congruent to its preimage.

Students should already be comfortable with the ideas of translations, reflections, and rotations. They have to use rigid motions to not only transform figures and predict the effect it has on a specific figure, but also to determine when two figures are congruent.

Congruence between two geometric objects can be defined as a rigid motion on the coordinate plane that maps one object onto another. Students should be able to recite this this definition verbatim. Or, more importantly, they should actually *understand it*.

The key here is to differentiate between congruence and carrying one object onto another. While two reflected objects can't always be carried onto one another, they can still be congruent because all that separates them is reflection—a rigid motion. Make sense?

Once students fully understand how these topics fit together, they will be able to use transformations to not only find and prove properties of geometry, but to create patterns, including tessellations.

If your students are giving you a hard time with why these things are important, remind them that if they ever want to dabble in design, whether it's fashion, interior, or video game, they have to identify and create patterns. At the root of patterns is, you guessed it, geometry. That'll get them thinking about transformations and congruence the next time they watch *Project Runway* or play *Call of Duty*.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GCO.7	**Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.**		
DESCRIPTION	Use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.		
ESSENTIAL QUESTION(S)	How can I use rigid motions to understand congruence?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Identify corresponding angles and sides of two triangles. Identify corresponding pairs of angles and sides of congruent triangles after rigid motions.	Justify congruency of two triangles using transformations. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if corresponding pairs of sides and corresponding pairs of angles are congruent. Use the definition of congruence in terms of rigid motions to show that if the corresponding pairs of sides and corresponding pairs of angles of two triangles are congruent, then the two triangles are congruent.	

EXPLANATIONS AND EXAMPLES

A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.

Congruence of triangles

Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.

Students should know that two triangles are congruent if there is a rigid motion that maps one onto the other. All corresponding pairs of all sides and all angles *must* be congruent, which means the two triangles are essentially equal. The only triangles that don't work like this are love triangles; each one is different in its own heartbreaking way.

Of course, if you've read through the standards before this, you know that rigid motions and congruence aren't limited to triangles. So why do triangles get their own standard? What makes them so special?

Since triangles are defined by 3 sides and 3 angles, knowing a limited number of each is often enough to find the rest of the missing information. In short, congruence is easier to prove with triangles. In fact, triangles have their own postulates that are designed to prove triangle congruence with limited information specifically. Fancy shmancy.

So if all corresponding sides and angles of two triangles are congruent, the two triangles themselves are congruent. In applying the rules of congruence and rigid motions to triangles, students drill down one level deeper: not only are the triangles themselves congruent, but the corresponding parts are congruent, as well. This opens up a whole new world of how to discover more about the use of triangles in the real world.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GCO.8	**Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.**		
DESCRIPTION	Use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria; ASA, SSS, and SAS		
ESSENTIAL QUESTION(S)	How can I use rigid motions to understand congruence?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Informally use rigid motions to take angles to angles and segments to segments (from 8th grade). Formally use dynamic geometry software or straightedge and compass to take angles to angles and segments to segments.	Explain how the criteria for triangle congruence (ASA, SAS, SSS) follows from the definition of congruence in terms of rigid motions.	

EXPLANATIONS AND EXAMPLES

Basically, this standard requires students to fall in love with triangles. They should be so head-over-heels that triangles become tri-*angels*. (That's an angel that's three times as heavenly.)

Of course, falling in love with triangles isn't easy. If your students who think geometry is a four-letter word, they may not want to enter in any spelling bees in the near future. But more than that, they can use three-letter words to save themselves from drowning in the oddly symmetrical sea of geometric congruence.

Specifically, we're talking about the three-letter words SSS, SAS, and ASA. They're actually acronyms for "Side Side Side," "Side Angle Side," and "Angle Side Angle." But we don't judge.

These 3 acronyms represent different combinations of sides and angles that must correspond in order for two triangles to be congruent: all corresponding sides, two corresponding sides and their included angles, or two corresponding angles and any corresponding side. If two triangles meet just *one* of these three criteria, they're congruent.

Students should know not only *what* these three rules are and how they work, but also *when* to use which one based on the situation. Sometimes more than one is applicable!

We also recommend showing *why* these rules work. A good way to do this is to find *all* angles and side lengths of two triangles while only given enough to satisfy the SSS, SAS, or ASA rules. Students should understand that when even one of these rules applies, the triangles are congruent not because the rule "says so," but because it's a quick way of realizing that all side lengths and angles are congruent.

Basically, they should understand that the concept of congruence hasn't changed. You've just given them shortcuts. All the more reason to fall in love with triangles, we think. In fact, don't be surprised if your students have already picked a date for the wedding.

(Source: www.shmoop.com)

CLUSTER	**3. Prove geometric theorems**
BIG IDEA	• **Shapes and objects can be classified and analyzed by defining attributes.**

STANDARD AND DECONSTRUCTION

GCO.9	**Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.**
DESCRIPTION	Prove theorems pertaining to lines and angles. Prove vertical angles are congruent. Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angels are congruent. Prove points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

ESSENTIAL QUESTION(S)	What strategy can be used to prove geometric theorems?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☐ 1 ☐ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Identify and use properties of vertical angles, parallel lines with transversals, all angle relationships, corresponding angles, alternate interior angles, perpendicular bisector, equidistant from endpoint.	Prove vertical angles are congruent. Prove corresponding angles are congruent when two parallel lines are cut by a transversal and converse. Prove alternate interior angles are congruent when two parallel lines are cut by a transversal and converse. Prove points are on a perpendicular bisector of a line segment are exactly equidistant from the segment's endpoint.	
EXPLANATIONS AND EXAMPLES	Whether students think you're *acute* or *obtuse*, the important thing is that they all know you're *right*. Not only can you prove it to them, you'll get them to prove it for themselves. Yep, it's time to talk about theorems about lines and angles, which all point to the same end goal: proving congruence. For many students, proofs are the most grueling part of geometry. It's best to ease them in by reminding them that proofs are just logical statements that flow from one to the next. They aren't a secret alien language, no more so than algebra or trigonometry. Honest. Students should know that a theorem is different from a postulate because it is a formula or statement deduced from a chain of reasoning, or a series of other proofs already accepted. A postulate, on the other hand, is an assumed truth based on rational geometric principles. Different students prefer different types of proofs. Some take a liking to paragraph proofs so they can provide a stream-of-consciousness rant and have it be an acceptable answer. Others prefer the two-column proof so that all their arguments are clearly laid out in front of them. Be lenient as to the format of the proof at first, but make sure your students know what the point of a proof actually is and what their goal is in writing one.		

EXPLANATIONS AND EXAMPLES (continued)	Knowing basic information and definitions about lines and angles is crucial, since they'll be the building blocks of your students' arguments. For instance, many theorems concerning lines and angles will make use of the fact that a straight line is 180°. Vertical angles come to mind. You can let students know that once one proof is worked out, they can use the result in future proofs. Much like Pokémon, proofs are meant to be collected and used when tackling difficult problems. So we can use the vertical angle proof to prove another theorem about alternate interior angles. Can Pikachu do that? We didn't think so. *(Source: www.shmoop.com)*

STANDARD AND DECONSTRUCTION

GCO.10	**Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180º; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.**
DESCRIPTION	Prove theorems pertaining to triangles. Prove the measures of interior angles of a triangle have a sum of 180º. Prove base angles of isosceles triangles are congruent. Prove the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length. Prove the medians of a triangle meet at a point.
ESSENTIAL QUESTION(S)	What strategy can be used to prove geometric theorems?
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.
DOK Range Target for Instruction & Assessment	☐ 1 ☐ 2 ☒ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Identify the hypothesis and conclusion of a theorem.	Design an argument to prove theorems about triangles. Analyze components of the theorem. Prove theorems about triangles.	

EXPLANATIONS AND EXAMPLES	Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles. If your students have gotten tired of lines and angles, they're in luck. Tell them that triangles are nothing like lines and angles. Instead, tell them triangles are new, edgy, and they've got some good points…three of them, actually. Conveniently leave out the fact that triangles are just three line segments joined together. Students should know the basic definitions that come with triangles and how to classify them based on angles and sides. When they can use the words "equiangular" and "isosceles" in everyday conversation, you'll know you've done a good job. Students should also be comfortable with the angles of a triangle, both interior and exterior. They should know that all the interior angles of a triangle add up to 180°, and they should know how to prove it. It's better to introduce these concepts to them by using concepts they should already know, like parallel lines and transversals. But that's just the tip of the triangular iceberg. There's way more inside triangles than just three interior angles. For instance, we can fill a triangle with medians, line segments that join the vertices of a triangle to the midpoints of its opposite sides. We can also connect the midpoints of each side in the triangle to form a similar triangle that's half the size of the original one.

EXPLANATIONS AND EXAMPLES (continued)	Finally, students shouldn't get lost with all the theorems and postulates. They all build on each other, and it's best to keep track of these proofs and postulates so that students don't get confused. Also, students should know that using proofs and theorems they've already learned isn't cheating; it's *applying* the skills they've learned, and it's highly encouraged. If studying triangles still seems more confusing than the Bermuda Triangle, give plenty of examples and draw on knowledge that's already been introduced. It's difficult to learn something new without understanding the basics, so go back and re-derive some proofs if needed. You'll get grumbles in the moment, but they'll thank you for it later. Better than being stuck in the Bermuda Triangle, anyway. *(Source: www.shmoop.com)*

STANDARD AND DECONSTRUCTION

GCO.11	**Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.**
DESCRIPTION	Prove theorems pertaining to parallelograms. Prove opposite sides are congruent. Prove opposite angles are congruent. Prove the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

ESSENTIAL QUESTION(S)	What strategy can be used to prove geometric theorems?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☐ 1 ☐ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Classify types of quadrilaterals. Explain theorems for parallelograms and relate to figure.	Use the principle that corresponding parts of congruent triangles are congruent to solve problems. Use properties of special quadrilaterals in a proof.	
EXPLANATIONS AND EXAMPLES	Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms. You are cuddled up with a cup of joe at your favorite coffee shop, Joe's Joe, to do some grading. You decide to take break when you overhear a couple of high school students debating over one of life's toughest questions to answer: which types of cheese and bread make the best grilled cheese sandwich? The answer is obvious to you. It's pumpernickel, mozzarella, and cheddar. But these students are making arguments for choices that go beyond personal preference. You are impressed with their critical thinking when they agree on a different answer: American and Swiss on rye. And in fact, they've managed convinced you of it, too. The critical thinking skills used by these students are much like those that students must learn when they prove mathematical theorems. They should learn how to link what they know in a logical string to prove or disprove an argument. They should have already done this with lines and triangles, so the next logical step would be to head right into the world of parallelograms. In order to prove theorems about parallelograms, students might want to know what a parallelogram is. (Spoiler alert: it's a quadrilateral with opposite sides that are parallel.) From there, students will want to come up with an argument to prove, whether it's that opposite sides of a parallelogram are congruent, opposite angles are congruent, or that the diagonals bisect each other.		

EXPLANATIONS AND EXAMPLES	You wouldn't order a grilled cheese just to sit there and stare at it. (Marveling at the beauty of the perfect grilled cheese is excusable and even understandable, but not tasting its delicious cheesy goodness is near blasphemous.) Just the same, students didn't learn about parallel lines, transversals, congruent triangles, and complementary angles for nothing. They should use the knowledge they already have and apply it to parallelograms. If students are struggling, tell them that pictures are always an excellent way to start proofs. That way, they should at least be able to get off the ground to start with. Refreshing their memory about the theorems and definitions they'll be using might be helpful as well. Also, using the two-column proof format might help students organize their thoughts better, at least in the beginning. Those paragraph proofs can get messy to the point of uselessness. Of course, that's not always the case for everything. When it comes to grilled cheese sandwiches, for example, messiness has got nothing to do with it. *(Source: www.shmoop.com)*

CLUSTER	**4. Make geometric constructions**
BIG IDEA	• **The location of lines, angles and geometric shapes within a plane provide geometric interpretations of mathematical situations.**

STANDARD AND DECONSTRUCTION

GCO.12	**Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.**		
DESCRIPTION	Copy a segment. Copy an angle. Bisect a segment. Bisect an angle. Construct perpendicular lines, including the perpendicular bisector of a line segment. Construct a line parallel to a given line through a point not on the line.		
ESSENTIAL QUESTION(S)	What tools can be used to make geometric constructions?		
MATHEMATICAL PRACTICE(S)	HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Explain the construction of geometric figures using a variety of tools and methods.	Apply the definitions, properties, and theorems about line segments, rays, and angles to support geometric constructions. Apply properties and theorems about parallel and perpendicular lines to support constructions.	Perform geometric constructions using a variety of tools and methods, including: Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

EXPLANATIONS AND EXAMPLES

Perform geometric constructions using a variety of tools and methods, including: Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line

Explanations and Examples:

Students may use geometric software to make geometric constructions.

Examples:

- Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.
- Construct the circumcenter of a given triangle.

For this standard, students are expected to be able to draw. Okay, so it is a little more complicated than that, but that's essentially the gist of it.

EXPLANATIONS AND EXAMPLES (contInued)	Like aspiring artists everywhere who send for their free art talent test and studiously copy the turtle or pirate, your students will be very intentional and deliberate as they construct lines and angles. Art students use their charcoal and oil pastels; geometry students will whip out their compasses and straightedges. Art students take suggestions to make their drawings more lifelike; geometry students will rely on properties, postulates, theorems, and corollaries to make their drawings more rigid. Art students know that practice makes progress; geometry students will figure this out soon enough. The standard itself lists a few examples of both the tools students might be presented with and the tasks they should be able to perform. However, it takes more than fancy tools to make these constructions. Students should keep definitions, properties, and theorems about line segments, rays, angles, and parallel and perpendicular lines safe in their back pockets in order to support their drawings. Students will benefit from plenty of opportunities to practice each skill. Even more helpful will be different contexts for each one. For example, students might first construct an angle bisector on a single angle, then the angle bisectors for all three angles of a triangle to show that they are concurrent, and finally explore whether the angle bisectors for other polygons are also concurrent. Once they have mastered using a compass and straightedge for a particular construction, students will benefit from exploring using other tools and methods, such as paper folding or using a mirror, to complete the same task. This will reinforce their understanding of the rules of the shape they're working with. Even though artists are ultimately free to create whatever art they want, remind your students that they still follow general rules. That way, students can't claim that you're stifling their creativity when you're actually giving them tools to nurture it. They'll be acing those geometry exams (and art talent tests) before they know it. *(Source: www.shmoop.com)*

STANDARD AND DECONSTRUCTION

GCO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
DESCRIPTION	Construct an equilateral triangle so that each vertex of the equilateral triangle is on the circle. Construct a square so that each vertex of the square is on the circle. Construct a regular hexagon so that each vertex of the regular hexagon is on the circle.

ESSENTIAL QUESTION(S)	What tools can be used to make geometric constructions?		
MATHEMATICAL PRACTICE(S)	HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☐ 1 ☐ 2 ☒ 3 ☐ 4		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:			Construct an equilateral triangle, square, and regular hexagon inscribed in a circle.
EXPLANATIONS AND EXAMPLES	A dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. Students may use geometric simulation software to model transformations. Students may observe patterns and verify experimentally the properties of dilations. Your students may understand generally what the term similar means, but it can be very specific when it comes to geometry. For instance, while Tia and Tamera Mowry are very similar (you know, since they're identical twins and all), they aren't geometrically similar. How on earth could that be? In geometry, similar objects are exactly the same shape, but not necessarily the same size. So one object could be smaller than a pea and another could be larger than Antarctica, but if they have the exact same shape, they're similar. How curious. Still, the idea of being "exactly the same shape" is a little vague. How can we be sure that two objects of different sizes still have the same shape? That's where dilations, centers, and scale factors come in. *(Source: www.shmoop.com)*		

DOMAIN:

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY (G-SRT)

HIGH SCHOOL

GEOMETRY

DOMAIN	Similarity, Right Triangles and Trigonometry (SRT)
CLUSTERS	1. Understand similarity in terms of similarity transformations. 2. Prove theorems involving similarity. 3. Define trigonometric ratios and solve problems involving right triangles. 4. Apply trigonometry to general triangles
ACADEMIC VOCABULARY	angle, circle , perpendicular lines, parallel lines, line segments, point, line, arc, rigid motion, congruent, angle-side-angle(asa), side-angle-side (sas), side-side-side (sss), inscribed, scale factor, dilation, aa similarity, theorem, law of sines, law of cosines

CLUSTER	1. Understand similarity in terms of similarity transformations.
BIG IDEA	• Similarity is essential to understanding geometric shapes and patterns in math and every day life.

STANDARD AND DECONSTRUCTION

GSRT.1	**Verify experimentally the properties of dilations given by a center and a scale factor.** **a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.** **b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.**
DESCRIPTION	a. Given a center and a scale factor, verify experimentally, that when dilating a figure in a coordinate plane, a segment of the pre-image that does not pass through the center of the dilation, is parallel to it's image when the dilation is preformed. However, a segment that passes through the center remains unchanged. b. Given a center and a scale factor, verify experimentally, that when performing dilations of a line segment, the pre-image, the segment which becomes the image is longer or shorter based on the ratio given by the scale factor.

ESSENTIAL QUESTION(S)	How can transformations help me to understand similarity?
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.5. Use appropriate tools strategically.
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☐ 3 ☐ 4

SUBSTANDARD DECONSTRUCTED	**GSRT.1.a: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.**		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Concepts/Skills: Define image, pre-image, scale factor, center, and similar figures as they relate to transformations. Identify a dilation stating its scale factor and center.	Verify experimentally that a dilated image is similar to its pre-image by showing congruent corresponding angles and proportional sides. Verify experimentally that a dilation takes a line not passing through the center of the dilation to a parallel line by showing the lines are parallel. Verify experimentally that dilation leaves a line passing through the center of the dilation unchanged, by showing that it is the same line.	

SUBSTANDARD DECONSTRUCTED	GSRT.1b: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Explain that the scale factor represents how many times longer or shorter a dilated line segment is than its pre-image.	Verify experimentally that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. Verify experimentally that the dilation of a line segment is longer or shorter in the ratio given by the scale factor.	

EXPLANATIONS AND EXAMPLES

A dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Students may use geometric simulation software to model transformations. Students may observe patterns and verify experimentally the properties of dilations.

Your students may understand generally what the term similar means, but it can be very specific when it comes to geometry. For instance, while Tia and Tamera Mowry are very similar (you know, since they're identical twins and all), they aren't geometrically similar. How on earth could that be?

In geometry, **similar** objects are exactly the same *shape*, but not necessarily the same *size*. So one object could be smaller than a pea and another could be larger than Antarctica, but if they have the exact same shape, they're similar. How curious.

Still, the idea of being "exactly the same shape" is a little vague. How can we be sure that two objects of different sizes still have the same shape? That's where dilations, centers, and scale factors come in.

(Source: www.shmoop.com)

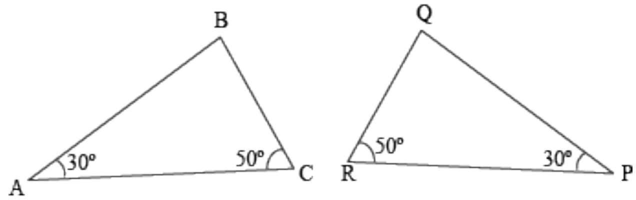
STANDARD AND DECONSTRUCTION

GSRT.2	**Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.**
DESCRIPTION	Use the idea of dilation transformations to develop the definition of similarity. Given two figures determine whether they are similar and explain their similarity based on the equality of corresponding angles and the proportionality of corresponding sides.

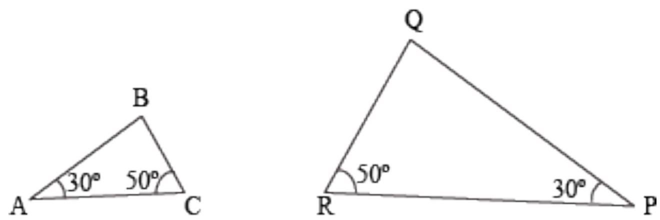
ESSENTIAL QUESTION(S)	How can transformations help me to understand similarity?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Explain that triangles are similar f all pairs of corresponding angles are congruent and all corresponding pairs of sides are proportional.	Given two figures, decide if they are similar by using the definition of similarity in terms of similarity transformations.	
EXPLANATIONS AND EXAMPLES	A similarity transformation is a rigid motion followed by a dilation. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. Students should already be familiar with transformations, at least to the extent that they know that it takes more than an, "Abracadabra!" to change or move a shape. It helps if they know what translation, reflection, rotation, and dilation are, though. Magic tricks are fun and all, but not exactly useful when determining similarity. Students should already know that if they can perform any rigid transformation to carry one shape exactly onto another, the two shapes are congruent. If we throw dilation into the mix (whether it's contraction or expansion), we can safely say that the two shapes are similar. While those basic rules apply to pretty much any two-dimensional shape you can imagine (although it might be a pain to verify that two dodecagons are similar), triangles get special treatment. In fact, triangles are like the royal family of geometry...only without the corgis.		

EXPLANATIONS AND EXAMPLES (continued)

Because they're so small and easy, we can use triangles to really get to the heart of what similarity is. Let's say we have two triangles, *ΔABC* and *ΔPQR*, that are congruent to each other.



Then, we can contract *ΔABC* so that it's half the size of ΔPQR. (That's a scale factor of 0.5. Hint, hint.) Dilation is still a similarity transformation, so even though they aren't congruent anymore, the two triangles are still similar.



Students should realize that even though the side lengths of *ΔABC* have changed, its angles have not. They might say something like, "Hey, those two triangles have similar-looking angle measures. They're just not the same size." If you hear something like that, it's a good sign.

This means two triangles are similar when all their corresponding angles are equal. In fact, students can take this to the next level. Whenever any two polygons of differing sizes have the same set of angle measures within them, the polygons are **similar.**

So that's angles. What about side lengths? Our scale factor tells us how we relate side lengths of similar triangles to each other. Ratios of corresponding side lengths should always give the same scale factor. If two shapes are similar, these ratios of corresponding side lengths will all equal the same number: the scale factor.

A good way to make sure students have cemented all this knowledge is to have the students measure the sides of similar shapes with a ruler and the angles with a protractor. Then, give another set of shapes that are *almost* similar. Students can then realize that the measures of the angles in the figure affect the scale ratio they calculate using the side lengths.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GSRT.3	**Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.**
DESCRIPTION	Use the properties of similarity transformations to develop the criteria for proving similar triangles; AA.
ESSENTIAL QUESTION(S)	How can transformations help me to understand similarity?
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recall the properties of similarity transformations.	Establish the AA criterion for similarity of triangles by extending the properties of similarity transformations to the general case of any two similar triangles.	

EXPLANATIONS AND EXAMPLES

Rotation, translation, reflection, contraction, and dilation. Poor shapes. It seems like any time they change, they get shunned.

Your students, on the other hand, won't be shunned for mastering the Angle-Angle criterion for establishing similarity between two triangles. To do that though, it only makes sense that they should be familiar with the notions of similarity, congruence, and different types of angles. (We're talking alternate interior angles, alternate exterior angles, corresponding angles, vertical angles, and more.) It will also behoove them to keep all of those theorems about congruent triangles in mind. You know, all the ones with S's and A's.

Students should also be familiar with the definitions—*definishuns*—of the similarity transformations that result in similar triangles: all the congruence transformations (translation, reflection, and rotation) and those that preserve shape (expansion and contraction).

The first three should be pretty familiar to students, who probably learned about them beginning back in kindergarten. They probably didn't know them by their fancy names, but slide, turn, and flip are pretty common vocab words for five year-olds, along with pasketti and petty larceny.

If students are having a hard time remembering which transformation is which, they might secretly enjoying getting out of their seats to act out each change. Make it a game and have their peers guess which transformation they're performing. You might just be amazed at how talented their performances can be.

Demonstrating mastery of the Angle-Angle similarity postulate requires a little talent as well. Students will have to be able to identify two pairs of congruent angles in two triangles. They should also know that since all the angles in a triangle add up to 180°, knowing two angles actually means knowing three. Sometimes it's as easy as calculating the measure of the unlabeled angle, but more often than not, students will need to draw on their knowledge of theorems about congruent, complementary, and supplementary angles.

Make sure that they're up to speed on all those theorems, and that they know the rules for writing proofs, and they should be good to go.

(Source: www.shmoop.com)

CLUSTER	**2. Prove theorems involving similarity**
BIG IDEA	• **Shapes and objects can be classified and analyzed by defining attributes.**

STANDARD AND DECONSTRUCTION

GSRT.4	**Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.**
DESCRIPTION	Use AA, SAS, SSS similarity theorems to prove triangles are similar. Use triangle similarity to prove other theorems about triangles o Prove a line parallel to one side of a triangle divides the other two proportionally, and it's converse o Prove the Pythagorean Theorem using triangle similarity.

ESSENTIAL QUESTION(S)	What strategy can be used to prove similarity theorems?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☐ 1 ☐ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recall postulates, theorems, and definitions to prove theorems about triangles.	Prove theorems involving similarity about triangles.	

EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

When Harry and Helen started out, they hung things on blank walls and picked out tiles together. But, like all couples who break up, they had to split up their stuff in a way that they felt was fair and proportional.

Proportional splitting seems to be a trend amongst triangles when they are cut apart. Oddly, it's a common thread in the theorems about said triangles as well—making sure the parts are distributed proportionally when they do get cut apart.

Whether it's the Side Splitter Theorem claiming that a line parallel to one side divides the other two sides proportionally, or the Angle Bisector Theorem insisting that the resultant side segment pieces are proportional when one angle is bisected, triangles care more than any shape we know about fairness.

Students should know that in triangles and in life, fair doesn't necessarily mean equal. The pieces of the triangle are proportional, but that doesn't always mean congruent. They each get the right amount according to their own proportional needs. (Does this make them Marxist?)

As for the Pythagorean theorem, well, that's not implicitly about splitting things up, but if we intend to prove it using similar triangles (which we do), then we will be explicitly splitting things up. We're talking about the fact that to prove similarity, we should look at *ratios* of side lengths rather than the lengths themselves.

Students will probably want to keep in mind the importance of being similar. Not to each other—though they will certainly be on top of the mismatched socks trend anyway—but the similarity between triangles that are, by definition, similar.

If they forget the basics of triangle similarity, give them a quick review of corresponding angles, corresponding sides, congruence transformations, and similarity transformations. These are pretty much the fundamental blocks with which students can build proofs about triangles.

Furthermore, students should definitely be able to name the three tests for identifying similar triangles: AA, SSS, and SAS. Remind them that when we're talking about similarity, those S's are more about the right proportions than they are about equality. Just like Harry and Helen.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GSRT.5	**Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.**		
DESCRIPTION	Using similarity theorems, prove that two triangles are congruent. Prove geometric figures, other than triangles, are similar and/or congruent.		
ESSENTIAL QUESTION(S)	What strategy can be used to prove similarity theorems?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recall congruence and similarity criteria for triangles.	Use congruency and similarity theorems for triangles to solve problems. Use congruency and similarity theorems for triangles to prove relationships in geometric figures.	

EXPLANATIONS AND EXAMPLES

Similarity postulates include SSS, SAS, and AA.

Congruence postulates include SSS, SAS, ASA, AAS, and H-L.

Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

Tom Hanks as Forrest in *Forrest Gump*, Tom Hanks as Ray Peterson in *The 'Burbs*, and Tom Hanks as Chuck Noland in *Cast Away.* Congruent or similar?

Well, they are technically different characters in very different movies. So they'd probably best be described as similar. But based on Hanks's fixation with urinating on camera, they certainly have congruent characteristics.

To truly test for congruence versus similarity, we'd need to evaluate them based on more specific terms. Like which side of the discussion about choosing an even numbered-urinal they come down on. Or their angle on preventing splash back.

Enough pee jokes? Yes, thanks.

The criteria to determine whether triangles are congruent or similar include proportionality or congruency of side lengths and congruency of angles. Fortunately, students don't have to check all three sides and all three angles every time they want to test for congruency or similarity. That would make the process rather inefficient.

And if we've said it once, we've said it a million times: mathematicians thrive on efficiency.

There are several ways to combine the side length proportionality or congruency and angle congruency to check for triangle congruency or similarity. They're various combinations and permutations of the letters S (for side) and A (for angle) and students should already be familiar with them. In fact, they should have mastered these congruency versus similarity tests for triangles.

The Tom Hanks character congruency versus similarity test, however, still needs some work.

(Source: www.shmoop.com)

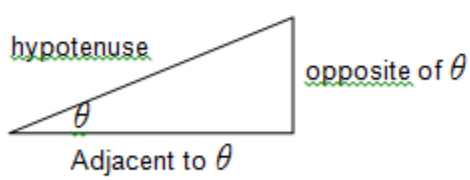
CLUSTER	**3. Define trigonometric ratios and solve problems involving right triangles**
BIG IDEA	• **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.** • **Shapes and objects can be classified and analyzed by defining attributes.**

STANDARD AND DECONSTRUCTION

GSRT.6	**Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.**		
DESCRIPTION	Using a corresponding angle of similar right triangles, show that the relationships of the side ratios are the same, which leads to the definition of trigonometric ratios for acute angles.		
ESSENTIAL QUESTION(S)	How do trigonometric ratios help in solving problems involving right triangles?		
MATHEMATICAL PRACTICE(S)	HS.MP.6. Attend to precision. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Name the sides of right triangles as related to an acute angle. Recognize that if two right triangles have a pair of acute, congruent angles, that the triangles are similar.	Compare common ratios for similar right triangles and develop a relationship between the ratio and the acute angle leading to the trigonometry ratios.	

EXPLANATIONS AND EXAMPLES

Students may use applets to explore the range of values of the trigonometric ratios as θ ranges from 0 to 90 degrees.



$$\text{sine of } \theta = \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{cosecant of } \theta = \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{cosine of } \theta = \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{secant of } \theta = \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{tangent of } \theta = \tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \text{cotangent of } \theta = \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

The old army colonel and his oft-hiccupping son. The revered Native American Sohcahtoa. And Henry, who is unable to add tens or hundreds.

Just as the cast-off playthings on the Island of Misfit Toys want nothing more than to find a child to love, this lovable clan of well-meaning folk desires nothing more than to help your students remember the definitions of trigonometric ratios for acute angles.

EXPLANATIONS AND EXAMPLES (continued)	Yep, they're all mnemonics used to help students remember that sine equals opposite over hypotenuse, cosine equals adjacent over hypotenuse, and tangent equals opposite over adjacent. The old army colonel and his son often hiccup. Silly old Henry can't add hundreds, tens, or anything. SOHCAHTOA And of course, there are about a bajillion more. Take your pick. Or, rather, let your students take their pick. It doesn't really matter to us which one they use, so long as they can remember which ratio is which. Of course, we'd also like them to remember that, in similar triangles, corresponding sides are proportional and corresponding angles are congruent, which is where we get those trigonometric ratios in the first place. Students would benefit from an investigation in which they find and compare these ratios. Perhaps give them a set of nested similar right triangles, each of which share one angle, and have them name the ratios of side lengths for each one. The key here is that students work with the ratios in similar triangles so they can see that, even when the side lengths are different, the ratio of their lengths stays the same for a congruent angle. Growing or shrinking a triangle's side lengths will not affect the ratio. Only changing the size of the angle will cause the sine or cosine measurement of the angle to change. After understanding the concepts of sine, cosine, and tangent, students should be able to use these as operators in order to find side lengths of right triangle given a side length and an angle. Then, they should be able to use the inverse trigonometric functions in order to find angle measures from side lengths. *(Source: www.shmoop.com)*

STANDARD AND DECONSTRUCTION

GSRT.7	**Explain and use the relationship between the sine and cosine of complementary angles.**		
DESCRIPTION	Explore the sine of an acute angle and the cosine of its complement and determine their relationship.		
ESSENTIAL QUESTION(S)	How do trigonometric ratios help in solving problems involving right triangles?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Geometric simulation software, applets, and graphing calculators can be used to explore the relationship between sine and cosine.	Explain how the sine and cosine of complementary angles are related to each other.	

EXPLANATIONS AND EXAMPLES

Geometric simulation software, applets, and graphing calculators can be used to explore the relationship between sine and cosine.

Who doesn't love a nice compliment? Love your new hair color! Those mismatched socks are so fashionable! OMG Vinny, your idea to get matching his-and-hers tramp stamps instead of wedding rings is so romantic!

Oops, wrong kind of compliment. We're talking here about complimentary angles. You know, the free ones. Like the how the chocolates they leave on your pillow at the Hilton are complimentary, right?

Wrong again. We're talking about angles that add up to 90°. As in, *complement* with an e. (Yes, that's it!) Right triangles are chock-full of them. Thanks to the Triangle Angle Sum Theorem, we know that the two acute angles in a right triangle add up to 90°.

Complementary angles belong together, much like sine and cosine. In fact, complementary angles are the peanut butter and jelly of geometry, and sine and cosine are the mate-for-life lobsters of trigonometry. And in math class, complementary angles and sine and cosine go great together. Too bad PB&J doesn't pair so well with lobster. (We sense an interesting Top Chef Quickfire challenge in the works.)

As it turns out, when we're dealing with a pair of complementary angles, the sine of one angle is equal to the cosine of the other angle (and vice versa). This is a very handy fact that students can use to evaluate or otherwise simplify complicated trigonometric equations.

Knowing that sine and cosine are related in this way, we can also find similar relationships between tangent and cotangent, and between secant and cosecant. (While they're not explicitly mentioned in the standard, they are derived from the sine-cosine relationship and may well come up as extensions of the basic content.)

In order to succeed with this standard, as well as its extension into the relationships between other trigonometric identities, students will need to be familiar with the ratios that define them. As long as they're familiar with sine, cosine, and tangent, knowing that cosecant, secant, and cotangent are the respective reciprocals should be enough to point them in the right direction.

Students will also need to be able to compute simple calculations that involve adding and subtracting up to 90, but we're going to hope that, since they're in your geometry class, they've already mastered that skill. Well, "master" might be too strong a word…

And finally, they'll probably need to be able to tell the difference between *compliment* and *complement.* (Really, it's not rocket science.)

(Source: www.shmoop.com)

MATHEMATICS

STANDARD AND DECONSTRUCTION

GSRT.8	**Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.**
DESCRIPTION	Apply both trigonometric ratios and Pythagorean Theorem to solve application problems involving right triangles.
ESSENTIAL QUESTION(S)	How do trigonometric ratios help in solving problems involving right triangles?
MATHEMATICAL PRACTICE(S)	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Concepts/Skills: Recognize which methods could be used to solve right triangles in applied problems. Solve for an unknown angle or side of a right triangle using sine, cosine, and tangent.		Apply right triangle trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

EXPLANATIONS AND EXAMPLES

Students may use graphing calculators or programs, tables, spreadsheets, or computer algebra systems to solve right triangle problems.

On the one hand, your students know the basic trigonometric ratios like the back of their hands. And on the other hand, they could use the Pythagorean theorem with one hand tied behind their backs. Now, it's time for them to put their hands together and use the two concepts to solve applications or word problems.

This standard focuses on having students apply their knowledge of trig ratios and the Pythagorean theorem to solve problems other than the usual, "Find the missing angle/side." Often, they'll find it helpful to draw a model or visually represent the given information in order to organize their thoughts and clearly show their work. This often involves creating and solving mathematical models.

And how, exactly, will students achieve and demonstrate mastery in regards to this standard? Give them plenty of word problems or applications to solve using trig ratios and the Pythagorean theorem. Challenge them to come up with their own problems for their peers to solve. Encourage them to find examples from their own lives of problems that can be solved with these concepts.

With enough practice solving applied problems, your students will start to see right triangles and trig functions everywhere. (We mean that figuratively. If they really do see right triangles and trig functions everywhere, they should consult an optometrist.)

(Source: www.shmoop.com)

Example:

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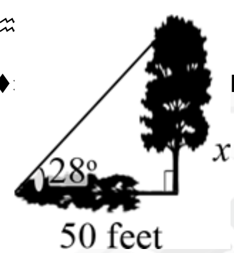


CLUSTER	**4. Applying trigonometry to general triangles**
BIG IDEA	• **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.** • **Shapes and objects can be classified and analyzed by defining attributes.**

STANDARD AND DECONSTRUCTION

GSRT.9	**(+) Derive the formula A=½ ab sin (C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.**
DESCRIPTION	For a triangle that is not a right triangle, draw an auxiliary line from a vertex, perpendicular to the opposite side and derive the formula, A=½ ab sin (C), for the area of a triangle, using the fact that the height of the triangle is, h=a sin(C).

ESSENTIAL QUESTION(S)	How does trigonometry help in solving problems involving general triangles?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recall right triangle trigonometry to solve mathematical problems.		Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side

EXPLANATIONS AND EXAMPLES

Area 51, originally intended to be a testing site for the U-2 spy plane, has grown in pop culture and infamy to outlandish proportions. The alleged site of myriad government and military cover-ups, this location is well known around world for its rumored possession and testing of alien technologies and specimens.

Many parents scoff at the idea that the government needs to cover up any evidence of extraterrestrial life, claiming their very own teenage offspring have clearly been replaced with aliens. We here at Shmoop are keeping our mouths shut. After all, that kind of information is classified for a reason.

As for Area 51, well, we disavow any knowledge of that topic as well.

But finding the area of a triangle is public knowledge that we are willing and able to discuss. Even young spacelings—er, students—are familiar with the topic. By second grade, they have begun their study of area by counting the number of squares inside a shape. By sixth grade, they've calculated area with formulas. And now, they need to be able to derive the formula for finding the area of a triangle.

Before they become convinced that the standard expects them to master an alien concept, have them work through a few sample problems. Start with the original formula they learned ($A = \frac{1}{2}bh$) and a few right triangles. Then demonstrate the derivation of the fancier sine-based version and relate it to the simpler one.

Being able to derive the formula is one skill that will be crucial for those students choosing to enroll in more advanced math courses, like analytic geometry, applied calculus, and intergalactic algebra.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GSRT.10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.		
DESCRIPTION	Using trigonometry and the relationship among sides and angles of any triangle, such as sin(C)=(h/a) , prove the Law of Sines. Using trigonometry and the relationship among sides and angles of any triangle and the Pythagorean Theorem to prove the Law of Cosines. Use the Laws of Sines to solve problems. Use the Laws of Cosines to solve problems.		
ESSENTIAL QUESTION(S)	How does trigonometry help in solving problems involving general triangles?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☒ 3 ☐ 4		
Learning Expectations	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use the Laws of Sines and Cosines this to find missing angles or side length measurements.	Prove the Law of Sines. Prove the Law of Cosines. Recognize when the Law of Sines or Law of Cosines can be applied to a problem and solve problems in context using them.	

EXPLANATIONS AND EXAMPLES

The world is full of signs. Traffic signs, billboards, official John Hancocks, sign language, and plenty more. By the way, what's your sign?

Whether they're Capricorns or Cancers, students should be familiar with a very different kind of sign. We're talking about none other than sine and cosine and the laws that govern them.

The Laws of Sines and Cosines allow students to easily solve triangles. Depending on what information they are given, students can use these laws to find the missing angles or side lengths of a triangle. As you might expect, that means students have to know these laws and understand what they mean. Deriving and proving these laws might be an excellent start.

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

EXPLANATIONS AND EXAMPLES (continued)

Students will also need to be fluent in the language of proofs, whether writing a two-column or paragraph proof, and being able to justify their statements with legitimate reasons. They might enjoy a break from the rigor of regular class work to play a matching game in which they have to pick out which theorem, property, or definition justifies a given statement.

Of course, students will also need a working knowledge of the whole collective of other theorems and definitions related to triangles and trigonometry, including but most definitely not limited to the Angle Sum Theorem for triangles, the definitions of sine and cosine, and their relationship among complementary angles.

Satisfying this standard is especially important for students planning to take more advanced mathematics courses. This is one of the few standards that have been earmarked as critical for students intending to enroll in higher math classes. If anything, that's a sign of how critical these laws are.

All these signs might just open up your eyes.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

GSRT.11	**(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).**		
DESCRIPTION	Understand and apply the Law of Sines and the Law of Cosines to find unknown measures in right triangles. Understand and apply the Law of Sines and the Law of Cosines to find unknown measures in non-right triangles.		
ESSENTIAL QUESTION(S)	How does trigonometry help in solving problems involving general triangles?		
MATHEMATICAL PRACTICE(S)	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.4. Model with mathematics		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Determine from given measurements in right and non-right triangles whether it is appropriate to use the Law of Sines or Cosines.	Apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	

EXPLANATIONS AND EXAMPLES

Students have already proven and used the Laws of Sines and Cosines, but now they're expected to extend their use of the laws and apply them to applications or word problems.

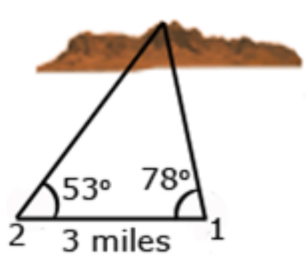
How will students master this standard? The same way they get to Carnegie Hall: practice, practice, practice.

It is in fact through solving many, many application problems using the Laws of Sines and Cosines that students will build their understanding of and confidence in applying these laws to solve problems. And as they become more confident and better understand what they're doing, they will be more prepared to demonstrate mastery of the standard. Sort of a feedback loop of mathematical brilliance, if you will.

Practice, practice, practice will also expose students to the many kinds of problems that can be solved through use of these laws, which will enable them to see the many uses that the laws have in different disciplines: not only math, but also engineering, surveying, orienteering, architecture, and so on.

Example:

- • Tara wants to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78o. From the second position, the angle between the mountain and the first position is 53o. How can Tara determine the distance of the mountain from each position, and what is the distance from each position?



DOMAIN:

CIRCLES (GC)

HIGH SCHOOL
GEOMETRY

MATHEMATICS

DOMAIN	Circles
CLUSTERS	1. Understand and apply theorems about circles. 2. Find arc lengths and areas of sectors of circles.
ACADEMIC VOCABULARY	angle, circle , perpendicular lines, parallel lines, line segments, point, line, arc, rigid motion, congruent, angle-side-angle(asa), side-angle-side (sas), side-side-side (sss), inscribed, scale factor, dilation, aa similarity, theorem, law of sines, law of cosines

CLUSTER	1. Understand and apply theorems about circles
BIG IDEA	• Congruence and similarity can be used to prove geometric relationships and to solve problems.

STANDARD AND DECONSTRUCTION

G.C.1	**Prove that all circles are similar.**
DESCRIPTION	Using the fact that the ratio of diameter to circumference is the same for circles, prove that all circles are similar.

ESSENTIAL QUESTION(S)	What properties of circles determine their similarity?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☐ 1 ☐ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize when figures are similar.	Compare the ratio of the circumference of a circle to the diameter of the circle. Discuss, develop, and justify the ratio of the circumference of a circle to the diameter of the circle for several circles. Determine that this ratio is constant for all circles.	

EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

What goes around comes around. Well, according to Justin Timberlake, anyway.

Students should understand that a circle is a closed curve on a plane. What's special about circles is that all the points on a circle are equidistant from the center. That means circles are defined by two details: position (the center of the circle) and size (the distance from the center to a point on the circle).

Students should know that position isn't a problem when we're talking about similarity or congruence transformations. If we have two congruent figures at different positions, translating them will easily map one on top of the other. But what about size?

Students should know that unlike polygons that have dimensions independent of one another (base and height, for instance), a circle's size depends only on one measurement: the radius r. Sure, we can look at the diameter d or the circumference C or even the area A, but they all depend on r as well. (Now would be a good time to tell them that $d = 2r$ and $C = 2\pi r$ and $A = \pi r^2$.)

Since all aspects of a circle's size depend on r, we can change the size of any circle simply by dilating the radius by a constant scale factor. Students should already know that dilations, whether they're expansions or contractions, are similarity transformations. We're changing the *size* of the circle, but not its shape.

If dilation changes a circle's size and translation can change its position, we can easily map one circle onto another using only those two transformations. Since both dilation and translation are similarity transformations, you can safely tell students that all circles are similar. In fact, we don't even need reflections or rotations to help us out. (Try rotating and reflecting circles and see how they change. Hint: they don't.)

Students can also use two circles and proportions to determine similarity. Two similar shapes will have a constant ratio when corresponding sides are compared. Circles don't have sides, but they do have radii, diameters, and circumferences that we can compare. If students do that, they'll quickly realize that a small circle with radius r and a larger circle with radius r' are similar by a constant scale factor.

EXPLANATIONS AND EXAMPLES (continued)

Students can also use two circles and proportions to determine similarity. Two similar shapes will have a constant ratio when corresponding sides are compared. Circles don't have sides, but they do have radii, diameters, and circumferences that we can compare. If students do that, they'll quickly realize that a small circle with radius r and a larger circle with radius r' are similar by a constant scale factor.

$$\frac{r}{r'} = \frac{d}{d'} = \frac{C}{C'}$$

Also, we could show in the same way that the area ratio equals the square of the radius ratio.

$$\frac{r^2}{r'^2} = \frac{A}{A'}$$

The circumferences, diameters, and radii of our two circles are all in proportion, which means they're similar. Essentially, this states that circles come in a variety of sizes, but they'll always be the exact same shape: a circle.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

G.C.2	**Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.**
DESCRIPTION	Using definitions, properties, and theorems, identify and describe relationships among inscribed angles, radii, and chords. Include central, inscribed, and circumscribed angles. Understand that inscribed angles on a diameter are right angles. Understand that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

ESSENTIAL QUESTION(S)	How do you show the relationships amongst properties of circles by applying theorems?
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Identify inscribed angles, radii, chords, central angles, circumscribed angles, diameter, tangent. Recognize that inscribed angles on a diameter are right angles. Recognize that radius of a circle is perpendicular to the radius at the point of tangency.	Examine the relationship between central, inscribed, and circumscribed angles by applying theorems about their measures.	

EXPLANATIONS AND EXAMPLES

When it comes to circles, students often think that they don't have to worry about angles anymore. After all, a circle is a curve. No corners means no angles, right? Right?

You may have to break the news to them gently because actually, circles have more angles than an anglerfish. (Note: the precise number of angles in an anglerfish has yet to be determined.)

Before students can identify and describe the various angles in a circle, they should be familiar with what these angles are. Students should also be familiar with the concept of arc measurement and how it relates to the measure of the different kinds of angles. Throw some chords in there, too, and we aren't talking about your guitar skills (though we're sure you can rock and roll with the best of 'em).

Students should know that a **central angle** is formed by two radii (where the vertex of the angle is the center of the circle), an **inscribed angle** is an angle formed by two chords (where the vertex of the angle is some point on the circle), and a **circumscribed angle** has a vertex outside the circle and sides that intersect with the circle.

To solidify the relationships between arc measure and angle measure, we suggest bringing a pizza to class. Explain that usually, we cut pizzas so that each slice of pizza has a particular central angle. The amount of crust (arc measure) depends on the central angle of the pizza slice.

If you have the whole pizza to yourself, you could even cut a huge slice from one side to the other, resulting in an inscribed angle. The measure of that angle will also affect how much crust you get on your slice.

EXPLANATIONS AND EXAMPLES (continued)

Somewhere in between the discussions of angles and pizza toppings, throw in the formulas for finding the measures of inscribed and circumscribed angles when given the central angle or arc measure. Most important of these is that an inscribed angle's measure is equal to half the measure of its intercepted arc.

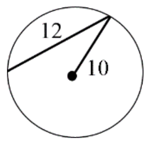
After students have mastered these angles and arc measures, talk about **tangents** and **secants** of circles, lines that intersect a circle at one and two points, respectively. It's also important for students to know that a point on a circle can only have one tangent and that tangents are always perpendicular to the radius of a circle.

Once you're done explaining all these relationships, feel free to gulp the lesson down with a big slice of pepperoni. Bust out your guitar chops while you're at it, and have a party.

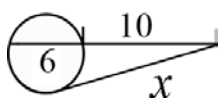
(Source: www.shmoop.com)

Examples:

- Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.



- Find the unknown length in the picture below.



STANDARD AND DECONSTRUCTION

G.C.3	**Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.**
DESCRIPTION	Construct inscribed circles of a triangle. Construct circumscribed circles of a triangle. Using definitions, properties, and theorems, prove properties of angles for a quadrilateral inscribed in a circle.
ESSENTIAL QUESTION(S)	How do you show the relationships amongst properties of circles by applying theorems?
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☒ 3 ☐ 4

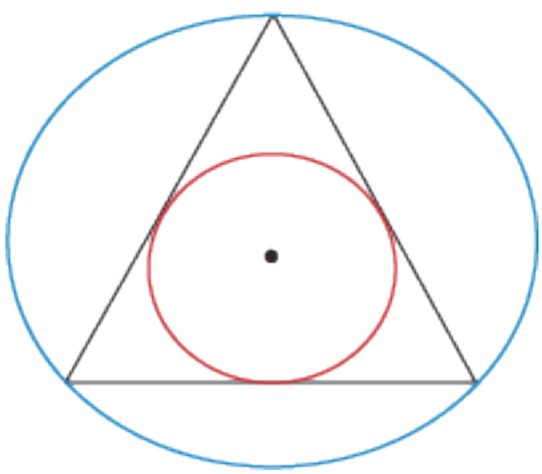
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define inscribed and circumscribed circles of a triangle. Recall midpoint and bisector definitions. Define a point of concurrency.	Prove properties of angles for a quadrilateral inscribed in a circle.	Students may use geometric simulation software to make geometric constructions.

EXPLANATIONS AND EXAMPLES

At the heart of it, the study of mathematics is the study of relationships. Before jumping into *Romeo and Juliet,* though, let's be clear that the relationships of interest in mathematics are those between mathematical objects. Not between two star-cross'd lovers.

But in terms of depth, Shakespeare's got nothing compared to the beauty with which Euclid explored the deep and intrinsic connections between circles and triangles. Yeah, we're talking about a triangle's inscribed and circumscribed circles.

Students should know that an **inscribed circle** is the largest circle that can fit on the *inside* of a triangle, with the three sides of the triangle tangent to the circle. A **circumscribed circle** is one that contains the three vertices of the triangle. Students should know the difference between and be able to construct both of these circles..



CIRCLES

EXPLANATIONS AND EXAMPLES (continued)

Students should understand that while circles have one defined center, triangles try to outdo them by having *four* different centers: the incenter, the circumcenter, the centroid, and the orthocenter. Imitation is the sincerest form of flattery and all, but do you think maybe they're compensating for something?

Students should know the definitions of these centers and how each one differs from the others. But how do we get to the true center of the triangle? The world may never know.

We can draw any triangle by plotting three points on a circle. Students can take this one point further, plot four points on a circle, connect them, and make a *cyclic quadrilateral*. (They're called that because the vertices are all on the circle, in kind of a, well, cycle.)

Students be able to prove theorems about cyclic quadrilaterals, the most important of which is that opposite angles in a cyclic quadrilateral add to 180°. Or, if students are wearing their fancy pants today, they can say that opposite angles in a cyclic quadrilateral are supplementary. We would say that too, but our fancy pants are at the dry cleaners.

This only touches on the deep and fascinating love triangle between circles, triangles, and quadrilaterals. So maybe these shapes are less *Romeo and Juliet* and more *A Midsummer Night's Dream*. Well, the Lysander-Helena-Demetrius bit, anyway.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

G.C.4	(+) Construct a tangent line from a point outside a given circle to the circle.		
DESCRIPTION	Construct a tangent line from a point outside a given circle to the circle.		
ESSENTIAL QUESTION(S)	How do you show the relationships amongst properties of circles by applying theorems?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define tangent, radius, perpendicular bisector, and midpoint. Identify the center of the circle.	Synthesize the theory that applies to circles that tangents drawn from a common external point are congruent. Synthesize the theory that applies to circles that a radius is perpendicular to a tangent at the point of tangency.	Construct the perpendicular bisector of the line segment between the center, C to the outside point, P. Construct arcs on circle C from the midpoint Q, having length of CQ. Construct a tangent line.

EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to make geometric constructions.

It's construction time, so tell your students to put on their hard hats. Actually, don't. You'll only get groans and eye rolls.

Students should already know that constructions involve straightedges and compasses rather than jackhammers and drills. Well, it depends on what you mean by drills. Hopefully they've become adept enough in using these instruments because they'll have to put them both to use.

If students don't already know the properties of circles and tangents before this construction, they should take away a few main points from it:

Tangents drawn to a circle are perpendicular to the circle's radius at the point of tangency.

Two tangents drawn to a circle from the same point outside the circle are equal. You can have students do this construction and measure the segments from the point of tangency to the shared point.

Two tangents drawn to a circle from the same point outside the circle make an angle that, when bisected, includes the circles center. Students can construct the angle bisector and see for themselves.

Tangents to a circle at either end of a diameter are parallel.

Given a circle with center B and a point A outside the circle, the construction of a tangent to $\odot B$ that goes through *A* is *relatively simple*. What's important is that students also understand the properties of circles and their tangents in order to make these constructions.

(Source: www.shmoop.com)

CIRCLES

CLUSTER	**2. Find arc lengths and areas of sectors of circles.**
BIG IDEA	**• Shapes and objects can be classified and analyzed by defining attributes.**

STANDARD AND DECONSTRUCTION

G.C.5	**Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.**
DESCRIPTION	Use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius, identifying the constant of proportionality as the radian measure of the angle. Find the arc length of a circle. Using similarity, derive the formula for the area of a sector. Find the area of a sector in a circle.

ESSENTIAL QUESTION(S)	How do I determine arc lengths and areas of sectors of circles?		
MATHEMATICAL PRACTICE(S)	HS.MP.2 Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recall how to find the area and circumference of a circle. Recall that all circles are similar. Explain that 1° = ∏/180 radians. Determine the constant of proportionality (scale factor).	Justify the radii of any two circles (r1 and r2) and the arc lengths (s1 and s2) determined by congruent central angles are proportional. Verify that the constant of a proportion is the same as the radian measure, Θ, of the given central angle. Conclude s = r Θ.	

EXPLANATIONS AND EXAMPLES

Students can use geometric simulation software to explore angle and radian measures and derive the formula for the area of a sector.

It's easy for students to memorize formulas and in fact, many of them have no problem with that. Formulas are their ticket to the Math Train, which gets them from algebra to geometry and eventually, to calculus. They continue plugging and chugging their way along without a care in the world until they hit a blockade: Derivation Station.

Uh oh. All they've done up until now is memorize. How ever will they get to Calculus City?

They shouldn't worry their little heads. Derivation Station is just about taking formulas we already know, applying them logically, and coming up with a new formula. In this case, it's about arc length. (Tip: make sure students know this is not the same as arc measurement!)

Students should first draw a circle with radius r and a central angle of θ. Visually, it should be clear that the arc length s depends on both the central angle and the radius. If we look at the circumference C as one bit arc length, we can see that its central angle is 360° (or 2π radians). If $C = 2\pi r$, then the arc length should equal the central angle (in radians) times the radius, or $s = \theta r$.

If students aren't as familiar with radians, make sure to take some time and explain it to them. It can be tricky going from degrees to radians, but you can use the circumference equation to translate from one to another. (We have 2π as our factor but it must come from 360° somehow. If we say $360k = 2\pi$ and solve for k, we have our conversion factor from degrees to radians. Yippee.).

(Source: www.shmoop.com)

CIRCLES

EXPLANATIONS AND EXAMPLES (continued)

Students should also be able to derive the formula for the sector of a circle. Just like we used the $C = 2\pi r$ to find arc length, we can use $A = \pi r^2$ to find the area of a sector. This time, the 360 degrees translates to π, not 2π. That means we're dividing the angle by 2, so our formula should be $A = \frac{1}{2}\theta r^2$.

The key thing to note here is that θ must be in radians. You can explain to them that a radian is about 57.3°, that it's the angle at which the ratio of the arc length to the radius is 1:1, and that it's an Australian experimental music band. That last one is optional, by the way.

Make sure the entire concept of radians isn't lost on your students. If you sense that radians confuse them more than they should, have them measure the radius of a circle and then mark an arc of the same length. Students can then measure the central angles intercepting that arc with their protractors. Even if they do this for circles of various sizes, the angle measure shouldn't change. That's a radian.

(Source: www.shmoop.com)

DOMAIN:

EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS (GPE)

HIGH SCHOOL
GEOMETRY

MATHEMATICS

LEXILE GRADE LEVEL BANDS: 9TH GRADE: 1050L TO 1260L, 10TH GRADE: 1080L TO 1335L, 11TH – 12TH GRADES: 1185L TO 1385L

DOMAIN	**Expressing Geometric Properties With Equations (GPE)**
CLUSTERS	**1. Translate between the geometric description and the equation for a conic section.** **2. Use coordinates to prove simple geometric theorems algebraically.**
ACADEMIC VOCABULARY	angle, circle , perpendicular lines, parallel lines, line segments, point, line, arc, rigid motion, congruent, angle-side-angle(asa), side-angle-side (sas), side-side-side (sss), inscribed, scale factor, dilation, aa similarity, theorem, law of sines, law of cosines

CLUSTER	**1. Translate between the geometric description and the equation for a conic section.**
BIG IDEA	• **Variables, expressions, and equations are algebraic representations of mathematical situations that dictate the unknown to be solved in real-world problems.** • **The location of Lines, angles and geometric shapes within a plane provide geometric interpretations of mathematical situations.**

STANDARD AND DECONSTRUCTION

G.GPE.1	**Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.**		
DESCRIPTION	Use the Pythagorean Theorem to derive the equation of a circle, given the center and the radius. Given an equation of a circle, complete the square to find the center and radius of a circle.		
ESSENTIAL QUESTION(S)	What strategy can be used to translate between the geometric description of a circle and an equation of a circle?		
MATHEMATICAL PRACTICE(S)	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define a circle. Use Pythagorean Theorem. Complete the square of a quadratic equation.	Derive equation of a circle using the Pythagorean Theorem, given coordinates of the center and length of the radius. Determine the center and radius by completing the square.	

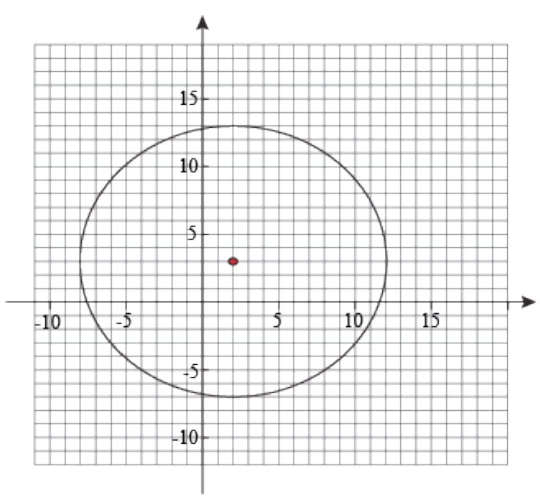
EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to explore the connection between circles and the Pythagorean Theorem.

h, the circle. The basis of so many delicious baked goods: donuts, pies, cookies, and cakes. Always regular and similar to all of its circle friends. It tries so hard to fit in with the crowd, and yet we insist on labeling it, giving it an equation, shoving it into a 360 degree oven for 12 to 14 minutes, and trying our hardest to let it cool before cramming its deliciousness down our gullets.

Students should know that the equation for a circle is $x^2 + y^2 = r^2$. You're right, that does look eerily familiar. Sort of like the Pythagorean theorem ($a^2 + b^2 = c^2$.) Wouldn't it be great if that was all the students needed to know?

If only it were that simple. The equation for a circle is actually $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle. But when a circle is centered on the origin, or (0, 0), the equation simplifies to $x^2 + y^2 = r^2$. So actually, comparing it to the Pythagorean theorem comes in quite handy after all.



EXPLANATIONS AND EXAMPLES (continued)

This circle is centered at (2, 3) with a radius 10, meaning that students have to adjust the left side of the standard circle equation. The adjustment should reflect how we would have to move the circle so that it is centered on the origin; in this case, two to the left and down three. Our equation ends up being $(x – 2)^2 + (y – 3)^2 = r^2$.

To find the radius, students can draw a right triangle inside the circle and use the Pythagorean theorem to find the length of the hypotenuse. It just so happens that the hypotenuse is also the radius of our circle. Since our hypotenuse here is 10, our equation is $(x – 2)^2 + (y – 3)^2 = 100$.

If students are having a hard time remembering whether to do add or subtract their h's and k's, they can simply think, "How would I move the center to the origin?" For example, if their circle was centered at (-5, -17), they'd have to adjust the left side by saying "plus 5" or "plus 17", since they'd have to move the center up and to the right, both of which are positive motions, or as we like to call them, good vibrations.

Perhaps your students find themselves in the lucky situation of having been given an equation and told to find the center and radius of the circle that equation describes. They will need to start by completing the square in the equation so that they can convert it into the standard form.

If they are having a hard time solving these kinds of problems, they might need a quick refresher on how to complete the square. Also, they should note that they're actually completing two squares, since both x and y are squared. For a standard about circles, we're sure using a lot of squares.

(Source: www.shmoop.com)

Examples:

- Write an equation for a circle with a radius of 2 units and center at (1, 3).
- Write an equation for a circle given that the endpoints of the diameter are (-2, 7) and (4, -8).
- Find the center and radius of the circle $4x2 + 4y2 - 4x + 2y – 1 = 0$.

STANDARD AND DECONSTRUCTION

G.GPE.2	**Derive the equation of a parabola given a focus and a directrix.**		
DESCRIPTION	Given a focus and directrix, derive the equation of a parabola. Given a parabola, identify the vertex, focus, directrix, and axis of symmetry, noting that every point on the parabola is the same distance from the focus and the directrix.		
ESSENTIAL QUESTION(S)	How can a focus and directrix help in the derivation of the equation of a parabola?		
MATHEMATICAL PRACTICE(S)	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define a parabola including the relationship of the focus and the equation of the directrix to the parabolic shape.	Derive the equation of a parabola given the focus and directrix.	

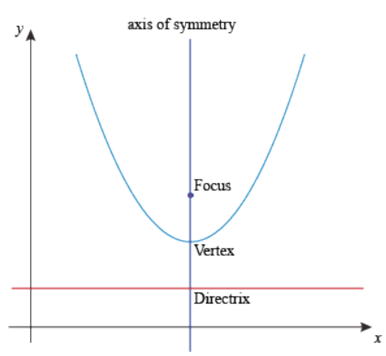
EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to explore parabolas.

What do your satellite dish, the path that Katniss Everdeen's arrow would follow if she aimed up in the sky, and the water swirling around inside a flushing toilet all have in common? Parabolas. Yes, you heard us right. The shape you get from slicing a cone perpendicularly to its base shows up in plenty of unusual real-world places.

Students should already know that parabolas, like pretty much everything else in the Mathland, can be labeled with their own unique equations in the form $y = ax^2 + bx + c$. But giving your students the values of *a, b*, and *c* would just be too easy, don't you think? Instead, have them find the equation when given a focus and directrix.

Students should know that a focus is a point near a parabola and a directrix is a line near a parabola. So what's so special about them? In Mathese, the parabola is the set of all the points that are the same distance between two things: a given point and a given line. In picture form, that would be something like this.



EXPLANATIONS AND EXAMPLES (continued)

Students should know that because the distances are equal, they find the equation of a parabola by applying the bulky equation below, where x_f and y_f are the coordinates of the focus and either x_d or y_d can be substituted with the value of the directrix. The other value becomes its respective variable (so either x_d becomes x, or y_d becomes y).

$$\sqrt{(x - x_f)^2 + (y - y_f)^2} = \sqrt{(x - x_d)^2 + (y - y_d)^2}$$

To help students become more efficient, tell them to square both sides and then expand them. Then, they can solve for y to get the equation of the parabola. No sweat (unless they happen to be on a treadmill or something).

Students can also use the $(x - h)^2 = 4p(y - k)$ formula for a horizontal directrix and $(y - k)^2 = 4p(x - h)$ for a vertical directrix. They should know that (h, k) are the coordinates of the parabola's vertex and p is the distance from the focus to the vertex. Remind students that p will be negative when the focus is either under or to the left of the directrix.

Sideways parabolas may exist in the real world when the satellite dish gets knocked over or when you stand on a platform and throw a boomerang. Hopefully, the watery flushing toilet parabola never goes sideways.

That would be gross.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

G.GPE.3	**(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.**
DESCRIPTION	Given the foci, derive the equation of an ellipse, noting that the sum of the distances from the foci to any fixed point on the ellipse is constant, identifying the major and minor axis. Given the foci, derive the equation of a hyperbola, noting that the absolute value of the differences of the distances form the foci to a point on the hyperbola is constant, and identifying the vertices, center, transverse axis, conjugate axis, and asymptotes.

ESSENTIAL QUESTION(S)	What strategy can be used to drive the equations of ellipses and hyperbolas?		
MATHEMATICAL PRACTICE(S)	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use definitions of conic sections and Pythagorean Theorem to derive equations.	Derive the equations of ellipses given the foci, using the fact that the sum of distances from the foci is constant. Derive the equations of hyperbolas given the foci, using the fact that the difference of distances from the foci is constant.	

EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to explore conic sections.

For many students, solving problems in Geometry land can sometimes feel like a business trip to Algebra land. (It isn't a leisurely vacation, that's for sure!) Working with the equations of ellipses and hyperbolas requires fluency in two dialects of Mathese: both Geospeak and Algetongue.

Focus, vertex, asymptote, transverse, conjugate, standard position, Pythagorean theorem—all-important geometric terms that students need to know. On the other hand, square, simplify, solve for *x*, and complete the square are crucial algebraic skills they need to be able to perform.

If their trip to Geometry land proves successful, they should know the equations that describe both ellipses (which look like elongated circles) and hyperbolas (which look like a pair of parabolas). For ellipses or hyperbolas with center *(h, k)*, a major axis of a, and a minor axis of b, the formulas are:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

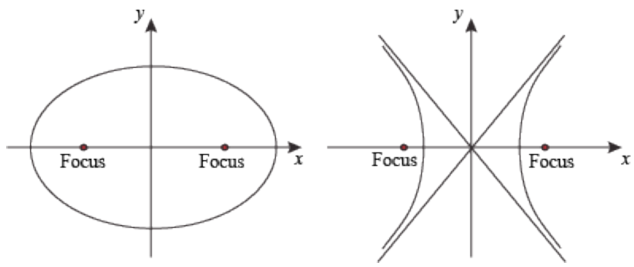
$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

Students should know that when the major axes are vertical rather than horizontal, the a^2 and b^2 values are switched. Whichever axis is elongated gets the larger a underneath it.

Chances are students will also want to know the distance formula. After all, the standard itself features the word "distances." Lucky for them, the distance formula is still the same as it always was.

EXPLANATIONS AND EXAMPLES (continued)

Students should understand that while parabolas have a single focus, ellipses and hyperbolas have *two* of them. (The plural of focus is "foci" so don't let that trip them up.) The distance from one focus to any point on the ellipse and back to the other focus will always be constant. The same applies to hyperbolas, only instead of adding the distances, we subtract them.



Given a set of two foci, students should be able to find the equation of an ellipse. They can do this by setting up the equation in the form of two distances calculations (the distance from one focus to *(x, y)* and the other focus to that same point) that equal a constant, or by finding the values of *h, k, a,* and *b*.

It also helps to imagine a right triangle inside the ellipse with vertices at the center, one of the foci, and at the vertical vertex. The length of the hypotenuse is half the constant distance c (the other half is the hypotenuse of the triangle on the other side), the length of the base helps find the location of the foci, and the length of the vertical leg is *b* (the same *b* as in the equations above). Students can also find *a* by dividing the constant distance by 2.

You might consider representing the constant distance as a string loosely connected to two foci. Using the string to guide your marker, draw the ellipse and explain the relationship between the values and distances. Bringing in triangles will help mathematically cement these relationships. Students should familiarize themselves with all the intricacies of Ellipseville so that they can actually get around (and around and around).

The itinerary for Hyperbola City should look about the same. Students should be able to perform essentially the same calculations, finding the equation when given the foci. The only difference is that instead of a sum of two distances, we're talking about a difference.

If students are struggling, they might benefit from:

- reviewing the algebra used to simplify big equations, especially completing the square
- considering how the Pythagorean Theorem is applied here
- focusing on when and where to use addition and subtraction
- referring to a labeled drawing of the shape in question

(Source: www.shmoop.com)

CLUSTER	**2. Find arc lengths and areas of sectors of circles.**
BIG IDEA	• **The location of Lines, angles and geometric shapes within a plane provide geometric interpretations of mathematical situations.**

STANDARD AND DECONSTRUCTION

G.GPE.4	**Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).**
DESCRIPTION	Use coordinate geometry to prove geometric theorems algebraically; such as prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).

ESSENTIAL QUESTION(S)	What strategies use coordinates to prove geometric theorems algebraically?		
MATHEMATICAL PRACTICE(S)	HS.MP.3 Reason abstractly and quantitatively.		
DOK Range Target for Instruction & Assessment	☐ 1 ☐ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recall previous understandings of coordinate geometry including distance, midpoint and slope formulas, equation of a line, definitions of parallel and perpendicular lines.	Use coordinates to prove simple geometric theorems algebraically . Derive the equation of a line through 2 points using similar right triangles. Derive simple proofs involving circles.	

EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to model figures and prove simple geometric theorems.

Students will need to know several major formulas and equations in order to prove they can hold their own against Descartes. Here are a few of the most important equations:

- the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- the midpoint formula: $(x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
- the equation of a circle: (x – h)2 + (y – k)2 = r2
- the equation of an ellipse: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$
- the equations of all the rest of the shapes/functions that can be plotted on the coordinate plane (parabolas, hyperbolas, etc.)

They'll also need to be pretty fluent in the properties of geometric shapes. If they don't know that the sum of any two sides of a triangle is always greater than the remaining side, or that all four sides of a rhombus are congruent, they may or may not be in some serious trouble.

To prove a geometric theorem about a shape on the coordinate plane, it's not enough for students to know these properties and formulas. They'll need to be able to *apply* them to the coordinate plane as well.

Most statements students need to prove will somehow involve *angles* and *lengths*. For instance, to prove that a given figure is a square, students could prove that the slopes of adjacent sides are perpendicular (slope formula), and that all sides are the exact same length (distance formula).

EXPLANATIONS AND EXAMPLES (continued)	There are infinite (well, it might seem that way) possibilities for algebraically proving the many properties of the many shapes in the coordinate plane. While students don't have to know *all* the ways to prove a particular geometric theorem, they should be able to come up with at least one. *(Source: www.shmoop.com)* Example: • Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram.

STANDARD AND DECONSTRUCTION

G.GPE.5	**Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).**		
DESCRIPTION	Using slope, prove lines are parallel or perpendicular. Find equations of lines based on certain slope criteria such as; finding the equation of a line parallel or perpendicular to a given line that passes through a given point.		
ESSENTIAL QUESTION(S)	What strategies use coordinates to prove geometric theorems algebraically?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recognize that slopes of parallel lines are equal. Recognize that slopes of perpendicular lines are opposite reciprocals. Find the equation of a line parallel to a given line that passes through a given point. Find the equation of a line perpendicular to a given line that passes through a given point.	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.	

EXPLANATIONS AND EXAMPLES

Lines can be horizontal, vertical, or neither.

Students may use a variety of different methods to construct a parallel or perpendicular line to a given line and calculate the slopes to compare the relationships.

If you live within driving distance of one of the top five ski resorts in the lower forty-eight, hitting the slopes can be a year round pastime. If you don't, it's still possible to ski year round,

Regardless of their skills on the slopes, we hope students won't experience epic fails when it comes to the slopes. Well, geometric slopes, anyway.

Students are expected to know that lines in the same plane can fall into one of three categories: parallel, perpendicular, or neither. Given two lines, they should be able to prove which of these three categories is applicable. They should also know how to find the equation of a line that is parallel or perpendicular to a given line.

Your students may need a quick refresher on some slope basics. You know, point your toes together, watch out for trees, and don't eat yellow snow. That kind of stuff.

A line's slope, of course, is the ratio of rise over run. In the slope-intercept form of a line's equation, it is the coefficient on the x term. Positively sloped lines point up, while negatively sloped ones point down. The greater the magnitude of the slope, the steeper the line.

Parallel lines, or what second graders would refer to as "train track lines," have the same slope and never cross each other. Perpendicular lines, on the other hand, cross at a right angle and feature slopes that are opposite reciprocals of each other.

When your students feel confident enough, tell 'em to grab their boots and poles. Time to hit the slopes!

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

G.GPE.6	**Find the point on a directed line segment between two given points that partitions the segment in a given ratio.**
DESCRIPTION	Given two points, find the point on the line segment between the two points that divides the segment into a given ratio.

ESSENTIAL QUESTION(S)	What strategies use coordinates to prove geometric theorems algebraically?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Recall the definition of ratio. Recall previous understandings of coordinate geometry.	Given a line segment (including those with positive and negative slopes) and a ratio, find the point on the segment that partitions the segment into the given ratio.	

EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to model figures or line segments.

Ross and Rachel. Rachel and Ross. Every Friday morning, we gathered around the break room coffee maker to discuss what it would take to split them up. Just like the 18-page wedge that finally came between the two old friends, we can also identify the coordinates of a partition point that splits up a directed line segment.

The three main methods that students can use to find the partition point are as distinct from each other as sitcom character archetypes. Students should be able to use the midpoint formula (if the ratio of the parts is 1:1), the section formula, and the distance formula.

If we're being honest, though, the midpoint formula is just a special case of the section formula. But it comes first in the list because it's the easiest (unlike the game show Pyramid). Just find the averages of the x and y coordinates to find the midpoint, which gives the partition point these coordinates.

The section formula is just a fancier version of the midpoint formula. If a line segment has endpoints (x_1, y_1) and (x_2, y_2), and a partition point *P* will separate the line segment into a ratio of *m:n*, then students should plug the numbers into the section formula to find the coordinates of *P*.

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Essentially, the midpoint formula is to finding averages as the section formula is to finding weighted averages. Given the endpoints of a line segment, students should be able to use both formulas to find the midpoint *M* and the partition point *P* at a specified ratio.

Students should also be able to determine the ratio of a partition using the distance formula. They need to remember that we are talking about *directed* line segments here. So it does matter on which side of the partition point the bigger segment lies. Remind them to keep track of which segments they're looking at, and hopefully they won't get mad at you. It's not as though you called them boring or anything.

(Source: www.shmoop.com)

EXPLANATIONS AND EXAMPLES (continued)	Example: • Given A (3, 2) and B(6, 11), Find the point that divides the line segment AB two-thirds of the way from A to B.. The point two-thirds of the way from A to B has x-coordinate two-thirds of the way from 3 to 6 and y coordinate two-thirds of the way from 2 to 11. So, (5, 8) is the point that is two-thirds from point A to point B. Find the midpoint of line segment AB.

STANDARD AND DECONSTRUCTION

G.GPE.7	**Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.**		
DESCRIPTION	Given two points, find the point on the line segment between the two points that divides the segment into a given ratio.		
ESSENTIAL QUESTION(S)	Use coordinate geometry and the distance formula to find the perimeters of polygons and the areas of triangles and rectangles.		
MATHEMATICAL PRACTICE(S)	HS.MP.2. Reason abstractly and quantitatively. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use the coordinates of the vertices of a polygon to find the necessary dimensions for finding the perimeter. Use the coordinates of the vertices of a triangle to find the necessary dimensions (base, height) for finding the area. Use the coordinates of the vertices of a rectangle to find the necessary dimensions (base, height) for finding the area.	Formulate a model of figures in contextual problems to compute area and/or perimeter.	

EXPLANATIONS AND EXAMPLES

Students may use geometric simulation software to model figures.

If there's one thing that mathematicians love, it's efficiency. And the distance formula is nothing if not efficient. It is so efficient that while we're going the distance, we can even go for speed.

Students are expected to be able to use the distance formula to find the distance between two coordinate points and then apply that information and know-how to calculate the perimeter and area of various polygons.

They should note that the distance formula is derived from the Pythagorean theorem. We suggest squaring both sides of the distance formula so it's a little easier to see that both equations set the sum of two squared terms equal to a third squared term.

Basically, the distance formula assumes that the distance we're measuring is the hypotenuse of a right triangle. The base of the triangle is denoted as a, or $(x_2 - x_1)$. The height is b, or $(y_2 - y_1)$. And the hypotenuse, *c*, is the distance, *D*. Now that that's all cleared up, let's go watch some E!

As one might expect, the distance formula is pretty handy for computing the distance between two coordinate points. Of course, it's best saved for when those points are not located along the same vertical or horizontal, because then simple subtraction would be the most efficient way to find the distance between them. No, the distance formula is more aptly used when the points are spaced diagonally.

(Source: www.shmoop.com)

EXPLANATIONS AND EXAMPLES (continued)	Students should know that if we compute the distances between the points around a polygon, then the distances can be added to find the polygon's perimeter. Students should know that this works for all polygons. Area is a little different, since it depends on the actual shape. Students should know that pretty much all polygons on the coordinate plane can be split into rectangles and triangles. As such, students should know how to calculate the areas of rectangles and triangles. Adding the areas of the individual pieces should give them the area of the entire shape. We recommend giving students a variety of shapes so they can apply their knowledge and problem-solving skills rather than automatically plugging points into a formula. If students are ever confused, they can always plot points on the coordinate plane, too. After all, this is still geometry, and a picture can speak volumes (er…areas). *(Source: www.shmoop.com)*

DOMAIN:

GEOMETRIC MEASUREMENT AND DIMENSION (GMD)

HIGH SCHOOL
GEOMETRY

MATHEMATICS

LEXILE GRADE LEVEL BANDS: 9TH GRADE: 1050L TO 1260L, 10TH GRADE: 1080L TO 1335L, 11TH – 12TH GRADES: 1185L TO 1385L

DOMAIN	Geometric Measurement and Dimension (GMD)
CLUSTERS	1. Explain volume formulas and use them to solve problems. 2. Visualize relationships between two-dimensional and three-dimensional objects.
ACADEMIC VOCABULARY	angle, circle , perpendicular lines, parallel lines, line segments, point, line, arc, rigid motion, congruent, angle-side-angle(asa), side-angle-side (sas), side-side-side (sss), inscribed, scale factor, dilation, aa similarity, theorem, law of sines, law of cosines

CLUSTER	1. Explain volume formulas and use them to solve problems.
BIG IDEA	• Measurable attributes of objects can be described mathematically by standard units. • Visualize relationships between two-dimensional and three-dimensional objects.

STANDARD AND DECONSTRUCTION

G.GMD.1	**Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.**		
DESCRIPTION	Explain the formulas for the circumference of a circle and the area of a circle by determining the meaning of each term or factor. Explain the formulas for the volume of a cylinder, pyramid and cone by determining the meaning of each term or factor.		
ESSENTIAL QUESTION(S)	How do the properties of circles provide a basis for explaining volume formulas and solving problems?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use dissection arguments, Cavalieri's principle, and informal limit arguments.	Give an informal argument for the formulas for the circumference and area of a circle. Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone.	
EXPLANATIONS AND EXAMPLES	Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. What is love? It's a question that has haunted humankind for centuries. Scientists, artists, and philosophers have all tried to find the answer, so maybe it's time for mathematicians to take a crack at it. Like geometry, love spans across all dimensions. Like a circle, it continues forever and forever. Like volume, it fills us up to the brim with happiness. So naturally, if your students want to understand love, they have to understand the various dimensions when it comes to circles. (Okay, we admit that last one was a stretch.) The first dimension of circles includes radius, diameter, and circumference. Students should easily see the circumference of a circle as a special type of perimeter. Students should also understand that the ratio of the circumference to the diameter is always equal to the yummiest number: π. If they need help visualizing this, they should cut a piece of string at the length of the circumference and compare it with the measurement of the diameter. The ratio of circumference to diameter should equal π (about 3.14). That's where we get the formula $C = \pi d = 2\pi r$. For the second dimension, order a pizza. Pepperoni, cheese, veggie, whatever you want. The messier, the better, we say. If we cut the pizza up into 8 slices and line them up in a row, one up, one down, we can create something that looks more or less like a lumpy parallelogram. Looks kind of funky, but no less delicious. If we cut the pizza into infinitely small slices, the "waves" on the top and bottom of these "parallelograms" would essentially disappear. The base of the parallelogram would be half the circumference of the circle (πr) and the height would be r. If we substitute these values into the area of a parallelogram $A = bh$, we'd end up with the area of a circle: $A = \pi r2$.		

MEASUREMENT & DIMENSION

EXPLANATIONS AND EXAMPLES (continued)	You thought the authentic Italian pizza would satisfy your appetite, but it's only made you hungrier. How about an all expense paid trip to Italy? (Sign us up.) To examine volume, we can look at the Leaning Tower of Pisa for some help. If we built the Upright Tower of Pisa (with the exact same height as the original), both towers would have the same volume. That's because all horizontal cross sections of both towers will be identical at any given height. Students should understand this concept as Cavalieri's principle, and be able to extend it to other solid figures. (No, this concept doesn't only apply to the Tower of Pisa.) It also helps if students know the volume formulas for prisms, cylinders, cones, pyramids, and spheres. Perhaps love has its roots in Italy. After all, Romeo and Juliet were from Italy, Italian is the most romantic of the romance languages, and we just explained all three dimensions of geometry using pizza and Pisa. And now that we think of it, Italians have explained love in two simple words: that's amore. *(Source: www.shmoop.com)*

STANDARD AND DECONSTRUCTION

G.GMD.2	**(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.**		
DESCRIPTION	Using Cavalieri's Principle, provide informal arguments to develop the formulas for the volume of spheres and other solid figures.		
ESSENTIAL QUESTION(S)	How do the properties of circles provide a basis for explaining volume formulas and solving problems?		
MATHEMATICAL PRACTICE(S)	HS.MP.3. Construct viable arguments and critique the reasoning of other HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:		Give an informal argument using Cavalieri's principle for the formula for the volume of a sphere and other solid figures.	

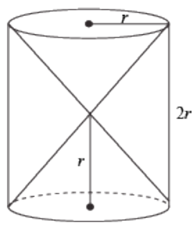
EXPLANATIONS AND EXAMPLES

Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

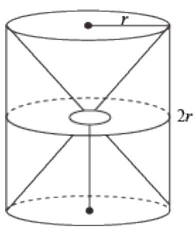
From ecosystems to the inner lives of cells, biology has instilled fear in middle and high school students all across the world. No activity caused more dread, however, than the inevitable biology dissection, which can send animal rights activists into a fury and queasy students into unconsciousness.

Fortunately, dissections in math are far less controversial (but only a little less messy).

Students should be familiar with the basic concept of Cavalieri's principle, especially when applied to oblique solids. What they might not know is that we can use Cavalieri's principle to find the volume of a sphere. We'll start with a cylinder with radius r and a height of 2*r*. Inside, let's put two cones, each with a height and a radius of *r*.



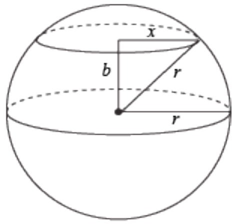
If we pass a plane through the top cone such that the inner circle has a radius of *b*, we can calculate the area of the shaded ring as $\pi(r^2 - b^2)$.



MEASUREMENT & DIMENSION

EXPLANATIONS AND EXAMPLES (continued)

Now let's look at our sphere of radius r.



The cross section that is *b* up from the great circle. We can call the radius of the cross section circle x and make a right triangle with legs *b* and *x* and hypotenuse *r*. We know that $x^2 + b^2 = r^2$ (thanks, Pythagoras), or $x^2 = r^2 – b^2$. If we multiply both sides by π, we should get the cross section's area, $A = \pi x2 = \pi(r^2 – b^2)$.

Look familiar? This is the same as the area of our ring from above. And since we would get the same thing for any b, then according to Cavalieri's principle, the volume of our sphere is equal to the volume of the solid between the cones and our cylinder which means $V = 2\pi r^3 – \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$.

Students can follow a similar process for other regular solid figures. For example, can they apply Cavalieri's principle to find the formula for the volume of a regular tetrahedron?

Working through some of these derivations should help students better understand the volume formulas. These volume formulas didn't come out of nowhere, and knowing how to derive them will not only give them a more solid (pun intended) understanding of volume, but it will also make memorizing these formulas unnecessary. If they're ever stuck, they'll have the tools to *derive* the formula instead.

Students might complain about how messy these derivations are, but it's nowhere near as bad as a cutting a frog open. Yuck.

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

G.GMD.3	**Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.**		
DESCRIPTION	Solve problems using volume formulas for cylinders, pyramids, cones, and spheres.		
ESSENTIAL QUESTION(S)	How do the properties of circles provide a basis for explaining volume formulas and solving problems?		
MATHEMATICAL PRACTICE(S)	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.2. Reason abstractly and quantitatively.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Utilize the appropriate formula for volume, depending on the figure.	Use volume formulas for cylinders, pyramids, cones, and spheres to solve contextual problems.	

EXPLANATIONS AND EXAMPLES

Missing measures can include but are not limited to slant height, altitude, height, diagonal of a prism, edge length, and radius.

Just about everything is in 3D nowadays. We're not saying that math is a trendsetter or anything, but these volume formulas speak for themselves.

$$V_{\text{cylinder}} = \pi r^2 h$$

$$V_{\text{pyramid}} = \frac{1}{3}Bh$$

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

Basically, as long as students know these formulas, what they mean, and how to use them, they should be good to go.

Students should also know that oblique solids deserve obliqual—er, *equal*—treatment. Cavalieri tells us that even though oblique solids are tilted, we can calculate their volumes using these same formulas. Well, he would tell us that if he were still alive.

If students need some hands on activities to really cement these formulas into their brains, you could always bring empty (and clean!) plastic containers from home. Mustard bottles, pickle jars, cake pans, or your favorite coffee mug.

Have them measure these containers and approximate their volumes using the formulas. Then, have them check their results by measuring the volume of water the containers actually hold. (Warning: your classroom may or may not become a splash zone.)

(Source: www.shmoop.com)

MEASUREMENT & DIMENSION

CLUSTER	**2. Visualize relationships between two-dimensional and three-dimensional objects.**
BIG IDEA	• **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

G.GMD.4	**Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two- dimensional objects.**		
DESCRIPTION	Given a three- dimensional object, identify the shape made when the object is cut into cross-sections. When rotating a two- dimensional figure, such as a square, know the three-dimensional figure that is generated, such as a cylinder. Understand that a cross section of a solid is an intersection of a plane (two- dimensional) and a solid (three-dimensional).		
ESSENTIAL QUESTION(S)	What method can be used to visualize two- and three- dimensional objects?		
MATHEMATICAL PRACTICE(S)	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use strategies to help visualize relationships between two-dimensional and three-dimensional objects.	Relate the shapes of two-dimensional cross-sections to their three-dimensional objects. Discover three-dimensional objects generated by rotations of two-dimensional objects.	
EXPLANATIONS AND EXAMPLES	Students may use geometric simulation software to model figures and create cross sectional views. If your students think they're too old for Play-Doh, they should think again. It's colorful fun for ages 2 to 102! If cheeky students give you any lip, just ask them if they'd rather be doing proofs. Works every time. Have them make 3D solids with the Play-Doh. Start by having them make cubes, pyramids, cones, spheres, and cylinders. To look at cross-sections of these solids, have them cut these Play-Doh solids in different ways and see what kinds of 2D shapes they form. (If rulers aren't sharp enough, borrow some clean scalpels from the biology teacher. You don't want frog guts all over your classroom.) This interactive activity helps students visualize what happens when we intersect a plane with a solid. The cross-sections we get can be squares, rectangles, triangles, or circles, and students should be able to link these 2D and 3D shapes to each other. (Now might also be a good time to tell them about conic sections, too.) They can then examine cross-sections of whatever amorphous, amoeba-shaped solids their imaginative little minds can come up with. While spinning Play-Doh is more trouble than it's worth, you can get some construction paper or index cards to relate the two-dimensional back the three-dimensional. The key is to get students to identify the 3D shape formed when a polygon or circle is rotated along an axis. If students can visualize these rotations and cross-sections, you've done your job. You might be left with a bit of Play-Doh and paper to clean up, but it's nothing compared to those gutted frogs left for that biology teacher. *(Source: www.shmoop.com)*		

DOMAIN:

MODELING WITH GEOMETRY (GMG)

HIGH SCHOOL
GEOMETRY

MATHEMATICS

DOMAIN	**Modeling With Geometry (MG)**
CLUSTERS	**1. Apply Geometric Concepts in Modeling Situations.**
ACADEMIC VOCABULARY	angle, circle , perpendicular lines, parallel lines, line segments, point, line, arc, rigid motion, congruent, angle-side-angle(asa), side-angle-side (sas), side-side-side (sss), inscribed, scale factor, dilation, aa similarity, theorem, law of sines, law of cosines

CLUSTER	**1. Apply geometric concepts in modeling situations.**
BIG IDEA	• **Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence.**

STANDARD AND DECONSTRUCTION

G.MG1	**Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).**
DESCRIPTION	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

ESSENTIAL QUESTION(S)	How can geometric concepts be used to describe objects found in nature?		
MATHEMATICAL PRACTICE(S)	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Use measures and properties of geometric shapes to describe real-world objects.	Given a real world object, classify the object as a known geometric shape – use this to solve problems in context.	

EXPLANATIONS AND EXAMPLES

Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

Like some of the best student-written metaphors around, this standard is all about students comparing things to other things. Of course, nothing really compares to the feelings of Geschpooklichkeit that we experience just thinking about it.

Perhaps even easier than the TV Guide crossword puzzle, students will find this standard to be as much about common sense as it is about geometry. Describing objects using the measures and properties of geometric shapes just means knowing what shapes look like and applying this knowledge to describe real-life objects.

Here are a few examples of tasks that students should be able to do:

- Recognize that the dice used in Monopoly are cubes, while the letter tiles in Scrabble are rectangular prisms
- Calculate the amount of floor left uncovered by the circular area rug in a rectangular living room
- Identify the person in the police line-up who robbed the convenience store based solely on the shadow the perp cast while fleeing the scene
- Describe a person's face as a perfect ellipse, like a circle that has been squished by a Thigh Master

It's really about helping students see geometry in the world around them. Angles, circles, squares, and spheres aren't hidden in the pages of a geometry textbook. They're everywhere we look.

If you want to get your students to start thinking a bit more abstractly, we suggest mentioning optical illusions and the Parisian lawn version of the Death Star. (It might just be a globe, but we'll need more evidence.)

Now toss a truncated cone (a lamp shade) over your head and we'll get this party started!

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

G.MG2	**Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).**		
DESCRIPTION	Use the concept of density when referring to situations involving area and volume models, such as persons per square mile.		
ESSENTIAL QUESTION(S)	How can geometric concepts be used to describe objects found in nature?		
MATHEMATICAL PRACTICE(S)	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Define density.	Apply concepts of density based on area and volume to model real-life situations.	

EXPLANATIONS AND EXAMPLES

Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

How many monkeys can fit in a barrel? How many slams are there in an old screen door? How many licks does it take to get to the Tootsie Roll center of a Tootsie Pop?

If your students are packed into your classroom like sardines, they may begin to question the school board's answer to, "How many students can we squeeze into a classroom?" But they won't make much ground complaining about the lack of elbowroom. Instead, they can reflect on the population density of their state compared with the neighbors, the salinity of the aquarium in the library, or how nutrient-dense the soil is in the bean field across the street.

Students should understand that density is a ratio of mass (or heat or people or things) to area or volume. The real world boasts many situations modeled by density concepts, but understanding doesn't come from chucking objects into water to see whether they sink or float. Instead, students can take a look at all the different examples of density all around us.

- Ohio asks its mail carriers to help count cottontail rabbits in order to determine the population density of that species of wildlife throughout the state. no lie.
- The USDA provides reports about the national yield of bushels of corn harvested per acre planted.
- Geologists record the quantity of volcanic eruptions (or other rude awakenings) that occurs in the Ring of Fire. Er, uh…we meant the Pacific Ring of Fire.

Like the professor trying to inspire his students to kick back and enjoy a few cold, uh, Yoohoo's with friends, students should try to come up with their own examples of how many things can fit into different spaces and the implications of those densities.

Students must also be able to use density to calculate other quantities related to it and interpret these answers in terms of their contexts. Using the right units, among other things, is good way to check that an answer makes sense.

If we're looking for mass, our answer should be in units of grams or pounds. If we're looking for the number of licks it takes to get to the center of a Tootsie Pop, we should have units of licks per Tootsie Pop. What is that number? The world may never know.!

(Source: www.shmoop.com)

STANDARD AND DECONSTRUCTION

G.MG3	**Apply geometric methods to solve design problems (e.g. designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).**
DESCRIPTION	Solve design problems by designing an object or structure that satisfies certain constraints, such as minimizing cost or working with a grid system based on ratios (i.e., The enlargement of a picture using a grid and ratios and proportions).

ESSENTIAL QUESTION(S)	How can geometric concepts be used to describe objects found in nature?		
MATHEMATICAL PRACTICE(S)	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.		
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☒ 3 ☒ 4		
Learning Expectations	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/applications.
Students should be able to:	Describe a typographical grid system.	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	

EXPLANATIONS AND EXAMPLES

Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

Now we're getting to the fun part of geometry: playing with tangrams. Using those little plastic pieces to model neat designs, like a fox, a house, or even a swan. Unfortunately, those probably aren't the kind of design problems your students will be working on. We're talking about more about solving multi-step word problems.

Really, these types of problems differ depending on what students might be learning. It could involve calculating the maximum area of a cow's grazing plot that can be contained by 600 meters of fencing (22,500 square meters). Maybe they'll have to find the volume of a pizza box when given a surface area of 320 square inches, a height of 3 inches, and a square base (300 cubic inches). Or maybe they'll just stick to tangrams.

Whatever kinds of problems students face, they will need at least a basic knowledge of the shapes and the formulas that describe them. At the very least.

Students will also need the know-how to solve systems of equations, the mental fortitude to simplify algebraic expressions, and the plucky spirit of adventure to resourcefully draw on the vast pool of their geometric enlightenment as they pick and choose exactly which formulas will serve them best, and modify them as needed.

Tangrams have got nothing on these design problems.

(Source: www.shmoop.com)

MODELING

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