COMMON CORE State Standards

DECONSTRUCTED for CLASSROOM IMPACT

SIXTH GRADE
MATHEMATICS

The COMMON CORE Institute

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Introduction

The Common Core Institute is pleased to offer this grade-level tool for educators who are teaching with the Common Core State Standards.

The Common Core Standards Deconstructed for Classroom Impact is designed for educators by educators as a two-pronged resource and tool 1) to help educators increase their depth of understanding of the Common Core Standards and 2) to enable teachers to plan College & Career Ready curriculum and classroom instruction that promotes inquiry and higher levels of cognitive demand.

What we have done is not all new. This work is a purposeful and thoughtful compilation of preexisting materials in the public domain, state department of education websites, and original work by the Center for College & Career Readiness. Among the works that have been compiled and/or referenced are the following: Common Core State Standards for Mathematics and the Appendix from the Common Core State Standards Initiative; Learning Progressions from The University of Arizona's Institute for Mathematics and Education, chaired by Dr. William McCallum; the Arizona Academic Content Standards; the North Carolina Instructional Support Tools; and numerous math practitioners currently in the classroom.

We hope you will find the concentrated and consolidated resource of value in your own planning. We also hope you will use this resource to facilitate discussion with your colleagues and, perhaps, as a lever to help assess targeted professional learning opportunities.

Understanding the Organization

The Overview acts as a quick-reference table of contents as it shows you each of the domains and related clusters covered in this specific grade-level booklet. This can help serve as a reminder of what clusters are part of which domains and can reinforce the specific domains for each grade level.

Key Changes identifies what has been moved to and what has been moved from this particular grade level, as appropriate. This section also includes **Critical Areas of Focus**, which is designed to help you begin to approach how to examine your curriculum, resources, and instructional practices. A review of the **Critical Areas of Focus** might enable you to target specific areas of professional learning to refresh, as needed.

Math Fluency Standards	
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 Add/subtract within 100 (pencil & paper)
3	Multiply/divide within 100 Add/subtract within 1000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations
7	Solve $px + q = r$, $p(x + q) = r$
8	Solve simple 2 x 2 systems by inspection

For each domain is the domain itself and the associated clusters. Within each domain are sections for each of the associated clusters. The cluster-specific content can take you to a deeper level of understanding. Perhaps most importantly, we include here the **Learning Progressions.** The **Learning Progressions** provide context for the current domain and its related standards. For any grade except SIXTH GRADE, you will see the domain-specific standards for the current grade in the center column. To the left are the domain-specific standards for the preceding grade and to the right are the domain-specific standards for the following grade. Combined with the **Critical Areas of Focus,** these **Learning Progressions** can assist you in focusing your planning.

For each cluster, we have included four key sections: Description, Big Idea, Academic Vocabulary, and Deconstructed Standard.

The cluster **Description** offers clarifying information, but also points to the **Big Idea** that can help you focus on that which is most important for this cluster within this domain. The **Academic Vocabulary** is derived from the cluster description and serves to remind you of potential challenges or barriers for your students.

Each standard specific to that cluster has been deconstructed. There **Deconstructed Standard** for each standard specific to that cluster and each **Deconstructed Standard** has its own subsections, which can provide you with additional guidance and insight as you plan. Note the deconstruction drills down to the substandards when appropriate. These subsections are:

- Standard Statement
- Standard Description
- Essential Question(s)
- Mathematical Practice(s)
- DOK Range Target for Learning and Assessment
- Learning Expectations
- Explanations and Examples

As noted, first are the **Standard Statement** and **Standard Description**, which are followed by the **Essential Question(s)** and the associated **Mathematical Practices**. The **Essential Question(s)** amplify the **Big Idea**, with the intent of taking you to a deeper level of understanding; they may also provide additional context for the **Academic Vocabulary**.

The **DOK Range Target for Learning and Assessment** remind you of the targeted level of cognitive demand. The **Learning Expectations** correlate to the DOK and express the student learning targets for student proficiency for KNOW, THINK, and DO, as appropriate. In some instances, there may be no learning targets for student proficiency for one or more of KNOW, THINK or DO. The learning targets are expressions of the deconstruction of the Standard as well as the alignment of the DOK with appropriate consideration of the **Essential Questions.**

The last subsection of the **Deconstructed Standard** includes **Explanations and Examples**. This subsection might be quite lengthy as it can include additional context for the standard itself as well as examples of what student work and student learning could look like. **Explanations and Examples** may offers ideas for instructional practice and lesson plans.

Standards for Mathematical Practice in 6th Grade

Each of the explanations below articulates some of the knowledge and skills expected of students to demonstrate grade-level mathematical proficiency.

PRACTICE	EXPLANATION
Make sense and persevere in problem solving.	Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?" "Does this make sense?" and "Can I solve the problem in a different way?" Students can explain the mathematical relationships. Students can check answers to problems using a different method.
Reason abstractly and quantitatively.	Students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operation.
Construct viable arguments and critique the reasoning of others.	Students construct arguments using verbal or written explanations accompanied by appropriate mathematical representations in words, numbers, or graphical representations. They further refine their mathematical communication skills through discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like, "How did you get that?" "Why is that true?" "Does that always work?" They can explain their thinking to others and respond to others' thinking.
Model with mathematics.	Students model problem situations using an appropriate approach. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use opportunities to connect and explain the connections between the different representations.
Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful.
Attend to precision.	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning.
Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems.
Look for and express regularity in repeated reasoning.	Students use repeated reasoning to understand algorithms and make generalizations about patterns. They may construct other examples and models that confirm their generalization.

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

OVERVIEW

Ratios and Proportional Relationships (RP)

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System (NS)

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations (EE)

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry (G)

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability (SP)

- Develop understanding of statistical variability.
- Summarize and describe distributions.

Mathematical Practices (MP)

MB 1. Make sense of problems and persevere in solving them.

MB 2. Reason abstractly and quantitatively.

MB 3. Construct viable arguments and critique the reasoning of others.

MB 4. Model with mathematics.

MB 5. Use appropriate tools strategically.

MB 6. Attend to precision.

MB 7. Look for and make use of structure.

MB 8. Look for and express regularity in repeated reasoning.

KEY CHANGES

NEW TO SIXTH GRADE	▪ Unit rate (6.RP.3b) ▪ Measurement unit conversions (6.RP 3d) ▪ Number line – opposites and absolute value (6.NS.6a, 6.NS.7c) ▪ Vertical and horizontal distances on the coordinate plane (6.NS.8) ▪ Distributive property and factoring (6.EE.3) ▪ Introduction of independent and dependent variables (6.NS.9) ▪ Volume of right rectangular prisms with fractional edges (6.G.2) ▪ Surface area with nets (only triangle and rectangle faces) (6.G.4) ▪ Dot plots, histograms, box plots (6.SP.4) ▪ Statistical variability (Mean Absolute Deviation (MAD) and Interquartile Range (IQR)) (6.G.5c)
MOVED FROM SIXTH GRADE	▪ Multiplication of fractions (moved to 5th grade) ▪ Scientific notation (moved to 8th grade) ▪ Transformations (moved to 8th grade) ▪ Area and circumference of circles (moved to 7th grade) ▪ Probability (moved to 7th grade) ▪ Two-step equations (moved to 7th grade) ▪ Solving one- and two-step inequalities (moved to 7th grade)

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

KEY CHANGES

CRITICAL AREAS OF FOCUS

1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.

Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers.

Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Writing, interpreting, and using expressions and equations.

Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Developing understanding of statistical thinking.

Building on and reinforcing their understanding of numbers, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

5. Reasoning about relationships among shapes to determine area, surface area, and volume.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

DOMAIN:

RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)

SIXTH GRADE

MATHEMATICS

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

DOMAIN:	Ratios and Proportional Relationships (RP)
CLUSTERS:	1. Understand ratio concepts and use ratio reasoning to solve problems.

RATIOS AND PROPORTIONS		
FIFTH	**SIXTH**	**SEVENTH**
RATIO AND PROPORTION, AND PERCENTS		
Ratio Boxes and Unit Ratio/Rate	**Ratio Boxes and Unit Ratio/Rate**	**Ratio Boxes and Unit Ratio/Rate**
	6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	7.RP.2.b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
	6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.	7.RP.2.c Represent proportional relationships by equations.
	6.RP.3.b Solve unit rate problems including those involving unit pricing and constant speed.	7.RP.2.d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
	6.RP.3.a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	7.RP.2.a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
	6.RP.3.d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
Percents	**Percents**	**Percents**
	6.RP.3.c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

CLUSTER	**1. Understand ratio concepts and use ratio reasoning to solve problems.**
DESCRIPTION	**This is the first time that students have formally studied ratio and proportion. The study of ratios and proportional reasoning extends students' work in measurement and multiplication and division in elementary grades. Ratios and proportional reasoning are the foundation for further study in mathematics, science and are useful in everyday life. Students should work with ratios to solve situations in daily life.**
BIG IDEA:	• **Proportionality is a numerical relationship that increases or decreases at constant rate.**
ACADEMIC VOCABULARY	**ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-to- whole, percent**

STANDARD AND DECONSTRUCTION

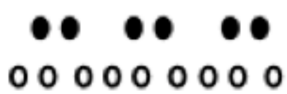
6.RP.1 — **Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."***

DESCRIPTION

A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish).

Example 1: A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms:

6 /9, 6 to 9 or 6:9. If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as



These values can be regrouped into 2 black circles (goldfish) to 3 white circles (guppies), which would reduce the ratio to, 2/ 3, 2 to 3, or 2:3.

Students should be able to identify and describe any ratio using "For every _____ there are _____" In the example above, the ratio could be expressed saying, "For every 2 goldfish, there are 3 guppies."

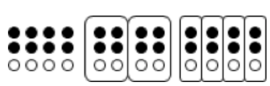
ESSENTIAL QUESTION(S)	How is a ratio different than a fraction?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Write ratio notation- a:b, a to b, a/b. Know order matters when writing a ratio. Know ratios can be simplified. Know ratios compare two quantities; the quantities do not have to be the same unit of measure. Recognize that ratios appear in a variety of different contexts; part-to-whole, part-to-part, and rates.	Generalize that all ratios relate two quantities or measures within a given situation in a multiplicative relationship. Analyze your context to determine which kind of ratio is represented.	

EXPLANATIONS AND EXAMPLES

A ratio is a comparison of two quantities that can be written as A to B or A:B.

A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. Understanding rate, however, is complicated and there is no universally accepted definition. When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationships between quantities before they are expected to work with rates numerically.

A comparison of 8 black circles to 4 white circles can be written as the ratio of 8:4 and can be regrouped into 4 black circles to 2 white circles (4:2) and 2 black circles to 1 white circle (2:1).



Students should be able to identify all these ratios and describe them using "For every…there are…"

STANDARD AND DECONSTRUCTION

6.RP.2	**Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.** ***For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar,so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."1***

DESCRIPTION

A unit rate expresses a ratio as part-to-one, comparing a quantity in terms of one unit of another quantity. Common unit rates are cost per item or distance per time. Students are able to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates (i.e. miles / hour and hours / miles) are reciprocals as in the second example below. At this level, students should use reasoning to find these unit rates instead of an algorithm or rule.

In 6th grade, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.

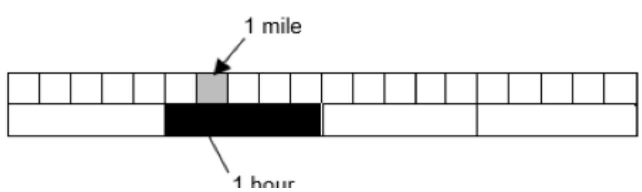
"This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."1

Example 1: There are 2 cookies for 3 students. What is the amount of cookie each student would receive? (i.e. the
unit
unit

Example 2:

On a bicycle Jack can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance Jack can travel in 1 hour and the amount of time required to travel 1 mile)?

Solution: Jack can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{\frac{1}{5}\,hr}{1\,mi}$. Students can represent the relationship between 20 miles and 4 hours.



ESSENTIAL QUESTION(S)	Why is a unit rate important for solving problems?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Identify and calculate a unit rate. Use appropriate math terminology as related to rate.	Analyze the relationship between a ratio a:b and a unit rate a/b where b ≠ 0.	

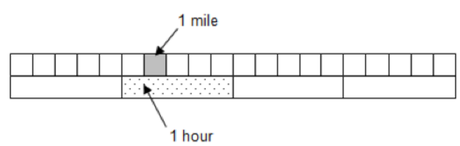
EXPLANATIONS AND EXAMPLES

A unit rate compares a quantity in terms of one unit of another quantity. Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates.

In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.

Examples: On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)?

Solution: You can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of an hour to travel each mile written as $\frac{\frac{1}{5}\,\text{hr}}{1\,\text{mi}}$. Students can represent the relationship between 20 miles and 4 hours.



A simple modeling clay recipe calls for 1 cup corn starch, 2 cups salt, and 2 cups boiling water. How many cups of corn starch are needed to mix with each cup of salt?

STANDARD AND DECONSTRUCTION

6.RP.3 **Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.**

DESCRIPTION

Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is not expected at this level. When working with ratio tables and graphs, whole number measurements are the expectation for this standard.

Example 1: At Books Unlimited, 3 paperback books cost \$18. What would 7 books cost? How many books could be purchased with \$54?

Solution: To find the price of 1 book, divide \$18 by 3. One book costs \$6. To find the price of 7 books, multiply \$6 (the cost of one book times 7 to get \$42.) To find the number of books that can be purchased with \$54, multiply \$6 times 9 to get \$54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (ie. 1 • 7 = 7; 6 • 7 = 42). Red numbers indicate solutions.

Number of Books (n)	Cost (C)
1	6
3	18
7	42
9	54

Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.

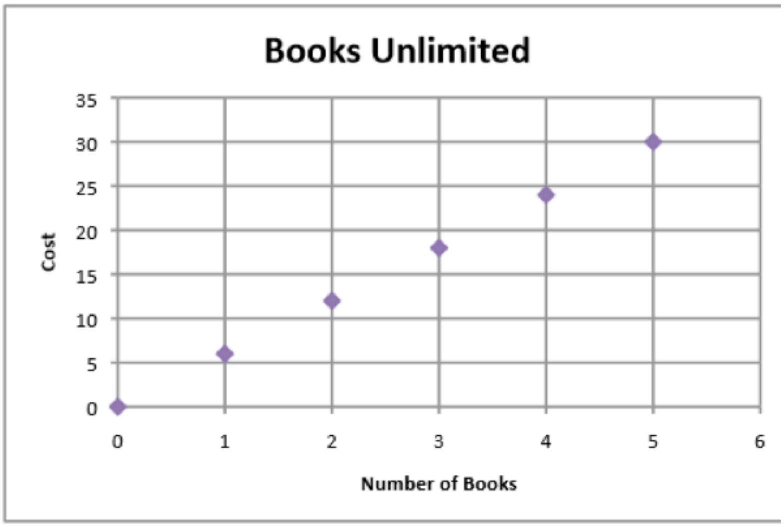
Number of Books (n)	Cost (C)
4	20
8	40

RATIOS AND PROPORTIONAL

DESCRIPTION (continued)

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be C = 6n, while the equation for the second bookstore is C = 5n. The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.

Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:



Example 2: Ratios can also be used in problem solving by thinking about the total amount for each ratio unit. The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 3: Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

ESSENTIAL QUESTION(S)	What strategies can you apply to solve real world problems using a ratio or unit rate?
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.7. Look for and make use of structure.

SUBSTANDARD DECONSTRUCTION	6.RP.3a: Make tables of equivalent ratios relating quantities with whole-number measurements, find the missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Make a table of equivalent ratios using whole numbers. Find the missing values in a table of equivalent ratios. Plot pairs of values that represent equivalent ratios on the coordinate plane.	Use tables to compare proportional quantities.	

SUBSTANDARD DECONSTRUCTION	6.RP.3b: Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:		Apply the concept of unit rate to solve real-world problems involving unit pricing. Apply the concept of unit rate to solve real-world problems involving constant speed. Solve real-world and mathematical problems involving ratio and rate.	

SUBSTANDARD DECONSTRUCTION	6.RP.3c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Know that a percent is a ratio of a number to 100. Find a percent of a number as a rate per 100.	Solve real-world problems involving finding the whole, given a part and a percent.	

SUBSTANDARD DECONSTRUCTION	6.RP.3d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:		Apply ratio reasoning to convert measurement units in real-world and mathematical problems.	

EXPLANATIONS AND EXAMPLES

Example: Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution: There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet:

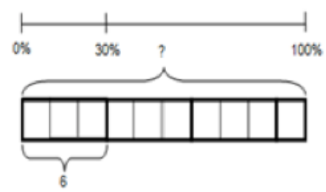
1. 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or
2. 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

Example: Compare the number of black to white circles. If the ratio remains the same, how many black circles will you have if you have 60 white circles?

Solution:

Black	4	40	20	60	?
White	3	30	15	45	60

Example: If 6 is 30% of a value, what is that value? (Solution: 20)



Example: A credit card company charges 17% interest on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If your bill totals $450 for this month, how much interest would you have to pay if you let the balance carry to the next month? Show the relationship on a graph and use the graph to predict the interest charges for a $300 balance.

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

DOMAIN:

THE NUMBER SYSTEM (NS)

SIXTH GRADE

MATHEMATICS

DOMAIN:	The Number System
CLUSTERS:	**1. Apply fractions and extend previous understandings of multiplication and division to divide fractions.** **2. Compute fluently with multi-digit numbers and find common factors and multiples.** **3. Apply and extend previous understandings of numbers to the system of rational numbers.**

THE NUMBER SYSTEM		
FIFTH	**SIXTH**	**SEVENTH**
PLACE VALUE AND DECIMALS		
Decimal Numbers, Integer Exponents, and Scientific Notation	**Decimal Numbers, Integer Exponents, and Scientific Notation**	**Decimal Numbers, Integer Exponents, and Scientific Notation**
5.NBT.3.a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).	6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithms.	
5.NBT.3.b Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.	
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.		
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.		
5.NBT.4 Use place value understanding to round decimals to any place.		
5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, and explain.		

THE NUMBER SYSTEM		
FIFTH	**SIXTH**	**SEVENTH**
MULTIPLICATION AND DIVISION		
Factors and Multiples	**Factors and Multiples**	**Factors and Multiplesn**
5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.	6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	
5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	6.NS.2 Fluently divide multi-digit numbers using the standard algorithm	
5.NF.4.a Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b.	6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.	
5.NF.4.b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.		
5.NF.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence		

Factors and Multiples	Factors and Multiples	Factors and Multiplesn
5.NF.5.a Interpret multiplication as scaling (resizing), by: Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.		

THE NUMBER SYSTEM		
FIFTH	**SIXTH**	**SEVENTH**
INTEGERS, NUMBER LINES, AND COORDINATE PLANES		
Integers on the Number Line	**Integers on the Number Line**	**Integers on the Number Line**
5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	7.NS.1.a Describe situations in which opposite quantities combine to make 0.
5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	6.NS.7.a Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.	
	6.NS.6.a Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., – (–3) = 3, and that 0 is its own opposite.	
	6.NS.7.c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.	
	6.NS.7.d Distinguish comparisons of absolute value from statements about order.	

Integers on the Number Line	Integers on the Number Line	Integers on the Number Line
	6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	
	6.NS.6.b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	
	6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	
RATIONAL AND IRRATIONAL NUMBERS		
Adding and Subtracting Rational Numbers	**Adding and Subtracting Rational Numbers**	**Adding and Subtracting Rational Numbers**
	6.NS.7.b Write, interpret, and explain statements of order for rational numbers in real-world contexts.	7.NS.1.b Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
	6.NS.6.c Find and position integers and other rational numbers on a horizontal or vertical number line diagram.	7.NS.1.c Understand subtraction of rational numbers as adding the additive inverse, p - q = p + (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
		7.NS.1.d Apply properties of operations as strategies to add and subtract rational numbers.

CLUSTER	**1. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
BIG IDEA	• Addition, Subtraction, Multiplication, and Division can be used with models, strategies, and their relationships with one another to solve real-world problems.
ACADEMIC VOCABULARY	reciprocal, multiplicative inverses, visual fraction model

STANDARD AND DECONSTRUCTION

6.NS.1

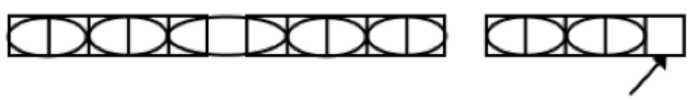
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. ***For example, create a, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.(In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?***

DESCRIPTION

In 5th grade students divided whole numbers by unit fractions and divided unit fractions by whole numbers. Students continue to develop this concept by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students develop an understanding of the relationship between multiplication and division.

Example 1:

Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "how many $\frac{2}{5}$ are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.



This section represents one-half of two-fifths

Students also write contextual problems for fraction division problems. For example, the problem, $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem:

DESCRIPTION
(continued)

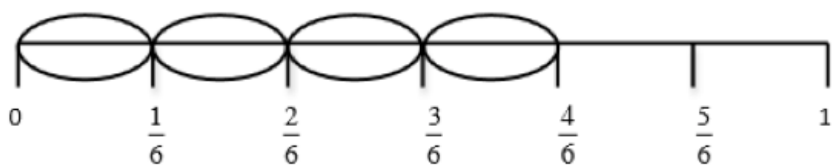
Example 2:

Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?

This problem can be modeled using a number line.

a. Start with a number line divided into thirds.

b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.

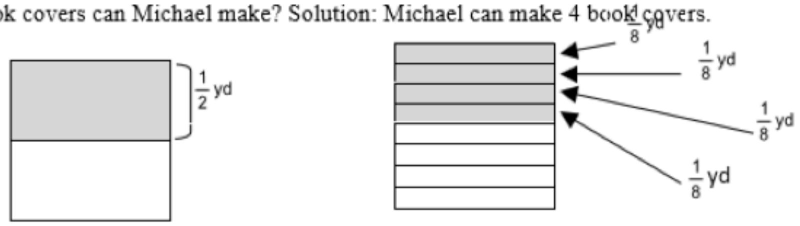


c. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

DESCRIPTION
(continued)

Example 3:

Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make? Solution: Michael can make 4 book covers.



Example 4:

Represent $\frac{1}{2}+\frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?

Explanation of Model:

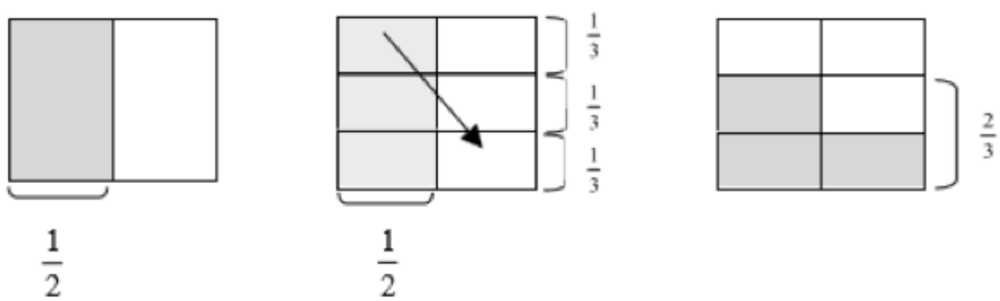
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only ¾ of the recipe can be made.



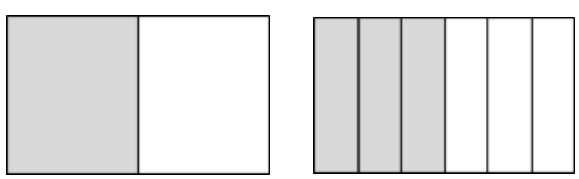
ESSENTIAL QUESTION(S)	Why can I use multiplication to solve a fractional division problem?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.7. Look for and make use of structure. 6.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Compute quotients of fractions divided by fractions (including mixed numbers).	Interpret quotients of fractions. Solving word problems involving division of fractions by fractions.	

EXPLANATIONS AND EXAMPLES

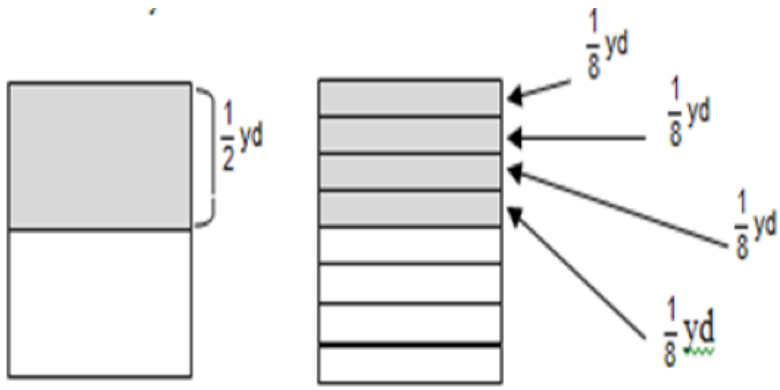
Contexts and visual models can help students to understand quotients of fractions and begin to develop the relationship between multiplication and division. Model development can be facilitated by building from familiar scenarios with whole or friendly number dividends or divisors. Computing quotients of fractions builds upon and extends student understandings developed in Grade 5. Students make drawings, model situations with manipulatives, or manipulate computer generated models.

Example: 3 people share 1/2 pound of chocolate. How much of a pound of chocolate does each person get?

Solution: Each person gets 1/6 lb. of chocolate.



EXPLANATIONS AND EXAMPLES
(continued)



Solution: Manny can make 4 book covers.

Context: You are making a recipe that calls for 2/3 cup of yogurt. You have 1/2 cup of yogurt from a snack pack. How much of the recipe can you make?

Explanation of Model:

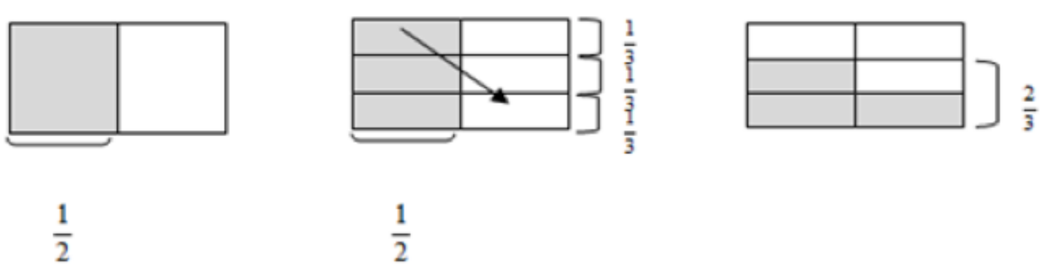
The first model shows 1/2 cup. The shaded squares in all three models show 1/2 cup.

The second model shows 1/2 cup and also shows 1/2 cups horizontally.

The third model shows 1/2 cup moved to fit in only the area shown by 2/3 of the model.

2/3 is the new referent unit (whole).

3 out of the 4 squares in the 2/3 portion are shaded. A 1/2 cup is only 3/4 of a cup portion, so you can only make ¾ of the recipe.



CLUSTER	**2. Compute fluently with multi-digit numbers and find common factors and multiples.**
BIG IDEA	• **Understanding of algorithms for all operations allow for the fluency needed to solve real-world problems efficiently and with precision.**
ACADEMIC VOCABULARY	**multi-digit**

STANDARD AND DECONSTRUCTION

6.NS.2	**Fluently divide multi-digit numbers using the standard algorithm.**

DESCRIPTION

In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm, continuing to use their understanding of place value to describe what they are doing. Place value has been a major emphasis in the elementary standards. This standard is the end of this progression to address students' understanding of place value.

Example 1: When dividing 32 into 8456, students should say, "There are 200 thirty-twos in 8456" as they write a 2 in the quotient. They could write 6400 beneath the 8456 rather than only writing 64.

$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$	There are 200 thirty twos in 8456.
$\begin{array}{r} 2 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \end{array}$	200 times 32 is 6400. 8456 minus 6400 is 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \end{array}$	There are 60 thirty twos in 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \\ \underline{1920} \\ 136 \end{array}$	60 times 32 is 1920. 2056 minus 1920 is 136.

DESCRIPTION (continued)

264 32)8456 6400 2056 1920 136 128	There are 4 thirty twos in 136. 4 times 32 is 128.
264 32)8456 6400 2056 1920 136 128 8	The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8. 8456 = 264 * 32 + 8

ESSENTIAL QUESTION(S)	How can using the standard algorithm for dividing multi-digit fractions create efficiency and accuracy?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.7. Look for and make use of structure. 6.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Fluently divide multi-digit numbers using the standard algorithm with speed and accuracy.		

EXPLANATIONS AND EXAMPLES

Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level.

As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students' language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, "There are 200 thirty-twos in 8456 " and could write 6400 beneath the 8456 rather than only writing 64.

2 32)8456	There are 200 thirty twos in 8456.
2 32)8456 −6400 2056	200 times 32 is 6400. 8456 minus 6400 is 2056.
26 32)8456 −6400 2056	There are 60 thirty twos in 2056.
26 32)8456 −6400 2056 −1920 136	60 times 32 is 1920. 2056 minus 1920 is 136.

264 32)8456 −6400 2056 −1920 136 −128	There are 4 thirty twos in 136. 4 times 32 is 128.
264 32)8456 −6400 2056 −1920 136 −128 8	The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8. 8456 = 264 * 32 + 8

STANDARD AND DECONSTRUCTION

6.NS.3	**Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.**		
DESCRIPTION	Procedural fluency is defined by the Common Core as "Skill in carrying out procedures flexibly, accurately, efficiently and appropriately." In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals were introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations. The use of estimation strategies supports student understanding of decimal operations. **Example:** First estimate the sum of 12.3 and 9.75. Solution: An estimate of the sum would be 12 + 10 or 22. Students could also state if their estimate is high or low. Answers of 230.5 or 2.305 indicate that students are not considering place value when adding.		
ESSENTIAL QUESTION(S)	How can using the standard algorithm for dividing multi-digit fractions create efficiency and accuracy?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.7. Look for and make use of structure. 6.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Fluently divide multi-digit numbers using the standard algorithm with speed and accuracy.		

EXPLANATIONS AND EXAMPLES

The use of estimation strategies supports student understanding of operating on decimals.

Example:

First, students estimate the sum and then find the exact sum of 14.4 and 8.75. An estimate of the sum might be 14 + 9 or 23. Students may also state if their estimate is low or high. They would expect their answer to be greater than 23. They can use their estimates to self-correct.

Answers of 10.19 or 101.9 indicate that students are not considering the concept of place value when adding (adding tenths to tenths or hundredths to hundredths) whereas answers like 22.125 or 22.79 indicate that students are having difficulty understanding how the four-tenths and seventy-five hundredths fit together to make one whole and 25 hundredths.

Students use the understanding they developed in 5th grade related to the patterns involved when multiplying and dividing by powers of ten to develop fluency with operations with multi-digit decimals.

STANDARD AND DECONSTRUCTION

6.NS.4	**Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).**

DESCRIPTION

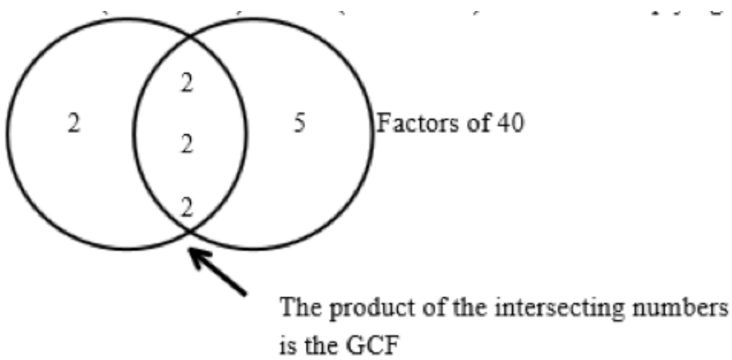
In elementary school, students identified primes, composites, and factor pairs (4.OA.4). In 6th grade, students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by:

1) Listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are relatively prime (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor.

2) Listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors

(2 • 2 • 2 = 8).

Factors of 16



DESCRIPTION (continued)

Students also understand that the greatest common factor of two prime numbers is 1.

Example 1: What is the greatest common factor (GCF) of 18 and 24?

Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.

Example 2: Use the greatest common factor and the distributive property to find the sum of 36 and 8.

36 + 8 = 4 (9) + 4(2) 44 = 4 (9 + 2) 44 = 4 (11) 44 = 44

Example 3: Ms. Spain and Mr. France have donated a total of 90 hot dogs and 72 bags of chips for the class picnic. Each student will receive the same amount of refreshments. All refreshments must be used.

a. What is the greatest number of students that can attend the picnic?

b. How many bags of chips will each student receive?

c. How many hotdogs will each student receive?

Solution:

a. Eighteen (18) is the greatest number of students that can attend the picnic (GCF).

b. Each student would receive 4 bags of chips.

c. Each student would receive 5 hot dogs.

Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by 1) listing the multiples of 6 (6, 12, 18, 24, 30, …) and 8 (8, 26, 24, 32, 40…), then taking the least in common from the list (24); or 2) using the prime factorization.

Step 1: Find the prime factors of 6 and 8. 6 = 2 • 3 8 = 2 • 2 • 2

Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2

Step 3: Multiply the common factors and any extra factors: 2 • 2 • 2 • 3 or 24 (one of the twos is in common; the other twos and the three are the extra factors.

Example 4: The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days. Both schools are serving pizza today. In how may days will both schools serve pizza again?

Solution: The solution to this problem will be the least common multiple (LCM) of 15 and 20. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 15 and a multiple of 20. One way to find the least common multiple is to find the prime factorization of each number: $2^2 \cdot 5 = 20$ and $3 \cdot 5 = 15$. To be a multiple of 20, a number must have 2 factors of 2 and one factor of 5 $(2 \cdot 2 \cdot 5)$. To be a multiple of 15, a number must have factors of 3 and 5. The least common multiple of 20 and 15 must have 2 factors of 2, one factor of 3 and one factor of 5 $(2 \cdot 2 \cdot 3 \cdot 5)$ or 60.

ESSENTIAL QUESTION(S)	How can finding greatest common factors and multiples create efficiency when solving problems?		
MATHEMATICAL PRACTICE(S)	6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Fluently identify the factors of two whole numbers less than or equal to 100 and determine the greatest common factor. Fluently identify the multiples of two whole numbers less than or equal to 12 and determine the least common multiple	Apply the distributive property to rewrite addition problems by factoring out the greatest common factor	

EXPLANATIONS AND EXAMPLES

What is the greatest common factor (GCF) of 24 and 36? How can you use factor lists or the prime factorizations to find the GCF?

Solution: 22 3 = 12. Students should be able to explain that both 24 and 36 have 2 factors of 2 and one factor of 3, thus 2 x 2 x 3 is the greatest common factor.)

What is the least common multiple (LCM) of 12 and 8? How can you use multiple lists or the prime factorizations to find the LCM?

Solution: 23 3 = 24. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 12 and a multiple of 8. To be a multiple of 12, a number must have 2 factors of 2 and one factor of 3 (2 x 2 x 3). To be a multiple of 8, a number must have 3 factors of 2 (2 x 2 x 2). Thus the least common multiple of 12 and 8 must have 3 factors of 2 and one factor of 3 (2 x 2 x 2 x 3).

Rewrite 84 + 28 by using the distributive property. Have you divided by the largest common factor? How do you know?

Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.

27 + 36 = 9 (3 + 4)

63 = 9 x 7

63 = 63

31 + 80

There are no common factors. I know that because 31 is a prime number, it only has 2 factors, 1 and 31. I know that 31 is not a factor of 80 because 2 x 31 is 62 and 3 x 31 is 93.

CLUSTER	**3. Apply and extend previous understandings of numbers to the system of rational numbers.**
BIG IDEA	• **The set of real numbers correspond to a unique point on the infinite number line, where precision is improved by using rational and irrational numbers.** • **Measures and expressions can be compared directly by their relative values.** • **The location of lines, angles, and geometric shapes within a plane provide geometric interpretations of mathematical situations.**
ACADEMIC VOCABULARY	**rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, x-axis, y-axis, coordinates.**

STANDARD AND DECONSTRUCTION

6.NS.5	**Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.**
DESCRIPTION	Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. **Example 1:** a. Use an integer to represent 25 feet below sea level. b. Use an integer to represent 25 feet above sea level. c. What would 0 (zero) represent in the scenario above? Solution: a. -25 b. +25 c. 0 would represent sea level

ESSENTIAL QUESTION(S)	• What is the relationship between a positive and a negative integer in relationship to zero? • How can finding greatest common factors and multiples create efficiency when solving problems?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	Know: Concepts/Skills	Think	Do
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Identify an integer and its opposite.	Use integers to represent quantities in real world situations. Explain where 0 fits into a situation represented by integers.	

EXPLANATIONS AND EXAMPLES

Example 1:

a. Use an integer to represent 25 feet below sea level

b. Use an integer to represent 25 feet above sea level.

c. What would 0 (zero) represent in the scenario above?

Solution:

a. -25

b. +25

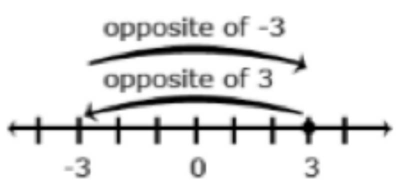
c. 0 would represent sea level

STANDARD AND DECONSTRUCTION

6.NS.6	**Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.**

DESCRIPTION

In elementary school, students worked with positive fractions, decimals and whole numbers on the number line and in quadrant 1 of the coordinate plane. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as "the opposite of 4" which would be negative 4. In this example, – (–6.4) would be read as "the opposite of the opposite of 6.4" which would be 6.4. Zero is its own opposite.



Example 1:

What is the opposite of $2\frac{1}{2}$? Explain your answer?

Solution:

$-2\frac{1}{2}$ because it is the same distance from 0 on the opposite side.

Students worked with Quadrant I in elementary school. As the x-axis and y-axis are extending to include negatives, students begin to with the Cartesian Coordinate system. Students recognize the point where the x-axis and y-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (-,+)

Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (2,1) and (2,1), the y coordinates differ only by signs, which represents a reflection across the x-axis. A change is the x-coordinates from (-2,4) to (2,4), represents a reflection across the y-axis. When the signs of both coordinates change, [(2,4) changes to (2,4)]., the ordered pair has been reflected across both axes.

DESCRIPTION
(continued)

Example1:

Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the *x*-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original point and the reflected point?

$\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ $(0.25, 0.75)$

Solution:

The coordinates of the reflected points would be $\left(\frac{1}{2}, 3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, 3\right)$ $(0.25, 0.75)$. Note that the *y*-coordinates are opposites.

Example 2:

Students place the following numbers would be on a number line: $-4.5, 2, 3.2, -3\frac{3}{5}, 0.2, -2, \frac{11}{2}$. Based on number line placement, numbers can be placed in order.

Solution:

The numbers in order from least to greatest are:

$-4.5, -3\frac{3}{5}, -2, 0.2, 2, 3.2, \frac{11}{2}$

Students place each of these numbers on a number line to justify this order.

ESSENTIAL QUESTION(S)	What is the relationship between a positive and a negative integer in relationship to zero on a number line?
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics.

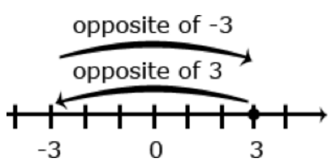
SUBSTANDARD DECONSTRUCTION	6.NS.6a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3 and that 0 is its own opposite.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Recognize opposite signs of numbers as locations on opposite sides of 0 on the number line.	Reason that the opposite of the opposite of a number is the number itself.	

SUBSTANDARD DECONSTRUCTION	6.NS.6b: Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Recognize the signs of both numbers in an ordered pair; indicate which quadrant of the coordinate plane the ordered pair will be located	Reason that when only the x value in a set of ordered pairs are opposites, it creates a reflection over the y axis. Recognize that when only the y value in a set of ordered pairs are opposites, it creates a reflection over the x axis. Reason that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	

SUBSTANDARD DECONSTRUCTION	6.NS.6c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Find and position integers and other rational numbers on a horizontal or vertical number line diagram. Find and position pairs of integers and other rational numbers on a coordinate plane.		

EXPLANATIONS AND EXAMPLES

Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids.



Example:

Graph the following points in the correct quadrant of the coordinate plane. If you reflected each point across the x-axis, what are the coordinates of the reflected points? What similarities do you notice between coordinates of the original point and the reflected point?

$\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ $(0.25, -0.75)$

STANDARD AND DECONSTRUCTION

6.NS.7	Understand ordering and absolute value of rational numbers.

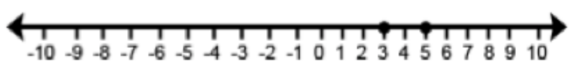
DESCRIPTION

Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line.

Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. Operations with integers are not the expectation at this level.

In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.

Case 1: Two positive numbers

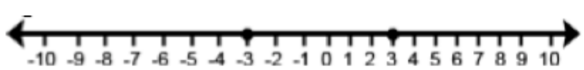


5 > 3

5 is greater than 3.

3 is less than 5.

Case 2: One positive and one negative number

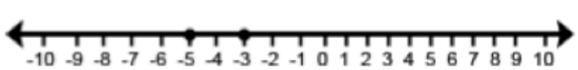


3 > -3

Positive 3 is greater than negative 3.

Negative 3 is less than positive 3.

Case 3: Two negative numbers



-3 > -5

Negative 3 is greater than negative 5.

Negative 5 is less than negative 3.

DESCRIPTION (continued)	Example 1: Write a statement to compare – 4 ½ and –2. Explain your answer. Solution: – 4 ½ < –2 because – 4 ½ is located to the left of –2 on the number line. Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.

ESSENTIAL QUESTION(S)	What is a situation where you would want the absolute value instead of the value of a rational number?
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics.

SUBSTANDARD DECONSTRUCTION	6.NS.7a: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:		Interpret statements of inequality as statements about relative position of two numbers on a number line diagram	

SUBSTANDARD DECONSTRUCTION	6.NS.7b: Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3°C > -7°C to express the fact that -3°C is warmer than -7°C.*		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Order rational numbers on a number line.	Write, interpret, and explain statements of order for rational numbers in real-world contexts.	

SUBSTANDARD DECONSTRUCTION	6.NS.7c: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $\|-30\| = 30$ to describe the size of the debt in dollars.*		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Identify absolute value of rational numbers.	Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.	

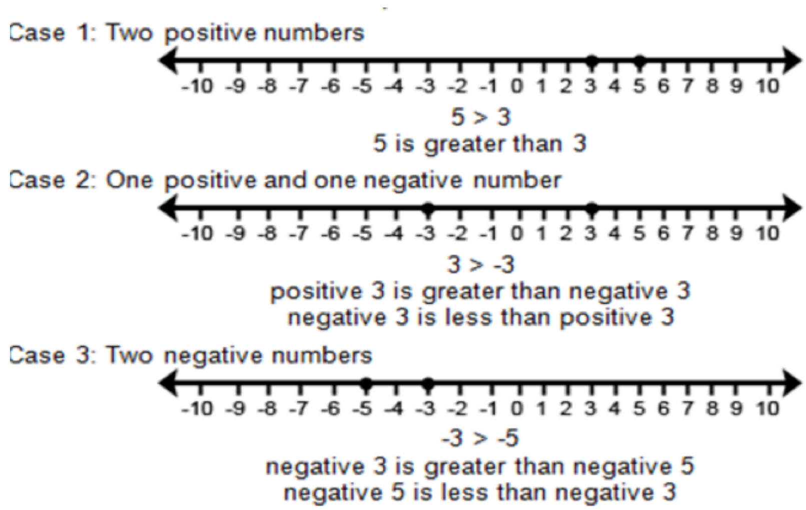
SUBSTANDARD DECONSTRUCTION	6.NS.7d: Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:		Distinguish comparisons of absolute value from statements about order and apply to real world contexts.	

EXPLANATIONS AND EXAMPLES

Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers.

In working with number line models, students internalize the order of the numbers; larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between the numbers.

EXPLANATIONS AND EXAMPLES
(continued)



Comparative statements generate informal experience with operations and lay the foundation for formal work with operations on integers in grade 7.

STANDARD AND DECONSTRUCTION

6.NS.8	**Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.**
DESCRIPTION	Students find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). Example 1: What is the distance between (–5, 2) and (–9, 2)? Solution: The distance would be 4 units. This would be a horizontal line since the y-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances 9 and 5. (| 9 | - | 5 |). <u>Example 2:</u> What is the distance between $(3, -5\frac{1}{2})$ and $(3, 2\frac{1}{4})$? *Solution:* The distance between $(3, -5\frac{1}{2})$ and $(3, 2\frac{1}{4})$ would be $7\frac{3}{4}$ units. This would be a vertical line since the *x*-coordinates are the same. The distance can be found by using a number line to count from $-5\frac{1}{2}$ to $2\frac{1}{4}$ or by recognizing that the distance (absolute value) from $-5\frac{1}{2}$ to 0 is $5\frac{1}{2}$ units and the distance (absolute value) from 0 to $2\frac{1}{4}$ is $2\frac{1}{4}$ units so the total distance would be $5\frac{1}{2} + 2\frac{1}{4}$ or $7\frac{3}{4}$ units. Students graph coordinates for polygons and find missing vertices based on properties of triangles and quadrilaterals.

THE NUMBER SYSTEM

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

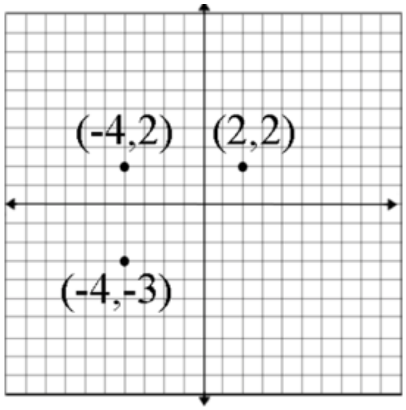
ESSENTIAL QUESTION(S)	How can the understanding of integers on a number line help solve problems when using the coordinate graph?
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Calculate absolute value. Graph points in all four quadrants of the coordinate plane.	Solve real-world problems by graphing points in all four quadrants of a coordinate plane. Given only coordinates, calculate the distances between two points with the same first coordinate or the same second coordinate using absolute value	

EXPLANATIONS AND EXAMPLES

Example:

If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?



To determine the distance along the x-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |4| or 4 units to the left of 0 and 2 is |2|or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4|+ |2|.

DOMAIN:

EXPRESSIONS AND EQUATIONS (EE)

SIXTH GRADE

MATHEMATICS

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

DOMAIN	Expressions and Equations
CLUSTERS	1. Apply and extend previous understandings of arithmetic to algebraic expressions. 2. Reason about and solve one-variable equations and inequalities. 3. Represent and analyze quantitative relationships between dependent and independent variables.

EXPRESSIONS AND EQUATIONS

FIFTH	SIXTH	SEVENTH
Exploring arithmetic and geometric patterns/sequences	**Exploring arithmetic and geometric patterns/sequences**	**Exploring arithmetic and geometric patterns/sequences**
Working with Expressions	**Working with Expressions**	**Working with Expressions**
5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.		
5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.		
	6.EE.2.a Write expressions that record operations with numbers and with letters standing for numbers.	
	6.EE.2.b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.	
	6.EE.2.c Evaluate expressions at specific values of their variables. Include expressions embedded in formulas or equations from real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	

EXPRESSIONS AND EQUATIONS		
FIFTH	**SIXTH**	**SEVENTH**
Exploring arithmetic and geometric patterns/sequences	**Exploring arithmetic and geometric patterns/sequences**	**Exploring arithmetic and geometric patterns/sequences**
Working with Expressions	**Working with Expressions**	**Working with Expressions**
	6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	
	6.EE.3 Apply the properties of operations to generate equivalent expressions.	
	6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).	
		7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
		7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

CLUSTER	**1. Apply and extend previous understandings of arithmetic to algebraic expressions.**
BIG IDEA	• The set of real numbers correspond to a unique point on the infinite number line, where precision is improved by using rational and irrational numbers. • Arithmetic processes are governed by the relationships within a set of numbers that are always true. • Variables, expressions, and equations are algebraic representations of mathematical situations that dictate the unknown to be solved in real-world problems. • Relationships between two sets of numbers can be described by mathematical rules, where a function is a unique rule that has a one-to-one correspondence.
ACADEMIC VOCABULARY	exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables

STANDARD AND DECONSTRUCTION

6.EE.1	Write and evaluate numerical expressions involving whole-number exponents.
DESCRIPTION	Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e. $\frac{1}{2}^5$ can be written $\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2}$ which has the same value as $\frac{1}{32}$). Students recognize that an expression with a variable represents the same mathematics (ie. x^5 can be written as $x \bullet x \bullet x \bullet x \bullet x$) and write algebraic expressions from verbal expressions. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. <u>Example 1:</u> What is the value of: • 0.2^3 *Solution:* 0.008 • $5 + 2^4 \bullet 6$ *Solution:* 101 • $7^2 - 24 \div 3 + 26$ *Solution:* 67 <u>Example 2:</u> What is the area of a square with a side length of $3x$? *Solution:* $3x \bullet 3x = 9x^2$ <u>Example 3:</u> $4^x = 64$ *Solution:* $x = 3$ because $4 \bullet 4 \bullet 4 = 64$

EXPRESSIONS AND EQUATIONS

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

ESSENTIAL QUESTION(S)	Why are orders of operations important when calculating an expression?		
MATHEMATICAL PRACTICE(S)	6.MP.2 Reason abstractly and quantitatively.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Write numerical expressions involving whole number exponents. Ex. 34 = 3 x 3 x 3 x 3 Solve order of operation problems that contain exponents. Ex. $3 + 2^2 - (2 + 3) = 2$		

EXPLANATIONS AND EXAMPLES

Example:

Write the following as a numerical expressions using exponential notation:

- The area of a square with a side length of 8 m: (Solution:)

- The volume of a cube with a side length of 5 ft: (Solution:)

- Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own: (Solution: mice)

Evaluate:

4^3 (Solution: 64)

$5 + 2^4 \times 6$ (Solution 101)

$7^2 - 24 \div 3 + 26$ (Solution: 67)

STANDARD AND DECONSTRUCTION

6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers.
DESCRIPTION	Students write expressions from verbal descriptions using letters and numbers, understanding order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, n" could be represented with 5n and that a number and letter written together means to multiply. All rational numbers may be used in writing expressions when operations are not expected. Students use appropriate mathematical language to write verbal expressions from algebraic expressions. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. **Example Set 1**: Students read algebraic expressions: • r + 21 as "some number plus 21" as well as "r plus 21." • n • 6 as "some number times 6" as well as "n times 6." • s ÷ 6 as "as some number divided by 6" as well as "s divided by 6." **Example Set 2**: Students write algebraic expressions: • 7 less than 3 times a number: Solution: 3x – 7 • 3 times the sum of a number and 5: Solution: 3 (x + 5) • 7 less than the product of 2 and a number: Solution: 2x – 7 • Twice the difference between a number and 5: Solution: 2(z – 5) • The quotient of the sum of x plus 4 and 2: Solution: x + 4 Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Variables are letters that represent numbers. There are various possibilities for the number they can represent. Consider the following expression: x2 + 5y + 3x + 6 The variables are x and y. There are 4 terms, x2, 5y, 3x, and 6. There are 3 variable terms, x2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x2 is 1, since x2 = 1x2. The term 5y represent 5y's or 5 • y. There is one constant term, 6. The expression represents a sum of all four terms.

ESSENTIAL QUESTION(S)	How are letters used in an algebraic expression instead of numbers?
MATHEMATICAL PRACTICE(S)	exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables

SUBSTANDARD DECONSTRUCTION	6.EE.2a: Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Use numbers and variables to represent desired operations.	Translating written phrases into algebraic expressions. Translating algebraic expressions into written phrases.	

SUBSTANDARD DECONSTRUCTION	6.EE.2b: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient). Identify parts of an expression as a single entity, even if not a monomial.		

SUBSTANDARD DECONSTRUCTION	6.EE.2c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.*		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Substitute specific values for variables. Evaluate algebraic expressions including those that arise from real-world problems. Apply order of operations when there are no parentheses for expressions that include whole number exponents.		

EXPLANATIONS AND EXAMPLES

It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Examples:

- r + 21 as "some number plus 21 as well as "r plus 21"
- n 6 as "some number times 6 as well as "n times 6"
- s ÷ 6 as "some number divided by 6" as well as "s divided by 6"

Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions.

Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable.

Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes.

Consider the following expression:

The variables are x and y.

There are 4 terms, x2, 5y, 3x, and 6.

There are 3 variable terms: x2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x2 is 1, since x2 = 1 x2. The term 5y represent 5 y's or 5 * y.

There is one constant term, 6.

The expression shows a sum of all four terms.

Examples:

7 more than 3 times a number (Solution: 3x +7)

3 times the sum of a number and 5 (Solution: 3(x+5))

7 less than the product of 2 and a number (Solution: 2x-7))

Twice the difference between a number and 5 (Solution: 2(z-5))

Evaluate 5(n + 3) – 7n, when n =

The expression c + 0.07c can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25.

The perimeter of a parallelogram is found using the formula p = 2l + 2w. What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches?

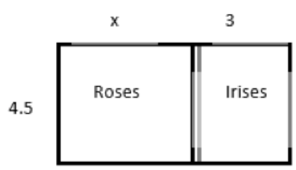
STANDARD AND DECONSTRUCTION

6.EE.3	**Apply the properties of operations to generate equivalent expressions.** ***For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.***

DESCRIPTION

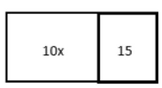
Students use the distributive property to write equivalent expressions. Using their understanding of area models from elementary grades, students illustrate the distributive property with variables. Properties are introduced throughout elementary grades (3.OA.5); however, there has not been an emphasis on recognizing and naming the property. In 6th grade, students are able to use the properties and identify them by name as used when justifying solution methods (see example 4).

Example 1: Given that the width is 4.5 units and the length can be represented by x + 3, the area of the flowers below can be expressed as 4.5(x +) or 4.5x + 13.5.



When given an expression representing area, students need to find the factors.

Example 2: The expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x + 3)



Example 3: Students use their understanding of multiplication to interpret 3 (2 + x) as 3 groups of (2 + x). They use a model to represent x, and make an array to show the meaning of 3(2 + x). They can explain why it makes sense that 3(2 + x) is equal to 6 + 3x.

An array with 3 columns and x + 2 in each column:

Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y must be 3y. They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that y + y + y = 3y.

Example 4: Prove that y + y + y = 3y

Solution: y + y + y = y • 1 + y • 1 + y • 1   Multiplicative Identity

y • (1 + 1 + 1) Distributive Property

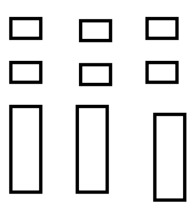
y • 3 = 3y   Commutative Property

ESSENTIAL QUESTION(S)	What strategies can be used to create two equivalent expressions?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Generate equivalent expressions using the properties of operations.	Apply the properties of operations to generate equivalent expressions.	

EXPLANATIONS AND EXAMPLES

Students use their understanding of multiplication to interpret 3(2 + x). For example, 3 groups of (2 + x). They use a model to represent x, and make an array to show the meaning of 3(2 + x). They can explain why it makes sense that 3(2 + x) is equal to 6 + 3x.

An array with 3 columns and x + 2 in each column:



Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y must be 3y. They also the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that y + y + y = 3y:

$y + y + y = y \,.\, 1 + y \,.\, 1 + y \,.\, 1 = y \,.\, (1 + 1 + 1) = y \,.\, 3 = 3y$

STANDARD AND DECONSTRUCTION

6.EE.4 **Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).** ***For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.***

DESCRIPTION

Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, 3x + 4x are like terms and can be combined as 7x; however, 3x + 4x2 are not like terms since the exponents with the x are not the same.

This concept can be illustrated by substituting in a value for x. For example, 9x – 3x = 6x not 6. Choosing a value for x, such as 2, can prove non-equivalence.

$9(2) - 3(2) = 6(2)$	however	$9(2) - 3(2) \stackrel{?}{=} 6$
$18 - 6 = 12$		$18 - 6 \stackrel{?}{=} 6$
$12 = 12$		$12 \neq 6$

Students can also generate equivalent expressions using the associative, commutative, and distributive properties.

They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example 1:

Are the expressions equivalent? Explain your answer?

4m + 8 4(m+2) 3m + 8 + m 2 + 2m + m + 6 + m

Solution:

Expression	Simplifying the Expression	Explanation
$4m + 8$	$4m + 8$	Already in simplest form
$4(m+2)$	$4(m+2)$ $4m + 8$	*Distributive property*
$3m + 8 + m$	$3m + 8 + m$ $3m + m + 8$ $4m + 8$	*Combined like terms*
$2 + 2m + m + 6 + m$	$2m + m + m + 2 + 6$ $4m + 8$	*Combined like terms* *Combined like terms*

ESSENTIAL QUESTION(S)	Why is equivalency important when solving problems?
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Recognize when two expressions are equivalent.	Prove that two equations are equivalent no matter what number is substituted.	

EXPLANATIONS AND EXAMPLES

Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example:

Are the expressions equivalent? How do you know?

4m + 8 4(m+2) 3m + 8 + m 2 + 2m + m + 6 + m

Solution:

Expression	Simplifying the Expression	Explanation
4m + 8	*4m + 8*	Already in simplest form
4(m+2)	*4(m+2)* *4m + 8*	*Distributive property*
3m + 8 + m	*3m + 8 + m* *3m + m + 8* *(3m + m) + 8* *4m + 8*	*Combined like terms*
2 + 2m + m + 6 + m	*2 + 2m + m + 6 + m* *2 + 6 + 2m + m + m* *(2 + 6) + (2m + m + m)* *8 + 4m* *4m + 8*	*Combined like terms*

EXPRESSIONS AND EQUATIONS

CLUSTER	**2. Reason about and solve one-variable equations and inequalities.**
BIG IDEA	• **Equations and inequalities can be solved using arithmetic and algebraic rules and equivalence.** • **Variables, expressions, and equations are algebraic representations of mathematical situations that dictate the unknown to be solved in real-world problems.**
ACADEMIC VOCABULARY	**inequalities, equations, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, profit, exceed**

STANDARD AND DECONSTRUCTION

6.EE.5 **Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.**

DESCRIPTION

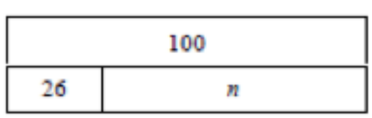
In elementary grades, students explored the concept of equality. In 6th grade, students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true.

Example 1:

Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation 26 + n = 100 where n is the number of papers the teacher gives to Joey. This equation can be stated as "some number was added to 26 and the result was 100." Students ask themselves, "What number was added to 26 to get 100?" to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem:

- Reasoning: 26 + 70 is 96 and 96 + 4 is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations: n + 26 = 100, 100 - n = 26, 100 - 26 = n. Select the equation that helps to find n easily.
- Use knowledge of inverse operations: Since subtraction "undoes" addition then subtract 26 from 100 to get the numerical value of n.
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.



EXPLANATIONS AND EXAMPLES
(continued)

Solution:

Students recognize the value of 74 would make a true statement if substituted for the variable.

26 + n = 100

26 + 74 = 100

100 = 100 ✓

Example 2:

The equation 0.44 s = 11 where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution.

Solution:

There are 25 stamps in the booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11.

By substituting 25 in for s and then multiplying, I get 11.

0.44(25) = 11

11 = 11

Example 3:

Twelve is less than 3 times another number can be shown by the inequality 12 < 3n. What numbers could possibly make this a true statement?

Solution:

Since 3 • 4 is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, 5 3/4, and 200. Given a set of values, students identify the values that make the inequality true.

STANDARD AND DECONSTRUCTION

6.EE.6	**Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.**

DESCRIPTION

Students write expressions to represent various real-world situations.

Example Set 1:

-Write an expression to represent Susan's age in three years, when a represents her present age. (Solution: a + 3)

-Write an expression to represent the number of wheels, w, on any number of bicycles. (Solution: 2n)

-Write an expression to represent the value of any number of quarters, q. (Solution: 0.25q)

Given a contextual situation, students define variables and write an expression to represent the situation.

Example 2:

The skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people.

n = the number of people

100 + 5n

No solving is expected with this standard; however, 6.EE.2c does address the evaluating of the expressions.

Students understand the inverse relationships that can exist between two variables. For example, if Sally has 3 times as many bracelets as Jane, then Jane has the amount of Sally. If S represents the number of bracelets Sally has, the s or s/3 represents the amount Jane has.

Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Example Set 3:

-Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.

Solution: 2c + 3 where c represents the number of crayons that Elizabeth has

-An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.

Solution: 28 + 0.35t where t represents the number of tickets purchased

-Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.

Solution: 15h + 20 = 85 where h is the number of hours worked

DESCRIPTION (continued)	-Describe a problem situation that can be solved using the equation 2c + 3 = 15; where c represents the cost of an item Possible solution: Sarah spent $15 at a craft store. -She bought one notebook for $3. -She bought 2 paintbrushes for x dollars. If each paintbrush cost the same amount, what was the cost of one brush? - Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. Solution: $5.00 + n

ESSENTIAL QUESTION(S)	How is a real-world problem with an unknown represented in as an algebraic expression?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics. 6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	Relate variables to a context. Write expressions when solving a real world or mathematical problem.	

EXPLANATIONS AND EXAMPLES

Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Examples:

*Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.

(Solution: 2c + 3 where c represents the number of crayons that Elizabeth has.)

*An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent. (Solution: 28 + 0.35t where t represents the number of tickets purchased)

*Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.

(Solution: 15h + 20 = 85 where h is the number of hours worked)

*Describe a problem situation that can be solved using the equation 2c + 3 = 15; where c represents the cost of an item

*Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. (Solution: $5.00 + n)

STANDARD AND DECONSTRUCTION

6.EE.7	**Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q* for cases in which *p, q* and *x* are all non-negative rational numbers.**

DESCRIPTION

Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, x + 4, any value can be substituted for the x to generate a numerical answer; however, in the equation x + 4 = 6, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.

Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include fractions and decimals with non-negative solutions. Students recognize that dividing by 6 and multiplying by 1/6 produces the same result. For example, x/6 = 9 and 1/6x = 9 will produce the same result.

Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem.

Example 1: Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

$56.58		
T	T	T

Sample Solution: Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled T is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation 3T = $56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 is only 30 but less than $20 each because 20 x 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That's $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15+3+0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58."

Example 2:

Julio gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julie has left.

20		
1.9	6.50	money left over (m)

Solution: 20 = 1.99 + 6.50 + x. x = $11.51

ESSENTIAL QUESTION(S)	Why is equality important when solving a real life problem algebraically?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Define inverse operation. Know how inverse operations can be used in solving one-variable equations.	Apply rules of the form x + p = q and px = q, for cases in which p, q, and x are all non-negative rational numbers, to solve real world and mathematical problems; with only one unknown quantity. Develop a rule for solving one-step equations using inverse operations with non-negative rational coefficients. Solve and write equations for real-world mathematical problems containing one unknown.	

EXPLANATIONS AND EXAMPLES

Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.

Example:

*Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

$56.58		
J	J	J

Sample Solution: Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation 3J = $56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 is only 30 but less than $20 each because 20 x 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That's $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15+3+0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58."

*Julio gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julio has left.

(Solution: 20 = 1.99 + 6.50 + x, x = $11.51)

20		
1.99	6.50	money left over (m)

STANDARD AND DECONSTRUCTION

6.EE.8

Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < *c* have infinitely many solutions; represent solutions of such inequalities on number line diagrams

DESCRIPTION

Many real-world situations are represented by inequalities. Students write inequalities to represent real world and mathematical situations. Students use the number line to represent inequalities from various contextual and mathematical situations.

Example 1:

The class must raise at least $100 to go on the field trip. They have collected $20. Write an inequality to represent the amount of money, m, the class still needs to raise. Represent this inequality on a number line.

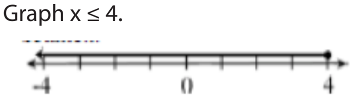
Solution:

The inequality m ≥ $80 represents this situation. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.

A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

Example 2:

Graph $x \leq 4$.

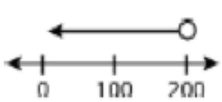


Example 3:

The Flores family spent less than $200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line.

Solution:

200 > x, where x is the amount spent on groceries.



ESSENTIAL QUESTION(S)	How can a numerical constraint be represented algebraically?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Identify the constraint or condition in a real-world or mathematical problem in order to set up an inequality. Recognize that inequalities of the form x > c or x < c have infinitely many solutions.	Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Represent solutions to inequalities or the form x > c or x < c, with infinitely many solutions, on number line diagrams.	

EXPLANATIONS AND EXAMPLES

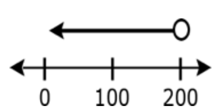
Examples:

Graph x ≤ 4.

*Jonas spent more than $50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line.

*Less than $200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line.

Solution: 200 > x



CLUSTER 3. Represent and analyze quantitative relationships between dependent and independent variables.

BIG IDEA Variables, expressions, and equations are algebraic representations of mathematical situations that dictate the unknown to be solved in real-world problems.

ACADEMIC VOCABULARY dependent variables, independent variables, discrete data, continuous data

STANDARD AND DECONSTRUCTION

6.EE.9 **Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.** ***For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.***

DESCRIPTION

The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.

Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts.

Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) *Relationships should be proportional with the line passing through the origin*. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and/or a table of values.

Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective.

Example 1:

What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

Solution:

y = 2.5x

ESSENTIAL QUESTION(S)	What type of problem would have independent and dependent variables?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.7. Look for and make use of structure. 6.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Define independent and dependent variables. Use variables to represent two quantities in a real-world problem that change in relationship to one another.	Write an equation to express one quantity (dependent) in terms of the other quantity (independent). Analyze the relationship between the dependent variable and independent variable using tables and graphs. Relate the data in a graph and table to the corresponding equation.	

EXPLANATIONS AND EXAMPLES

Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.

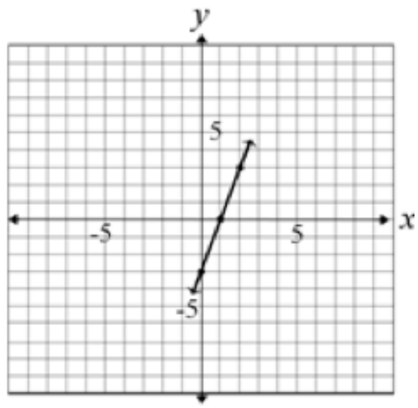
Examples:

*What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

EXPLANATIONS AND EXAMPLES
(continued)

*Use the graph below to describe the change in y as x increases by 1.



*Susan started with $1 in her savings. She plans to add $4 per week to her savings. Use an equation, table, and graph to demonstrate the relationship between the number of weeks that pass and the amount in her savings account.

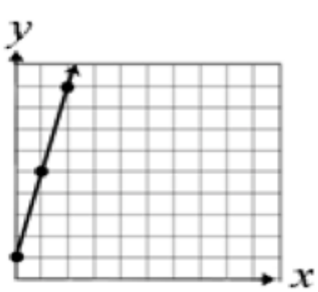
Language: Susan has $1 in her savings account. She is going to save $4 each week.

Equation: $y = 4x + 1$

Table:

x	y
0	1
1	5
2	9

Graph:



DOMAIN:

GEOMETRY (G)

SIXTH GRADE

MATHEMATICS

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

DOMAIN	Geometry (G)
CLUSTERS	1. Solve real-world and mathematical problems involving area, surface area, and volume.
BIG IDEA	• Variables, expressions, and equations are algebraic representations of mathematical situations that dictate the unknown to be solved in real-world problems. • Equations and inequalities that can be solved using arithmetic and algebraic rules and equivalence. • Measurable attributes of objects can be described mathematically by standard units. • The location of lines, angles, and geometric shapes within a plane provide geometric interpretations of mathematical situations.
ACADEMIC VOCABULARY	area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism

GEOMETRY		
FIFTH	**SIXTH**	**SEVENTH**
LENGTH, AREA, AND VOLUME		
Decimal Numbers, Integer Exponents, and Scientific Notation	**Decimal Numbers, Integer Exponents, and Scientific Notation**	**Decimal Numbers, Integer Exponents, and Scientific Notation**
	6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
	6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right	7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
		7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
	6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	

STATISTICS AND PROBABILITY		
FIFTH	**SIXTH**	**SEVENTH**
VARIATION, DISTRIBUTION, AND MODELING		
Describing the Distribution of a Set of Data	**Describing the Distribution of a Set of Data**	**Describing the Distribution of a Set of Data**
	6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	
	6.SP.4 Part 1 Display numerical data in plots on a number line, including dot plots, histograms.	
	6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.	
	6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	
	6.SP.5.c Summarize numerical data sets in relation to their context, such as by: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	
	6.SP.4 Part 2 [Display numerical data in plots on a box plot[s]. 6.SP.5.b Summarize numerical data sets in relation to their context, such as by describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	
	6.SP.5.a Summarize numerical data sets in relation to their context, such as by reporting the number of observations.	

STATISTICS AND PROBABILITY		
FIFTH	**SIXTH**	**SEVENTH**
VARIATION, DISTRIBUTION, AND MODELING		
Describing the Distribution of a Set of Data	**Describing the Distribution of a Set of Data**	**Describing the Distribution of a Set of Data**
	6.SP.5.d Summarize numerical data sets in relation to their context, such as by relating choice of measures of center and variability to the shape of the data distribution in context.	
		7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
		7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

STANDARD AND DECONSTRUCTION

6.G.1 **Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.**

DESCRIPTION

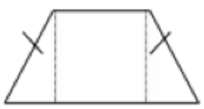
Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of why the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for all students.

Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½ bh or (b x h)/2.

The following site helps students to discover the area formula of triangles.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L577

Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.


Isosceles trapezoid


Right trapezoid

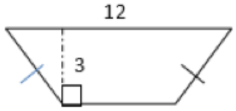
Note: Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure.

Example 1: Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.

Solution: Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90°angle and would be used to find the area using:

$A = \frac{1}{2} bh$
$A = \frac{1}{2}$ (3 units)(4 units)
$A = \frac{1}{2}$ 12 units^2
$A = 6$ units^2

Example 2: Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



DESCRIPTION
(continued)

Solution: The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units2. The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be ½ (2.5 units)(3 units) or 3.75 units 2 (squared).

$$\begin{array}{r l} 21 & \text{units}^2 \\ 3.75 & \text{units}^2 \\ \underline{+3.75} & \underline{\text{units}^2} \\ 28.5 & \text{units}^2 \end{array}$$

Example 3: A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

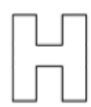
Solution: The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches2. The area of the new rectangle is 48 inches2. The area increased 4 times (quadrupled). Students may also create a drawing to show this visually.

Example 4:
The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?

Solution:
Change the dimensions of the bulletin board to inches (4 feet = 48 inches; 3 feet = 36 inches). The area of the board would be 48 inches x 36 inches or 1728 inches^2. The area of one index card is 12 inches^2. Divide 1728 inches^2 by 24 inches^2 to get the number of index cards. 72 index cards would be needed.

Example 5:
The sixth grade class at Hernandez School is building a giant wooden H for their school. The "H" will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

1. How large will the H be if measured in square feet?
2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?



Solution:
1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft^2. The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft^2. There are two of these pieces.
   The area of the "H" would be 100 ft^2 – 18.75 ft^2 – 18.75 ft^2, which is 62.5ft^2.
   A second solution would be to decompose the "H" into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft^2 and the area of the smaller rectangle would be 12.5 ft^2. Therefore the area of the "H" would be 25 ft^2 + 25 ft^2 + 12.5 ft^2 or 62.5ft^2.
2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut **two pieces of wood in half to create four pieces 5 ft. by 2.5 ft.** These pieces will make the two taller rectangles. A **third piece would be cut to measure 5ft. by 2.5 ft.** to create the middle piece.

DESCRIPTION (continued)

Solution: The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units2. The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be ½ (2.5 units)(3 units) or 3.75 units 2 (squared).

Example 6:
A border that is 2 ft wide surrounds a rectangular flowerbed 3 ft by 4 ft. What is the area of the border?

Solution:
Two sides 4 ft. by 2 ft. would be 8ft^2 x 2 or 16 ft^2
Two sides 3 ft. by 2 ft. would be 6ft^2 x 2 or 12 ft^2
Four corners measuring 2 ft. by 2 ft. would be 4ft^2 x 4 or 16 ft^2

The total area of the border would be 16 ft^2 + 12 ft^2 + 16 ft^2 or **44ft^2**

ESSENTIAL QUESTION(S)	How can my understanding of finding areas of rectangles and triangles help find the area of another shape?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure. 6.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Recognize and know how to compose and decompose polygons into triangles and rectangles.	Compare the area of a triangle to the area of the composted rectangle. Apply the techniques of composing and/or decomposing to find the area of triangles, special quadrilaterals, and polygons to solve mathematical and real-world problems. Discuss, develop, and justify formulas for triangles and parallelograms.	

EXPLANATIONS AND EXAMPLES

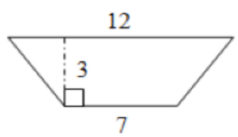
Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites. Students can use tools such as the Isometric Drawing Tool on NCTM's Illuminations site to shift, rotate, color, decompose and view figures in 2D or 3D.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=125

Examples:

*Find the area of a triangle with a base length of three units and a height of four units.

*Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



*A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

*The area of the rectangular school garden is 24 square units. The length of the garden is 8 units. What is the length of the fence needed to enclose the entire garden?

*The sixth grade class at Hernandez School is building a giant wooden H for their school. The H will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

*How large will the H be if measured in square feet?

*The truck that will be used to bring the wood from the lumber yard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many pieces and what dimensions) are needed to complete the project?

STANDARD AND DECONSTRUCTION

6.G.2	**Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas *V = l w h* and *V = b h* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.**

DESCRIPTION

Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The use of models was emphasized as students worked to derive the formula V = Bh (5.MD.3, 5.MD.4, 5.MD.5) The unit cube was 1 x 1 x 1.

In 6th grade the unit cube will have fractional edge lengths. (ie. ½ • ½ • ½) Students find the volume of the right rectangular prism with these unit cubes.

Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students *derive* the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6).

In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes.

Example 1:

A right rectangular prism has edges of $1\frac{1}{4}$", 1" and $1\frac{1}{2}$". How many cubes with side lengths of $\frac{1}{4}$ would be needed to fill the prism? What is the volume of the prism?

Solution:

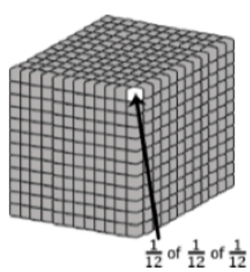
The number of $\frac{1}{4}$" cubes can be found by recognizing the smaller cubes would be $\frac{1}{4}$" on all edges, changing the dimensions to $\frac{5}{4}$", $\frac{4}{4}$" and $\frac{6}{4}$". The number of one-fourth inch unit cubes making up the prism is 120 (5 x 4 x 6).

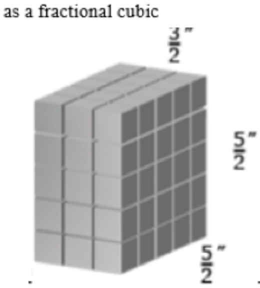
Each smaller cube has a volume of $\frac{1}{64}$ ($\frac{1}{4}$" x $\frac{1}{4}$" x $\frac{1}{4}$"), meaning 64 small cubes would make up the unit cube.

Therefore, the volume is $\frac{5}{4}$ x $\frac{6}{4}$ x $\frac{4}{4}$ or $\frac{120}{64}$ (120 smaller cubes with volumes of $\frac{1}{64}$ or $1\frac{56}{64}$ → 1 unit cube with 56 smaller cubes with a volume of $\frac{1}{64}$.

Example 2:

The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ft^3.





Example 3:

The model shows a rectangular prism with dimensions $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{5}{2}$ inches. Each of the cubic units in the model is $\frac{1}{2}$in. on each side. Students work with the model to illustrate $\frac{3}{2}$ x $\frac{5}{2}$ x $\frac{5}{2}$ = (3 x 5 x 5) x $\frac{1}{8}$. Students reason that a small cube has volume of $\frac{1}{8}$ in^3 because 8 of them fit in a unit cube.

ESSENTIAL QUESTION(S)	What is an efficient strategy for calculating the area, surface area, and volume of a cube?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure. 6.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Know how to calculate the volume of a right rectangular prism.	Apply volume formulas for right rectangular prisms to solve real-world and mathematical problems involving rectangular prisms with fractional edge lengths.	Model the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths.

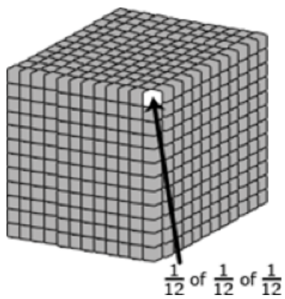
EXPLANATIONS AND EXAMPLES

Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations. (**http://illuminations.nctm.org/ActivityDetail.aspx?ID=6**)

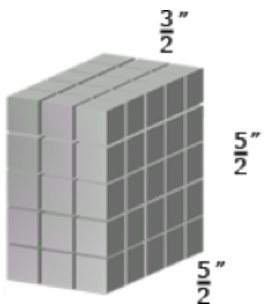
In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes.

Examples:

*The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of 1/12 ft3



*The models show a rectangular prism with dimensions 3/2 inches, 5/2 inches, and 5/2 inches. Each of the cubic units in the model is 1/8 in3. Students work with the model to illustrate 3/2 x 5/2 x 5/2 = (3 x 5 x 5) x 1/8. Students reason that a small cube has volume 1/8 because 8 of them fit in a unit cube.



STANDARD AND DECONSTRUCTION

6.G.3	**Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.**
DESCRIPTION	Students are given the coordinates of polygons to draw in the coordinate plane. If both x-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the y-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane.

ESSENTIAL QUESTION(S)	How can the coordinate graph be used as a strategy for finding area, surface area, or volume in a real-life problem?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Draw polygons in the coordinate plane. Use coordinates (with the same x-coordinate or the same y-coordinate) to find the length of a side of a polygon.	Apply the technique of using coordinates to find the length of a side of a polygon drawn in the coordinate plane to solve real-world and mathematical problems.	

EXPLANATIONS AND EXAMPLES

Example:

On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

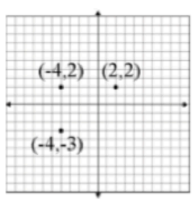
*What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?

*What shape is formed by connecting the three locations? The city council is planning to place a city park in this area. How large is the area of the planned park?

This standard can be taught in conjunction with 6.G.1 to help students develop the formula for the triangle by using the squares of the coordinate grid. Given a triangle, students can make the corresponding square or rectangle and realize the triangle is ½.

Students progress from counting the squares to making a rectangle and recognizing the triangle as ½ to the development of the formula for the area of a triangle.

Example 1: If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.



Solution:
To determine the distance along the x-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |-4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4| + |2| . The length is 6 and the width is 5.

The fourth vertex would be (2, -3).
The area would be 5 x 6 or 30 units^2.
The perimeter would be 5 + 5 + 6 + 6 or 22 units.

<u>Example 2:</u>
On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?

Solution:
1. The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same y-coordinate. The distance between the x-coordinates is 2 (from -2 to 0).
2. The three locations form a right triangle. The area is 2 mi^2.

STANDARD AND DECONSTRUCTION

6.G.4 **Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.**

DESCRIPTION

A net is a two-dimensional representation of a three-dimensional figure. Students represent three- dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure.

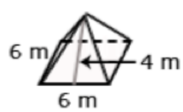
Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

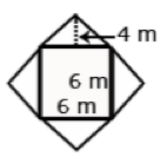
Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations **(http://illuminations.nctm.org/ActivityDetail.aspx?ID=205)**.

Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Example 1: Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?

Example 2: Create the net for a given prism or pyramid, and then use the net to calculate the surface area.





ESSENTIAL QUESTION(S)	How can a net drawing help me calculate the area of a figure?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure. 6.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Know that 3-D figures can be represented by nets.	Represent three-dimensional figures using nets made up of rectangles and triangles. Apply knowledge of calculating the area of rectangles and triangles to a net, and combine the areas for each shape into one answer representing the surface area of a three-dimensional figure. Solve real-world and mathematical problems involving surface area using nets.	

EXPLANATIONS AND EXAMPLES

Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

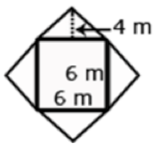
Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations **(http://illuminations.nctm.org/ActivityDetail.aspx?ID=205)**.

Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Examples:

*Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?

*Create the net for a given prism or pyramid, and then use the net to calculate the surface area.



DOMAIN:

STATISTICS AND PROBABILITY (SP)

SIXTH GRADE

MATHEMATICS

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

DOMAIN	Statistics and Probability (SP)
CLUSTERS	1. Develop understanding of statistical variability.
BIG IDEA	• Asking appropriate questions to collect data from sources can lead to analyzing and interpreting real world situations and problems. • Interpretation of a data set can be described by using appropriate measures of center and spread.
ACADEMIC VOCABULARY	statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean

STATISTICS AND PROBABILITY		
FIFTH	**SIXTH**	**SEVENTH**
VARIATION, DISTRIBUTION, AND MODELING		
Describing the Distribution of a Set of Data	**Describing the Distribution of a Set of Data**	**Describing the Distribution of a Set of Data**
	6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	
	6.SP.4 Part 1 Display numerical data in plots on a number line, including dot plots, histograms.	
	6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.	
	6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	
	6.SP.5.c Summarize numerical data sets in relation to their context, such as by: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	

STATISTICS AND PROBABILITY		
FIFTH	**SIXTH**	**SEVENTH**
VARIATION, DISTRIBUTION, AND MODELING		
Describing the Distribution of a Set of Data	**Describing the Distribution of a Set of Data**	**Describing the Distribution of a Set of Data**
	6.SP.4 Part 2 [Display numerical data in plots on a box plot[s].	
	6.SP.5.b Summarize numerical data sets in relation to their context, such as by: Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	
	6.SP.5.a Summarize numerical data sets in relation to their context, such as by: Reporting the number of observations.	
	6.SP.5.d Summarize numerical data sets in relation to their context, such as by: Relating choice of measures of center and variability to the shape of the data distribution in context.	
		7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
		7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

DOMAIN	1. Statistics and Probability
CLUSTERS	Develop understanding of statistical variability.
BIG IDEA	• Asking appropriate questions to collect data from sources can lead to analyzing and interpreting real world situations and problems. • Interpretation of a data set can be described by using appropriate measures of center and spread.
ACADEMIC VOCABULARY	statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean

CLUSTER	1. Develop understanding of statistical variability.
BIG IDEA	• Variables, expressions, and equations are algebraic representations of mathematical situations that dictate the unknown to be solved in real-world problems.
ACADEMIC VOCABULARY	statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean

STANDARD AND DECONSTRUCTION

6.SP.1	**Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.**
DESCRIPTION	Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, "How tall am I?" is not a statistical question because there is only one response; however, the question, "How tall are the students in my class?" is a statistical question since the responses anticipates variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values. Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking, "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: "How many hours per week on average do students at Jefferson Middle School exercise?"

ESSENTIAL QUESTION(S)	What are the characteristics of a statistical question?		
MATHEMATICAL PRACTICE(S)	6.MP.1. Make sense of problems and persevere in solving them. 6.MP.3. Construct viable arguments and critique the reasoning of others. 6.MP.6. Attend to precision.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Recognize that data can have variability. Recognize a statistical question (examples versus non-examples).	Apply the technique of using coordinates to find the length of a side of a polygon drawn in the coordinate plane to solve real-world and mathematical problems.	

EXPLANATIONS AND EXAMPLES

Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents).

Questions can result in a narrow or wide range of numerical values. For example, asking classmates "How old are the students in my class in years?" will result in less variability than asking "How old are the students in my class in months?"

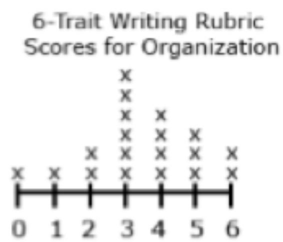
Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking, "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: "How many hours per week on average do students at Jefferson Middle School exercise?"

To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers.

STANDARD AND DECONSTRUCTION

6.SP.2	**Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.**
DESCRIPTION	The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread, and overall shape with dot plots, histograms, and box plots.

Example 1: The dot plot shows the writing scores for a group of students on organization. Describe the data.



Solution: The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.68. If all students scored the same, the score would be 3.68.

NOTE: Mode as a measure of center and range as a measure of variability are not addressed in the CCSS and as such are not a focus of instruction. These concepts can be introduced during instruction as needed.

ESSENTIAL QUESTION(S)	How can data be described?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Know that a set of data has a distribution. Describe a set of data by its center. Describe a set of data by its spread and overall shape.		

EXPLANATIONS AND EXAMPLES

The two dot plots show the 6-trait writing scores for a group of students on two different traits: organization and ideas. The center, spread, and overall shape can be used to compare the data sets. Students consider the context in which the data were collected and identify clusters, peaks, gaps, and symmetry. Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.

STANDARD AND DECONSTRUCTION

6.SP.3 **Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.**

DESCRIPTION

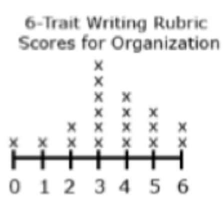
Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (ie. midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic.

Example 1: Consider the data shown in the dot plot of the six trait scores for organization for a group of students.

• How many students are represented in the data set?

• What is the mean and median of the data set? What do these values mean? How do they compare?

• What is the range of the data? What does this value mean?



Solution:

• 19 students are represented in the data set.

• The mean of the data set is 3.5. The median is 3.

• The mean indicates that if the values were equally distributed, all students would score a 3.5.

• The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower.

• The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.

ESSENTIAL QUESTION(S)	What are different ways data can be described?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Recognize there are measures of central tendency for a data set. Recognize there are measures of variances for a data set. Recognize measures of central tendency for a data set; summarize the data with a single number. Recognize measures of variation for a data set; describe how its values vary with a single number.		

EXPLANATIONS AND EXAMPLES

When using measures of center (mean, median, and mode) and range, students are describing a data set in a single number. The range provides a single number that describes how the values vary across the data set. The range can also be expressed by stating the minimum and maximum values.

Example:

Consider the data shown in the dot plot of the six trait scores for organization for a group of students.

-How many students are represented in the data set?

-What are the mean, median, and mode of the data set? What do these values mean? How do they compare?

-What is the range of the data? What does this value mean?

CLUSTERS	**2. Summarize and describe distributions.**
BIG IDEA	Tables, charts, and graphs allows for analyzing data efficiently and effectively. Interpretation of a data set can be described by using appropriate measures of center and spread.
ACADEMIC VOCABULARY	box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier

STANDARD AND DECONSTRUCTION

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

DESCRIPTION

Students display data graphically using number lines. Dot plots, histograms, and box plots are three graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others.

Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions may vary from it.

A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape of a distribution. Students understand that the size of the box or whiskers represents the middle 50% of the data.

Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations.

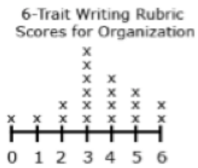
Box Plot Tool: http://illuminations.nctm.org/ActivityDetail.aspx?ID=77

Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

EXPLANATIONS AND EXAMPLES

Example 1: Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

Solution:



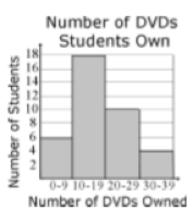
Example 2: Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution: A histogram using 5 intervals (bins) 0-9, 10-19, …30-39) to organize the data is displayed below.

Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

Example 3: Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?



EXPLANATIONS AND EXAMPLES (continued)

Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

Example 3: Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Solution:

Five number summary

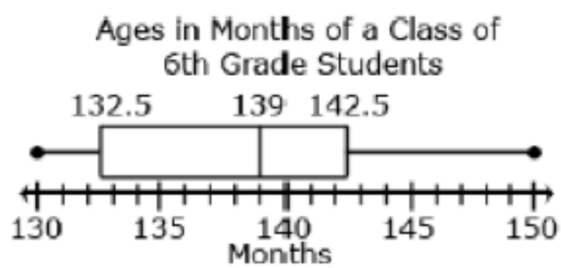
Minimum: 130 months

Quartile 1 (Q1): (132 + 133) ÷ 2 = 132.5 months

Median (Q2): 139 months

Quartile 3 (Q3): (142 + 143) ÷ 2 = 142.5 months

Maximum: 150 months



This box plot shows that

-of the students in the class are from 130 to 132.5 months old.

- of the students in the class are from 142.5 months to 150 months old.

- of the class are from 132.5 to 142.5 months old.

- The median class age is 139 months.

SIXTH GRADE

LEXILE GRADE LEVEL BANDS: 925L TO 1120L

ESSENTIAL QUESTION(S)	How does the type of data determine the best choice of visual representation?		
MATHEMATICAL PRACTICE(S)	6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics. 6.MP.5. Use appropriate tools strategically. 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Assessment Types	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling applications.
Students should be able to:	Identify the components of dot plots, histograms, and box plots. Find the median, quartile, and interquartile range of a set of data.	Analyze a set of data to determine its variance.	Create a dot plot to display a set of numerical data. Create a histogram to display a set of numerical data. Create a box plot to display a set of numerical data.

EXPLANATIONS AND EXAMPLES

In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by others students or contained in reference materials. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations.

Box Plot Tool: http://illuminations.nctm.org/ActivityDetail.aspx?ID=77

Histogram Tool: http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it.

Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. Box plots are generated from the five number summary of a data set consisting of the minimum, maximum, median, and two quartile values. Students can readily compare two sets of data if they are displayed with side by side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data.

Examples:

*Nineteen students completed a writing sample that was scored using the six traits rubric. The scores for the trait of organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

EXPLANATIONS AND EXAMPLES
(continued)

• Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

A histogram using 5 ranges (0-9, 10-19, …30-39) to organize the data is displayed below.

• Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

EXPLANATIONS AND EXAMPLES
(continued)

Five number summary

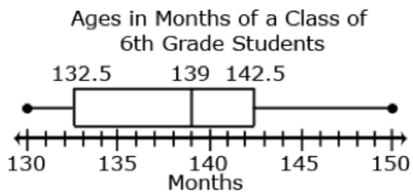
Minimum – 130 months

Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months

Median (Q2) – 139 months

Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months

Maximum – 150 months



This box plot shows that

- ¼ of the students in the class are from 130 to 132.5 months old
- ¼ of the students in the class are from 142.5 months to 150 months old
- ½ of the class are from 132.5 to 142.5 months old
- the median class age is 139 months.

STANDARD AND DECONSTRUCTION

6.SP.5	Summarize numerical data sets in relation to their context.
DESCRIPTION	

Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), the number of observations, and summary statistics. Summary statistics include quantitative measures of center (median and median) and variability (interquartile range and mean absolute deviation) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles.

Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable).

Measures of center given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of an ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it.

The mean is the arithmetic average; the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point.

Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point).

Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean. Students find a missing value in a data set to produce a specific average.

Example 1: Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below:

Project 1: 18

Project 2: 15

Project 3: 16

Project 4: ??

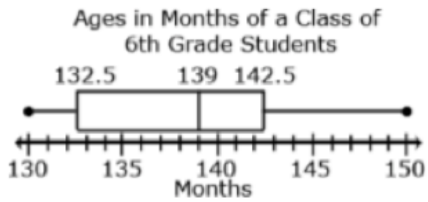
What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning.

Solution: One possible solution is to calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 (68 – 49 = 19).

Measures of Variability Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers. Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference from reading a box plot.

DESCRIPTION
(continued)

Example 1: What is the IQR of the data below? :



Solution: The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations. Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

Example 2: The following data set represents the size of 9 families: 3, 2, 4, 2, 9, 8, 2, 11, 4. What is the MAD for this data set?

Solution: The mean is 5. The MAD is the average variability of the data set.

To find the MAD:

1. Find the deviation from the mean.
2. Find the absolute deviation for each of the values from step 1.
3. Find the average of these absolute deviations.

The table below shows these calculations:

Data Value	Deviation from Mean	Absolute Deviation
3	-2	2
2	-3	3
4	-1	1
2	-3	3
9	4	4
8	3	3
2	-3	3
11	6	6
4	-1	1
	MAD	26/9 = 2.89

DESCRIPTION (continued)	
	This value indicates that an average family size varies 2.89 from the mean of 5. Students understand how the measures of center and measures of variability are represented by graphical displays. Students describe the context of the data, using the shape of the data, and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

SUBSTANDARD DECONSTRUCTION	6.SP.5a: Reporting the number of operations.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Learning Expectations:	Know: Concepts/Skills	Think	Do
Students should be able to:	Report the number of observations in a data set or display.		

SUBSTANDARD DECONSTRUCTION	6.SP.5b: Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	Know: Concepts/Skills	Think	Do
Students should be able to:	Describe the data being collected, including how it was measured and its units of measurement.		

SUBSTANDARD DECONSTRUCTION	6.SP.5.c: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.		
DOK Range Target for Instruction & Assessment	☐ 1 ☐ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:	Calculate quantitative measures of center. Calculate quantitative measures of variance. Identify outliers.		

SUBSTANDARD DECONSTRUCTION	6.SP.5d: Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Learning Expectations:	**Know: Concepts/Skills**	**Think**	**Do**
Students should be able to:		Determine the effect of outliers on quantitative measures of a set of data. Choose the appropriate measure of central tendency to represent the data. Analyze the shape of the data distribution and the context in which the data were gathered to choose the appropriate measures of central tendency and variability and justify why this measure is appropriate in terms of the context.	

EXPLANATIONS AND EXAMPLES (continued)

Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics include quantitative measures of center, spread, and variability including extreme values (minimum and maximum), mean, median, mode, range, quartiles, interquartile ranges, and mean absolute deviation.

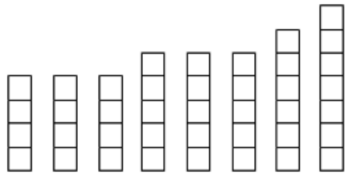
The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

Understanding the Mean

The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean.

For example, students could generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their feet to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points which are whole numbers between 1 and 10 that are easy to model with counters or stacking cubes.

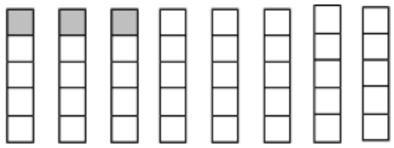
Students generate a data set by drawing eight student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen, there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data set could be represented with stacking cubes.



EXPLANATIONS AND EXAMPLES (continued)

Students can model the mean by "leveling" the stacks or distributing the blocks so the stacks are "fair". Students are seeking to answer the question "If all of the students had the same number of letters in their name, how many letters would each person have?"

One block from the stack of six and two blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have five blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5.

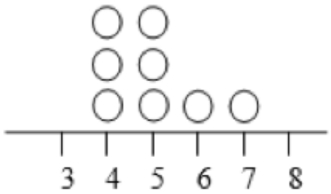


If it was not possible to make the stacks exactly even, students could begin to consider what part of the extra blocks each stack would have.

Understanding Mean Absolute Deviation

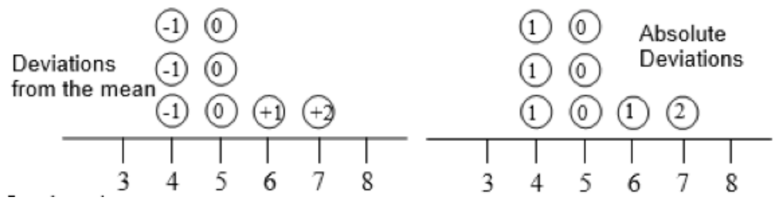
The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and that the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets.

In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.



EXPLANATIONS AND EXAMPLES
(continued)

To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.



Continued on the next page.

Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
John	4	-1	1
Luis	4	-1	1
Mike	4	-1	1
Carol	5	0	0
Maria	5	0	0
Karen	5	0	0
Sierra	6	+1	1
Monique	7	+2	2
Total	40	0	6

The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be 6 ÷ 8 or ¾ or 0.75. The mean absolute deviation is a small number, indicating that there is little variability in the data set.

Consider a different data set also containing 8 names: Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita. Summarize the data set and its variability. How does this compare to the first data set?

The mean of this data set is still 5. $\frac{(3 + 3 + 3 + 7 + 7 + 7)}{8} = \frac{40}{8} = 5$

EXPLANATIONS AND EXAMPLES
(continued)

Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
Sue	3	-2	2
Joe	3	-2	2
Jim	3	-2	2
Amy	3	-2	2
Sabrina	7	+2	2
Timothy	7	+2	2
Adelita	7	+2	2
Monique	7	+2	2
Total	40	0	16

The mean deviation of this data set is 16 ÷ 8 or 2. Although the mean is the same, there is much more variability in this data set.

Understanding Medians and Quartiles_

Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles (Q3 – Q1). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed.

Consider the first data set again. Recall that the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. The data set can be represented in a numerical list. To find the median and quartile, the values are placed in order from least to greatest.

5 4 5 4 7 6 4 5 ⟶ 4 4 4 5 5 5 6 7

The middle value in the ordered data set is the median. If there is an even number of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4th and 5th values which are both 5. Students find quartile 1 (Q1) by examining the lower half of the data. Again there are 4 values which is an even number of values. Q1 would be the average of the 2nd and 3rd value in the data set or 4. Students find quartile 3 (Q3) by examining the upper half of the data. Q3 would be the average of the 6th and 7th value in the data set or 5.5. The mean of the data set was 5 and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 (5.5 – 4). The interquartile range is small, showing little variability in the data.

4 4 4 5 5 5 6 7

Q1 = 4 | Q3 = 5.5

Median = 5

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