HiSet Prep - Name



Don't move to next pages until you get okayed by teacher stamp

Page Number	Description - Red Book - STECK VAUGHN	Teacher Stamp
Pages 82-85	Working with Ratios, unit rates and proportions Proportional Reasoning - setting up and solving proportions	
Pages 138-145	Percentage and Proportion Problems Proportional Reasoning - setting up and solving proportions	
Pages 215-218	Variables and Algebraic Expressions - SUBSTITUTION - REVERSE GUESS AND CHECK	
Pages 220-226	Algebra and Variables SUBSTITUTION - REVERSE GUESS AND CHECK	
Pages 238-239	Word Problems - Algebra and Variables SUBSTITUTION - REVERSE GUESS AND CHECK	
Pages 248-249	Inequalities.	
Pages 254-265	The Cartesian Grid - Slope intercept and graphing lines.	
Pages 280-285	Geometry and Volume problems. Isolating the variable - SUBSTITUTION - REVERSE GUESS AND CHECK	
Pages 286-293	Angles and their relationships	2000
Pages 312-313 320-323	Pythagorean Relationships and Triangles	
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Working with Ratios

A **ratio** is a comparison of two numbers. A ratio can be written with the word *to*, with a colon (:), or as a fraction. Always write the terms of the ratio in the same order as the problem compares them.

Example 1 A painter mixes 4 quarts of white paint and 2 quarts of blue paint. What is the ratio of white paint to blue paint?

The ratio of white paint to blue paint can be written as 4 to 2, 4:2, or $\frac{4}{2}$.

Just as you can reduce fractions to lowest terms, you can simplify ratios to their lowest terms. The ratio $\frac{4}{2}$ can be simplified to $\frac{2}{1}$, which means for every 2 quarts of white paint, there is 1 quart of blue paint. Ratios are written as a fraction even if the denominator is 1.

Ratios have much in common with fractions, but there is an important difference. A ratio may look like an improper fraction, but it should not be changed to a whole or mixed number because it is a comparison of two items, not just a part of a whole.

Ratios often express rates. A ratio with the denominator 1 is called a **unit rate**. Unit rates are often expressed using the word *per*.

Example 2 If Barbara earns \$180 in 15 hours, how much does she earn per hour?

Write the ratio of earnings to hours. Then divide to find the unit rate.

$$\frac{\text{dollars earned}}{\text{hours}} = \frac{\$180}{15} = \frac{180 \div 15}{15 \div 15} = \frac{\$12}{1 \text{ hr}}$$

Barbara earns \$12 for every 1 hour she works. In other words, she earns **\$12 per hour.**

Some ratio problems require more than one step. You may not be given both of the numbers you need to write a ratio. Instead, you may have to solve for one of the numbers.

Example 3 Kimball Discount has 25 employees. Of those 25 employees, 15 are women. What is the ratio of the number of men to the number of women employees?

You need to write a ratio comparing the number of men to the number of women. You have been given the total number of employees and the number of women employees.

Step 1 Subtract to find the number of men. 25 - 15 = 10 men

Step 2 Write the ratio of men to women. $\frac{\text{men}}{\text{women}} = \frac{10}{15}$

Step 3 Simplify the ratio. $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

The ratio of men to women employees at Kimball Discount is $\frac{2}{3}$.



Writing the ratio in words will help you keep the numbers in the correct order. The words will also help you remember the meaning of the numbers. Including labels in your final ratio is also helpful.

A. Write each ratio as a fraction in lowest terms.

- 1. 18 wins to 6 losses
- 2. 80 full-time workers to 100 part-time workers
- 3. 16 fiction books to 12 non-fiction books
- 4. \$21 to \$9
- 5. 69 female employees to 90 total employees

B. Find each unit rate.

- 11. 400 miles in 5 hours
- 12. \$216 in 18 hours
- 13. 54 calories in 6 grams of fat
- 14. 432 people for 36 teams
- 15. \$10 for 10 pounds of grass seed

. Solve as directed.

- 21. The Monarchs won 30 games and lost 6 games.
 - a. What is the ratio of games won to games lost?
 - b. What is the ratio of games lost to games played?
 - c. What is the ratio of games won to games played?
 - d. What is the ratio of games lost to games won?
- 22. In a factory, there are 35 union workers and 14 non-union workers.
 - a. What is the ratio of non-union to union workers?
 - b. What is the ratio of union workers to the total number of workers?
 - c. What is the ratio of the number of non-union workers to the total number of workers?
 - d. What is the ratio of union workers to non-union workers?

- 6. 16 minutes to 30 minutes
- 7. 85 miles to 100 miles
- 8. 35 customers to 7 sales
- 9. 10 adults to 120 children
- 10. 15 wins to 20 games played
- 16. 5400 oranges in 60 bags
- 17. 1024 feet in 16 seconds
- 18. \$150 for 25 yards of fabric
- 19. 135 pages in 3 hours
- 20. 460 calories in 2 servings
- 23. On a quiz, Denzel answered 16 questions correctly and 4 questions incorrectly.
 - a. What is the ratio of correct to incorrect answers?
 - b. What is the ratio of correct answers to total questions?
 - c. What is the ratio of incorrect answers to total questions?
- 24. A manufacturer produced 1000 electric switches. Of these, 50 were found to have defects.
 - a. What is the ratio of defective switches to total switches?
 - b. What is the ratio of defective to good switches?
 - c. What is the ratio of good switches to total switches?

Answers start on page 381.

Solving Proportions

In a proportion,

ratios must be written in the same order. In

this example,

and hours on

the bottom. Use labels to

keep track of

the order.

both ratios have

miles on the top

the terms in both

When two ratios are written as equal ratios, the equation is called a **proportion**. Think about the following statement:

If Paul earns \$8 in 1 hour, then he will earn \$56 in 7 hours. Example 1

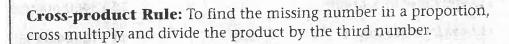
From the information in the sentence, you can write a proportion. Use cross products to make sure the ratios are equal.

$$\frac{8}{1}$$
 $\frac{36}{7}$

$$\frac{8}{1} \xrightarrow{56} \text{ Cross products: } 1 \times 56 = 8 \times 7$$

$$56 = 56$$

As you can see from the example above, every proportion has four terms. In a proportion problem, one of the four terms is missing. The proportion can be solved using this rule:



Example 2 Gayla drove 165 miles in 3 hours. At the same rate, how far can she drive in 5 hours?

In this problem, you are comparing miles to hours. Set up two equal ratios. Write x to stand for the missing term.

$$\frac{\text{miles}}{\text{hours}} = \frac{165}{3} = \frac{x}{5}$$

Find the cross product. Step 1

$$165 \times 5 = 825$$

Divide by 3, the remaining term. Step 2

$$825 \div 3 = 275$$

Gayla can drive 275 miles in 5 hours.



You can easily solve proportion problems using a calculator. Enter the numbers and operations in this order:

$$165 \times 5 \div 3 = 275.$$

Some proportion problems state a ratio using a colon. Read carefully to understand what the numbers in the ratio represent.

At a school board meeting, the ratio of parents to teachers Example 3 is 3:2. If there are 72 parents at the meeting, how many teachers are there?

The ratio 3:2 compares parents to teachers. $\frac{\text{parents}}{\text{teachers}} = \frac{3}{2} = \frac{72}{x}$ Step 1 Write the second ratio in the same order.

$$\frac{\text{parents}}{\text{teachers}} = \frac{3}{2} = \frac{72}{x}$$

Find the cross product, and divide by the Step 2 remaining term.

$$2 \times 72 = 144$$

 $144 \div 3 = 49$

There are **48 teachers** at the meeting.

A. Solve for the missing term in each proportion.

1.
$$\frac{2}{3} = \frac{x}{15}$$

$$6. \ \frac{15}{24} = \frac{5}{x}$$

11.
$$\frac{49}{7} = \frac{x}{10}$$

$$2. \ \frac{28}{12} = \frac{14}{x}$$

7.
$$\frac{12}{15} = \frac{24}{x}$$

12.
$$\frac{32}{8} = \frac{x}{15}$$

3.
$$\frac{9}{10} = \frac{x}{20}$$

8.
$$\frac{14}{6} = \frac{7}{x}$$

13.
$$\frac{18}{6} = \frac{3}{x}$$

4.
$$\frac{5}{6} = \frac{x}{18}$$

9.
$$\frac{115}{30} = \frac{x}{6}$$

14.
$$\frac{6}{120} = \frac{5}{x}$$

5.
$$\frac{3}{4} = \frac{9}{x}$$

10.
$$\frac{5}{20} = \frac{8}{x}$$

15.
$$\frac{64}{8} = \frac{x}{5}$$

B. For each situation, the first ratio has been written for you. Write the second ratio to complete the proportion and solve.

16. A recipe that serves 8 people calls for 2 cups of milk. How many cups of milk will be needed for 36 servings?

$$\frac{\text{servings}}{\text{cups of milk}} \quad \frac{8}{2} = \frac{?}{?}$$

18. A person uses about 315 calories to jog 3 miles. How many calories will be used in a 10-mile jog?

$$\frac{\text{calories}}{\text{miles}} \quad \frac{315}{3} = \frac{?}{?}$$

17. Sandra can drive 32 miles on 2 gallons of gasoline. How far can she drive on a full tank of 13 gallons?

$$\frac{\text{miles}}{\text{gallons}} \quad \frac{32}{2} = \frac{?}{?}$$

19. An architect is planning a city parking lot. For every 12 commuters, the parking lot will need 5 parking spaces. How many parking spaces will be needed for 132 commuters?

$$\frac{\text{commuters}}{\text{parking spaces}} \quad \frac{12}{5} = \frac{?}{?}$$

C. Write a proportion and solve for the missing term.

20. The scale on a map says that 2 inches equal 150 miles. If two cities are actually 750 miles apart, how many inches apart will they be on the map?

22. Ky bought 2 gallons of paint for \$27. How much would he spend for 10 gallons?

21. The ratio of full-time to part-time employees at Kelly Manufacturing is 5:3. If there are 48 part-time employees at the plant, how many full-time workers are there?

23. The ratio of wins to losses for the Bulldogs was 7:2. If the team won 21 games, how many did they lose?

GED PRACTICE

Directions: Choose the one best answer to each question. Use your calculator when indicated.



- The sweater shown above is on sale for 20% off. If the price tag lists the original price, how much would you save by buying it on sale?
 - (1) \$ 7
 - (2) \$ 9
 - (3) \$12
 - (4) \$14
 - (5) \$15



- 2. Sidney's insurance paid 90% of the cost of getting his car fixed. If the repair bill was \$625, how much did the insurance pay?
 - (1) \$437.50
 - (2) \$468.75
 - (3) \$500.00
 - (4) \$562.50
 - (5) \$605.15



- 3. Aldora earns \$1344 per month. If 2.5% of her earnings goes to state income tax, how much does she pay per month in state income tax?
 - (1) \$ 20.16
 - (2) \$ 26.88
 - (3) \$ 33.60
 - (4) \$ 42.20
 - (5) \$336.00

- 4. On a test, a student got 80% of the items correct. If the student got 56 items correct, how many items were on the test?
 - (1) 64
 - (2) 70
 - (3)72
 - (4) 84
 - (5) 90
- 5. The Bulldogs won 18 games out of 45. What percent of their games did the Bulldogs win?
 - (1) 40%
 - (2) 45%
 - (3) 50%
 - (4) 55%
 - (5) 60%



- 6. Eighty percent of the Usagi Express Company's employees are drivers. If there are 300 drivers in the company, how many employees work for Usagi Express?
 - (1) 320
 - (2) 335
 - (3) 342
 - (4) 365.
 - (5) 375



To solve a percent problem set up a proportion:

 $\frac{Part}{Base} = \frac{Rate\%}{100\%}$

Determine which element is missing: the base, part, or rate; substitute the numbers into the proportion and solve.

Answers start on page 397.

- 4. Solve using pencil and paper.
 - 1. Find 3% of 500.
 - 2. What is 15% of \$950?
 - 3. Find 90% of 72.
 - 4. Find 85% of 140.
 - 5. Find 125% of \$220. (*Hint:* Since 125% is greater than 100%, the part is greater than \$220.)
 - 6. What is 150% of 184?

- 7. What is 75% of 80?
- 8. Find 5% of \$200.
- 9. Find 8% of \$1600.
- 10. What is 55% of \$20?
- 11. Find $5\frac{1}{2}\%$ of 300.
- 12. What is $33\frac{1}{3}\%$ of \$600?



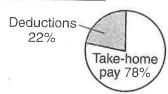
- B. Use your calculator to solve these problems. Round answers to the nearest hundredth or cent.
- 13. Find 6% of \$84.50.
- 14. Find 4% of 1278.
- 15. What is 210% of \$158.75?
- 16. What is 45% of \$10.80?

- 17. Find $8\frac{3}{4}\%$ of \$575.00.
- 18. What is $1\frac{1}{2}\%$ of \$50.00?
- 19. What is 7% of 49.5?
- 20. Find 135% of \$17.50.

C. Solve.

21. During the winter, Green Art, a landscaping service, contacted 1200 homeowners. Of these, 15% hired the company to do work at their homes. How many homeowners hired Green Art?

Use the following graph to answer Questions 22 and 23.



- 22. Angelo earns \$2080 per month. What amount is deducted from his pay each month?
- 23. Of Angelo's \$2080 monthly earnings, how much is his take-home pay?

- 24. Paul bought a sofa for \$690. He paid 8% sales tax on the purchase. What was the total amount of his purchase? (*Hint:* This problem has two steps. Find the amount of the sales tax, and add it to the price of the sofa.)
- 25. For a party of 8 or more, a restaurant automatically adds a 15% tip to the bill. The bill for a birthday party of 20 diners comes to \$186.35. What is the amount of the tip rounded to the nearest cent?
- 26. Carol's utility bill for April of this year is 150% of her bill for the same month last year. If her April bill last year was \$74.22, what is the amount of her bill for April this year?

Answers start on page 398.

A. Solve using pencil and paper.

- 1. What percent is 123 of 820?
- 2. What percent is \$4.50 of \$6.00?
- 3. \$18.00 is what percent of \$22.50?
- 4. 350 is what percent of 2000?

- 5. The number 240 is what percent of 60?
- 6. What percent of 4000 is 120?
- 7. \$5 is what percent of \$160?
- 8. What percent of \$40.00 is \$72.00?



B. Use your calculator to solve these problems.

- 9. \$3.50 is what percent of \$175.00?
- 10. What percent is 326 of 1304?
- 11. What percent of 5000 is 225?
- 12. \$144 is what percent of \$150?
- 13. What percent of \$110.00 is \$82.50?
- 14. 40 is what percent of 16?

C. Find the percent of increase or decrease.

- 15. Original amount: 1500 New amount: 1725
- 16. Original amount: \$520.00 New amount: \$582.40

- 17. Original amount: 280 New amount: 70
- 18. Original amount: \$1200 New amount: \$1140

D. Solve.

- 19. Rae supervises a loading dock. She needs to have 140 cartons loaded into a truck for delivery. By lunch, 119 cartons have been loaded. What percent of the job is finished?
- 20. Jorge stocked the store shelves with 25 cases of soft drinks. Of these, 10 are cola drinks, 8 are root beer, and 7 are orange soda. Of the total cases of soft drinks, what percent is cola? (*Hint:* There is extra information.)
- 21. Stacey was given a raise. Her new monthly pay is \$1508. Previously, her monthly pay was \$1450. What percent raise did she receive?
- 22. Ahmad took a test with 50 items. He answered 44 of the items correctly. What percent of the test items did he answer correctly?

Refer to the following information to answer Questions 23 and 24.



- 23. Smith's Electronics has the camcorder shown above on sale. To the nearest percent, by what percent did the store discount the camcorder?
- 24. Lucia receives a \$44 commission for each camcorder that she sells. What percent of the sale price is her commission?

Answers start on page 398.

A. Solve using pencil and paper.

- 1. \$54 is 60% of what amount?
- 2. 720 is 9% of what number?
- 3. 6% of what amount is \$1.92?
- 4. 17% of what number is 85?
- 5. \$495 is 90% of what amount?
- 6. 270% of what number is 810?

- 7. 115% of what amount is \$207?
- 8. \$1020 is 85% of what amount?
- 9. \$0.21 is 7% of what amount?
- 10. 125% of what number is 660?
- 11. \$6.30 is 15% of what amount?
- 12. 6000 is $66\frac{2}{3}\%$ of what number?



B. Use your calculator to solve these problems.

- 13. \$3.75 is 75% of what amount?
- 14. 15% of what number is 9.6?
- 15. 35% of what amount is \$157.50?
- 16. \$26.88 is 24% of what amount?
- C. Solve.
 - 21. Jeff's insurance paid \$7500 for his surgery. If the insurance paid 80% of the total bill, what was the amount of the total bill?
 - 22. Last year Florence paid 15% of her annual income in federal income tax. If her tax bill was \$3555, what was her annual income?

Refer to the following information to answer Question 23.

89% of CAA Members Vote to Elect Elena Richards

- 23. If 178 members voted for Elena, how many did <u>not</u> vote for her? (*Hint:* You need to find the base, but the base is not the final answer.)
- 24. Lori knew that 25% of the store's customers in May made a purchase. If 207 customers made a purchase, how many visited the store?

- 17. \$62.40 is 104% of what amount?
- 18. 3.8% of what amount is \$0.76?
- 19. \$679.35 is $5\frac{1}{4}$ % of what amount?
- 20. \$1.26 is $3\frac{1}{2}$ % of what amount?
 - 25. Carmen bought a new jacket for \$98. The sale price was 70% of the original price. What was the original price?

Refer to the following table to answer Questions 26 through 28.

	tom Furniture Cat or New Items	alog Sales
Product	Number of Returns	Percent of Total Sales
Folding Step Chair	8	4%
Art Glass Table	3	20%
Rolling Table	15	12 1/2%

- 26. How many art glass tables were sold?
- 27. How many of the rolling tables that were sold were <u>not</u> returned to the furniture store?
- 28. What percent of the folding step chairs were not returned?

Answers start on page 399.

- A. Translate the following into algebraic expressions. Use the variable x to represent the unknown number in each expression unless otherwise noted.
 - 1. the difference between a number and 2
 - 2. twice a number, increased by 4
 - 3. 9 less than three times a number
 - 4. five times the sum of a number and -3
 - 5. the product of a number and 11
 - 6. 10 less than the product of 4 and a number
 - 7. the quotient of a number and 3
 - 8. 5 minus twice a number
- B. Evaluate these expressions as directed.
 - 16. What is the value of 3(x 6) + 2y
 - a. when x = -7 and y = 10?
 - b. when x = 5 and y = -2?
 - c. when x = 0 and y = 6?
 - d. when x = 3 and y = 3?
 - 17. What is the value of $x^2 y^2$
 - a. when x = 0 and y = 2?
 - b. when x = -2 and y = 1?
 - c. when x = 5 and y = -5?
 - d. when x = -1 and y = -2?
 - 18. What is the value of $\frac{(x+5)^2}{x-5}$
 - a. when x = 0?
 - b. when x = 1?
 - c. when x = 3?
 - d. when x = 4?
 - 19. What is the value of $8x + \frac{-2y}{-1}$
 - a. when x = 4 and y = 2?
 - b. when x = 9 and y = -4?
 - c. when x = -1 and y = 0?
 - d. when x = 5 and y = -5?

- 9. twice a number increased by the product of 3 and 8
- 10. the quotient of 3 minus a number and 6
- 11. 8 minus the sum of 15 and a number
- 12. 5 divided by the product of x and y
- 13. the product of 3 and x divided by the sum of x and y
- 14. x multiplied by itself, then increased by the product of 12 and y
- 15. the product of 2 and the difference of x and y'
- 20. What is the value of $\frac{(6+x)^2}{y}$
 - a. when x = 4 and y = '-1?
 - b. when x = 0 and y = 6?
 - c. when x = 0 and y = -6?
 - d. when x = 2 and y = 2?
- 21. What is the value of $x^2 + 2x 6$
 - a. when x = -3?
 - b. when x = 2?
 - c. when x = 4?
 - d. when x = 8?
- 22. What is the value of $-3y(y^2 + 2)$
 - a. when y = 2?
 - b. when y = 10?
 - c. when y = -9?
 - d. when y = 0?
- 23. What is the value of $\frac{2(x^2 + y)}{z}$
 - a. when x = 4, y = -2, and z = 7?
 - b. when x = 3, y = 0, and z = 6?
 - c. when x = -1, y = -9, and z = -1?

Simplifying Expressions



A number next to a letter means multiplication; all these mean "2 multiplied by y": 2y 2 × y 2 • y 2(y)

It may be helpful to rewrite an

expression

grouping like

terms next to

Remember: keep the sign that precedes each

term with that

addition sign.

term and add an

each other.

Simplifying an expression means combining like terms. A **term** is a number or the combination of a number and one or more variables or a variable raised to a power. **The factors** of a number are the values that, when multiplied together, result in that number.

Examples 5 x 2x xy $4x^2$ Factors 5 and 1 1 and x 2 and x x and y 4 and x^2

In an algebraic expression, a positive of negative sign is part of the term that follows it: the term "owns" the sign that comes before it. An addition sign is understood in front of the negative sign.

Example The expression $3x^2 - 7x + 14$ has three terms.

This expression can also be written as $3x^2 + (-7x) + (+14)$.

Like terms have the same variable or variables raised to the same power. Study these examples to identify like terms.

4x and 9x are like terms. Both terms contain x.

7xy and 8xy are like terms. Both terms contain xy.

4 and 6y are not like terms. An integer and y are different.

3x and 3y are not like terms. Variables x and y are different.

5y² and 6y are not like terms. The powers are different.

We combine like terms in an expression so that there is only one term containing that variable. Simplified expressions are easier to evaluate.

Example 1 Simplify 4x + 6y - 3x - 4y.

Step 1 Group the like terms. Group the x terms and group the y terms. (The sign travels with the term.)

Step 2 Combine like terms. 4x + 6y - 3x - 4y = 4x + 6y + -3x + -4y = 4x + 6y + -3x + (6y + -4y) = 4x + 6y + (6y + -4

In simplified form, 4x + 6y - 3x - 4y is equal to x + 2y.

The order of operations says to perform operations in parentheses first. However, in algebraic expressions parentheses often contain unlike terms that cannot be combined. To simplify an expression that contains parentheses, use the **distributive property** to remove the parentheses.

Distributive To multiply a factor by a sum of terms, multiply the factor by each term in parentheses. Then combine the products. 5(x + y) = 5x + 5y

Example 2 Simplify 2x(3x - 6) + 5x.

Step 1To remove parentheses multiply
each term in the parentheses by
the factor.2x(3x + -6) + 5x
2x(3x) + 2x(-6) + 5xStep 2Combine like terms. $6x^2 - 12x + 5x$

2x(3x - 6) + 5x is equal to $6x^2 - 7x$.

 $6x^2 - 7x$

A. Simplify each expression.

1.
$$7x - 8y + 9x$$

2.
$$5y^2 - 4y - 2y^2$$

$$3.4m - 9n - 3 + 6n$$

$$4. -5x + 16 - 8x - 14 + 10x$$

$$5, 9x - 6 + 8x^{2} + 13$$

$$6.25 - 3n + 16n$$

7.
$$12(x + 3y)$$

8.
$$5x(-y + 9)$$

9.
$$4(2x + y) - 3(x - 5)$$

10.
$$15 + 6(x - 4) + 8x$$

11.
$$3m + 2(m - n) - 5(m + n)$$

12.
$$x - 2(xy - y) + 4xy - x(3 + y)$$

B. Simplify. Then evaluate each expression as directed.

13. Find the value of
$$3x + 5(x + 9) - 4x$$
, when $x = -5$.

14. Find the value of
$$2m - 3(m + 5) - 15$$
, when $m = 10$.

15. Find the value of
$$xy + 4x(1 - y) + 2x$$
, when $x = -1$ and $y = 5$.

16. Find the value of
$$3y(2xz + 2) - 6xyz$$
, when $x = -4$, $y = -3$, and $z = 7$.

17. Find the value of
$$4(2x - y) - 3x + 2y$$
, when $x = 0$ and $y = -2$.

18. Find the value of
$$9a - 8b(2 + a) + 16b$$
, when $a = -4$ and $b = -1$.

C. Choose the best answer to each question.

19. The expression 4(x + 2y) - (x + y) is equal to which of the following expressions?

(1)
$$3x + y$$

(2)
$$3x + 3y$$

$$(3) 3x + 5y$$

$$(4) 3x + 7y$$

$$(5) 3x + 9y$$

20. The expression $8n - 2(n^2 + n) + 12$ is equal to which of the following expressions?

$$(1) 2n + 12$$

$$(2) 4n + 12$$

(3)
$$2n^2 + 10n + 12$$

$$(4) -2n^2 + 6n + 12$$

$$(5) -2n^2 + 6n + 24$$

21. Which of the following expressions is equal to the expression
$$-m(2m + 2n) + 3mn + 2m^2$$
?

$$(3) -4m^2 + mn$$

$$(4) 4m^2 + mn$$

$$(5) 4m^2 + 5mn$$

22. The expression 3(-4b) - 2(a - b - c) is equal to which of the following expressions?

$$(1) -2a - 10b - 2c$$

$$(2) -2a - 10b + 2c$$

$$(3) -2a -5b + 2c$$

$$(4) -2a - 4b - 2c$$

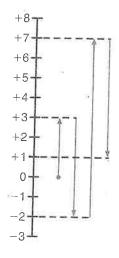
$$(5) 2a - 4b - 2c$$

GED STRATEGY Solving Word Problems

Using Number Lines



A number line can be vertical or horizontal. A vertical number line can help you show situations like temperature, the height of a building, or land above and below sea level.



As you have seen, number lines are useful for showing addition and subtraction operations. On the GED Mathematics Test, drawing a number line can help you solve problems that describe several increases and decreases.

- Example 1 An office building has 3 floors below and 8 floors above ground level. After entering the building on the ground level, a delivery person takes the elevator to four locations, where she makes deliveries at each stop. She travels 3 floors up, 5 floors down, 9 floors up, and 6 floors down. Where does she make the final delivery?
 - (1) 2 floors below ground level
 - (2) 1 floor below ground level
 - (3) ground level
 - (4) 1 floor above ground level
 - (5) 2 floors above ground level

You may have realized that ground level can be represented by the integer 0. The floors above ground level can be assigned positive numbers, and the floors below ground level can be represented by negative numbers.

One way to solve the problem is to draw a vertical number line that shows the integers from -3 to +8. Then start at 0 (ground level) and count out each change described in the problem. Count up 3, down 5, up 9, and down 6.

You finish counting at +1, so the correct answer is **option** (4) 1 floor above ground level.

Note: You can also solve this problem by writing and evaluating an algebraic expression: +3 - 5 + 9 - 6 =, which can be written as follows: (+3) + (-5) + (+9) + (-6)

To solve:
$$(+3) + (+9) + (-5) + (-6)$$

+12 + -11 = +1

- Example 2 On a number line, a certain number x is 5 units greater than the number found halfway between -5 and -1. What is the value of x?
 - (1) -8
 - (2) -3
 - (3) 0
 - (4) 2
 - (5) 5

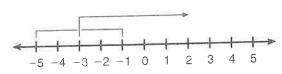


Remember, subtracting a positive number is the same as adding a negative number. For example,

$$-3-5=-3+(-5)$$

$$-4-2=-4+(-2)$$

Since you are not given a diagram, quickly sketch a number line, find the point halfway between -5 and -1, and count 5 spaces to the right.



The correct answer is **option (4) 2**.

GED PRACTICE

Directions: Choose the one best answer for each question.

- In a card game, Rita loses 1 point, gains 5 points, and loses 8 points. Jerry has 6 points. What is the difference in their scores?
 - (1) 4
 - (2) 6
 - (3) 8
 - (4) 10
 - (5) 12
- 2. On a number line, Max places a mark 3 units to the left of the point halfway between 1 and -3. On what point is Max's mark?
 - (1) 2
 - (2) 0
 - (3) -1
 - (4) -3
 - (5) -4
- 3. At 10 A.M., it is 5°F below zero. By 11 A.M., the temperature rises 6°F. If it drops 3°F by 1 P.M., what is the temperature at 1 P.M.?
 - $(1) -4^{\circ}F$
 - (2) -2°F
 - (3) 0°F
 - (4) 2°F
 - (5) 14°F

Question 4 refers to the following information.

Aaron's Dice Rolls

Round	Red Die	Green Die
3	4	6
4	2	1
5	6	4

- 4. In a dice game, each player rolls two dice, one red and one green. The number on the red die is added to the player's score from the previous round. The number on the green die is subtracted from the player's score. If Aaron had +4 points after the first two rounds, how many points does he have after five rounds?
 - (1) -5
 - (2) -3
 - (3) 1
 - (4) 3
 - (5) 5
- 5. On a number line, -2 is located halfway between which of the following points?
 - (1) -3 and 0
 - (2) -4 and 1
 - (3) -4 and -1
 - (4) -4 and 0
 - (5) -5 and 0

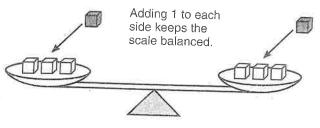
Answers start on page 416.



GED SKILL Equations

Solving One-Step Equations

An **equation** is a mathematical statement that shows that two quantities are equal. When an equation contains a variable, we use algebra to find the value of the variable. To **solve** an equation means to find the number that makes the statement true.



Solve the equation.	2x - 1 = 9
When x equals 5,	2(5) - 1 = 9
the statement is true.	10 - 1 = 9
	9 = 9

To solve an equation, you must keep the two sides of the equation equal. Think of an equation

as a balance scale. Whatever you do to one side of the scale, you must also do to the other side to keep the scale balanced.

The basic strategy in solving an equation is to isolate the variable, that is, get the variable alone on one side of the equation by performing imverse, or opposite, operations to both sides of the equation.

Remember: • Addition and subtraction are inverse operations. • Multiplication and division are inverse operations.

Example 1 Solve: x - 13 = 25.



You may be able to solve simple equations using mental math. However, writing out each step will improve your understanding of algebra and make it easier to check your work.

Step 1	Think about the meaning of the	x - 13 = 25
	equation, the operation, and its inverse:	
	here, the operation is subtraction; the	
	inverse operation is addition.	

Step 2 Perform the inverse operation to
$$x - 13 + 13 = 25 + 13$$

both sides of the equation to isolate $x = 38$
the variable and keep the equation
balanced. Here, add 13 to both sides.

Step 3 Check. Substitute your solution, 38, into the original equation for the variable
$$x$$
. $38 - 13 = 25$ $25 = 25$

The value 38 makes the equation true.

Example 2 Solve: 5x = -35.

Step 1 The operation is multiplication; the inverse of multiplication is division.

Step 2 Divide both sides by 5.

Step 3 Substitute and check.

$$5x = -35$$

$$\frac{5x}{5} = \frac{-35}{5}$$
The value 7 makes the equation true.

$$5x = -35$$

$$\frac{5x}{5} = -35$$

$$-35 = -35$$

Study these examples to see how subtraction and multiplication can be used as inverse operations to solve equations.

Examples

Subtract 40
$$x + 40 = 75$$
 $\frac{x}{9} = 4$ Multiply both from both $x + 40 - 40 = 75 - 40$ 1 sides by 9. sides. $x = 35$ 1 $\frac{x}{9} \times 9 = 4(9)$ $x = 36$

(Note that cancellation applies only to the left side of the equation.)

$$35 + 40 = 75$$

$$\frac{36}{9} = 4$$

GED SKILL FOCUS

A. Solve. Do not use a calculator.

1.
$$x - 15 = 4$$

2.
$$x - 7 = 3$$

$$3. \frac{x}{2} = 12$$

$$4. -6x = -42$$

5.
$$x + 9 = 22$$

$$6. -12x = 60$$

7.
$$x - 8 = -10$$

8.
$$\frac{x}{-3} = 18$$

9.
$$5x = -45$$

10.
$$9 + x = -18$$

$$11. -11x = -132$$

12.
$$7x = 63$$



B. Solve. You may use a calculator for these items.

13.
$$x - 94 = 52$$

14.
$$6.5 + x = 12.25$$

15.
$$0.25x = 12$$

16.
$$200x = 25$$

17.
$$-69 + x = 124$$

$$18. -3.6x = -17.28$$

19.
$$0.38 + x = 2.5$$

20.
$$6x = 3.3$$

C. Translate each question into an algebraic equation and solve.

- 21. -13 added to what number equals 20? (*Hint*: -13 + x = 20)
- 22. What number multiplied by 10 equals 900?
- 23. What number divided by 4 equals 60?
- 24. 5 subtracted from what number equals –14?

- 25. What number divided by 4 is 32?
- 26. What number multiplied by −6 equals 48?
- 27. 52 added to what number equals 100?
- 28. 4 subtracted from what number equals -17?

Answers start on page 417.

Solving Multi-Step Equations

Some equations involve more than one operation. Remember: your goal is to isolate the variable on one side of the equation. Also, when solving multi-step equations, use the reverse order of operations.

Example 1
$$5x - 10 = 35$$

Step 3 Check. Substitute the value for
$$x$$
.

$$5x - 10 + 10 = 35 + 10$$
$$5x = 45$$
$$\frac{5x}{5} = \frac{45}{5}$$

$$x = 9$$

 $5(9) - 10 = 35$
 $45 - 10 = 35$

35 = 35

The solution is x = 9.

Some equations may have variable terms on both sides. If so, you need to group all of the variable terms on one side of the equation.

Example 2
$$12x + 9 = 10x + 1$$

Step 1 Group the variable.
Subtract
$$10x$$
 from both sides.
Step 2 Subtract 9 from both sides.
Step 3 Divide both sides by 2.

$$12x - 10x + 9 = 10x - 10x + 1$$

$$2x + 9 - 9 = 1 - 9$$

$$2x = -8$$

$$\frac{2x}{2} \equiv \frac{-8}{2}$$

$$x = -4$$

The solution is x = -4.

Some equations contain parentheses. Remove the parentheses by multiplying each term within the parentheses by the factor.

Example 3
$$3(x + 1) = -12$$

The solution is x = -5.

Step 1
 Multiply both terms inside the parentheses by 3.

$$3(x + 1) = -12$$

 Step 2
 Subtract 3 from both sides.
 $3x + 3 = -12 - 3$
 $3x = -15$
 $3x = -15$

 Step 3
 Divide both sides by 3.

$$\frac{3x}{3} = \frac{-15}{3}$$
 $x = -5$

$$x = -5$$

 Step 4
 Check.

$$3(-5 + 1) = -12$$
 $(-15 + 3) = -12$



There is often more than one way to solve equations, such as first adding or subtracting the integer and then the variable. But, you must always follow the basic rules for solving equations: always add or subtract first; then multiply or divide.

-12 = -12



The integer for a lone variable is understood to be 1: x = 1x and x = -1x

The first step in solving some equations is to **combine like terms**. Always simplify each side of the equation before solving.

Example 4
$$2x + 5 - 3x = 8 + 2$$

Step 1 Simplify by combining like terms.
$$2x + 5 - 3x = 8 + 2$$

 $2x - 3x = -x$; $8 + 2 = 10$ $-x + 5 = 10$

Step 2 Isolate the variable. Here, subtract 5
$$-x + 5 - 5 = 10 - 5$$
 from each side. $-x = 5$

Step 3 Solve for
$$x$$
. Multiply each side by -1 $-x(-1) = 5(-1)$ so that you solve for x , not $-x$. $x = -5$

Step 4 Check.
$$2(-5) + 5 - 3(-5) = 8 + 2$$
$$-10 + 5 - (-15) = 10$$

$$-10 + 5 + 15 = 10$$

$$10 = 10$$

The solution is -5.

GED SKILL FOCUS

Solve each equation.

1.
$$6x + 7 = 37$$

$$2. 4x + 5x - 10 = 35$$

$$3. 3x - 6x + 2 = -4x$$

$$4. 6 - x + 12 = 10x + 7$$

5.
$$5x + 7 - 4x = 6$$

6.
$$9x + 6x - 12x = -7x + 2x - 12 + 5x$$

7.
$$7x + 3 = 31$$

8.
$$3x - 8 = 28$$

9.
$$8x + 6 = 5x + 9$$

10.
$$11x - 10 = 8x + 5$$

11.
$$-2x - 4 = 4x - 10$$

12.
$$5x + 8 = x - 8$$

13.
$$11x - 12 = 9x + 2$$

14.
$$5(x + 1) = 75$$

15.
$$5(x-7)=5$$

16.
$$6(2 + x) = 5x + 15$$

17.
$$4x + 5 = 21$$

18.
$$2x - 5x + 11 = 38$$

19.
$$3x - 8 = x + 4$$

$$20. 7(x - 2) = 21$$

$$21. 5x - 13x + 2x = -70 + x$$

$$22. 8x + 12 = 44 + 4x$$

23.
$$2(x + 4) = 14 + x$$

24.
$$5x + 3 = 8(x - 3)$$

25.
$$2(x + 2x) - 6 = 30$$

$$26. 11x + 12 = 9x - 32$$

$$27. \ 3(x-9) - 2 = -35$$

28.
$$3(4x + 3) = -9(-x + 2)$$

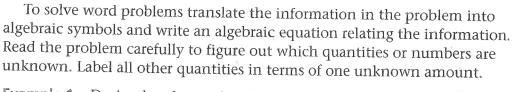
29.
$$x + 11 + 3x = 20 + 7x$$

$$30. \ 4(2x + 5) + 4 = 3(5x - 6)$$

GED STRATEGY Solving Word Problems

Translating Problems into Equations

As a general rule, let x equal the quantity you know the least about; this is usually the amount to which the other amount is compared or related.



- During lunch one day, a cafe sold 8 more turkey sandwiches than ham sandwiches. If there were 32 sandwiches sold in all, how many were ham sandwiches?
- Identify the unknown amount(s); assign the variable. Step 1 There are 2 unknown amounts: the number of ham sandwiches and the number of turkey sandwiches. Pick one to be the unknown x. Here, let x = the number of ham sandwiches.
- Label the other quantities in terms of x. Since there were Step 2 8 more turkey sandwiches than ham, let x + 8 equal the number of turkey sandwiches. If you had let x = the number of turkey sandwiches, then the number of ham sandwiches would be x - 8.
- Write an equation. You know 32 sandwiches in all were sold. Step 3 Thus, the sum of the number of turkey sandwiches (x + 8) and the number of ham sandwiches (x) is 32: (x + 8) + x = 32.

The equation (x + 8) + x = 32 can be used to solve the problem.

Some algebra items on the GED Mathematics Test are set-up problems. Instead of solving the equation, you choose a correct method to solve the problem. To work with these items, analyze the situation and write an equation. Then compare your equation to the answer choices.

Rae's weekly income is \$250 less than twice her husband's.



There is often more than one way to write an equation:

$$x + (x + 8) = 32$$

and $2x + 8 = 32$

are the same as

$$(x + 8) + x = 32.$$

Together they earn \$890 per week. Which equation could be used to find her husband's weekly earnings (x)?

$$(1) (2x + 250) + x = 890$$

$$(2) (2x - 250) + x = 890$$

$$(3) (2x + 250) - x = 890$$

$$(4) (2x - 250) - x = 890$$

$$(5) 2x + 2x - 250 = 890$$

Step 2 Rae's income is \$250 less than twice x, or 2x - 250.

Step 3 The sum of their incomes is \$890.

Step 4 Since none of the choices match, rearrange your equation and compare again.

earnings
Let
$$2x - 250 = \text{Rae's}$$

earnings
 $x + 2x - 250 = 890$
 $x + 2x - 250 = 890$
is the same as

(2x - 250) + x = 8:

Let x = husband's

Option (2) (2x - 250) + x = 890 is correct.

irections: Choose the one best answer to each question.

 Birnham Mills has 360 employees. The number of production employees is twelve more than three times the number of employees who work in management. Which equation could be used to find the number of management employees?

$$(1) 3x + 12 = 360$$

(2)
$$4x = 360$$

$$(3) 3x - 12 + x = 360$$

$$(4) (3x + 12) - x = 360$$

$$(5) x + (3x + 12) = 360$$

2. At a gym Frank did a certain number of pushups. Tom did 12 more than Frank. The total number both men did was 66. Which equation could be used to find the number of pushups Frank did?

(1)
$$x(x + 12) = 66$$

(2)
$$x + 12x = 66$$

(3)
$$2x + 12 = 66$$

$$(4) \ 2x = 66 + 12$$

(5)
$$x + 12 = 66 + x$$

3. Eva got two parking tickets. The fine for the second ticket was \$4 less than twice the fine for the first ticket. If the fines total \$65, which equation could be used to find the amount of the first fine?

(1)
$$3x = 65 - 4$$

(2)
$$2(x-4) = 65$$

(3)
$$x(x-4) = 65$$

$$(4) x + (2x - 4) = 65$$

$$(5) \ 2(2x-4)=65$$

4. Eight times a number, divided by 4, equals two times that number. Which of the following equations could be used to find the number?

(1)
$$\frac{8y}{4} = 2y$$

$$(2) 8\left(\frac{4}{y}\right) = 2y$$

(3)
$$\frac{8}{4y} = 2y$$

(4)
$$8v(4) = 2$$

(5)
$$\frac{8y}{4y} = 2y$$

5. The number of girls signed up for a sports program is 12 fewer than twice the number of boys (x). If 60 children are signed up for the program, which of the following equations could be used to find the number of boys?

(1)
$$2x - 12 = 60$$

(2)
$$2(x + x - 12) = 60$$

(3)
$$x + 2x = 60 - 12$$

$$(4) x + 2(x - 12) = 60$$

(5)
$$3x = 60 + 12$$

6. An adult ticket is twice the cost of a child's ticket. Angela paid \$28 for two adult tickets and three children's tickets. Which of the following equations could be used to find the price of a child's ticket?

(1)
$$x + 2x = 28$$

(2)
$$3x + 2(2x) = 28$$

(3)
$$2(x + 2x) = 28$$

$$(4) \ \ 3(2x) + 2x = 28$$

(5)
$$3x + 2x = 28$$

Solving Algebraic Equations

Algebra word problems describe the relationship among the numbers is a situation. To solve an algebra problem, translate the information into algebraic symbols, write and solve an equation, and check your answer.

Example 1 The total of three **consecutive numbers** is 189. What is the greatest of the three numbers?

Step 1 Identify the unknown amounts; assign *x*. Let *x* represent the smallest number.

Step 2 Label the other quantities in terms of x. If x is the smallest number, the next two numbers are x + 1 and x + 2.

Step 3 Write an equation. The sum of x + (x + 1) + (x + 2) = 189 the three numbers is 189. x + x + 1 + x + 2 = 189

Step 4 Combine like terms and x + x + x + 1 + 2 = 189 solve for x. 3x + 3 = 189

3x = 186x = 62

Step 5 Solve the problem. The value of *x* represents the smallest of three consecutive numbers: the three numbers are 62, 63, and 64.

Step 6 Check your answer. Read the problem again to make sure your answer is reasonable. Since 62 + 63 + 64 = 189, the answer makes sense.

The correct answer is 64.

In some problems, a chart is useful to clarify the information and show relationships between and among the quantities.

Example 2 Ralph is three times as old as his daughter Ella. In ten years, he will be only two times as old as Ella. How old is he now?

There are four unknowns: two present ages and two future ages.

	Ella's Age	Ralph's Age		
Now	A: <i>x</i>	B: 3x		
In 10 years	C: x + 10	D: 3x + 10		

A: Let x equal Ella's age now.

B: Ralph is 3 times as old as Ella, so his age now is 3x.

C: In ten years, Ella will be x + 10 years old.

D: In ten years, Ralph will be 3x + 10 years old.

We know Ralph's age in ten years (3x + 10) 3x + 10 = 2(x + 10) will be 2 times Ella's age in 10 years (x + 10). 3x + 10 = 2x + 20 Then solve for x. 3x - 2x + 10 - 10 = 2x - 2x + 20 - 10 x = 10

Use the chart to find the answer to the problem.

Substitute 10 for the variable x in each box.

3x = 3(10) = 30

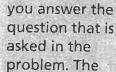
Ella's age now is 10.

x + 10 = 10 + 10 = 203x + 10 = 3(10) + 10 = 40

Ralph's age now is 30. As a check, note that in 10 years, Ella will be 20

As a check, note that in 10 years, Ella will be 20 and Ralph will be 40, twice as old as Ella.

Ralph is now 30 years old.



Make sure that

value of x may

answer to the question.

not be the

ranslate each statement into an algebraic equation. Then solve.

- 1. The sum of a number and twice that number is 15. What is the number?
- 2. The sum of 7 and twice a certain number is 10 more than the number. What is the number?
- 3. One number is 5 less than another. The sum of the two numbers is 181. Find the two numbers.
- 4. When the sum of 8 and a certain number is increased by 12, the result is the same as the product of 3 and the number. What is the number?
- 5. The sum of two consecutive numbers is 49. What is the lesser number?
- 6. George is 5 times as old as his son. In 15 years he will be only twice as old as his son. How old will his son be in 15 years?
- 7. The sum of three consecutive even numbers is 30. What is the greatest of the three even numbers? (*Hint*: Let x represent the first even number. Let x + 2 represent the second even number.)
- 8. Nora is 4 years older than Diana. Two years from now Nora will be twice as old as Diana. How old is Diana now?
- 9. A bank teller had 125 \$10 and \$5 bills to start a day. If the total value of the bills was \$1000, how many \$5 bills did the teller have? [Hint: If x equals the number of \$5 bills, then 5x equals the value of the \$5 bills. The number of \$10 bills is 125 x, and the total value of the \$10 bills is 10(125 x).]

- 10. Twice a number divided by 4 is 16. What is the number?
- 11. The sum of three consecutive odd numbers is 315. What are the numbers? (*Hint*: If x represents the first odd number, the second number is x + 2.)
- 12. One number is two more than three times another. The sum of the numbers is 26. What is the lesser number?
- 13. Armando works part-time at a pet store. This week he earned \$18 less than 4 times the amount he earned last week. His total earnings for the two weeks were \$262. How much did he earn this week?
- 14. There are two consecutive numbers. The sum of the lesser number and three times the greater number is 103. Find the two numbers.
- 15. A shoe store sold 340 pairs of shoes in one day. The number of pairs of athletic shoes sold was 4 more than twice the number of pairs of dress shoes sold. How many pairs of athletic shoes were sold?
- 16. A school sold 200 tickets to a play. Ticket prices were \$8 per adult and \$5 per child. If the total sales for the tickets came to \$1414, how many children's tickets were sold? (*Hint:* If x adult tickets were sold, then 200 x children's tickets were sold.)
- 17. Erika's uncle is three times as old as she is now. Four years ago, he was four times as old as she was then. How old is Erika now?

GED STRATEGY Solving Word Problems

Using Distance and Cost Formulas

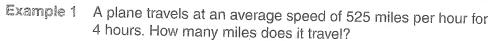
A formula is a special type of equation. A formula relates information to solve a certain kind of problem. When you take the GED Mathematics Test, you will be given a page of formulas to use in solving problems.

Two important formulas are the distance and cost formulas.

Distance distance = rate \times time or d = rt

total cost = (number of units) \times (cost per unit) or c = mrTotal Cost

To use formulas, first choose the formula that shows how the facts in the problem are related. Then substitute the known quantities and solve.



- (1)60.0
- (2)131.25
- (3) 240.0
- (4) 525.0
- (5) 2100.0

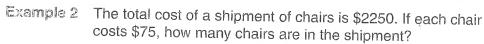
Use the distance formula where d = distance, Step 1 d = rtr = rate (average speed), and t = time.

Step 2 Substitute the known quantities. $d = 525 \times$ Step 3 **Solve** for *d*.

d = 2100

The plane traveled 2100 miles.

You can solve for any variable in a formula if you know the values of the other variables. Substitute the values you know for the variables in the problem. Then use inverse operations to solve for the unknown variable.



- (1)30
- (2)75
- (3)225
- (4)2,250
- (5) 168,750

equation by \$75.

- Use the cost formula, where c = total cost, Step 1 c = nrn = number of units, and r = cost per unit, to solve for n, the number of chairs.
- Substitute the known quantities. Step 2 \$2250 = n(\$75) $\frac{\$2250}{\$75} = \frac{n(\$75)}{\$75}$ Step 3 Solve for n. Divide both sides of the

There are option (1) 30 chairs in the shipment.



The variables in a formula use related units of measure: if the rate is in miles per hour, the distance will be in miles, and the time will be in hours.



You can use a formula to solve for any of its variables. Using c = nr, you can also find $n = \frac{c}{r}$ or $r = \frac{c}{a}$

30 = n

A. Write the square roots. Do not use a calculator.

1.
$$\sqrt{16}$$

4.
$$\sqrt{9}$$

7.
$$\sqrt{25}$$

2.
$$\sqrt{0}$$

5.
$$\sqrt{49}$$

8.
$$\sqrt{1}$$

3.
$$\sqrt{100}$$

6.
$$\sqrt{121}$$

9.
$$\sqrt{144}$$

B. Find the length of the side of each square.

10.
$$x = ?$$

$$A = 36$$
sq cm

12.
$$x = ?$$

$$A = 16$$
sq yd

14.
$$x = ?$$

$$A = 81$$
sq m

11.
$$x = ?$$

$$A = 100$$
sq ft

13.
$$x = ?$$

$$A = 49$$
sq in

15.
$$x = ?$$

$$A = 64$$
sq cm



C. Use a calculator to find the square roots. Round your answers to the nearest hundredth.

16.
$$\sqrt{28}$$

19.
$$\sqrt{6}$$

22.
$$\sqrt{130}$$

17.
$$\sqrt{95}$$

20.
$$\sqrt{324}$$

23.
$$\sqrt{169}$$

18.
$$\sqrt{32}$$

21.
$$\sqrt{44}$$

24.
$$\sqrt{228}$$

- D. Choose the best answer to each question.
 - 25. The square root of 22 is between which of the following pairs of numbers?
 - (1) 2 and 3
 - (2) 3 and 4
 - (3) 4 and 5
 - (4) 5 and 6
 - (5) 21 and 22

- 26. The area of a square platform is about 72 square feet. The length in feet of each side is between which two measurements?
 - (1) 4 and 5
 - (2) 5 and 6
 - (3) 6 and 7
 - (4) 7 and 8
 - (5) 8 and 9

GED STRATEGY Solving Word Problems -

Working Backwards

Each multiple-choice question on the GED Mathematics Test has five answer options. You must choose the best answer for each question.

For most questions, it is faster to solve the problem directly. Read the problem carefully, decide what the question asks, choose the operations to use, and solve. Always make sure your answer makes sense. Then, look at the five options to see whether your answer is among them.

For some algebra problems, however, working backwards from the answer choices may save time. Most algebra problems ask you to solve for a particular variable. Ordinarily, you would write an equation and solve it. However, it may be faster to try each answer choice in the given situation to see which one is true.

Example 1 The sum of three consecutive numbers is 30. What are the numbers?

- (1) 6, 7, and 8
- (2) 8, 9, and 10
- (3) 9, 10, and 11
- (4) 11, 12, and 13
- (5) 14, 15, and 16

It is not necessary to write an equation to solve this problem. You know the numbers add up to 30, so simply add the numbers for each option. You can quickly eliminate options (4) and (5) since 10 + 10 + 10 = 30, and all the numbers in these options are greater than 10. Clearly, options (4) and (5) total more than 30. Quickly add the numbers for the first three options.

Option (1):

$$6 + 7 + 8 = 21$$

Option (2):

$$8 + 9 + 10 = 27$$

Option (3):

$$9 + 10 + 11 = 30$$

Option (3) 9, 10, and 11 is the correct answer.

Example 2 A test has two parts. Each part is worth 50 points. When Jan took the test, she earned 10 points more on the first part than on the second part. Her total score was 86. What did she score on each part of the test?

- (1) 46 and 50
- (2) 38 and 48
- (3) 36 and 46
- (4) 44 and 42
- (5) 32 and 54

You can narrow your choice to options (2) and (3) since these are the only answer choices with numbers 10 points apart. **Option (2) 38 and 48** is the only pair of numbers that is ten points apart and totals 86 points.



Use your knowledge of averages to solve problems asking for consecutive numbers. If the sum of 3 numbers is 30, the average is 10. Look for a choice with a middle value of 10.

GED PRACTICE

Directions: Choose the one best answer to each question.

- 1. The sum of three consecutive numbers is 45. What are the three numbers?
 - (1) 10, 11, and 12
 - (2) 12, 13, and 14
 - (3) 13, 14, and 15
 - (4) 14, 15, and 16
 - (5) 15, 16, and 17
- 2. Jess and David took an 800-mile driving trip. Jess drove 200 miles more than David did on the trip. How many miles did David drive?
 - (1) 200
 - (2) 250
 - (3) 300
 - (4) 400
 - (5) 500
- 3. The sum of two consecutive numbers is 95. What are the two numbers?
 - (1) 40 and 41
 - (2) 42 and 43
 - (3) 47 and 48
 - (4) 52 and 53
 - (5) 57 and 58
- 4. Four consecutive numbers total 38. What are the four numbers?
 - (1) 7, 8, 9, and 10
 - (2) 8, 9, 10, and 11
 - (3) 9, 10, 11, and 12
 - (4) 10, 11, 12, and 13
 - (5) 11, 12, 13, and 14

- 5. Bill works two jobs. Last week, he worked 30 hours in all. If he worked two hours more at one job than he worked at the other, how many hours did he work at each job?
 - (1) 9 and 11
 - (2) 13 and 15
 - (3) 14 and 16
 - (4) 19 and 21
 - (5) 24 and 26
- 6. Marta scored a total of 93 points on her English test. She scored 5 points lower on the writing part of the test than on the reading part. What were her scores on each part of the test?
 - (1) 43 and 48
 - (2) 44 and 49
 - (3) 45 and 50
 - (4) 47 and 52
 - (5) 54 and 59
- 7. Evelyn drove a total of 334 miles on Monday and Tuesday. She drove 50 miles farther on Tuesday than she did on Monday. How many miles did she drive each day?
 - (1) 92 and 142
 - (2) 125 and 175
 - (3) 142 and 192
 - (4) 234 and 284
 - (5) 284 and 334

GED STRATEGY Solving Word Problems

Applying Patterns and Functions



To identify a mathematical pattern, find how one term becomes the one following it. That is, what function is applied to the first term to obtain the next? Then test your function on the rest of the pattern.

y = 3x - 4 y = 3x - 4OUT

A mathematical **pattern** is an arrangement of numbers or terms formed by following a particular rule. If you know the rule, you can find other terms in the pattern.

Example 1 Find the eighth term in the sequence: 1, 7, 13, 19, 25, ...

- **Step 1 Identify the rule used to make the pattern.** Study the sequence. Each number in the sequence is six more than the preceding number. The rule is "add 6."
- Step 2 Apply the rule to continue the pattern. The number 25 is the 5th term in the pattern. You need to find the 8th term.

The correct answer is 43.

An algebraic rule is sometimes called a **function**. You can think of a function as a machine that performs certain operations. For each number that enters the machine (x), there will be only one number that comes out (y). The function shown here multiplies a number by 3 and then subtracts 4. We can write the function as an equation: y = 3x - 4.

Example 2 For the function, y = 3x - 4, what numbers are needed to complete the table below?

х	-3	-2	-1	0	1	2
у	-13	-10	-7		-1	

- (1) -6 and 2
- (2) -5 and 2
- (3) -5 and 3
- (4) -4 and 2
- (5) -4 and 4

Use substitution to find the value of y. Replace x with 0 and 2. Solve for y.

$$y = 3x - 4$$
 $y = 3x - 4$
 $y = 3(0) - 4$ $y = 3(2) - 4$
 $y = 0 - 4$ $y = 6 - 4$
 $y = -4$ $y = 2$

The missing numbers are -4 and 2. Check your work by testing the pattern. Each value for y is 3 more than the number before it. Continue the pattern to make sure -4 and 2 are correct.

Option 1	Option 2	Option 3	Option 4	Option 5
$x^2 - 7x = 60$	$x^2 - 7x = 60$			
$3^2 - 7(3) = 60$	$5^2 - 7(5) = 60$	$6^2 - 7(6) = 60$	$10^2 - 7(10) = 60$	$12^2 - 7(12) = 60$
9 - 21 = 60	25 - 35 = 60	36 - 42 = 60	100 - 70 = 60	144 - 84 = 60
$-12 \neq 60$	$-10 \neq 60$	$-6 \neq 60$	$30 \neq 60$	60 = 60
false	false	false	false	true

Note: The symbol ≠ means "is not equal to."

Option (5) is the correct answer. If you have time, you could substitute -5, the other value from option 5, to check your work.

$$(-5)^2 - 7(-5) = 60$$
 $25 + 35 = 60$ $60 = 60$ true

GED PRACTICE

Directions: Choose the one best answer for each question.

- 1. In the equation $x^2 + 72 = 18x$, what are the possible values for x?
 - (1) -9 and -8
 - (2) -9 and 8
 - (3) -6 and 12
 - (4) 8 and 9
 - (5) 12 and 6
- 2. If $2x^2 10x + 12 = 0$, what is one possible value for x?
 - (1) -4
 - (2) -3
 - (3) 3
 - (4) 6
 - (5) 12
- 3. In the equation $x^2 x = 12$, what are the possible values for x?
 - (1) 6 and -2
 - (2) 4 and -3
 - (3) 3 and -4
 - (4) 2 and -6
 - (5) -3 and -4

- 4. If $x^2 + 13x = -40$, what are the possible values for x?
 - (1) 10 and 4
 - (2) 8 and 5
 - (3) -4 and -10
 - (4) -5 and -8
 - (5) -6 and -7
- 5. In the quadratic equation $9x^2 36 = 0$, which pair of solutions makes the equation true?
 - (1) 9 and -4
 - (2) 6 and -6
 - (3) 4 and -9
 - (4) 3 and -12
 - (5) 2 and -2
- 6. In the equation $2x^2 x = 45$, what is one possible value of x?
 - (1) 9
 - (2) 5
 - (3) 3
 - (4) -5
 - (5) -9

Solving and Graphing Inequalities



To remember the meanings of the symbols > and <, just remember that the symbol points to the lesser amount.

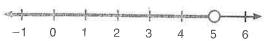
An inequality means two algebraic expressions are not equal. Other inequality symbols in addition to greater than and less than symbols are:

$$\geq$$
 means "is greater than or equal to" $4 \geq 2$
 \leq means "is less than or equal to" $7 \leq 9$

In an inequality, a variable may have many values that make the statement true. Consider x < 5. The numbers 4, 3, 2, 1, and so on are all possible values for x. We can graph the possible solutions to an inequality.

Example 1 Graph the solution set of the inequality x < 5.

On a number line, every number to the left of 5 is a solution. Graph the solution by drawing a solid line on the number line. An empty circle at the



number 5 shows that 5 itself is not included as a solution. Five is not "less than" 5.

Example 2 Graph the solution set of the inequality $x \ge -3$.



The solution set of the inequality $x \ge -3$ includes the number -3 and all numbers to the right of -3. The circle at -3 is filled in to show that -3 is included as a solution.



An inequality can be solved much like an equation. The same number can be added to or subtracted from both sides of an inequality.

Example 3 Solve 2x + 7 < x + 10.

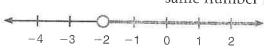
$$x < 3$$

2(2) + 7 < 2 + 10
11 < 12 is true

x + 7 < 10

The solution to the inequality is x < 3.

Both sides of an inequality can also be multiplied or divided by the same number to simplify the inequality. But there is an important rule to remember: If you multiply or divide an inequality by a



negative number, the inequality sign must be reversed.

Example 4 Solve: 3x - 4 < 5x.

$$3x - 5x - 4 < 5x - 5x$$

Step 1 Subtract
$$5x$$
 from both sides.

Step 3 Divide by
$$-2$$
, and reverse the inequality sign.

Step 4 Check using a number greater than
$$-2$$
.

The solution to the inequality is
$$x > -2$$
.

$$3x - 5x - 4 < 5x - 5x$$

$$-2x-4<0$$

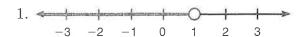
$$-2x - 4 + 4 < 0 + 4$$

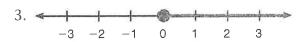
$$\frac{-2x}{-2} < \frac{4}{-2}$$

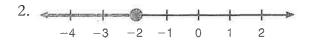
$$x > -2$$

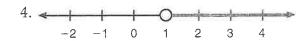
$$3(2) - 4 < 5(2)$$

A. Write the inequality for each number line.



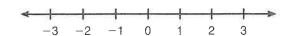




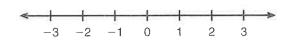


B. Solve and graph the solution set for each inequality.

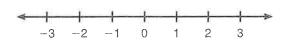
5.
$$2x < 6$$



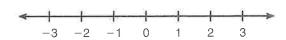
8.
$$8x < 7x$$



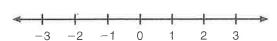
6.
$$x + 1 > 0$$



9.
$$4x - 2 < 3x$$



7.
$$5x \le 3x - 4$$



10.
$$3x - 1 \ge 2$$



C. Solve the following inequalities.

11.
$$3x - 7 < 2x + 1$$

12.
$$5x + 2 > 4x + 1$$

13.
$$6x - 4 \le 3x + 2$$

14.
$$3(x + 1) \ge x + 4x - 5$$

15. 5 + 8(x - 2) < x + 3

16.
$$x + 12 < 5(x + 8)$$

17.
$$2x + (4 - 3x) \le 21$$

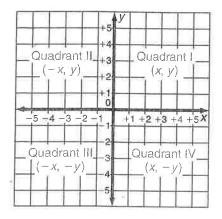
18.
$$7x - 3x - x < 3x + 2x + 10$$

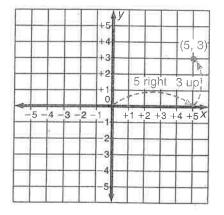
- D. Solve.
 - 19. When five times a number is added to 6, the result is less than 4 times that same number added to 10. What is the solution to the inequality?
- 20. The three sides of a triangle must add up to a number less than or equal to 65 inches. One side is 21 inches. Another side is 18 inches. What is the longest the third side can be (in inches)?

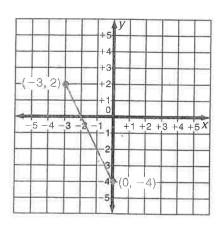
Lesson 22

GED SKILL The Coordinate Plane

Coordinate Graphs







Imagine a blank sheet of paper with one dot on it. How could you describe the exact location of the dot? You might use the edges of the paper to give directions. For example, you could say that the dot was four inches from the top edge and three inches from the left edge. A coordinate graph works the same way.

A **coordinate graph** is a system for finding the location of a point on a flat surface called a **plane**. A coordinate graph is formed from two axis lines that cross at a point called the **origin**. The horizontal line is the **x-axis** and the vertical line is the **y-axis**. Both lines are marked as number lines with the origin at zero. The axes divide the graph into four **quadrants**.

Each point on the grid is named by two numbers, an **x-coordinate** and a **y-coordinate**. The x-coordinate is always written first; the y-coordinate is always written second. Together, the coordinates are called an **ordered pair** and are enclosed in parentheses, separated by a comma.

Example 1 Plot the point (5,3) on a coordinate grid.

- Step 1 Start at the origin—coordinates (0,0). Move the number of units of the x-coordinate in the appropriate direction, here 5 units to the right, the positive x direction.
- Step 2 From that point, move the number of units of the y-coordinate in the appropriate direction, here 3 units upwards, the positive y direction.

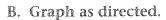
The location of point (5,3) is shown by **the dot** on the grid.

You may be asked to use coordinates to draw a line segment or a figure on a coordinate system.

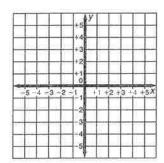
- Example 2 Draw a line segment on the coordinate grid connecting points (0,-4) and (-3,2).
- Step 1 Plot point (0,-4). Start at the origin. The 0 indicates that no move along the *x*-axis is required. Move -4 on the *y*-axis (down from the origin), and plot the point.
- Step 2 Plot point (-3,2). Start at the origin; move 3 units to the left on the *x*-axis, then 2 units up; plot the point.
- **Step 3** Connect the points by drawing a line segment.

The location of the points and segment are shown on the grid.

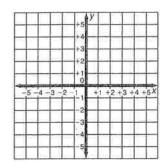
- A. Use the coordinate grid to answer the following questions.
 - 1. What are the coordinates of Point *A*?
 - 2. Which point is found at (-4,-2)?
 - 3. Which point is on the *y*-axis?
 - 4. What are the coordinates of Point *D*?
 - 5. Which point has a negative *x*-value and a positive *y*-value?
 - 6. For which points are the *x*-values greater than the *y*-values?
 - 7. Which points are found in Quadrant IV?



8. Plot the ordered pairs (1,-4), (0,5), and (-2,-3) on the coordinate graph below.

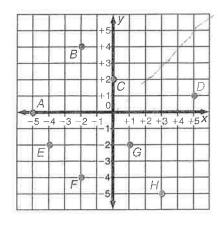


9. Draw a line segment connecting Point A at (4,3) and Point B at (2,-5).

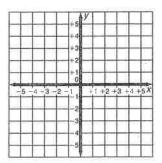




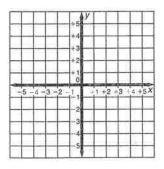
The ordered pairs (5,3) and (3,5) have different locations. Remember, the location on the x-axis (horizontal axis) is always named first. (x,y)



10. Graph the following points and connect them to form a square: Point D at (-3,3), Point E at (1,3), Point F at (1,-1), and Point G at (-3,-1).



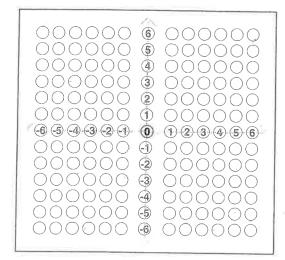
11. Connect each pair of points to form two intersecting line segments. Point J(-5,-4) and Point K(5,2) Point P(-3,4) and Point Q(0,-3)



Answers start on page 426.

GED STRATEGY Working with Special Formats

Plotting Ordered Pairs



Some questions on the GED Mathematics Test ask you to show the location of a point on a coordinate grid. A special coordinate grid will be provided on the answer sheet to record your answers. This grid uses bubbles to represent each ordered pair on the coordinate plane.

The answer grid shown to the left is a sample of this GED answer format. Since positive and negative numbers are shown on the axis lines, there are no bubbles for points that lie directly on either the x- or y-axis. The origin (0,0) is not labeled, but you know that it lies at the intersection of the x- and y-axes.

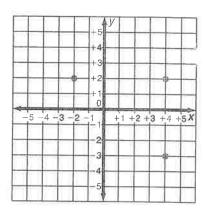
To use the grid to record an answer, carefully bubble in the circle at the correct location. Be careful not to make stray marks on the grid.

Most questions that test your understanding of the coordinate grid will refer to a diagram or graph.

The x- and y-axes on a graph may not be labeled with numbers. Remember that the origin on either scale (the point where the axes cross) is always 0.

Example

Jesse has graphed three points on the coordinate grid at right. The points will become three corners of a rectangle. Where must he place the fourth point to complete the rectangle? Graph your answer on the coordinate grid.



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		00000 (a) 00000 (a) 00000 (a) 00000 (a) 00000 (a) 00000 (a) 00000 (a) 00000 (a)	00000 00000 00000 00000 00000 00000 0000

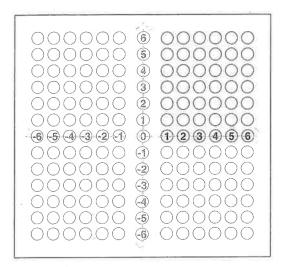
- Complete the rectangle to find the location of the fourth point. You know that a rectangle has four sides. The opposite sides must be the same length. From the three points already shown on the graph, you know the length and the width of the rectangle. The remaining point must be located at point (-2,-3).
- Step 2 Bubble in your answer on the coordinate grid. Starting at the origin, count two units left and three down. Fill in the circle neatly and completely.

The grid to the left shows the correct location of the fourth point.

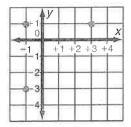
GED PRACTICE

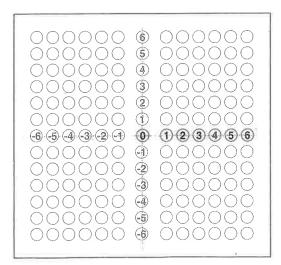
Directions: Grid in the answer to each question on the answer grid provided.

A point has an x-coordinate of 4 and a y-coordinate of -1. Show the location of the point on the coordinate grid.

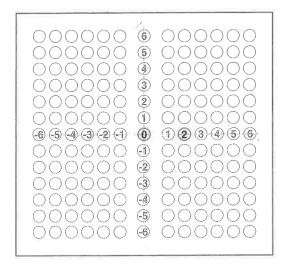


On the coordinate plane below, three points are drawn to mark the corners of a square. Graph the location of the fourth corner needed to complete the square.

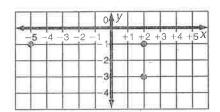




3. A point has an *x*-coordinate of -5 and a *y*-coordinate of 3. Show the location of the point on the coordinate grid.



4. On the coordinate plane below, three points are drawn to mark the corners of a rectangle. Graph the location of the fourth corner needed to complete the figure.



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000000 @	000000

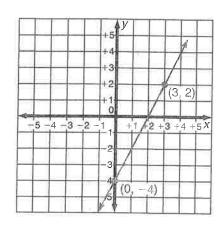
Graphing Equations

You know that some equations have two different variables. For example, the equation y = 2x - 4 has two variables, x and y. For each specific value substituted for x, there is a unique value of y. One way to show the possible solutions for such an equation is to draw the graph of the equation on a coordinate grid.

The equation y = 2x - 4 is called a **linear equation** because its graph forms a straight line. To draw the line, you need to know at least two points on the line.

Example 1 Graph the equation y = 2x - 4.

Step 1



Identify a point on the line.	
Choose any value for x ; 0 is	Let $x = 0$
usually ideal. Substitute <i>x</i> into	y = 2(0) - 4
the equation. Solve for <i>y.</i>	y = 0 - 4 = -4
This ordered pair makes the	(0, -4)
equation true.	,

- Step 2 Find another point on the line. Let x = 3 Choose another value for x and solve for the new y. Write the coordinates. y = 2(3) 4 y = 6 4 = 2 (3,2)
- Step 3 Locate both ordered pairs on a grid and draw a line through them.

The graph of the equation is shown to the left. Every point on the line satisfies, or solves, the equation.



Although you only need two points to draw the graph of a line, it is always a good idea to find a third point to check your work.

Some questions refer to the graph of an equation, but they can be solved without drawing the graph.

Example 2 Which of the following points lies on a graph of the equation x = 3 + y?

- (1) (4,-1)
- (2) (3,1)
- (3) (2,-1)
- (4) (1,2)
- (5) (0,-4)

Instead of drawing the graph, substitute the coordinates from each answer choice for x and y in the equation. The correct option is the ordered pair that makes the equation true.

Only **option** (3) (2,-1) makes the equation true.

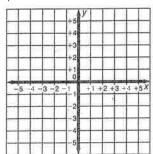
$$2 = 3 + (-1)$$

$$2 = 2$$

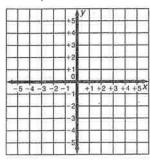
GED SKILL FOCUS

A. Graph each equation on a coordinate grid.

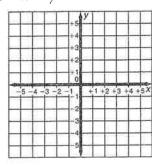
1.
$$y = 3x - 4$$



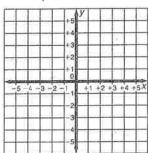
3.
$$x - 2y = 1$$



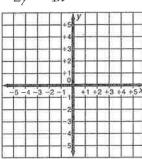
5.
$$-x = y + 2$$



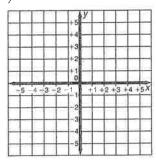
$$2. 2x + y = 5$$



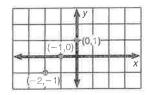
$$4. -2y = 4x$$



6.
$$y = 6 - 3x$$

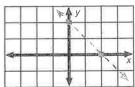


- 3. Choose the best answer to each question.
 - 7. The points graphed on the grid below satisfy which of the following equations?



- (1) x y = 1
- (2) x y = -1
- (3) 2y x = 0
- (4) x = 0
- (5) y = 0
- 8. Which ordered pair is a solution of x y = 1?
 - (1) (-3, -4)
 - (2) (-3,-2)
 - (3) (-1,0)
 - (4) (0,1)
 - (5) (1,-2)

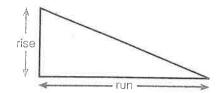
9. Which equation of a line is shown on the graph?



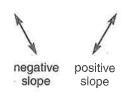
- (1) x + y = -4
- (2) x + y = -2
- (3) x + y = 0
- (4) x + y = 2
- (5) x + y = 4
- 10. What is the missing *x*-value if (?,1) is a solution of -4x + 7y = 15?
 - $(1) \frac{-11}{2}$
 - (2) -2
 - (3) 2
 - $(4) \frac{19}{7}$
 - $(5) \frac{11}{2}$

Answers start on page 427.

Finding the Slope of a Line

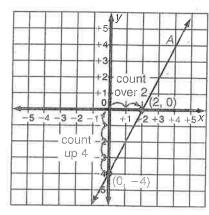


Slope is a number that measures the steepness of a line. In everyday life, we use slope to calculate the steepness of a ramp or flight of stairs. We may determine the slope of a roof or a roadway. Slope is the ratio of *rise to run*, where rise is a measurement of vertical distance and run is a measurement of horizontal distance.



Slope can be positive or negative. All lines that rise as they move from left to right have a positive slope. All lines that fall as they move from left to right have a negative slope. If you have a graph of a line, you can find its slope by examining the line and counting grid units to find the rise and run.

Example 1 Find the slope of Line *A*.



- Step 1 Select two points on the line. Starting at either point, count the number of units up or down to reach the level of the second point. This is the rise of the line. Here, start at (0,-4) and count up 4.
- Step 2 From this intermediate point, count the units left or right to reach the second point. This is the run of the line. Here, count 2 to the right.
- Step 3 Write the slope as the fraction rise $\frac{4}{2}$ over run.
- Step 4 Decide whether the slope is positive or negative. Here, the slope is positive since the line rises as it moves from left to right.

The slope of Line A is +2.

You can also find the slope of a line by using an algebraic formula. It will be listed on the page of formulas you will be given when you take the GED Mathematics Test.

slope (m) of a line =
$$\frac{rise}{rum} = m = \frac{\gamma_2 - \gamma_1}{x_2 - x_1}$$
, where (x_1, y_1) and (x_2, y_2) are two points on the line



- **Step 1** Let one point be (x_1, y_1) and the other be (x_2, y_2) . In this case, let $(-1,2) = (x_1, y_1)$ and $(1,-4) = (x_2, y_2)$.
- Step 2 Substitute into the formula and solve.

$$m = \frac{-4 - 2}{1 - (-1)} = \frac{-6}{2} = -3$$

The slope of the line is -3.

Some unique features of slope to remember:

- The slope of any horizontal line, including the *x*-axis, is 0.
- * A vertical line, including the *y*-axis, has no slope.
- All lines with the same slope are parallel.

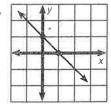


Use the grid-unit counting method whenever you are given a graph of the line. Use the formula method when it is not practical to draw a graph of the line.

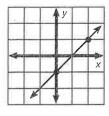
GED SKILL FOCUS

A. Find the slope of each line.

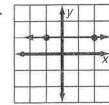




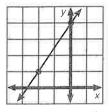
3.



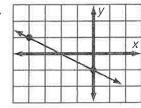
5.



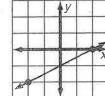
2.



4



6.



B. Find the slope of the line that passes through each pair of points.

7.
$$(1,-3)$$
 and $(0,1)$

9.
$$(4,5)$$
 and $(3,-4)$

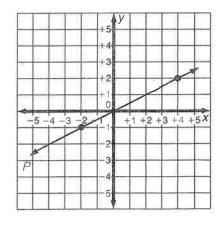
11.
$$(-3,-3)$$
 and $(-2,0)$

10.
$$(-4,2)$$
 and $(5,3)$

12.
$$(-6, -2)$$
 and $(3, 4)$

C. Solve.

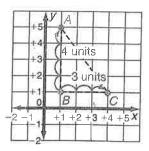
Use the following graph to answer question 13.



13. What is the slope of Line
$$P$$
?

- 14. The following points lie on Line R: (-4,3), (1,3), and (5,3). What is the slope of Line R?
- 15. Line X has a slope of 3. The line passes through Point Y at (-2,-1) and also passes through Point Z, which has an x-coordinate of 0. What are the coordinates of Point Z? (Hint: Substitute the given values in the slope formula.)
- 16. The following points lie on Line K: (-3,-3) and (5,2). What is the slope of Line K?

Finding the Distance Between Points



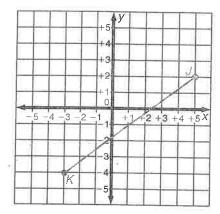
It is sometimes necessary to find the distance between two points on a coordinate grid. You can easily find the distance between points on a vertical or a horizontal line by counting. For example, on the grid at left, point *A* is 4 units from point *B*, and point *B* is 3 units from point *C*. Note that points *ABC* form a right triangle.

To find the distance between two points that are not on the same grid line such as points *A* and *C*, you can use a formula. This formula is on the GED formulas page that you will be given when you take the GED Mathematics Test.

distance between points =
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

To use the formula, you need to know the coordinates for the two points. Assign one point to be (x_1,y_1) and the other point to be (x_2,y_2) . Then substitute the values into the formula and solve. It does not matter which point you choose to be (x_1,y_1) .

Example 1 In the coordinate grid at left, what is the distance between points *J* and *K*?



Step 1 Find the coordinates of the points.

Step 2 Assign the variables.

Step 3 Substitute the coordinates into the formula and solve.

The distance between points *J* and *K* is **10 units**.

Point
$$J = (5,2)$$
; $K = (-3,-4)$

Let $J(5,2) = (x_1,y_1)$ Let $K(-3,-4) = (x_2$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-3 - 5)^2 + (-4 - 2)^2}$ $= \sqrt{(-8)^2 + (-6)^2}$ $= \sqrt{64 + 36}$

 $= \sqrt{100}$ = 10

The distance between points will rarely be a whole number. If you need to find the answer to a certain decimal place, use your calculator.

Example 2 Find the distance to the nearest tenth between point A at (0,-2) and point B at (1,4).

Step 1 Decide which point will be
$$(x_1, y_1)$$
.

Let $(0,-2) = (x_1,y_1)$ Let $(1,4) = (x_2,y_2)$

 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(1 - 0)^2 + (4 - (-2))^2}$ $= \sqrt{(1)^2 + (6)^2}$ $= \sqrt{1 + 36}$

Step 3 Estimate the square root.

Think: $6^2 = 36$ and $7^2 = 49$. Since 37 is between 36 and 49,

d is between 6 and 7.



Most calculators require you to enter the number first, then press the square root key.

37 SHIFT X2 6.08276253

Round to the nearest tenth. 6.08 rounds to 6.1. The distance between the points is about **6.1 units**.

GED SKILL FOCUS



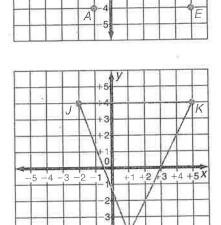
A. Find the distance between the following points. Round to the nearest tenth, if necessary. You may use a calculator.

Use the coordinate grid on the right to answer questions 1 through 5.

- 1. Points A and C
- 2. Points B and D
- 3. Points A and E
- 4. Points A and D
- 5. Points C and E

Use the coordinate grid on the right to answer questions 6 through 8.

- 6. A triangle is drawn by connecting points *J*, *K*, and *L*. What is the length of side *JL*?
- 7. What is the length of side *KL*?
- 8. What is the length of side JK?



+1 +2 +3 +4 +5 X

B. Solve as directed.

- 9. Point *X* is located at (9,3) on a coordinate grid. If a line were drawn directly to the origin, what would be the length of the line to the nearest tenth unit?
- 10. Two points are located at (2,5) and (-4,5). What is the distance between the points?
- 11. A line is drawn from point B at (0,6) to point C at (8,0). What is the length of the line to the nearest tenth unit?
- 12. Two points are located at (2,-1) and (2,4). What is the distance between the points?

Answers start on page 428.

GED STRATEGY Solving Word Problems:

Finding the Equation of a Line

You know how to graph an equation by finding ordered pairs that make the equation true and drawing a line through those points. You may be asked to work backwards from points to determine the equation of a line.

On the GED Mathematics Test the answer choices to such questions are written in a format called the **slope-intercept form of a line**. The variable m is the slope of the line. The variable b is the y-intercept, the point at which the line crosses the y-axis. The sum of mx + b when y = 0 is the x-intercept.

$$y = mx + b$$
, where $m =$ slope and $b = y$ -intercept

Example 1 What is the equation of the line shown on the graph?

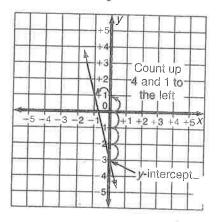
(1)
$$y = -2x - 3$$

(2)
$$y = -3x + 2$$

(3)
$$y = -4x + 2$$

(4)
$$y = -4x - 3$$

(5)
$$y = -4x + 3$$



Step 1 Find the y-intercept of the line. The line shown crosses the y-axis at (0,-3). Therefore:

the y-intercept is -3

Step 2 Find the slope of the line using the grid-unit counting method. Count from the *y*-intercept to another point on the line with whole numbers as the ordered pair, here, (-1,1). The line rises 4 units for every 1 unit of run to the left (a negative direction).

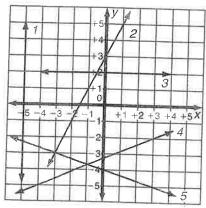
Step 3 Use the claracie to $\frac{1}{2}$

Step 3 Use the slope-intercept form to write the equation.

$$y = mx + b$$
 $y = -4x + (-3)$ or $y = -4x - 3$

The correct answer is **option** (4) y = -4x - 3.

You may need to rewrite an equation in slope-intercept form.



You can check

your work by

substituting the

coordinates of a

point on the line

for x and y in the

equation. If the

equation is true.

your answer is

correct.

Example 2 Which of the lines shows the graph of the equation 2x - y = -3?

Step 1 Rewrite the equation in slope-intercept form. y = mx + bIsolate the variables. -y = -3 - 2xEliminate the negative variable. y = 3 + 2xRearrange the terms. y = 2x + 3

The slope of the line is 2 and the *y*-intercept is **Step 2 Examine the lines on the graph.** Only Line 2 has a slope of positive 2 and passes through point (0,3).

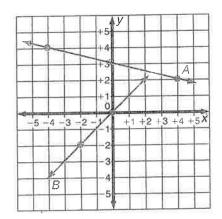
The correct answer is Line 2.

GED PRACTICE

Directions: Choose the one best answer to each question.

- 1. What are the coordinates of the *y*-intercept of the line y = -3x 2?
 - (1) (0,-5)
 - (2) (0,-3)
 - (3) (0,-2)
 - (4) (-2,0)
 - (5) (-3,0)

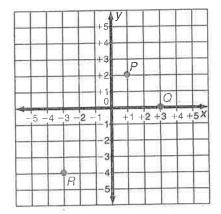
Questions 2 and 3 refer to the following graph.



- 2. What is the equation of Line A?
 - (1) y 4x = 3
 - (2) $y \frac{1}{4}x = -2$
 - (3) y = 4x
 - (4) $y + \frac{1}{4}x = -2$
 - (5) $y + \frac{1}{4}x = 3$
- 3. What is the equation of Line B?
 - (1) y = -x
 - (2) y = x
 - (3) y = 2x
 - (4) y = -2x
 - (5) y = x + 1

- 4. The point (0,-5) is the *y*-intercept of which of the following lines?
 - (1) y = 2x
 - (2) y = -x + 5
 - (3) y = 3x 5
 - (4) y = -2x 3
 - (5) Not enough information is given,

Questions 5 and 6 refer to the following graph.



- 5. What would be the equation of a line drawn through points *P* and *Q*?
 - (1) y = -x + 3
 - (2) y = -x 3
 - (3) y = 2x 3
 - (4) y = 3x + 2
 - (5) y = 3x + 3
- 6. Which of the following is the *y*-intercept of a line drawn through points *R* and *Q*?
 - (1) (-3,-4)
 - (2) (0,-2)
 - (3) (0,-3)
 - (4) (0,-4)
 - (5) (3,-4)

GED Mini-Test • Lessons 20-22

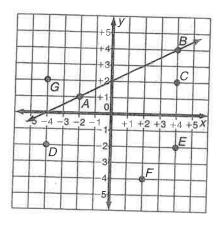
Directions: This is a thirty-minute practice test. After thirty minutes, mark the last item you finished. Then complete the test and check your answers. If most of your answers are correct, but you didn't finish, try to work faster next time.



Part 1

Choose the one best answer to each question. You MAY use your calculator.

Questions 1 and 2 refer to the following diagram.

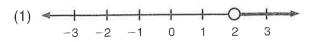


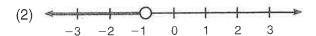
- 1. Which point lies at coordinates (4,-2)?
 - (1) Point C
 - (2) Point D
 - (3) Point E
 - (4) Point F
 - (5) Point G
- 2. What is the slope of the line that passes through points A and B on the graph?
 - $(1) \frac{1}{2}$
 - (2) $\frac{1}{3}$
 - (3) $\frac{1}{2}$
 - (4) $\frac{2}{3}$
 - (5) 2

- 3. Which of the following best represents the solution set to the inequality 5x + 2 < 6x + 3x + 10?
 - (1) x > -2
 - (2) x < -2
 - (3) x < 3
 - (4) x > -3
 - (5) x < -3
- 4. The sum of three consecutive odd numbers is 135. What are the three numbers?
 - (1) 41, 43, and 45
 - (2) 43, 45, and 47
 - (3) 44, 45, and 46
 - (4) 44, 46, and 48
 - (5) 45, 47, and 49
- 5. If the pattern continues, what is the next term in the sequence below?
 - 0 1
- 3
- 7
- 15

- (1) 25
- (2) 27
- (3) 29
- (4) 31
- (5) 33

6. Which graph shows the solution of x - 3 < -1?







7. In which of the following are the numbers arranged in order from least to greatest?

(1)
$$2.34 \times 10^2$$
, 5.2×10^2 , 4.7×10^{-1}

(2)
$$2.34 \times 10^2$$
, 4.7×10^{-1} , 5.2×10^2

(3)
$$4.7 \times 10^{-1}$$
, 5.2×10^{2} , 2.34×10^{2}

(4)
$$4.7 \times 10^{-1}$$
, 2.34×10^{2} , 5.2×10^{2}

(5)
$$5.2 \times 10^2$$
, 4.7×10^{-1} , 2.34×10^2

8. Which of the following shows the correct factors of the expression $x^2 - 12x + 36$?

(1)
$$(x + 6)(x + 6)$$

(2)
$$(x-6)(x-6)$$

(3)
$$(x-6)(x+6)$$

$$(4) (x-3)(x-12)$$

(5)
$$(x + 3)(x + 12)$$

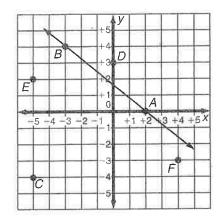
9. Which ordered pair is a solution of 5x - y = 1?

$$(1)$$
 $(1,-4)$

$$(3) (-1,6)$$

$$(4)$$
 $(-2,-11)$

Questions 10 and 11 refer to the following figure.



- 10. The graph of the equation $y = -\frac{3}{2}x + 3$ would pass through which of the following points on the coordinate grid?
 - (1) A and B
 - (2) A and C
 - (3) A, D, and F
 - (4) A and E
 - (5) C and D
- 11. What is the distance from point *C* to point *A* to the nearest whole unit?
 - (1) 7
 - (2) 8
 - (3) 9
 - (4) 10
 - (5) 11
- 12. The average human heart pumps 114,000 gallons of blood per day. Which of the following expressions represents that amount in scientific notation?

(1)
$$1.14 \times 10^{-5}$$

(2)
$$1.14 \times 10^{-4}$$

(3)
$$1.14 \times 10^3$$

(4)
$$1.14 \times 10^{5}$$

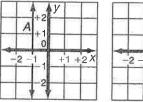
(5)
$$1.14 \times 10^6$$

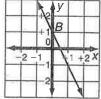
Part 2

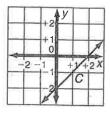
Directions: Choose the one best answer to each question. You may NOT use your calculator.

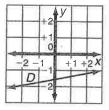
- 13. If $x^2 + x = 20$, what are the values for x that will make the equation true?
 - (1) -5 and 4
 - (2) -5 and -4
 - (3) 5 and 4
 - (4) 5 and -4
 - (5) Not enough information is given.
- 14. Aaron's weekly pay (p) can be represented by p = \$200 + \$6s, where s is the number of sales he makes in a week. How much will Aaron earn in a week if he makes 32 sales?
 - (1) \$192
 - (2) \$232
 - (3) \$238
 - (4) \$384
 - (5) \$392
- 15. Which ordered pair is a solution of 4x y = 3?
 - (1) (-5,2)
 - (2) (-1,1)
 - (3) (0,3)
 - (4) (1,-1)
 - (5) (2,5)
- 16. A square mural has an area of about 240 square feet. What is the approximate length of one side of the mural?
 - (1) between 18 and 19 feet
 - (2) between 17 and 18 feet
 - (3) between 16 and 17 feet
 - (4) between 15 and 16 feet
 - (5) between 14 and 15 feet

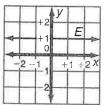
Question 17 refers to the following graphs.





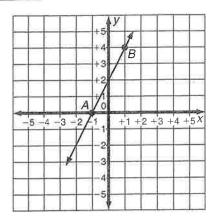






- 17. Which of the lines shown above has a negative slope?
 - (1) A
 - (2) B
 - (3) C
 - (4) D
 - (5) E
- 18. Which of the following is equal to the expression $\frac{x+4x}{x^2-2x}$?
 - (1) 2
 - (2) $\frac{5}{x-2}$
 - (3) x + 2
 - (4) 1 + 2x
 - (5) 2x
- 19. Which point is <u>not</u> on a graph of the line 2x y = -1?
 - (1) (-3,-5)
 - (2) (-1,-1)
 - (3) (1,3)
 - (4) (2,5)
 - (5) (3,6)

Question 20 refers to the following figure.



- 20. What is the equation of the line shown on the graph?
 - (1) y = 2x + 2
 - (2) y = 2x 1
 - (3) y = x + 2
 - (4) y = x 1
 - (5) $y = \frac{1}{2}x + 2$
- 21. A repair company charges a flat fee of \$40 plus \$30 for each hour (h) spent making a repair. Which of the following equations could be used to find the charge (c) for any service call?
 - (1) c = \$30h
 - (2) c = \$40h
 - (3) c = \$40 + \$30h
 - (4) c = \$40h + \$30h
 - (5) c = \$30 + \$40h
- 22. What is the slope of a line that passes through points at (-2,-2) and (-4,4)?
 - (1) -3
 - (2) -1
 - (3) $-\frac{1}{3}$
 - (4) $\frac{1}{3}$
 - (5) 3

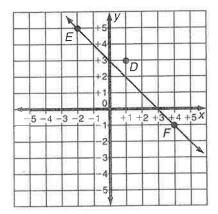
23. The following amounts were deposited in a savings account each month.

January February \$20 \$32 March April \$44 \$56

If the pattern continues, how much will be deposited in December?

- (1) \$112
- (2) \$140
- (3) \$144
- (4) \$152
- (5) \$164

Questions 24 and 25 refer to the following graph.



24. What is the equation of the line that passes through points *E* and *F*?

(1)
$$y = -3x - 3$$

(2)
$$y = 3x + 3$$

(3)
$$y = x + 3$$

(4)
$$y = -x + 3$$

(5)
$$y = -x - 3$$

- 25. What is the distance in units from point *F* to point *D*?
 - (1) 3
 - (2) 4
 - (3) 5
 - (4) 6
 - (5) 7

Unit 3 Cumulative Review Algebra

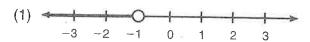
Part 1

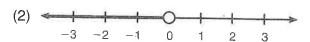
Directions: Choose the one best answer to each question. You MAY use your calculator.

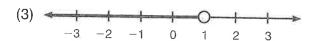
- 1. What is the value of the expression 4x 2y + xy, when x = -1 and y = 5?
 - (1) -26
 - (2) -19
 - (3) -9
 - (4) -1
 - (5)
- 2. Which of the following expressions is equal to 2 (x + 7)?
 - (1) x 9
 - (2) -x 5
 - (3) -x 9
 - (4) x + 9
 - (5) -x + 14
- 3. If -6(x + 1) + 4 = 8x 9, what is the value of x?
 - (1) $\frac{1}{2}$
 - (2) $-\frac{1}{2}$
 - (3) -1
 - (4) -3
 - (5) -5
- 4. A certain number decreased by two is equal to seven increased by the quotient of the same number and four. What is the number?
 - (1) -12
 - (2) -5
 - (3) 5
 - (4) 7
 - (5) 12

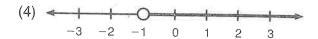
5. Which of the following graphs shows the solution set for the inequality?

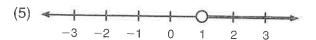
$$6 - 5x < 7x - 6$$











- 6. The product of a number (x) and -4 is 8 more than 2 added to -5 times that number. Which of the following equations could be used to find the value of x?
 - (1) -4x 8 = 2x + (-5x)
 - (2) -4x + 8 = -5x + 2
 - (3) -4x = -5x + 2 + 8
 - (4) -4x = 8 + 2 + (-5)
 - (5) -4x = 8 + x(-5 + 2)
- 7. What is the slope of the line on the graph?



- (1) 2
- (2) $\frac{3}{2}$
- (3) $\frac{2}{3}$
- $(4) -\frac{2}{3}$
- (5) $-\frac{3}{2}$



You can use either the x^2 or x^y key when solving volume problems on a calculator. Try the key strokes below to solve the cylinder problem in Example 3.

$$3.14 \times 4 \times 4 \times 10 = 502.4$$

$$3.14 \times 4 \times x^2 \times 10 = 502.4$$

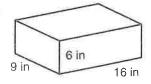
The volume of the cylinder is about **502.4 cubic centimeters**.

GED SKILL FOCUS

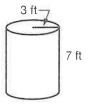


A. Find the volume of each figure to the nearest whole unit. You may use your calculator.

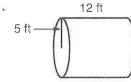
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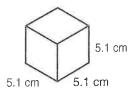
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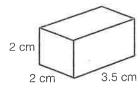
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2.



4.

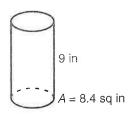


- B. Solve. You may use your calculator for question 9.
 - 6. What is the volume in cubic feet of a rectangular crate with these measurements: length = 4 feet, width = 5 feet, and height = 6 feet?
 - 7. The rectangular trash containers at an apartment complex are 10 feet long, 6 feet wide, and 5 feet high. How many cubic feet of trash does each container hold?
 - 8. A health club is going to build a swimming pool. The hole needs to be 100 feet long by 25 feet wide with a depth of 5 feet. How many cubic feet of dirt will be removed to dig the hole?



9. Paul needs to store a box that measures $1\frac{1}{2}$ yards on each side. What is the volume in cubic yards of the box?

Refer to the diagram below to answer Question 10.



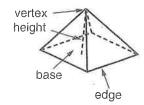
- 10. Which of the following expressions can be used to find the volume in cubic inches of the vase?
 - (1) (8.4)9
 - (2) (8.4²)(9)
 - (3) (3.14)(8.4)(9)
 - (4) (3.14)(8.4²)(9)
 - $(5) (3.14)(8.4)(9^2)$

Answers start on page 435.

Pyramids and Cones

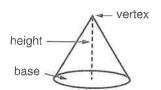
The three-dimensional solids studied so far—rectangular containers, cubes, and cylinders—all have two identical bases. You may think of these bases as the top and bottom sides of the figure. Pyramids and cones are also three-dimensional solids, but they have only one base.

Pyramid



A pyramid has one base with sides of the same length. A pyramid can have either a square or triangular base. However, the volume formula used on the GED Mathematics Test is for only a pyramid with a square base. The base is connected to a single point, called a vertex, by triangular faces (sides).

Cone



A come has one circular base and one vertex. A curved surface connects the base and vertex.

The volume of a pyramid or a cone is $\frac{1}{3}$ of the area of its base multiplied by its height.

$$V = \frac{1}{3} \times \text{Area of base} \times \text{height}$$

$$V = \frac{1}{3}Ah$$

pyramid Volume =
$$\frac{1}{3}$$
 × (area of square) × height = $\frac{1}{3}$ × (base edge)² × height

$$V = \frac{1}{3}s^2h$$

cone

Volume =
$$\frac{1}{3}$$
 × (area of circle) × height

$$V = \frac{1}{2} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times \text{radius}^2 \times \text{height}$$

$$V = \frac{1}{3} \pi r^2 h$$

Example 1 Find the volume of the pyramid in cubic feet.

h = 5 ft

Step 1 Find the area of the base (a square). Choose the formula. Substitute and solve.

$$A = s^2$$

 $A = 6^2 = 36 \text{ sq ft}$

Choose the volume formula. Step 2

ose the volume formula.
$$V = \frac{1}{3}Ah$$

$$V = \frac{1}{3}(36)(5)$$

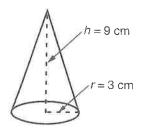
$$V = 60$$
 cu ft



You can use the fraction key to multiply by a fraction. Or multiply the area and height and divide by 3.

1 ab/c 3 \times 36 \times 5 = 60. or 36 \times 5 \div 3 = 60.

The volume of the pyramid is 60 cubic feet.



Example 2 Find the volume of the cone to the nearest cubic centimeter.

Step 1 Use the formula for finding the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$

Step 2 Substitute.

$$V = \frac{1}{3}(3.14)(3^2)(9)$$

Step 3 Solve.

$$V = \frac{1}{3}(3.14)(9)(9) = 84.78$$
, which rounds to 85 cu cm.

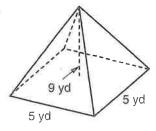
The volume of the cone is about 85 cubic centimeters.

GED SKILL FOCUS

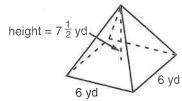


A. Find the volume of each figure. Round your answers to the nearest whole number. You may use a calculator.

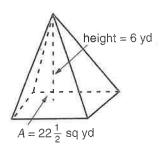
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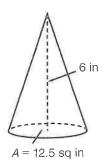
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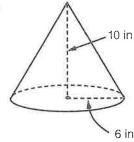
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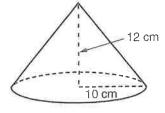
2.



4.

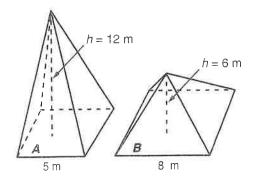


6.



B. Solve.

- 7. What is the volume in cubic centimeters of a cone with a height of 8 centimeters and a base area of 12.6 square centimeters? (*Hint:* You already know the area of the circular base. Use $V = \frac{1}{3}Ah$ instead of $V = \frac{1}{3}\pi r^2h$.)
- 8. The area of the base of pyramid A is 25 square meters. The area of the base of pyramid B is 64 square meters. In cubic meters, how much greater is the volume of pyramid B than the volume of pyramid A?

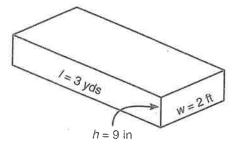


- 9. A pyramid has a square base with one edge labeled 9 centimeters and a height of 18.25 centimeters. Which of the following expressions can be used to find the volume of the pyramid in cubic centimeters?
 - (1) 92(18.25)
 - (2) 9³(18.25)
 - (3) $\frac{1}{3}$ (9)(18.25)
 - (4) $\frac{1}{3}$ (9²)(18.25)
 - (5) Not enough information is given.

GED STRATEGY Solving Word Problems

Converting Measurements

To solve perimeter, area, and volume problems, all the measurements must be written using the same units. Converting measurements involves using conversion factor ratios to change from one unit to another.



Example

What is the volume in <u>cubic inches</u> of the rectangular solid shown to the <u>left?</u>

- (1) 54 (2) 648 (3) 1,944
- (4) 23,328
- (5) 93,312

Step 1 Convert measurements. Convert the length and width to inches. (To review converting measurements, see pages 158–159.)
The length is 3 yards. Convert to inches using the conversion factor: 1 yard = 36 inches. Cross multiply to solve.

$$\frac{\text{yard}}{\text{inches}} \frac{1}{36} = \frac{3}{l} \quad l = 3(36)$$
 $l = 108 \text{ inches}$

The width is 2 feet. Convert to inches: 1 foot = 12 inches.

$$\frac{\text{foot}}{\text{inches}} \frac{1}{12} = \frac{2}{w} \quad w = 2(12) \qquad w = 24 \text{ inches}$$

Step 2 Choose the appropriate formula.

Substitute and solve.

$$V = lwh$$

 $V = 108 \times 24 \times 9 = 23,328$ cu in

The correct option is (4) 23,328.

You could also have solved this problem in terms of feet or yards.



More than one unit of measure may appear in the same problem. Convert all measurements to the same unit before using formulas.

In feet: In yards:

$$V = lwh$$

$$= 3 \text{ yd} \times 2 \text{ ft} \times 9 \text{ in}$$

$$= 9 \text{ ft} \times 2 \text{ ft} \times \frac{3}{4} \text{ ft}$$

$$= 13\frac{1}{2} \text{ cu ft}$$

$$V = lwh$$

$$= 3 \text{ yd} \times 2 \text{ ft} \times 9 \text{ in}$$

$$= 3 \text{ yd} \times \frac{2}{3} \text{ yd} \times \frac{1}{4} \text{ yd}$$

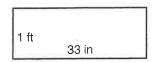
$$= \frac{1}{2} \text{ cu yd}$$

Often you must decide which measurement unit to use. Generally, it is best to convert to the smallest unit so that you will not have to work with fractions. Also, read the question. You usually save time by converting all measures to the unit of measure called for in the question.

GED PRACTICE

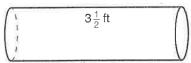
Directions: Choose the one best answer to each question.

Questions 1 and 2 refer to the following figure.



- 1. What is the perimeter of the rectangle in feet?
 - $(1) 6\frac{1}{2}$
 - (2) $7\frac{1}{2}$
 - (3) 8
 - (4) $8\frac{1}{4}$
 - (5) $8\frac{1}{2}$
- 2. What is the area of the rectangle in <u>square</u> inches?
 - (1) 364
 - (2) 386
 - (3) 396
 - (4) 412
 - (5) 432

Question 3 refers to the following diagram.

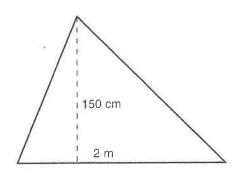


diameter of base is 12 inches

- 3. Which of the following expressions can be used to find the volume of the cylinder in cubic inches?
 - (1) 42(12)
 - (2) 3.14(6²)
 - $(3) \ 3.14(6^2)(3.5)$
 - (4) (3.14)(6²)(42)
 - $(5) (3.14)(12^2)(42)$

- 4. What is the approximate volume in <u>cubic feet</u> of a box with a length of 2½ ft, a width of 1 ft 6 in, and a height of 1 ft 9 in?
 - (1) between 3 and 4
 - (2) between 4 and 5
 - (3) between 5 and 6
 - (4) between 6 and 7
 - (5) between 7 and 8

Question 5 refers to the following diagram.



- 5. What is the area in square centimeters of the triangle in the figure?(Hint: 1 meter = 100 centimeters)
 - (1) 12,000
 - (2) 12,500
 - (3) 15,000
 - (4) 20,000
 - (5) 22,500
- 6. A cone has a height of 1 foot 4 inches and a base with an area of 18 square inches. What is the volume of the cone in cubic inches?
 - (1) 84
 - (2) 96
 - (3) 144
 - (4) 252
 - (5) 288

Answers start on page 436.

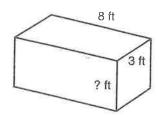
GED STRATEGY Solving Word Problems

Solving for Variables in Formulas

You have already worked with various geometric formulas. Formulas are a type of equation that contains variables used to solve measurement and geometry problems. The algebraic properties that you learned in Unit 3 can be used to solve formulas for a specific variable.

Remember that an equation shows two equal mathematical statements, called *expressions*. In order to keep the expressions equal, apply the same operation to both sides of the equation.

Example 1 The volume of the rectangular container is 120 cubic feet. What is the height of the container?

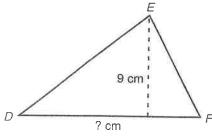


Step 1Choose the appropriate formula.
$$V = lwh$$
Step 2Substitute the known values. $120 = 8(3)h$ Step 3Simplify. $120 = 24h$ Step 4Solve for the unknown $\frac{120}{24} = \frac{24h}{24}$

The height of the container is 5 feet.

Often, the way to answer a set-up question on the GED Mathematics Test is to rewrite the equation to solve for a certain variable. The five multiple-choice options for such questions show possible ways to solve the problem. You have to choose the correct one as shown below.

Example 2 The area of triangle *DEF* is 72 sq cm. Which of the following expressions could be used to find the length of the base of triangle *DEF*?



- $(1) \frac{72}{2} (9)$
- (2) 2(9)(72)
- (3) $\frac{2(9)}{72}$
- (4) $\frac{2(72)}{9}$
- (5) $\frac{9}{2(72)}$

$$A = \frac{1}{2}bh$$

$$2 \times A = \frac{1}{2}bh \times 2$$
$$2A = bh$$

$$\frac{2A}{b} = b$$

$$\frac{2(72)}{9} = b$$

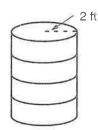
Option (4) $\frac{2(72)}{9}$ is correct.

GED PRACTICE

Directions: Choose the one best answer to each question.

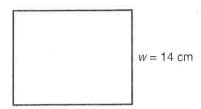
- 1. The area of a rectangular mural is 180 square feet. If the mural is 15 feet in length, what is its width in feet?
 - (1) 6
 - (2) 12
 - (3) 165
 - (4) 2700
 - (5) Not enough information is given.

Question 2 refers to the following diagram.



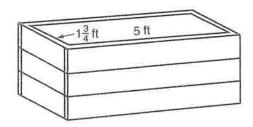
- A storage barrel has the shape of a cylinder. The volume of the barrel is about 81.64 cubic feet. Which of the following expressions could be used to find its height in feet?
 - $(1) \ \frac{81.64}{(3.14)(2^2)}$
 - (2) $\frac{(3.14)(2^2)}{81.64}$
 - $(3) \ \frac{81.64(2^2)}{(3.14)}$
 - (4) 81.64(3.14)(2²)
 - (5) 81.64 $(3.14)(2^2)$
- 3. A small section of a roof has the shape of a triangle. The total area of the section is 10.5 square feet. If the height of the section is 3.5 feet, what is the measure of the base in feet?
 - (1) 3
 - (2) 6
 - (3) 7
 - (4) 14
 - (5) 36.75

Question 4 refers to the following figure.



- 4. The perimeter of the rectangle is 64 cm. Which of the following expressions could be used to find the length of the rectangle?
 - $(1) \frac{64}{14}$
 - (2) $\frac{64}{2(14)}$
 - (3) 64 14
 - $(4) \frac{64-2(14)}{2}$
 - $(5) \ \frac{64 + 2(14)}{2}$

Question 5 refers to the following diagram.

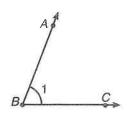


- 5. Mike plans to build a wooden flower box similar to the one shown in the diagram. He wants the box to have a volume of 17½ cubic feet. If base of the box has the dimensions shown above, what should the height of the box measure in feet?
 - (1) 2
 - (2) $2\frac{1}{2}$
 - (3) $3\frac{1}{2}$
 - (4) 10
 - (5) Not enough information is given.

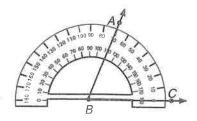
Lesson

GED SKILL Lines and Angles

Kinds of Angles



An angle is the space or opening between a pair of lines, called rays, that extend from a common point, the vertex. Angles can be named using three letters: a point on one ray, the vertex, and a point on the other ray. They can also be named by the letter of the vertex alone or by a number written inside the angle. The symbol for angle is \angle . This angle can be named $\angle ABC$, $\angle B$, or $\angle 1$.



Angles are measured in degrees (°) using a protractor. When ray BC is placed along the bottom of the protractor, the measure of the angle is read along the scale that starts at 0. The measure (m) of angle ABC equals 70° or in symbols, $m \angle ABC = 70^{\circ}$.

Angles are classified by their measures. These measures are based on the fact that a circle contains 360°. When measuring an angle, you are actually measuring the number of degrees (or part of the circle) contained in the opening between two rays.

A right angle measures exactly 90°.	An acute angle measures less than 90°.	An obtuse angle measures more than 90° but less than 180°.	A straight angle measures exactly 180°.	A reflex angle measures more than 180° but less than 360°.	
This symbol means the angle measures 90°.					

Angles are also related based on the sum of their measures. (Here's a memory aid: 90 comes before 180; c comes before s.)

- If the sum of the measures of two angles equals 90°, the angles are called complementary angles.
- If the sum of the measures of two angles equals 180°, the angles are called supplementary angles.

Example 1 If the measure of $\angle BXC$ is 25°, what is the measure of $\angle AXB$?

write the given measures on the diagram. Make a sketch of the facts in a

It often helps to

problem if there is no diagram.

The angles are complementary because $\angle AXC$ has a right angle symbol, and a right angle measures 90°.



Step 1 Write an equation.

Step 2 Substitute the known measures.

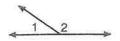
Step 3 Solve. Subtract 25 from both sides.

The measure of $\angle AXB$ is 65°.



 $m \angle AXB + m \angle BXC = 90^{\circ}$ $m \angle AXB + 25^{\circ} = 9($

 $m \angle AXB = 65^{\circ}$



Example 2 Angles 1 and 2 are supplementary angles. If the measure of $\angle 1$ is 35°, what is the measure of $\angle 2$?

Step 1 Write an equation.

 $m \angle 1 + m \angle 2 = 180^{\circ}$

Step 2 Substitute the known measures.

 $35^{\circ} + m \angle 2 = 180^{\circ}$

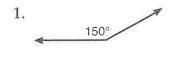
Step 3 Solve.

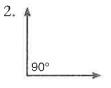
 $m\angle 2 = 145^{\circ}$

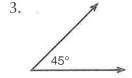
The measure of $\angle 2$ is **145°**.

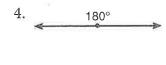
GED SKILL FOCUS

A. Label each angle as acute, obtuse, right, or straight.





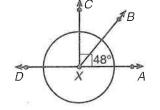




B. Solve.

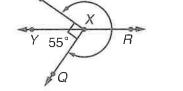
Refer to the figure below to answer Questions 5 through 8. $\angle AXB$ and $\angle BXC$ are complementary.

- 5. What is $m \angle BXC$?
- 6. What is $m \angle DXC$?
- 7. What is $m \angle BXD$?
- 8. Which angle forms a supplementary angle with $\angle DXB$?



Refer to the figure below to answer Questions 9 through 12. $\angle ZXY$ and $\angle YXQ$ are complementary.

- 9. What is $m \angle ZXY$?
- 10. What is $m \angle ZXR$?
- 11. What is $m \angle QXR$?



- 12. What is the measure of the reflex angle marked by the curved arrow?
- C. Choose the best answer to each question.
 - 13. The measure of $\angle A$ is 28°. The measure of $\angle B$ is 62°. Which of the following is true?
 - (1) $\angle A$ is complementary to $\angle B$.
 - (2) $\angle A$ and $\angle B$ are supplementary angles.
 - (3) $\angle A$ and $\angle B$ are obtuse angles.
 - (4) $\angle A$ and $\angle B$ are reflex angles.
 - (5) $\angle A$ is an acute angle; $\angle B$ is an obtuse angle.

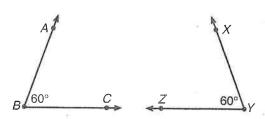
- 14. $\angle M$ and $\angle R$ are supplementary angles. The measure of $\angle M$ is 40°. What is the measure of $\angle R$?
 - $(1) 40^{\circ}$
 - (2) 50°
 - (3) 90°
 - (4) 140°
 - (5) 180°

Answers start on page 437.

Congruent and Vertical Angles

Angles can be related in other ways. Angles that have equal measures are **congruent angles**. In the figure below, angle ABC is congruent to angle XYZ. The angles are congruent even though they are not turned in the same direction. The symbol \cong means "is congruent to."

 $\angle ABC \cong \angle XYZ$



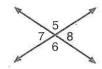


Learn the meaning of terms used in geometry. A simple problem is impossible if you don't know the vocabulary.

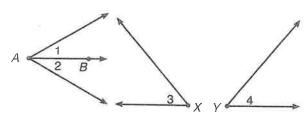
Some angles have a special relationship because of their location with respect to each other. When two lines intersect, or cross, four angles are formed. The angles that are across from each other are called **opposite angles**, or **vertical angles**. Each pair of vertical angles is congruent.

$$\angle 5 \cong \angle 6$$
 $m \angle 5 = m \angle 6$

$$\angle 7 \cong \angle 8$$
 $m \angle 7 = m \angle 8$



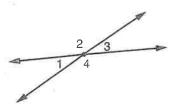
Angles can also be described as **adjacent** or **non-adjacent**. Adjacent angles have a common vertex and a common ray.



 $\angle 1$ and $\angle 2$ are adjacent. They share vertex A and ray AB (also written as \overrightarrow{AB}).

 $\angle 3$ and $\angle 4$ are non-adjacent angles.

Many geometry problems require you to apply logical reasoning to find congruent angles. Use your understanding of the properties of angles to solve geometry problems.



Example In the figure, angle 1 measures 30°. Find the measures of angles 2, 3, and 4.

- Step 1 Assign the known values. $m\angle 1 = 30^{\circ}$
- Step 2 Identify known relationships.

(a) supplementary angles $m \angle 1 + m \angle 2 = 180^{\circ}$

(b) vertical angles $m \angle 1 = m \angle 3; m \angle 2 = m \angle 4$

Step 3 Solve for unknown values.

(a)
$$m\angle 1 + m\angle 2 = 30^{\circ} + m\angle 2 = 180^{\circ}; m\angle 2 = 150^{\circ}$$

(b) $m\angle 3 = 30^{\circ}; m\angle 4 = 150^{\circ}$

The angle measures are $m \angle 2 = 150^{\circ}$, $m \angle 3 = 30^{\circ}$, and $m \angle 4 = 150^{\circ}$.

GED SKILL FOCUS

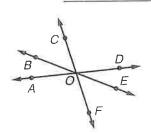
A. Refer to the figure shown at the right to answer Questions 1 through 9.

- 1. Name an angle adjacent to $\angle 3$.
- 2. Name the angle that is the vertical angle to $\angle 2$.
- 3. Name the angle that is the vertical angle to $\angle 6$.
- 4. Name the angle that is the vertical angle to $\angle 1$.
- 5. Name two angles that are adjacent to $\angle 1$.
- 6. Name two angles that are adjacent to $\angle 5$.
- 7. Name the angle congruent to $\angle 5$.
- 8. Name the angle congruent to $\angle 4$.
- 9. Name the angle congruent to $\angle 3$.



B. Solve.

Refer to the following information to answer Questions 10 through 15.



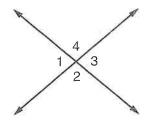
$$m \angle AOB = 30^{\circ}$$

$$m \angle EOF = 50^{\circ}$$

- 10. Since $m \angle EOF = 50^{\circ}$, what other angle also has a measure equal to 50° ?
- 11. The sum of the measures of $\angle AOB$, $\angle BOC$, and $\angle COD$ is 180°. What is the measure of $\angle COD$?
- 12. What is the measure of $\angle AOF$? (*Hint:* You already found the measure of the vertical angle to $\angle AOF$.)
- 13. What is the sum of the measures of $\angle DOE$ and $\angle EOF$?
- 14. What is the measure of the reflex angle for $\angle EOF$?
- 15. Which two angles are supplementary to $\angle DOF$?

- 16. Angle Q is congruent to its vertical angle, $\angle R$. Which of the following must be true?
 - (1) $\angle Q$ and $\angle R$ are complementary angles.
 - (2) $\angle Q$ and $\angle R$ are acute angles.
 - (3) $\angle Q$ and $\angle R$ are obtuse angles.
 - (4) $\angle Q$ and $\angle R$ are adjacent angles.
 - (5) $\angle Q$ and $\angle R$ have equal measures.

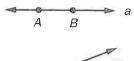
Refer to the figure below to answer Question 17.

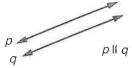


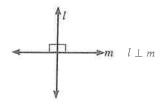
- 17. The measure of $\angle 4 = 100^{\circ}$. Which of the following statements is true?
 - (1) ∠4 and ∠2 are complementary
 - (2) $m\angle 2 = 100^{\circ}$
 - (3) $m \angle 1 = 100^{\circ}$
 - (4) $m \angle 1 + m \angle 3 = 100^{\circ}$
 - (5) $m \angle 4 + m \angle 2 = 180^{\circ}$

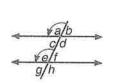
Answers start on page 437.

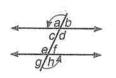
Lines and Angles

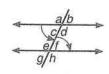


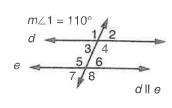












In geometry, arrows are drawn at both ends of a line to show that the line extends indefinitely in both directions. A line can be named by two points on the line. The line shown here is "line AB" (\overrightarrow{AB}) . The line can also be named by a lower case letter (line a).

Two lines in the same plane (flat surface) either intersect (cross) or are parallel. **Parallel lines** never intersect. They have exactly the same slope. The symbol || means "is parallel to."

As you have seen, intersecting lines form vertical angles. When two lines intersect to form right angles, the lines are **perpendicular**. The symbol for perpendicular lines is \bot .

Special pairs of angles are formed when a line, called a **transversal**, crosses two or more parallel lines. In the figure at right, the transversal intersects line AB and line CD. Notice that some of the angles formed are inside the parallel lines and some are outside.

You already know that vertical angles are equal in measure ($\angle a$ and $\angle d$; $\angle b$ and $\angle c$; $\angle e$ and $\angle h$; $\angle f$ and $\angle g$). The following pairs of angles are also always equal in measure (congruent).

Corresponding angles are in the same position with respect to the transversal. That is, they are on the same side of the transversal and either both above or both below the parallel lines

$$\angle b$$
 and $\angle f$

$$\angle c$$
 and $\angle g$

$$\angle d$$
 and $\angle h$

Alternate exterior angles are always outside the parallel lines and on opposite sides of the transversal.

$$\angle a$$
 and $\angle h$ $\angle b$ and $\angle g$

Alternate interior angles are always inside the parallel lines and on opposite sides of the transversal.

$$\angle c$$
 and $\angle f$ $\angle d$ and $\angle e$

If you know the measure of one angle, you can find the measures of the others. There are often several ways to determine the other angles.

Example

If the measure of $\angle 1$ is 110° , what is the measure of $\angle 6$?

Step 1 Identify an angle that is related to both the known and unknown angles.

Angles 1 and 5 are corresponding (congruent) angles. $m\angle 1 = m\angle 5$ Angles 5 and 6 are supplementary angles. $m\angle 5 + m\angle 6 = 180^{\circ}$

Step 2 Find the measure of the angle identified in Step 1. Since $m \angle 1 = 110^\circ$, and $m \angle 1 = m \angle 5$, $m \angle 5 = 110^\circ$

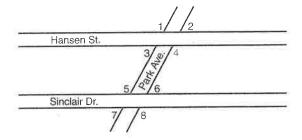
Step 3 Find the measure of the unknown angle. $m\angle 5 + m\angle 6 = 180^\circ$; $m\angle 5 = 110^\circ$ $m\angle 6 = 180^\circ - 110^\circ = 70^\circ$

The measure of $\angle 6$ is **70°**.

GED SKILL FOCUS

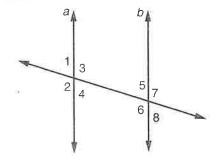
- A. Refer to the figure shown at the right to answer Questions 1 through 10. Lines p and mare parallel.
 - 1. Name the interior angles.
 - 2. Name the interior angle on the same side of the transversal as $\angle 6$.
 - 3. Name the exterior angles.
 - 4. Name the exterior angle on the same side of the transversal as $\angle 4$.
 - 5. Which angle corresponds to $\angle 1$?
 - 6. Which angle corresponds to $\angle 7$?
 - 7. Which angle corresponds to $\angle 8$?
 - 8. Which angle corresponds to $\angle 2$?
 - 9. Which angle is an alternate interior angle with $\angle 2$?
 - 10. Which angle is an alternate exterior angle with $\angle 5$?
- B. Choose the best answer to each question.

Refer to the diagram below to answer Ouestions 11 and 12.



- 11. Hansen Street runs parallel to Sinclair Drive. Which statement is true?
 - (1) $m \angle 1 = m \angle 7$
 - (2) $m\angle 2 + m\angle 6 = 90^{\circ}$
 - (3) $\angle 5$ is complementary to $\angle 6$.
 - (4) $\angle 4$ is adjacent to $\angle 6$.
 - (5) $m \angle 1 + m \angle 7 = 180^{\circ}$
- 12. The measure of ∠2 is equal to 60°. Which other angles must also measure 60°?
 - (1) Only ∠3
 - (2) $\angle 3$, $\angle 6$, and $\angle 7$
 - (3) $\angle 3$, $\angle 5$, and $\angle 8$
 - (4) $\angle 4$, $\angle 6$, and $\angle 8$
 - (5) Only ∠7

Refer to the diagram below to answer Questions 13 and 14.

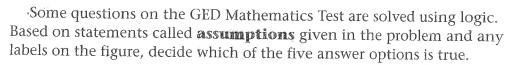


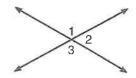
- 13. Line a is parallel to line b. Which of the following statements is true?
 - (1) $m \angle 2 + m \angle 5 = 180^{\circ}$
 - (2) $\angle 1$ is complementary to $\angle 2$.
 - (3) $\angle 6$ is supplementary to $\angle 7$.
 - (4) $\angle 2$ and $\angle 6$ are vertical angles.
 - (5) $\angle 1$ and $\angle 6$ are corresponding angles.
- 14. Which of the following are not supplementary angles?
 - (1) $\angle 1$ and $\angle 2$
 - (2) $\angle 1$ and $\angle 3$
 - (3) \angle 3 and \angle 7
 - (4) $\angle 3$ and $\angle 8$
 - (5) $\angle 4$ and $\angle 6$

Answers start on page 438.

GED STRATEGY Solving Word Problems =

Using Logical Reasoning





The sum of the measures of angles 1 and 2 is 180°. The sum of the measures of angles 2 and 3 is 180°. Based on this information, which of the following statements is true?

- (1) $m \angle 1 = 120^{\circ}$
- (2) $m\angle 2 = 60^{\circ}$
- (3) $m \angle 3 = 100^{\circ}$
- (4) $m \angle 1 = m \angle 2$
- (5) $m \angle 1 = m \angle 3$

Angles 1 and 3 are vertical angles so **option** (5) $m \angle 1 = m \angle 3$ is correct.



To have a better chance of choosing the correct option eliminate any answer options you know are incorrect.

Options (1), (2), (3), and (4) are incorrect because no information is provided about specific angle measures. Although the measure of angle 1 looks greater than the measure of angle 2 in the figure, do not base a conclusion on the appearance of a figure; you must rely only on supporting evidence. Option (4) would be correct only if the intersecting lines were perpendicular.

You often need to apply algebraic reasoning to solve geometry problems involving complementary and supplementary angles.

Example 2 The measure of an angle is twice the measure of its complement. What is the smaller angle's measure?

- $(1) 20^{\circ}$
- $(2) 30^{\circ}$
- (3) 45°
- (4) 90°
- (5) Not enough information is given.

Step 1 Assign the variables.

Let x = the measure of the smaller angle. Let 2x = the measure of the larger angle.

Step 2 Write an equation. The sum of the measures $x + 2x = 90^{\circ}$ of the two angles is 90° because the angles are complementary.

Step 3 Solve.

 $3x = 90^{\circ}$

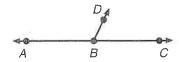
 $x = 30^{\circ}$

The smaller angle measures 30°. **Option (2)** is correct. If the question had asked for the larger angle, the answer would be $2(30^\circ) = 60^\circ$.

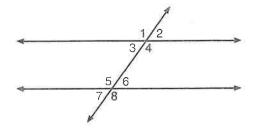
GED PRACTICE

Directions: Choose the one best answer to each question.

Question 1 is based on the following figure.

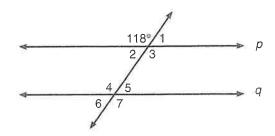


- 1. Angle *ABC* is a straight angle. Which of the following statements is true?
 - (1) $m \angle DBC = 90^{\circ}$
 - (2) $\angle ABD$ and $\angle DBC$ are supplementary.
 - (3) $m \angle ABD = 90^{\circ}$
 - (4) $\angle ABD$ and $\angle DBC$ are complementary.
 - (5) $m \angle ABC = 90^{\circ}$
- 2. Angle 1 is congruent to ∠5. Which of the following pairs of angles is also congruent?

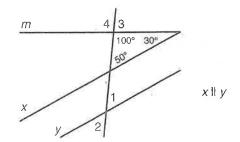


- (1) $\angle 1$ and $\angle 2$
- (2) $\angle 3$ and $\angle 4$
- (3) \angle 3 and \angle 7
- (4) ∠3 and ∠8
- (5) $\angle 7$ and $\angle 8$
- 3. One angle is 12° less than its complement. If the measure of the larger angle is x, which of the following statements must be true?
 - $(1) x + (x 12^\circ) = 180^\circ$
 - (2) $x + (x 12^\circ) = 90^\circ$
 - (3) $x + (12^{\circ} x) = 90^{\circ}$
 - (4) $x + (x + 12^\circ) = 180^\circ$
 - (5) $x 90^{\circ} = x + 12^{\circ}$

4. The measure of ∠7 is 116°. Which of the following is a true statement?



- (1) ∠1 and ∠5 are congruent.
- (2) $\angle 1$ and $\angle 4$ are supplementary.
- (3) $m \angle 1 + m \angle 4 = 180^{\circ}$
- (4) Lines p and q are not parallel.
- (5) Lines p and q are perpendicular.
- 5. Which conclusion is true?



- (1) $m \angle 1 = 50^{\circ}$
- (2) $m\angle 2 = 80^{\circ}$
- (3) $m \angle 3 = 50^{\circ}$
- (4) $m \angle 4 = 80^{\circ}$
- (5) $m\angle 4 = m\angle 3$
- 6. What is the measure of ∠AXB if it is four times the measure of its supplement, ∠BXC?
 - (1) 36°
 - (2) 45°
 - (3) 72°
 - (4) 135°
 - (5) 144°

Answers start on page 438.

GED Mini-Test • Lessons 23 and 24

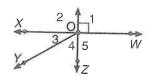
Directions: This is a thirty-minute practice test. After thirty minutes, mark the last item you finished. Then complete the test and check your answers. If most of your answers are correct, but you didn't finish, try to work faster next time.



Part 1

Directions: Choose the one best answer to each question. You MAY use your calculator.

Questions 1 through 3 refer to the figure.



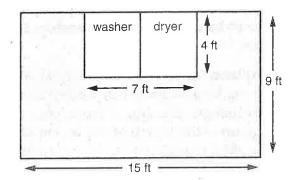
- Which of the following angles is supplementary, but not adjacent, to ∠2?
 - (1) ∠1
 - (2) ∠3
 - (3) ∠4
 - (4) ∠5
 - (5) Not enough information is given.
- 2. The measure of $\angle 3$ is 25°. What is the measure of $\angle WOY$?
 - (1) 65°
 - (2) 115°
 - (3) 135°
 - (4) 155°
 - (5) 165°
- 3. If an angle is supplementary to ∠XOZ, the angle must also be which of the following?
 - (1) an acute angle
 - (2) a right angle
 - (3) an obtuse angle
 - (4) a vertical angle
 - (5) congruent to ∠3

- 4. A circular pool has a circumference of about 40 meters. Which of the following expressions could be used to find the diameter of the pool in meters?
 - (1) 40π
 - (2) $\frac{40}{\pi}$
 - (3) $\frac{\pi}{40}$
 - (4) $\frac{2(40)}{\pi}$
 - (5) $\frac{\pi}{2(40)}$
- 5. One side of the square base of a container measures 4 inches. If the container can be filled to a height of 4 inches, how many cubic inches of liquid can the container hold?
 - (1) 12
 - (2) 16
 - (3) 20
 - (4) 32
 - (5) 64
- 6. A rectangular frame is $2\frac{1}{2}$ feet by 18 inches. What is the frame's perimeter in feet?
 - (1) $3\frac{3}{4}$
 - (2) 4
 - (3) 8
 - (4) $8\frac{3}{5}$
 - (5) 9

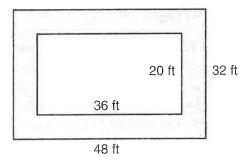
GED SKILL FOCUS

Solve.

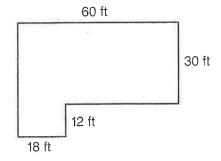
1. Emily plans to tile her laundry room floor, except for the space underneath the washer and dryer. Each tile covers 1 square foot of space. How many tiles will Emily need to do the job?



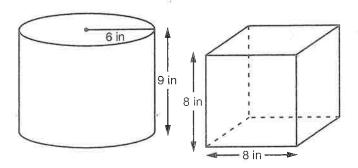
2. In the following figure, the shaded part shows the walkway around a garden. The garden measures 36 feet by 20 feet. How many square feet is the surface of the walkway?



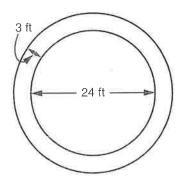
3. Construction workers will line the outside edge of a pool with tile strips, each 6 inches in length. How many strips are needed to go completely around the edge of the pool shown below?



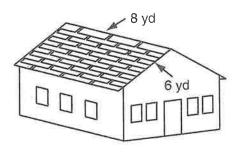
4. A toy maker is using a cylindrical container, shown below, to package a new toy. The toy is in the shape of a cube, 8 inches on a side. The remaining space inside the cylinder will be filled with protective filler. How many cubic inches of filler will be needed per package?



5. A circular fountain is 24 feet in diameter and has a three-foot wide gravel walkway around it. What is the area of the walkway in square feet?



6. Jerry is ordering shingles for the roof of the storage building shown below. A bundle of shingles covers 3 square yards of roof. How many bundles of shingles should Jerry order?

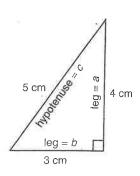


Answers start on page 444.



GED SKILL Working with Right Triangles

The Pythagorean Relationship



(117)

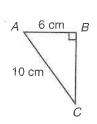
Watch for 3-4-5 right triangles in problems on the GED Math Test. Multiples are also commonly used: 6-8-10, 9-12-15, and 1.5-2-2.5.

The ancient Egyptians discovered a special property of triangles whose sides measure 3, 4, and 5 units. They learned that the angle opposite the longest side is <u>always</u> a right angle. The ancient Greeks learned why this relationship exists and named it the **Pythagorean Relationship** after the Greek mathematician Pythagoras.

The Pythagorean Relationship explains the special relationship between the **legs** (the two shorter sides) and the **hypotenuse** (the longest side) of a right triangle. It states that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. Note that a 3-4-5 triangle is always a right triangle, but a right triangle is not always a 3-4-5 triangle.

Pythagorean Relationship $a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse of a right triangle

You can use the Pythagorean Relationship to find a missing length of a right triangle.



Example 1 What is the length of side *BC* of the triangle shown to the left?

Identify the legs and the hypotenuse. Sides AB and BC are the legs. Side AC is the hypotenuse. The hypotenuse is always opposite the right angle. **Assign a variable.** Let b represent the measure of side BC, a leg, the missing measure.

Step 3 Use the Pythagorean Relationship.

$$a^{2} + b^{2} = c^{2}$$

$$6^{2} + b^{2} = 10^{2}$$

$$36 + b^{2} = 100$$

$$b^{2} = 64$$

$$b = \sqrt{64}$$

$$b = 8$$



To find the length of a side b using a calculator, rewrite the Pythagorean Relationship as $c^2 - a^2 = b^2$. Then input as follows:

10 x^2 - 6 x^2 = SHIFT x^2 8.

The length of the missing side is **8 centimeters**.

Given the lengths of the sides of a triangle, you can use the Pythagorean Relationship to determine whether or not the triangle is a right triangle.

Example 2 The lengths of the sides of a triangle are 5, 6, and 9 inches. Is the triangle a right triangle?

If the triangle is a right triangle, the longest side, 9 inches, must be the hypotenuse. Substitute the other measures for a and b. Let a = 5 and b = 6. Then solve for c. If the solution is 9, the triangle is a right triangle.

$$a^{2} + b^{2} = c^{2}$$

$$5^{2} + 6^{2} = c^{2}$$

$$25 + 36 = c^{2}$$

$$61 = c^{2}$$

$$\sqrt{61} = c \approx 7.8$$

Use your calculator on the final step, or work backwards. If the hypotenuse, c, is 9, then $c^2 = 81$.

The triangle in the example is **not** a right triangle.

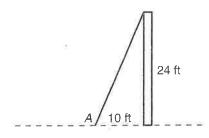
GED SKILL FOCUS

A. Each question gives the lengths of the three sides of a triangle. Write *Yes* if the triangle is right triangle. Write *No* if it is not.

8. 1
$$1\frac{1}{3}$$
 $1\frac{2}{3}$

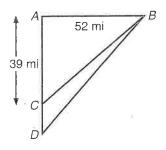
B. Solve as directed.

Refer to the diagram to answer Questions 13 and 14.



- 13. The pole is perpendicular to the ground. How many feet of wire are needed to reach from the top of the pole to point *A*?
- 14. If point *A* is moved 8 feet farther from the pole, how much more wire will be needed?

Refer to the diagram to answer Questions 15 and 16.

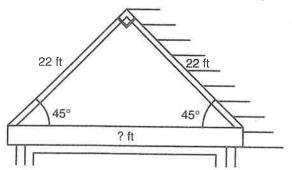


- 15. How far is it from point C to point B?
- 16. Point *D* is 11 miles from point *C*. To the nearest whole mile, how far is it from point *D* to point *B*?

GED STRATEGY Solving Word Problems =

Recognizing Applications of the Pythagorean Relationship

Many practical problems can be solved using the Pythagorean Relationship. Since diagonal bracing is often used to make structures stronger, construction situations often involve right triangles.



Example 1

A beam across the front of a garage needs to be replaced. Using the diagram at the left, what is the length of the beam to the nearest foot?

- (1) 26
- (2) 31
- (3) 44
- (4)968
- (5) Not enough information is given.



Don't assume that a triangle is a right triangle. Always look for evidence that the triangle must be a right triangle before applying the Pythagorean Relationship.

Did you realize that the front of the garage is a right triangle? The right angle is at the peak of the roof. The other angles each measure 45°. Remember that the sum of the angles in a triangle equals 180°. Even without the right angle symbol, you can prove that the peak of the roof is a right angle because $180^{\circ} - 45^{\circ} - 45^{\circ} = 90^{\circ}$. The beam across the front is the hypotenuse.

Use the Pythagorean Relationship.

$$a^{2} + b^{2} = c^{2}$$

$$22^{2} + 22^{2} = c^{2}$$

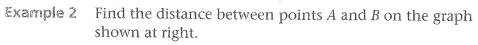
$$484 + 484 = c^{2}$$

$$968 = c^{2}$$

$$\sqrt{968} = c \approx 31.1$$

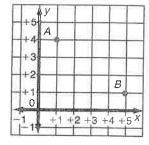
The wooden beam is about 31 feet in length. **Option 2 is correct.**

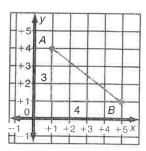
You know the distance formula for finding the distance between points on a coordinate grid. Using the Pythagorean Relationship may be easier.



- Step 1 Let the distance between the points be the hypotenuse of a right triangle. Draw in the legs of the triangle so that they form a right angle.
- **Step 2** Count units to find the lengths of the legs.
- You can solve for the length of the hypotenuse using the Pythagorean Relationship; however, this triangle is the common 3-4-5 triangle. Since the legs measure 3 and 4 units, the hypotenuse is 5.

The distance between points A and B is **5 units**.





GED Mini-Test • Lessons 25-27

Directions: This is a thirty-minute practice test. After thirty minutes, mark the last question you finished. Then complete the test and check your answers. If most of your answers are correct, but you didn't finish, try to work faster next time.

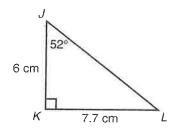


Part 1

Directions: Choose the one best answer to each question. You MAY use your calculator.

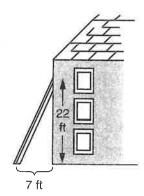
- On a map with a scale of 1.5 cm = 60 km,
 4.7 cm on the map represents how many actual kilometers?
 - (1) 40
 - (2) 117
 - (3) 188
 - (4) 282
 - (5) 423

Questions 2 and 3 refer to the following diagram.



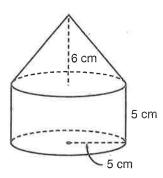
- 2. If $\triangle JKL$ is a right triangle, what is the measure of $\angle L$?
 - (1) 26°
 - (2) 38°
 - (3) 52°
 - (4) 90°
 - (5) 128°
- 3. Which of the following expressions can be used to find the measure of side \overline{JL} ?
 - (1) 6 + 7.7
 - $(2) (6)^2 + (7.7)^2$
 - (3) 6×7.7
 - (4) $\sqrt{6+7.7}$
 - (5) $\sqrt{(6)^2 + (7.7)^2}$

- 4. Which of the following is a right triangle?
 - (1) a triangle with sides of 4, 5, and 6
 - (2) a triangle with sides of 5, 7, and 9
 - (3) a triangle with sides of 6, 8, and 11
 - (4) a triangle with sides of 7, 9, and 12
 - (5) a triangle with sides of 7, 24, and 25
- 5. To the nearest foot, how long will the ladder need to be to reach a third floor window?



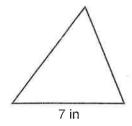
- (1) 18
- (2) 20
- (3) 22
- (4) 23
- (5) 25
- 6. A circular pool, 20 feet in diameter, has a 2-foot wide gravel walk around it. What is the approximate area of the walk in square feet?
 - (1) 100
 - (2) 138
 - (3) 144
 - (4) 314
 - (5) 452

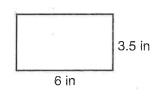
Question 7 refers to the following figure.



- 7. What is the volume to the nearest cubic centimeter of the object shown above?
 - (1) 79
 - (2) 157
 - (3) 393
 - (4) 471
 - (5) 550

Question 8 refers to the following figures.

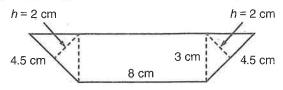




- 8. A rectangle and triangle have the same area. What is the height of the triangle in inches?
 - (1) 3
 - (2) 3.5
 - (3) 6
 - (4) 12
 - (5) 21
- 9. A 3-foot post casts a $4\frac{1}{2}$ -foot shadow at the same time that a telephone pole casts a shadow of 33 feet. What is the length in feet of the telephone pole?
 - (1) 18
 - (2) 22
 - (3) 28
 - (4) 33
 - (5) 99

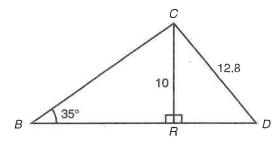
GED Mini-Test

Question 10 refers to the following figure.



- 10. What is the area in square centimeters of the figure shown?
 - (1) 27
 - (2) 28.5
 - (3) 33
 - (4) 37.5
 - (5) 42

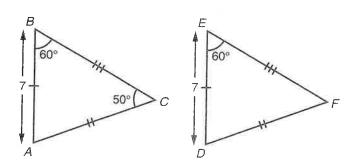
Questions 11 and 12 refer to the following figure.



- 11. To the nearest unit, what is the measure of side *RD*?
 - (1) 3
 - (2) 5
 - (3) 7
 - (4) 8
 - (5) 16
- 12. In $\triangle BRC$, what is the measure of $\angle BCR$?
 - (1) 35°
 - (2) 45°
 - (3) 55°
 - (4) 90°
 - (5) 135°

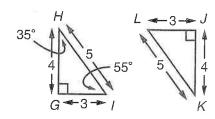
- 13. A four-sided figure has sides, in order, of 6,10, 6, and 10. There are no right angles.What is the figure?
 - (1) triangle
 - (2) square
 - (3) trapezoid
 - (4) rhombus
 - (5) parallelogram

Question 14 refers to the following figure.

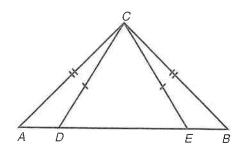


- 14. The two triangles are congruent. What is the measure of $\angle D$?
 - (1). 50°
 - (2) 70°
 - $(3) 110^{\circ}$
 - (4) 180°
 - (5) Not enough information is given.
- 15. The three sides of triangle *ABC* measure 12 feet, 16 feet, and 20 feet. Triangle *DEF* is similar to $\triangle ABC$. The shortest side of $\triangle DEF$ measures 15 feet. What are the lengths in feet of the other two sides of $\triangle DEF$?
 - (1) 18 and 23
 - (2) 18 and 25
 - (3) 19 and 23
 - (4) 20 and 24
 - (5) 20 and 25

16. What is the measure of $\angle L$?



- (1) 35°
- $(2) 45^{\circ}$
- $(3) 55^{\circ}$
- (4) 90°
- (5) Not enough information is given.
- 17. Sides AB and BC of $\triangle ABC$ each measure 10 inches. If $m \angle A = 60^{\circ}$ and $m \angle B = 60^{\circ}$, what is the length in inches of side AC?
 - (1) 5
 - (2) 6
 - (3) 10
 - (4) 14
 - (5) Not enough information is given.
- 18. In order to prove that $\triangle ACE \cong \triangle BCD$ by SSS, it is necessary to know which of the following?



- (1) $\overline{AD} \cong \overline{BC}$
- (2) $\overline{AE} \cong \overline{BD}$
- (3) $\angle DAC \cong \angle EBC$
- $(4) \ \angle CDE \cong \angle CED$
- (5) $\overline{DE} \cong \overline{EB}$

- 19. A wooden rectangular frame is 6 feet by 8 feet. A diagonal brace will be added to the back of the frame. What is the length in feet of the brace?
 - (1) 5
 - (2) 7
 - (3) 8
 - (4) 10
 - (5) 13
- 20. A circle has a radius of 7 inches. Which of the following is the best estimate of the circumference of the circle in inches?
 - (1) 15
 - (2) 25
 - (3) 40
 - (4) 50
 - (5) 150
- 1. Triangle *ABC* has sides measuring 2.9, 4.6, and 4.9 cm. Its angle measures are 78°, 36°, and 66°. Triangle *ABC* can be classified as which of the following two kinds of triangles?
 - (1) equilateral and acute
 - (2) isosceles and acute
 - (3) isosceles and obtuse
 - (4) scalene and acute
 - (5) scalene and obtuse
- 22. In a right triangle, the measure of one acute angle is five times larger than the measure of the other acute angle. Which of the following equations can be used to find the measure of the smaller angle?

$$(1) x + 5x + 90^{\circ} = 180^{\circ}$$

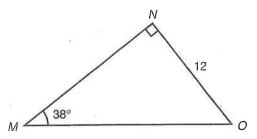
(2)
$$x + (5 + x) + 90^{\circ} = 180^{\circ}$$

(3)
$$x + 5x = 180^{\circ}$$

(4)
$$90x + 5x = 180^{\circ}$$

$$(5) 180^{\circ} - 90^{\circ} = 5x$$

Question 23 refers to the following figure.



23. Which of the following expressions could be used to find angle O?

$$(1) 38^{\circ} + 90^{\circ}$$

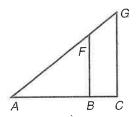
(2)
$$180^{\circ} - 90^{\circ} + 38^{\circ}$$

(3)
$$180^{\circ} - (90^{\circ} + 38^{\circ})$$

(4)
$$\sqrt{38^2 + 90^2}$$

(5) Not enough information is given.

Question 24 refers to the following figure.



24. If $\triangle ABF \sim \triangle ACG$, then which of the following is a true proportion?

$$(1) \ \frac{\overline{AF}}{\overline{AB}} = \frac{\overline{AB}}{\overline{AC}}$$

(2)
$$\frac{\overline{AB}}{\overline{AC}} = \frac{\overline{FB}}{\overline{GC}}$$

(3)
$$\frac{\overline{AF}}{\overline{AC}} = \frac{\overline{AC}}{\overline{AB}}$$

$$(4) \ \frac{\overline{AB}}{\overline{GC}} = \frac{\overline{AC}}{\overline{FB}}$$

(5)
$$\frac{\overline{AG}}{\overline{AC}} = \frac{\overline{AB}}{\overline{AF}}$$

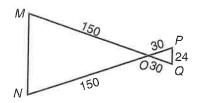
Unit 4 Cumulative Review Geometry



Part 1

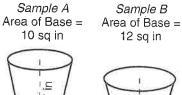
Directions: Choose the one best answer to each question. You MAY use your calculator.

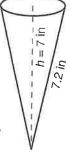
Question 1 refers to the following figure.

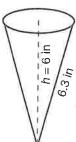


- 1. If \overline{PQ} and \overline{MN} are parallel, what is the length of \overline{MN} ?
 - (1) 48
 - (2) 60
 - (3) 90
 - (4) 120
 - (5) 150
- 2. What is the approximate area in square feet of a circular pool with a diameter of 8 feet?
 - (1) 50
 - (2) 48
 - (3) 40
 - (4) 20
 - (5) 16
- 3. The scale on a house plan is 1 inch = 2 feet. The drawing of the kitchen is 5 inches by 7 inches. What is the area of the actual kitchen in square feet?
 - (1) 12
 - (2) 24
 - (3) 35
 - (4) 70
 - (5) 140

Question 4 refers to the following information.

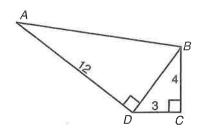






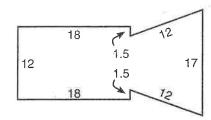
- 4. How many more cubic inches does Sample B hold than Sample A?
 - $(1) \frac{2}{3}$
 - (2) 3
 - (3) 5
 - (4) $6\frac{2}{3}$
 - (5) 9
- 5. To the nearest tenth meter, what is the circumference of a circle with a radius of 3 meters?
 - (1) 4.7
 - (2) 7.1
 - (3) 9.4
 - (4) 18.8
 - (5) 28.2
- 6. The inside volume of a refrigerator is 19.5 cubic feet. If the interior is 5.2 feet high and 1.5 feet deep, what is its width in feet?
 - (1) 2.5
 - (2) 2.9
 - (3) 3.75
 - (4) 13
 - (5) 15.2

Question 7 refers to the following figure.



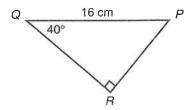
- 7. If $\angle ADB$ and $\underline{\angle C}$ are right angles, what is the length of \overline{AB} ?
 - (1) 5
 - (2) 10
 - (3) 13
 - (4) 16
 - (5) 17
- 8. The perimeter of a rectangle is 5 feet. The length is 6 inches longer than the width. What is the width in inches?
 - (1) 4
 - (2) 8
 - (3) 10
 - (4) 12
 - (5) 15

Question 9 refers to the following figure.



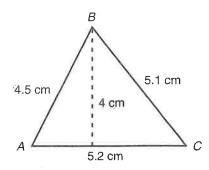
- 9. What is the perimeter of the figure?
 - (1) 72
 - (2) 75
 - (3) 90
 - (4) 92
 - (5) 101

Questions 10 and 11 refer to the following figure.



- 10. What is the measure of $\angle RPQ$?
 - (1) 40°
 - (2) 50°
 - (3) 60°
 - (4) 90°
 - $(5) 140^{\circ}$
- 11. Which of the following statements is true about △*PRQ*?
 - (1) The lengths of \overline{QR} and \overline{PR} are equal.
 - (2) The length of \overline{QP} is the shortest.
 - (3) The length of \overline{PR} is longer than the length of \overline{QR} .
 - (4) The length of \overline{QR} is longer than the length of \overline{PR} .
 - (5) The lengths of \overline{QR} , \overline{PR} , and \overline{QP} are equal.

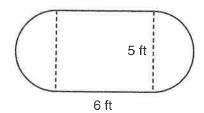
Question 12 refers to the following figure.



- 12. What is the area to the nearest square centimeter of $\triangle ABC$?
 - (1) 10
 - (2) 14
 - (3) 17
 - (4) 19
 - (5) 21

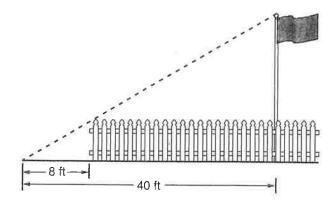
- 13. In a right triangle, the measure of one acute angle is 5 times the measure of the other acute angle. What is the measure of the smaller acute angle?
 - $(1) 10^{\circ}$
 - $(2) 15^{\circ}$
 - $(3) 20^{\circ}$
 - $(4) 25^{\circ}$
 - (5) 30°

Question 14 refers to the following figure.



- 14. A tabletop has the measurements shown in the figure. If the ends of the table are half-circles, what is the approximate area of the tabletop in square feet?
 - (1) 110
 - (2) 80
 - (3) 50
 - (4) 40
 - (5) 25
- 15. A cylinder-shaped candle mold has a base area of 12 square inches. The mold is 5 inches high. How many cubic inches of candle wax are needed to make 200 candles?
 - (1) 452,160
 - (2) 113,040
 - (3) 37,680
 - (4) 12,000
 - (5) 3,400

Question 16 refers to the following figure.



- 16. The shadows cast by a fence post and a flagpole are measured at the same time of day. If the fence post is 5 feet in height, what is the height in feet of the flagpole?
 - (1) 22
 - (2) 25
 - (3) 40
 - (4) 320
 - (5) Not enough information is given.
- 17. The scale on a map is 1 inch = 1.5 miles. The actual distance from Ivey to Garrett is 4 miles. How far apart in inches are these two cities on the map?
 - (1) less than 2
 - (2) $2\frac{1}{3}$
 - (3) $2\frac{2}{3}$
 - (4) 3
 - (5) 6
- 18. Angles M and N are complementary angles. If $\angle M$ measures 26°, what is the measure of $\angle N$?
 - (1) 26°
 - (2) 64°
 - (3) 90°
 - (4) 154°
 - (5) Not enough information is given.

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