1. A number is chosen randomly from the first billion natural numbers. The probability that the product of the number with its two immediate successors is divisible by 24 is closest to

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$

2. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{2n^2+2}{3n^2+1}$, then $\sum_{n=1}^{\infty} a_n$ is

(A) 0

(B) divergent

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

3. Each of the four entries of a 2×2 matrix is filled by independently choosing either 1 or -1 uniformly at random. What is the probablity that the matrix is singular?

(A) $\frac{1}{16}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

4. We need to fill a 3×3 matrix by either 0 or 1 such that each row has exactly one 0 and each column has exactly one 0. The number of ways we can do this is

(A) 8 (B) 6

(C) 4 (D) 2

5. Let $f : [0, 1] \to \mathbb{R}$ be a convex function with $f(0) = 0$. Which of the following is always true for f?

(A) f is differentiable

(B) f may not be differentiable but it is continuous

(C) $f(x) \geq xf'(x)$ for all $x \in [0, 1]$ if f is differentiable

(D) none of the above

6. If $f : \mathbb{R} \to \mathbb{R}$ is strictly quasi-concave, then it follows that

(A) f is not strictly convex

(B) f is not linear

(C) f is monotonic

(D) if f is quadratic, then the coefficient of $x^2 \leq 0$

7. Consider a function $f : [-1, 1] \to \mathbb{R}$ shown in Figure 1.

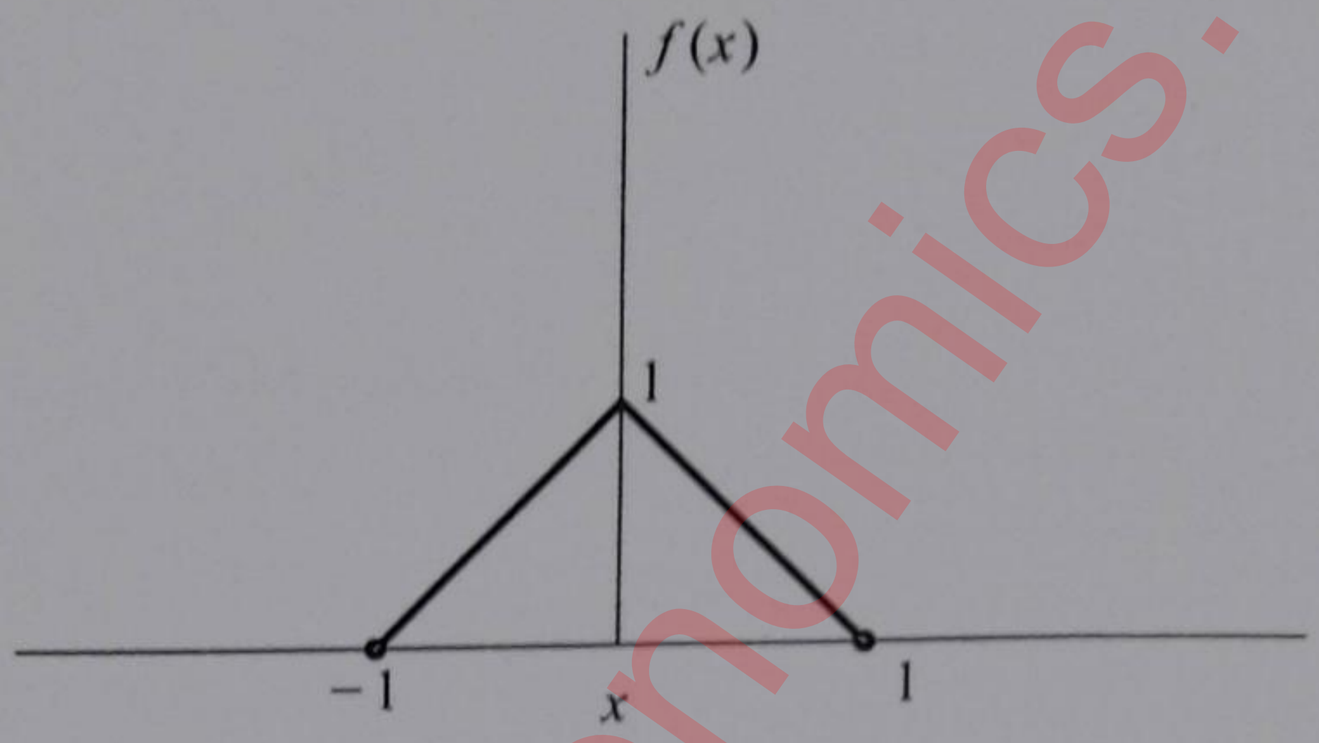

Figure 1: $f : [-1, 1] \to \mathbb{R}$

The value of $\int_{-1}^{1} f(x^2 - 1)dx$ equals

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

8. The rank of the matrix

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

is

(A) 0

(B) 1

(C) 2

(D) 3

9. Let V be the vector space of polynomials $p(x)$ of degree less than or equal to 2 that have real coefficients. Then T is a linear transformation from V to V if T is defined by

(A) $T(p(x)) = x + p(x)$

(B) $T(p(x)) = xp(x)$

(C) $T(p(x)) = \frac{dp(x)}{dx}$

(D) $T(p(x)) = \int p(x)dx$ where the constant of integration is taken to be zero.

10. X is a random variable that can take values only in $[0, 10]$. $P(X > 5) \leq \frac{2}{5}$ and $P(X < 1) \leq \frac{1}{2}$. Then

(A) $E(X) \geq 1$

(B) $E(X) \leq 5$

(C) $E(X) \geq 0.5$ and $E(X) \leq 8.5$

(D) None of the above is true

11. Using data from a sample of size n, the intercept and slope coefficients from an ordinary least squares regression of y on x, are a and b respectively. Which of the following is **false**?

(A) $\sum_{i=1}^{n}(y_i - a - bx_i)x_i = 0$

(B) $\frac{1}{n}\sum_{i=1}^{n} y_i = a + \frac{b}{n}\sum_{i=1}^{n} x_i$

(C) a and b are the solution to $\min_{\alpha,\beta} \sum_{i=1}^{n}(y_i - \alpha - \beta x_i)^2 = 0$

(D) a and b are the solution to $\min_{\alpha,\beta} \sum_{i=1}^{n} |y_i - \alpha - \beta x_i| = 0$

12. Given data $(-1, 1), (0, 0), (1, 1)$ on (x, y), the standard deviations of x and y and the correlation coefficient of x and y are respectively

(A) $\sigma_x = \sqrt{2}/\sqrt{3}, \quad \sigma_y = \sqrt{2}/3, \quad r = 0$

(B) $\sigma_x = 2/3, \quad \sigma_y = 2/9, \quad r = 0$

(C) $\sigma_x = 0, \quad \sigma_y = 0, \quad r$ is undefined

(D) None of the above

13. An island nation has two potential vaccine firms: denoted as 1 and 2. Both need to invest in R&D to manufacture vaccines. The cost of R&D for firms 1 and 2 are f_1 and f_2 respectively. Once R&D is done, the cost of per unit manufacturing of vaccine is drawn unformly from $[0, 1]$. The firms know their (fixed) cost of R&D but only know that the cost of per unit manufacturing is uniformly drawn from $[0, 1]$.

Total demand of vaccine is 1 unit and if firm $i \in \{1, 2\}$ supplies $q_i \in [0, 1]$ units and has a per unit cost of c_i, it incurs a manufacuring cost of $c_i q_i$ (along with f_i).

Suppose both firms invest in R&D but only the lowest per unit cost firm is chosen to supply the entire one unit of vaccine. What is the total expected cost of vaccination (expected cost is the fixed cost of R&D and expected cost of manufacturing)?

(A) $f_1 + f_2 + \frac{1}{2}$ (B) $f_1 + f_2 + \frac{1}{3}$
(C) $f_1 + f_2 + \frac{2}{3}$ (D) $f_1 + f_2 + \frac{3}{4}$

14. In question 13, suppose $f_1 < f_2 < \frac{1}{6}$. Consider two more alternatives:

    (b) Suppose both firms invest in R&D and both supply $\frac{1}{2}$ units of vaccine.

    (c) Only firm 1 invests in R&D and supplies the entire one unit of vaccine.

    Denote the total expected cost of vaccination from the alternatives (b) and (c) as C_2 and C_3 respectively. Denote the total expected cost of vaccination the alternative in Question 13 as C_1. Then, which of the following is true?

    (A) $C_1 < C_2 < C_3$ (B) $C_1 < C_3 < C_2$
    (C) $C_2 < C_1 < C_3$ (D) $C_3 < C_1 < C_2$

15. Consider a production function $z = 2x + 3y$. For what price ratio $\frac{p_x}{p_y}$, will a corner solution in y, i.e. $(x = 0)$ be possible, if the objective is to minimize the cost of producing a given positive quantity z_0 of z?

(A) $\frac{p_x}{p_y} = 2/3$ (B) $\frac{p_x}{p_y} > 2/3$

(C) $\frac{p_x}{p_y} < 2/3$ (D) $\frac{p_x}{p_y} \leq 2/3$

16. India and China produce only shirts and phones using only 2 factors of production: either higher skilled labour H or low skilled labour L. Shirts are high skill labour intensive while phones are low skill labour intensive. The production function for each good is identical in both countries. India and China have equal amounts of lower skilled labour, but India has a greater amount of higher skilled labour. Which good will India import?

(A) Shirts

(B) Phones

(C) Both Shirts and Phones

(D) Neither Shirts nor Phones

17. Continue with the same setup as in Question 16, but now China's population doubles. The overall welfare of the representative agent in India will:

(A) Increase

(B) Decrease

(C) Stay the same

(D) Increase or decrease

18. Virat and Mithali eat rice and drink milk in exactly the same quantities. The price of rice falls. In response, Virat increases the amount of milk but decreases the amount of rice he consumes. Mithali, on the other hand, increases both rice and milk consumption. Both Virat and Mithali spend all their income on eating rice or drinking milk. For Virat's behaviour to be consistent with standard, well-behaved indifference curves, his preferences over rice consumption imply that for him, rice must be a:

(A) Inferior good
(B) Giffen good
(C) Luxury good
(D) Normal good

19. Continue with the same setup as in Question 18, and choose the answer from below that will correctly fill in the blanks in the following sentence. With respect to rice, for Virat, the income effect must be ___ than the substitution effect while for Mithali the income effect must be ___ than the substitution effect if both Virat and Mithali have standard well-behaved preference relations.

(A) greater, lesser
(B) lesser, lesser
(C) greater, greater
(D) lesser, greater

20. Rohit spends all his money on dosas and filter coffee. He stays in Delhi where each dosa and filter coffee cost the same. He eats 15 dosas and drinks 35 filter coffees in a week. He gets a chance to move to either Chennai or Bangalore. In Chennai, he can just afford to have 40 dosas and 10 filter coffees in a week. Like in Delhi, each dosa and filter coffee cost the same. In Bangalore, he can just afford to have 10 dosas and 20 filter coffees in a week. Here, 2 filter coffees costs the same as 1 dosa. Where will Rohit prefer to stay?

(A) Delhi
(B) Chennai
(C) Bangalore
(D) Indifferent between Delhi and Chennai

21. Consider a duopoly with market demand $p = 10 - q$. The cost function of firm 1 is $7q_1$, and that of firm 2 is $2q_2$, where q_i is the quantity produced by firm i, $i = 1, 2$. In equilibrium, firm 2 charges a price of:

(A) 7
(B) 6
(C) 10
(D) 0

22. A cake of size 1 is to be divided among two individuals 1 and 2. Let x_i be the share of the cake going to individual i, $i = 1, 2$, where $0 \leq x_i \leq 1$. The utility functions are $u_1(x_1, x_2) = x_1$, and $u_2(x_1, x_2) = x_2 + |x_1 - x_2|$, where $|a|$ is the absolute value of a. The Pareto optimal cake divisions include:

(A) $(1, 0)$
(B) $(1/2, 1/2)$
(C) $(3/4, 1/4)$
(D) None of the above

23. Inventory investment can be expected to

(A) rise when the real interest rate rises, other things being equal
(B) not depend on the real interest rate, other things being equal
(C) fall when the real interest rate rises, other things being equal
(D) depend only on the change in real GDP

24. Consider the IS-LM model with the real interest rate, R, on the vertical axis and output, Y, on the horizontal axis. Now suppose that the central bank chooses R for the economy, based on its own assessment, at $R = \bar{R}$. In this case the LM curve will

(A) not exist
(B) will be horizontal at $R = \bar{R}$
(C) upward sloping like the usual LM curve
(D) None of the other options

25. If the short-run IS-LM equilibrium occurs at a level of income above the natural rate of output, in the long run output will return to the natural rate via

(A) an increase in the price level

(B) a decrease in the interest rate

(C) an increase in the money supply

(D) a downward shift of the consumption function

26. If the short-run aggregate supply curve is steep, the Phillips curve will be:

(A) flat

(B) steep

(C) backward-bending

(D) unrelated to the slope of the short-run aggregate supply curve

27. Consider a supply-demand diagram for the labor market with an upward sloping labor supply curve (L^s) and a downward sloping labor demand curve (L^d). Let the wage be on the vertical axis, and the level of employment (L) be on the horizontal axis. Suppose the wage is rigid above the equilibrium wage at $\bar{w}$, i.e., it fails to adjust to clear the labor market. Then a reduction in labor demand leads to

(A) A larger reduction in employment compared to the case if wages were flexible

(B) A smaller reduction in employment compared to the case if wages were flexible

(C) The same reduction in employment compared to the case if wages were flexible

(D) None of the other options.

28. Let the economy's production function be given by $Y = AK^{\frac{1}{3}}L^{\frac{2}{3}}$ where Y = output, $A > 0$ is the level of technology (also called total factor productivity), K is the capital stock, and L is the level of employment. Consider the standard supply-demand diagram for labor as in the previous question, except that there are no wage rigidities this time. The labor demand curve is given by ____; a negative total factor productivity shock leads to a ____in labor demand, and a ____in employment.

(A) $w = \frac{3}{2}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; upward shift; fall

(B) $w = \frac{3}{2}\frac{AK^{\frac{2}{3}}}{L^{\frac{1}{3}}}$; downward shift; fall

(C) $w = \frac{2}{3}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; downward shift; fall

(D) $w = \frac{1}{3}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; downward shift; rise

29. Suppose there are two countries, B and C, that have no trade and no financial transactions with any countries except each other. B imports a total of goods worth 10 million bollars from C, where a bollar is a unit of B's currency. B has no exports. Which of the following must be true?

(A) B has a capital account deficit

(B) C has a current account deficit

(C) C is buying assets from B.

(D) The exchange rate of collars per bollar is bigger than 1, where a collar is a unit of C's currency.

30. There are no capital controls between the US and the UK. If the interest rate is higher in the US than in the UK, then we can conclude that

(A) The US dollar is expected to appreciate with respect to the pound (the UK's currency)

(B) The pound is expected to appreciate with respect to the US dollar

(C) The interest rate in the US is expected to increase

(D) The interest rate in the US is expected to decrease