

## IIT-JAM 2020 (Physics)

### Paper Specific Instructions:

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, A, B and C. All sections are compulsory. Questions in each section are of different types.
2. Section – A contains a total of 30 Multiple Choice Questions (MCQ). Each MCQ type question has four choices out of which only one choice is the correct answer. Questions Q1.-Q30. Belong to this section and carry a total of 50 marks. Q1. - Q10. carry 1 mark each and Questions Q11. - Q30. carry 2 marks each.
3. Section – B contains a total of 10 Multiple Select Questions (MSQ). Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q31. - Q40. belong to this section and carry 2 marks each with a total of 20 marks.
4. Section – C contains a total of 20 Numerical Answer Type (NAT) questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q41. - Q60. belong to this section and carry a total of 30 marks. Q41. - Q50. carry 1 mark each and Questions Q51. - Q60. carry 2 marks each.
5. In all sections, questions not attempted will result in zero mark. In Section – A (MCQ), wrong answer will result in NEGATIVE marks. For all 1 mark questions,  $1/3$  marks will be deducted for each wrong answer. For all 2 marks questions,  $2/3$  marks will be deducted for each wrong answer. In Section – B (MSQ), there is NO NEGATIVE and NO PARTIAL marking provisions. There is NO NEGATIVE marking in Section – C (NAT) as well.
6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are NOT allowed in the examination hall.
7. The Scribble Pad will be provided for rough work.

## SECTION – A

### MULTIPLE CHOICE QUESTIONS (MCQ)

**Q1 – Q10 carry one mark each.**

Q1. Which one of the following functions has a discontinuity in the second derivative at  $x = 0$ , where  $x$  is a real variable?

- (a)  $f(x) = |x|^3$       (b)  $f(x) = x|x|$       (c)  $f(x) = \cos(|x|)$       (d)  $f(x) = |x|^2$

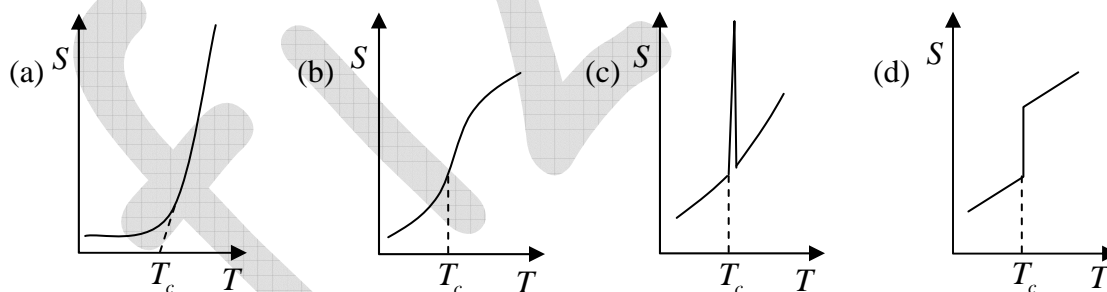
Q2. A collimated beam of laser light of wavelength  $514\text{nm}$  is normally incident on a smooth glass slab placed in air. Given the refractive indices of glass and air are 1.47 and 1.0, respectively, the percentage of light intensity reflected back is

- (a) 0      (b) 4.0      (c) 3.6      (d) 4.2

Q3. Two stationary point particles with equal and opposite charges are at some fixed distance from each other. The points having zero electric potential lie on:

- (a) A sphere      (b) A plane      (c) A cylinder      (d) Two parallel planes

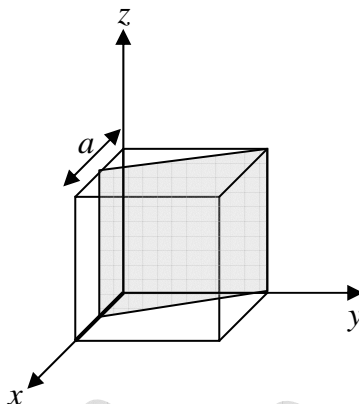
Q4. For a system undergoing a first order phase transition at a temperature  $T_c$ , which one of the following graphs best describes the variation of entropy ( $S$ ) as a function of temperature ( $T$ )?



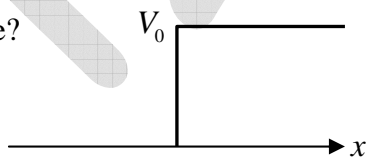
Q5. In a photoelectric effect experiment, a monochromatic light source emitting photons with energy greater than the work function of the metal under test is used. If the power of the light source is doubled, which one of the following statements is correct?

- (a) The number of emitted photoelectrons remains the same  
 (b) The stopping potential remains the same  
 (c) The number of emitted photoelectrons decreases  
 (d) The stopping potential doubles

- Q6. The figure below shows a cubic unit cell with lattice constant  $a$ . The shaded crystallographic plane intersects the  $x$ -axis at  $0.5a$ . The Miller indices of the shaded plane are



- (a)  $(210)$                       (b)  $(\bar{2}10)$                       (c)  $(110)$                       (d)  $(102)$
- Q7. For a particle moving in a central potential, which one of the following statements is correct?
- (a) The motion is restricted to a plane due to the conservation of angular momentum  
 (b) The motion is restricted to a plane due to the conservation of energy only  
 (c) The motion is restricted to a plane due to the conservation of linear momentum  
 (d) The motion is not restricted to a plane
- Q8. Consider the motion of a quantum particle of mass  $m$  and energy  $E$  under the influence of a step potential of height  $V_0$ . If  $R$  denotes the reflection coefficient, which one of the following statements is true?



- (a) If  $E = \frac{4}{3}V_0$ ,  $R = 1$                       (b)  $E = \frac{4}{3}V_0$ ,  $R = 0$   
 (c)  $E = \frac{1}{2}V_0$ ,  $R = 1$                       (d)  $E = \frac{1}{2}V_0$ ,  $R = 0.5$
- Q9. The Boolean function  $\overline{PQ}(\bar{P} + Q)(Q + \bar{Q})$  is equivalent to:
- (a)  $P$                       (b)  $\bar{P}$                       (c)  $\bar{P}Q$                       (d)  $P\bar{Q}$

Q10. Three point charges each carrying a charge  $q$  are placed on the vertices of an equilateral triangle of side  $L$ . The electrostatic potential energy of the configuration is:

- (a)  $\frac{1}{4\pi\epsilon_0} \frac{q^2}{L}$  (b)  $\frac{2}{4\pi\epsilon_0} \frac{q^2}{L}$  (c)  $\frac{3}{4\pi\epsilon_0} \frac{q^2}{L}$  (d)  $\frac{1}{\pi\epsilon_0} \frac{q^2}{L}$

**Q11 - Q30 carry two marks each.**

Q11. Which of the following statements is correct?

Given  $\binom{n}{m} = \frac{n!}{m!(n-m)!}$  is the binomial coefficient

- (a)  $\cos n\theta = \cos^n \theta - \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \binom{n}{4} \cos^{n-4} \theta \sin^4 \theta - \dots$   
 (b)  $\sin n\theta = \binom{n}{1} \cos^{n-1} \theta \sin \theta + \binom{n}{3} \cos^{n-3} \theta \sin^3 \theta + \dots$   
 (c)  $\cos n\theta = \cos^n \theta + \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \binom{n}{4} \cos^{n-4} \theta \sin^4 \theta + \dots$   
 (d)  $\sin n\theta = \cos^n \theta - \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \binom{n}{4} \cos^{n-4} \theta \sin^4 \theta - \dots$

Q12. The volume integral  $\int_V e^{-\left(\frac{r}{R}\right)^2} \bar{\nabla} \cdot \left( \frac{\hat{r}}{r^2} \right) d^3r$ , where  $V$  is the volume of a sphere of radius  $R$  centered at the origin, is equal to

- (a)  $4\pi$  (b) 0 (c)  $\frac{4}{3}\pi R^3$  (d) 1

Q13.  $\lim_{x \rightarrow 0^+} x^{-x}$  is equal to

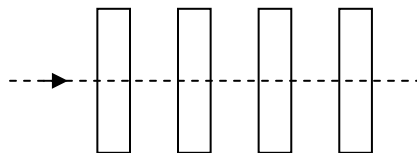
- (a) 0 (b)  $\infty$  (c)  $e$  (d) 1

Q14. A wheel is rotating at a frequency  $f_0$  Hz about a fixed vertical axis. The wheel stops in  $t_0$  seconds, with constant angular deceleration. The number of turns covered by the wheel before it comes to rest is given by

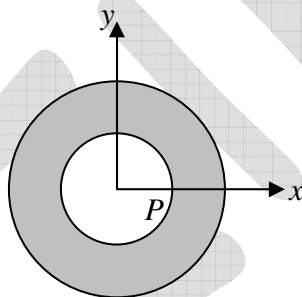
- (a)  $f_0 t_0$  (b)  $2f_0 t_0$  (c)  $\frac{f_0 t_0}{2}$  (d)  $\frac{f_0 t_0}{\sqrt{2}}$

- Q15. Two objects of masses  $m$  and  $2m$  are moving at speeds of  $v$  and  $v/2$ , respectively. After undergoing a completely inelastic collision, they move together with a speed of  $v/3$ . The angle between the initial velocity of the two objects is
- (a)  $60^\circ$  (b)  $120^\circ$  (c)  $45^\circ$  (d)  $90^\circ$
- Q16. Two planets  $P_1$  and  $P_2$  having masses  $M_1$  and  $M_2$  revolve around the Sun in elliptical orbits, with time periods  $T_1$  and  $T_2$  respectively. The minimum and maximum distances of planet  $P_1$  from the Sun are  $R$  and  $3R$  respectively, whereas for planet  $P_2$  these are  $2R$  and  $4R$ , respectively, where  $R$  is a constant. Assuming  $M_1$  and  $M_2$  are much smaller than the mass of the Sun, the magnitude of  $\frac{T_2}{T_1}$  is
- (a)  $\frac{2}{3}\sqrt{\frac{2M_1}{3M_2}}$  (b)  $\frac{3}{2}\sqrt{\frac{3M_2}{2M_1}}$  (c)  $\frac{3}{2}\sqrt{\frac{3}{2}}$  (d)  $\frac{2}{3}\sqrt{\frac{2}{3}}$
- Q17. The intensity of the primary maximum in a two-slit interference pattern is given by  $I_2$  and the intensity of the primary maximum in a three-slit interference pattern is given by  $I_3$ . Assuming the far-field approximation, same slit parameters and intensity of the incident light in both the cases,  $I_2$  and  $I_3$  are related as
- (a)  $I_2 = \frac{3}{2}I_3$  (b)  $I_2 = \frac{9}{4}I_3$  (c)  $I_2 = \frac{2}{3}I_3$  (d)  $I_2 = \frac{4}{9}I_3$
- Q18. A short rod of length  $L$  and negligible diameter lies along the optical axis of concave mirror at a distance of  $3m$ . The focal length of the mirror is  $1m$  and  $L \ll 1m$ . If  $L'$  is the length of image of the object in the mirror, then
- (a)  $\frac{L'}{L} = 4$  (b)  $\frac{L'}{L} = 2$  (c)  $\frac{L'}{L} = \frac{1}{16}$  (d)  $\frac{L'}{L} = \frac{1}{4}$

- Q19. A beam of unpolarized light of intensity  $I_0$  falls on a system of four identical linear polarizers placed in a line as shown in figure. The transmission axes of any two successive polarizers make an angle of  $30^\circ$  with each other. If the transmitted light has intensity  $I$ , the ratio  $\frac{I}{I_0}$  is



- (a)  $\frac{81}{256}$       (b)  $\frac{9}{16}$       (c)  $\frac{27}{64}$       (d)  $\frac{27}{128}$
- Q20. Consider an annular region in free space containing a uniform magnetic field in the  $z$ -direction, schematically represented by the shaded region in the figure. A particle having charge  $Q$  and mass  $M$  starts off from point  $P(a, 0, 0)$  in the  $+x$ -direction with constant speed  $v$ . If the radii of inner and outer circles are  $a$  and  $b$ , respectively, the minimum magnetic field required so that the particle returns to the inner circle is



- (a)  $\frac{Mv}{Q} \left( \frac{b^2 - a^2}{b} \right)^{-1}$       (b)  $\frac{Mv}{Q} \left( \frac{b^2 - a^2}{2b} \right)^{-1}$
- (c)  $\frac{Mv}{Q} \left( \frac{b^2 - a^2}{3b} \right)^{-1}$       (d)  $\frac{Mv}{Q} \left( \frac{b^2 - a^2}{4b} \right)^{-1}$
- Q21. A thin conducting square loop of side  $L$  is placed in the first quadrant of the  $xy$ -plane with one of the vertices at the origin. If a changing magnetic field  $\vec{B}(t) = \beta_0 (5zyt \hat{x} + zxt \hat{y} + 3y^2t \hat{z})$  is applied, where  $\beta_0$  is a constant, then the magnitude of the induced electromotive force in the loop is
- (a)  $4\beta_0 L^4$       (b)  $3\beta_0 L^4$       (c)  $2\beta_0 L^4$       (d)  $\beta_0 L^4$

Q22. In which one of the following limits the Fermi-Dirac distribution  $n_F(\varepsilon, T) = \left( e^{\frac{\varepsilon - \mu}{k_B T}} + 1 \right)^{-1}$

and Bose-Einstein distribution  $n_B(\varepsilon, T) = \left( e^{\frac{\varepsilon - \mu}{k_B T}} - 1 \right)^{-1}$  reduce to Maxwell-Boltzmann

distribution? (Here  $\varepsilon$  is the energy of the state,  $\mu$  is the chemical potential,  $k_B$  is the Boltzmann constant and  $T$  is the temperature)

(a)  $\mu = 0$  (b)  $(\varepsilon - \mu) \ll k_B T$

(c)  $(\varepsilon - \mu) \gg k_B T$  (d)  $\mu \gg k_B T$

Q23. Consider  $N$  classical particles at temperature  $T$ , each of which can have two possible energies 0 and  $\varepsilon$ . The number of particles in the lower energy level ( $N_0$ ) and higher energy level ( $N_\varepsilon$ ) levels are related by

( $k_B$  is the Boltzmann constant)

(a)  $\frac{N_0}{N_\varepsilon} = e^{\frac{-\varepsilon}{k_B T}}$  (b)  $\frac{N_0}{N_\varepsilon} = e^{\frac{\varepsilon}{k_B T}}$  (c)  $\frac{N_0}{N_\varepsilon} = 1 + e^{\frac{\varepsilon}{k_B T}}$  (d)  $\frac{N_0}{N_\varepsilon} = 1 + e^{\frac{-\varepsilon}{k_B T}}$

Q24. The root mean square (*rms*) speeds of Hydrogen atoms at 500 K,  $V_H$  and Helium atoms at 2000 K,  $V_{He}$  are related as

(a)  $V_H > V_{He}$  (b)  $V_H < V_{He}$  (c)  $V_H = V_{He}$  (d)  $V_H \gg V_{He}$

Q25. The normalized ground-state wave function of a one-dimensional quantum harmonic oscillator with force constant  $K$  and mass  $m$  is  $\psi_0(x) = \left( \frac{\alpha}{\pi} \right)^{1/4} e^{-\alpha x^2/2}$ , where  $\alpha = m\omega_0/\hbar$

and  $\omega_0^2 = K/m$ . Which one of the following is the probability of finding the particle outside the classically allowed region? (The classically allowed region is where the total energy is greater than the potential energy)

(a)  $\frac{2}{\sqrt{\pi}} \int_1^\infty y^2 e^{-y^2} dy$  (b)  $\frac{2}{\sqrt{\pi}} \int_1^\infty e^{-y^2} dy$

(c) 0.5 (d) 0



Q26. A linear operator  $\hat{O}$  acts on two orthonormal states of a system  $\psi_1$  and  $\psi_2$  as per following:  $\hat{O}\psi_1 = \psi_2$ ,  $\hat{O}\psi_2 = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$ . The system is in a superposed state defined by

$\psi = \frac{1}{\sqrt{2}}\psi_1 + \frac{i}{\sqrt{2}}\psi_2$ . The expectation value of  $\hat{O}$  in the state  $\psi$  is

(a)  $\frac{1}{2\sqrt{2}}(1+i(\sqrt{2}+1))$

(b)  $\frac{1}{2\sqrt{2}}(1-i(\sqrt{2}+1))$

(c)  $\frac{1}{2\sqrt{2}}(1+i(\sqrt{2}-1))$

(d)  $\frac{1}{2\sqrt{2}}(1-i(\sqrt{2}-1))$

Q27. Consider a one-dimensional infinite potential well of width  $a$ . The system contains five non-interacting electrons, each of mass  $m$ , at temperature  $T=0K$ . The energy of the highest occupied state is

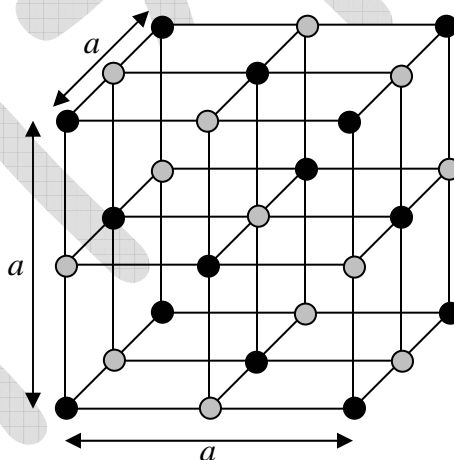
(a)  $\frac{25\pi^2\hbar^2}{2ma^2}$

(b)  $\frac{10\pi^2\hbar^2}{2ma^2}$

(c)  $\frac{5\pi^2\hbar^2}{2ma^2}$

(d)  $\frac{9\pi^2\hbar^2}{2ma^2}$

Q28. Consider the crystal structure shown in the figure, where black and grey spheres represent atoms of two different elements and  $a$  denotes the lattice constant. The Bravais lattice for this structure is



(a) Simple cubic

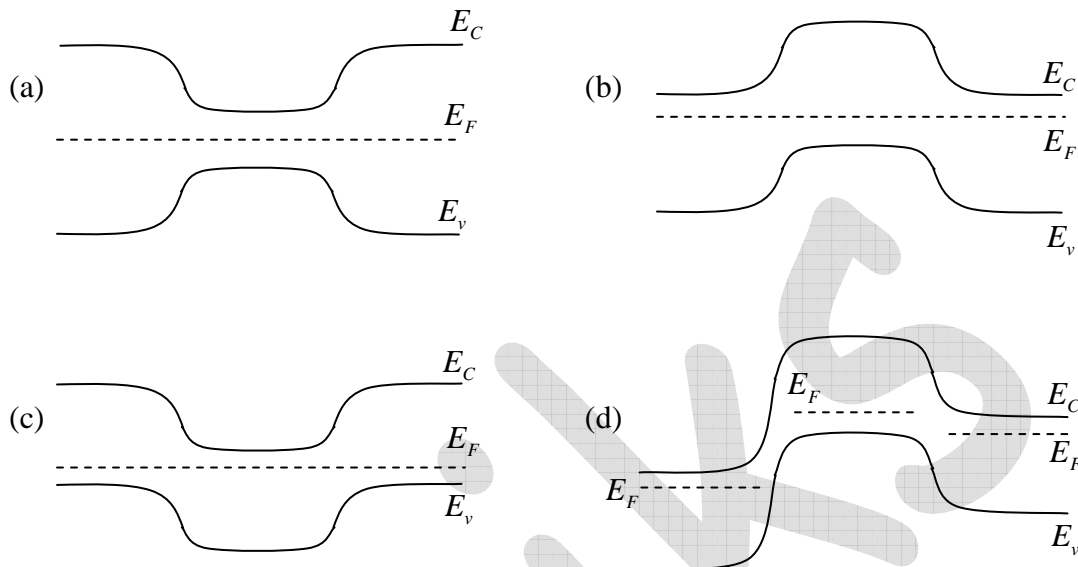
(b) Face-centered cubic

(c) Body-centered cubic

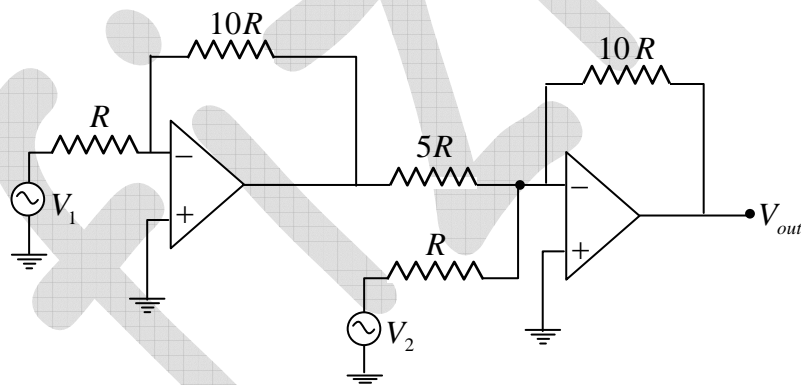
(d) Triclinic



- Q29. For an unbiased Silicon  $n-p-n$  transistor in thermal equilibrium, which one of the following electronic energy band diagrams is correct? ( $E_c$  = conduction band minimum,  $E_v$  = valence band maximum,  $E_F$  = Fermi level).



- Q30. In the circuit shown in the figure, both OPAMPs are ideal. The output for the circuit  $V_{out}$  is



- (a)  $20V_1 + 10V_2$   
 (b)  $-20V_1 + 10V_2$   
 (c)  $10V_1 - 20V_2$   
 (d)  $20V_1 - 10V_2$

## SECTION - B

## MULTIPLE SELECT QUESTIONS (MSQ)

**Q31 – Q40 carry two marks each.**

Q31. If  $P$  and  $Q$  are Hermitian matrices which of the following is/are true?

(A matrix  $P$  is Hermitian if  $P = P^\dagger$ , where the elements  $P_{ij}^\dagger = P_{ji}^*$ )

- (a)  $PQ + QP$  is always Hermitian
- (b)  $i(PQ - QP)$  is always Hermitian
- (c)  $PQ$  is always Hermitian
- (d)  $PQ - QP$  is always Hermitian

Q32. Consider a vector function  $\vec{u}(\vec{r})$  and two scalar functions  $\psi(\vec{r})$  and  $\phi(\vec{r})$ . The unit vector  $\hat{n}$  is normal to the elementary surface  $dS$ ,  $dV$  is an infinitesimal volume  $d\vec{l}$  is an infinitesimal line element and  $\frac{\partial}{\partial n}$  denotes the partial derivative along  $\hat{n}$ . Which of the following identities is/are correct?

- (a)  $\int_V \vec{\nabla} \cdot \vec{u} dV = \oint_S \vec{u} \cdot \hat{n} dS$ , where surface  $S$  bounds the volume  $V$
- (b)  $\int_V [\psi \nabla^2 \phi - \phi \nabla^2 \psi] dV = \oint_S \left[ \psi \frac{\partial \phi}{\partial n} - \phi \frac{\partial \psi}{\partial n} \right] dS$ , where surface  $S$  bounds the volume  $V$
- (c)  $\int_V [\psi \nabla^2 \phi - \phi \nabla^2 \psi] dV = \oint_S \left[ \psi \frac{\partial \phi}{\partial n} + \phi \frac{\partial \psi}{\partial n} \right] dS$ , where surface  $S$  bounds the volume  $V$
- (d)  $\oint_C \vec{u} \cdot d\vec{l} = \iint_S (\vec{\nabla} \times \vec{u}) \cdot \hat{n} dS$ , where  $C$  is the boundary of surface  $S$

Q33. A thin rod of uniform density and length  $2\sqrt{3}m$  is undergoing small oscillations about a pivot point. The time period of oscillation ( $T_m$ ) is minimum when the distance of the pivot point from the center-of-mass of the rod is  $x_m$ . Which of the following is/are correct?

- (a)  $x_m = 1m$
- (b)  $x_m = \frac{\sqrt{3}}{2}m$
- (c)  $T_m = \frac{2\pi}{\sqrt{3}}s$
- (d)  $T_m = \frac{2\pi}{\sqrt{5}}s$

Q34. Three sinusoidal waves off the same frequency travel with the same speed along the positive  $x$  - direction. The amplitudes of the waves are  $a$ ,  $\frac{a}{2}$  and  $\frac{a}{3}$  and the phase constants of the waves are  $\frac{\pi}{2}$ ,  $\pi$  and  $\frac{3\pi}{2}$ , respectively. If  $A_m$  and  $\varphi_m$  are the amplitude and phase constant of the wave resulting from the superposition of the three waves, which of the following is correct?

(a)  $A_m = \frac{5}{6}a$

(b)  $\varphi_m = \frac{\pi}{2} + \tan^{-1}\left(\frac{3}{4}\right)$

(c)  $A_m = \frac{7}{6}a$

(d)  $\varphi_m = \tan^{-1}\left(\frac{2}{3}\right)$

Q35. An object executes simple harmonic motion along the  $x$ -direction with angular frequency  $\omega$  and amplitude  $a$ . The speed of the object is  $4\text{ cm/s}$  and  $2\text{ cm/s}$  when it is at distances  $2\text{ cm}$  and  $6\text{ cm}$  respectively from the equilibrium position. Which of the following is/are correct?

(a)  $\omega = \sqrt{\frac{3}{8}}\text{ rad/s}$

(b)  $\omega = \sqrt{\frac{5}{6}}\text{ rad/s}$

(c)  $\omega = \sqrt{\frac{140}{3}}\text{ cm}$

(d)  $\omega = \sqrt{\frac{175}{6}}\text{ cm}$

Q36. For electric and magnetic field  $\vec{E}$  and  $\vec{B}$ , due to a charge density  $\rho(\vec{r}, t)$  and a current density  $\vec{J}(\vec{r}, t)$ , which of the following relations is/are always correct?

(a)  $\vec{\nabla} \times \vec{E} = 0$

(b)  $\vec{\nabla} \cdot \vec{B} = 0$

(c)  $\vec{\nabla} \cdot \vec{J} - \frac{\partial \rho}{\partial t} = 0$

(d)  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ , where  $\vec{F}$  is the force on a particle with charge  $q$  moving with velocity  $\vec{v}$

Q37. A spherical dielectric shell with inner radius  $a$  and outer radius  $b$ , has polarization

$$\vec{P} = \frac{k}{r^2} \hat{r}, \text{ where } k \text{ is a constant and } \hat{r} \text{ is the unit vector along the radial direction. Which}$$

of the following statements is/are correct?

(a) The surface density of bound charges on the inner and outer surfaces are  $-k$  and  $+k$ , respectively. The volume density of bound charges inside the dielectric is zero

(b) The surface density of bound charges is zero on both the inner and outer surfaces. The volume density of bound charges inside the dielectric is  $+k$

(c) The surface density of bound charges on the inner and outer surfaces are  $\frac{-k}{a^2}$  and  $\frac{k}{b^2}$ , respectively. The volume density of bound charges inside the dielectric is zero

(d) The surface density of bound charges is zero on both the inner and outer surfaces. The volume density of bound charges inside the dielectric is  $\frac{3k}{4\pi(b^2 - a^2)}$

Q38. One mole of an ideal gas having specific heat ratio ( $\gamma$ ) of 1.6 is mixed with one mole of another ideal gas having specific heat ratio of 1.4. If  $C_v$  and  $C_p$  are the molar specific heat capacities of the gas mixture at constant volume and pressure, respectively, which of the following is/are correct?

( $R$  denotes the universal gas constant).

(a)  $C_v = 2.08 R$

(b)  $C_p = 2.9 R$

(c)  $C_p = 1.48 C_v$

(d)  $C_p = 1.52 C_v$

Q39. Two relativistic particles with opposite velocities collide head-on and come to rest by sticking with each other. Which of the following quantities is/are conserved in the collision?

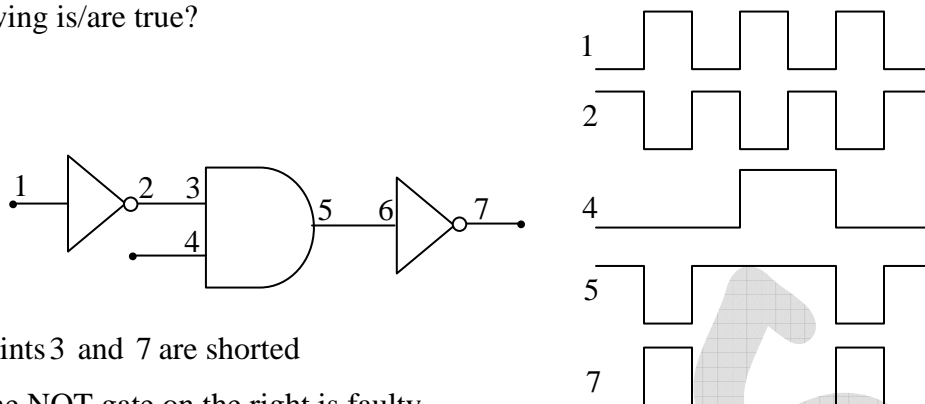
(a) Total momentum

(b) Total energy

(c) Total kinetic energy

(d) Total rest mass

- Q40. Figure shows a circuit diagram comprising Boolean logic gates and the corresponding timing diagrams show the digital signals at various points in the circuit. Which of the following is/are true?



- (a) points 3 and 7 are shorted
- (b) The NOT gate on the right is faulty
- (c) The AND gate is faulty and acts like a NOR gate
- (d) The AND gate is faulty and acts like an OR gate

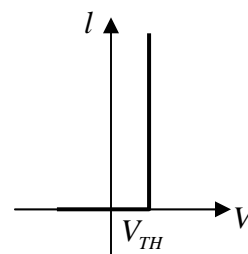
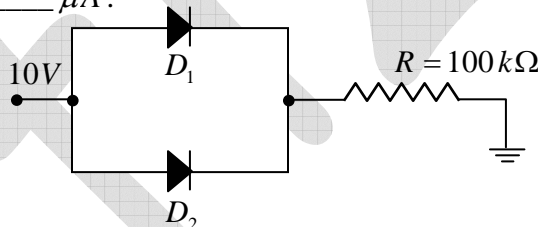
## SECTION - C

### NUMERICAL ANSWER TYPE (NAT)

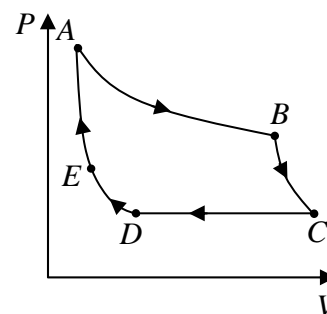
**Q41 - Q50 carry one mark each.**

- Q41. The line integral of the vector function  $u(x, y) = 2y\hat{i} + x\hat{j}$  along the straight line from  $(0, 0)$  to  $(2, 4)$  is \_\_\_\_\_
- Q42. Consider a thin bi-convex lens of relative refractive index  $n = 1.5$ . The radius of curvature of one surface of the lens is twice that of the other. The magnitude of larger radius of curvature in units of the focal length of the lens is \_\_\_\_\_ (Round off to 1 decimal place)
- Q43. Water flows in a horizontal pipe in a streamlined manner at an absolute pressure of  $4 \times 10^5 \text{ Pa}$  and speed of  $6 \text{ m/s}$ . If it exists the pipe at a pressure of  $10^5 \text{ Pa}$ , the speed of water at the exit point is \_\_\_\_\_ m/s (Round off to 1 decimal place)  
(The density of water is  $1000 \text{ kg/m}^3$ )
- Q44. Consider a retarder with refractive indices  $n_e = 1.551$  and  $n_o = 1.542$  along the extraordinary and ordinary axes, respectively. The thickness of this retarder for which a left circularly polarized light of wavelength  $600 \text{ nm}$  will be converted into a right circularly polarized light is \_\_\_\_\_  $\mu\text{m}$ .

- Q45. Using a battery a  $10\text{ pF}$  capacitor is charged to  $50\text{ V}$  and then the battery is removed. After that, a second uncharged capacitor is connected to the first capacitor in parallel. If the final voltage across the second capacitor is  $20\text{ V}$ , its capacitance is \_\_\_\_\_  $\text{pF}$ .
- Q46. Consider two spherical perfect blackbodies with radii  $R_1$  and  $R_2$  at temperatures  $T_1 = 1000\text{ K}$  and  $T_2 = 2000\text{ K}$ , respectively. They both emit radiation of power  $1\text{ kW}$ . The ratio of their radii  $R_1 / R_2$  is given by \_\_\_\_\_
- Q47. In a Compton scattering experiment, the wavelength of incident  $X$ -rays is  $0.500\text{ \AA}$ . If the Compton wavelength  $\lambda_c$  is  $0.024\text{ \AA}$ , the value of the longest wavelength possible for the scattered  $X$ -ray is \_\_\_\_\_  $\text{\AA}$  (specify up to 3 decimal places)
- Q48. A solid with FCC crystal structure is probed using  $X$ -rays of wavelength  $0.2\text{ nm}$ . For the crystallographic plane given by  $(2,0,0)$ , a first order diffraction peak is observed for a Bragg angle of  $21^\circ$ . The unit cell size is \_\_\_\_\_  $\text{nm}$  (Round off to 2 decimal places)
- Q49. The figure shows a circuit containing two diodes  $D_1$  and  $D_2$  with threshold voltages  $V_{TH}$  of  $0.7\text{ V}$  and  $0.3\text{ V}$ , respectively. Considering the simplified diode model, which assumes diode  $I$ - $V$  characteristic as shown in the plot on the right, the current through the resistor  $R$  is \_\_\_\_\_  $\mu\text{A}$ .



- Q50. An ideal gas undergoes an isothermal expansion along a path  $AB$ , adiabatic expansion along  $BC$ , isobaric compression along  $CD$ , isothermal compression along  $DE$ , and adiabatic compression along  $EA$ , as shown in the figure. The work done by the gas along the process  $BC$  is  $10\text{ J}$ . The change in the internal energy along process  $EA$  is  $16\text{ J}$ . The absolute value of the change in the internal energy along the process  $CD$  is \_\_\_\_\_  $\text{J}$



**Q51 - Q60 carry two marks each.**

Q51. If a function  $y(x)$  is described by the initial-value problem  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ , with initial conditions  $y(0) = 2$  and  $\left(\frac{dy}{dx}\right)_{x=0} = 0$ , then the value of  $y$  at  $x=1$  is \_\_\_\_\_

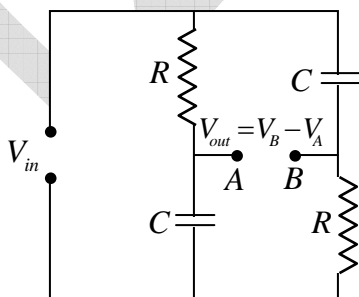
(Round off to 2 decimal places)

Q52. A vehicle of mass  $600\text{ kg}$  with an engine operating at constant power  $P$  accelerates from rest on a straight horizontal road. The vehicle covers a distance of  $600\text{ m}$  in 1 minute. Neglecting all losses, the magnitude of  $P$  is \_\_\_\_\_  $\text{kW}$ . (Round off to 2 decimal places)

Q53. The angular momentum of a particle relative to origin varies with time ( $t$ ) as  $\vec{L} = (4\hat{x} + at^2\hat{y})\text{ kg m}^2/\text{s}$ , where  $a = 1\text{ kg m}^2/\text{s}^3$ . The angle between  $\vec{L}$  and the torque acting on the particle becomes  $45^\circ$  after a time of \_\_\_\_\_  $\text{s}$ .

Q54. Two transverse waves  $y_1 = 5\cos(kx - \omega t)\text{ cm}$ , and  $y_2 = 5\cos(kx + \omega t)\text{ cm}$ , travel on a string along  $x$ -axis. If the speed of a point at  $x=0$  is zero at  $t=0\text{ s}$ ,  $0.25\text{ s}$  and  $0.5\text{ s}$ , then the minimum frequency of the waves is \_\_\_\_\_  $\text{Hz}$ .

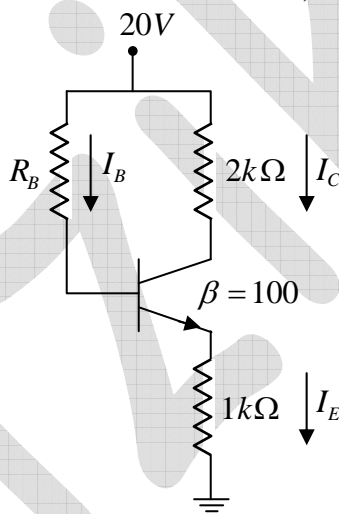
Q55. For the  $ac$  circuit shown in the figure,  $R = 100\text{ k}\Omega$  and  $C = 10\text{ pF}$ , the phase difference between  $V_{in}$  and  $V_{out}$  is  $90^\circ$  at the input signal frequency of \_\_\_\_\_  $\text{kHz}$ . (Round off to 2 decimal places)



Q56. The magnetic fields in tesla in the two regions separated by the  $z=0$  plane are given by  $\vec{B}_1 = 3\hat{x} + 5\hat{z}$  and  $\vec{B}_2 = \hat{x} + 3\hat{y} + 5\hat{z}$ . The magnitude of the surface current density at the interface between the two regions is  $\alpha \times 10^6\text{ A/m}$ . Given the permeability of the free space  $\mu_0 = 4\pi \times 10^{-7}\text{ N/A}^2$ , the value of  $\alpha$  is \_\_\_\_\_. (Round off 2 decimal places)



- Q57. A body at a temperature  $T$  is brought into contact with a reservoir at temperature  $2T$ . Thermal equilibrium is established at constant pressure. The heat capacity of the body at constant pressure is  $C_p$ . The total change in entropy of the body and the reservoir in units of  $C_p$  is \_\_\_\_\_. (Round off 2 decimal places)
- Q58. One mole of an ideal monoatomic gas at pressure  $P$ , volume  $V$  and temperature  $T$  is expanded isothermally to volume  $4V$ . Thereafter, the gas is heated isochorically (at constant volume) till its pressure becomes  $P$ . If  $R$  is the universal gas constant, the total heat transfer in the process, in units of  $RT$  is \_\_\_\_\_. (Round off 2 decimal places)
- Q59. In the transistor circuit given in the figure, the emitter-base junction has a voltage drop of  $0.7V$ . A collector-emitter voltage of  $14V$  reverse biases the collector. Assuming the collector current to be the same as the emitter current, the value of  $R_B$  is \_\_\_\_\_  $k\Omega$ .



- Q60. The radioactive nuclei  $^{40}K$  decay to  $^{40}Ar$  with a half-life of  $1.25 \times 10^9$  years. The  $\frac{^{40}K}{^{40}Ar}$  isotopic ratio for a particular rock is found to be 50. The age of the rock is  $m \times 10^7$  years. The value of  $m$  is \_\_\_\_\_. (Round off to 2 decimal places)