Indian Statistical Institute

MSQE (PEA)
Past Year Papers (2006-2024)

Contents

1 ISI PEA 2006

1. If $f(x) = \log\left(\frac{1+x}{1-x}\right), \quad 0 < x < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ equals

   A. $2f(x)$

   B. $\frac{f(x)}{2}$

   C. $(f(x))^2$

   D. none of these

2. If $u = \phi(x-y, y-z, z-x)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ equals

   A. 0

   B. 1

   C. u

   D. none of these

3. Let A and B be disjoint sets containing m and n elements, respectively, and let $C = A \cup B$. The number of subsets S of C that contain k elements and that also have the property that $S \cap A$ contains i elements is

   A. $\begin{pmatrix} m \\ i \end{pmatrix}$

   B. $\begin{pmatrix} n \\ i \end{pmatrix}$

   C. $\begin{pmatrix} m \\ k-i \end{pmatrix}\begin{pmatrix} n \\ i \end{pmatrix}$

   D. $\begin{pmatrix} m \\ i \end{pmatrix}\begin{pmatrix} n \\ k-i \end{pmatrix}$

4. The number of disjoint intervals over which the function $f(x) = |0.5x^2 - |\, x \,|\,|$ is decreasing is

   A. one

   B. two

   C. three

   D. none of these

5. For a set of real numbers $x_1, x_2, \ldots\ldots, x_n$, the root mean square (RMS) defined as $\text{RMS} = \left\{\frac{1}{N}\sum_{i=1}^{n} x_i^2\right\}^{1/2}$ is a measure of central tendency. If AM denotes the arithmetic mean of the set of numbers, then which of the following statements is correct?

   A. RMS < AM always

   B. RMS > AM always

   C. RMS < AM when the numbers are not all equal

D. RMS > AM when numbers are not all equal

6. Let $f(x)$ be a function of real variable and let Δf be the function $\Delta f(\mathrm{x}) = f(x+1) - f(x)$. For k > 1, put $\Delta^k f = \Delta\left(\Delta^{k-1} f\right)$. Then $\Delta^k f(x)$ equals

   A. $\sum_{j=0}^{k}(-1)^j \begin{pmatrix} k \\ j \end{pmatrix} f(x+j)$

   B. $\sum_{j=0}^{k}(-1)^{j+1} \begin{pmatrix} k \\ j \end{pmatrix} f(x+j)$

   C. $\sum_{j=0}^{k}(-1)^j \begin{pmatrix} k \\ j \end{pmatrix} f(x+k-j)$

   D. $\sum_{j=0}^{k}(-1)^{j+1} \begin{pmatrix} k \\ j \end{pmatrix} f(x+k-j)$

7. Let $I_n = \int_0^\infty x^n e^{-x} dx$, where n is some positive integer. Then I_n equals

   A. $n! - nI_{n-1}$

   B. $n! + nI_{n-1}$

   C. nI_{n-1}

   D. none of these.

8. If $x^3 = 1$, then $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ equals

   A. $(cx^2 + bx + a)\begin{vmatrix} 1 & b & c \\ x & c & a \\ x^2 & a & b \end{vmatrix}$

   B. $(cx^2 + bx + a)\begin{vmatrix} x & b & c \\ 1 & c & a \\ x^2 & a & b \end{vmatrix}$

   C. $(cx^2 + bx + a)\begin{vmatrix} x^2 & b & c \\ x & c & a \\ 1 & a & b \end{vmatrix}$

   D. $(cx^2 + bx + a)\begin{vmatrix} 1 & b & c \\ x^2 & c & a \\ x & a & b \end{vmatrix}$

9. Consider any integer $I = m^2 + n^2$, where m and n are any two odd integers. Then

   A. I is never divisible by 2

   B. I is never divisible by 4

   C. I is never divisible by 6

   D. none of these.

10. A box has 10 red balls and 5 black balls. A ball is selected from the box. If the ball is red, it is returned to the box. If the ball is black, it and 2 additional black balls are added to the box. The probability that a second ball selected from the box will be red is

    A. $\frac{47}{72}$
    B. $\frac{25}{72}$
    C. $\frac{55}{153}$
    D. $\frac{98}{153}$.

11. Let $f(x) = \frac{\log\left(1+\frac{x}{p}\right)-\log\left(1-\frac{x}{q}\right)}{x}, x \neq 0$. If f is continuous at $x = 0$, then the value of $f(0)$ is

    A. $\frac{1}{p} - \frac{1}{q}$
    B. $p + q$
    C. $\frac{1}{p} + \frac{1}{q}$
    D. none of these.

12. Consider four positive numbers x_1, x_2, y_1, y_2 such that $y_1, y_2 > x_1x_2$ Consider the number $S = (x_1y_2 + x_2y_1) - 2x_1x_2$. The number S is

    A. always a negative integer
    B. can be a negative fraction
    C. always a positive number
    D. none of these.

13. Given $x \geq y \geq z$, and $x + y + z = 12$, the maximum value of $x + 3y + 5z$ is

    A. 36
    B. 42
    C. 38
    D. 32 .

14. The number of positive pairs of integral values of (x, y) that solves $2xy - 4x^2 + 12x - 5y = 11$ is

    A. 4
    B. 1
    C. 2
    D. none of these.

15. Consider any continuous function $f : [0, 1] \to [0, 1]$. Which one of the following statements is incorrect?

    A. f always has at least one maximum in the interval [0,1]
    B. f always has at least one minimum in the interval [0,1]

C. $\exists$x $\in [0,1]$ such that $f(x) = x$

D. the function f must always have the property that $f(0) \in \{0,1\}$ $f(1) \in \{0,1\}$ and $f(0) + f(1) = I$

2 ISI PEA 2007

1. Let α and β be any two positive real numbers. Then $\lim_{x\to 0}\frac{(1+x)^{\alpha}-1}{(1+x)^{\beta}-1}$ equals

   A. $\frac{\alpha}{\beta}$

   B. $\frac{\alpha+1}{\beta+1}$

   C. $\frac{\alpha-1}{\beta-1}$

   D. 1

2. Suppose the number X is odd. Then X^2-1 is

   A. odd;

   B. not prime;

   C. necessarily positive;

   D. none of the above.

3. The value of k for which the function $f(x)=ke^{kx}$ is a probability density function on the interval $[0,1]$ is

   A. $k=\log 2$;

   B. $k=2\log 2$;

   C. $k=3\log 3$;

   D. $k=3\log 4$

4. p and q are positive integers such that p^2-q^2 is a prime number. Then, $p-q$ is

   A. a prime number;

   B. an even number greater than 2

   C. an odd number greater than 1 but not prime;

   D. none of these.

5. Any non-decreasing function defined on the interval $[a,b]$

   A. is differentiable on (a,b)

   B. is continuous in $[a,b]$ but not differentiable;

   C. has a continuous inverse;

   D. none of these.

6. The equation $\begin{vmatrix} x & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 8 & 1 \end{vmatrix} = 0$ is satisfied by

   A. $x=1$;

   B. $x=3$

   C. $x=4$

D. none of these.

7. If $f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}}$, then $f'(x)$ is

A. $\frac{x}{2f(x)-1}$

B. $\frac{1}{2f(x)-1}$.

C. $\frac{1}{x\sqrt{f(x)}}$;

D. $\frac{1}{2f(x)+1}$.

8. If $P = \log_x(xy)$ and $Q = \log_y(xy)$, then $P + Q$ equals

A. PQ

B. $\frac{P}{Q}$

C. $\frac{Q}{P}$

D. $\frac{PQ}{2}$

9. The solution to $\int \frac{2x^3+1}{x^4+2x} dx$ is

A. $\frac{x^4+2x}{4x^3+2}+$ constant;

B. $\log x^4 + \log 2x+$ constant;

C. $\frac{1}{2} \log |x^4 + 2x| +$ constant;

D. $\left|\frac{x^4+2x}{4x^3+2}\right| +$ constant.

10. The set of all values of x for which $x^2 - 3x + 2 > 0$ is

A. $(-\infty, 1)$

B. $(2, \infty)$

C. $(-\infty, 2) \cap (1, \infty)$

D. $(-\infty, 1) \cup (2, \infty)$

11. Consider the functions $f_1(x) = x^2$ and $f_2(x) = 4x^3 + 7$ defined on the real line. Then

A. f_1 is one-to-one and onto, but not f_2

B. f_2 is one-to-one and onto, but not f_1

C. both f_1 and f_2 are one-to-one and onto;

D. none of the above.

12. If $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$, $a > 0, b > 0$, then $f'(0)$ equals

A. $\left(\frac{b^2-a^2}{b^2}\right) \left(\frac{a}{b}\right)^{a+b-1}$

B. $\left(2 \log \left(\frac{a}{b}\right) + \frac{b^2-a^2}{ab}\right) \left(\frac{a}{b}\right)^{a+b}$

C. $2\log\left(\frac{a}{b}\right) + \frac{b^2-a^2}{ab}$

D. $\left(\frac{b^2-a^2}{ba}\right)$.

13. The linear programming problem

$$\begin{aligned} \max_{x,y} z &= 0.5x + 1.5y \\ \text{subject to:} \quad x + y &\leq 6 \\ 3x + y &\leq 15 \\ x + 3y &\leq 15 \\ x, y &\geq 0 \end{aligned}$$

has

A. no solution;

B. a unique non-degenerate solution;

C. a corner solution;

D. infinitely many solutions.

14. Let $f(x;\theta) = \theta f(x;1) + (1-\theta) f(x;0)$, where θ is a constant satisfying $0 < \theta < 1$ Further, both $f(x;1)$ and $f(x;0)$ are probability density functions $(p \cdot d, f.)$. Then

A. $f(x;\theta)$ is a $p.d.f.$ for all values of θ

B. $f(x;\theta)$ is a $p.d.f.$ only for $0 < \theta < \frac{1}{2}$

C. $f(x;\theta)$ is a $p.d.f.$ only for $\frac{1}{2} \leq \theta < 1$

D. $f(x;\theta)$ is not a $p.d.f.$ for any value of θ.

15. The correlation coefficient r for the following five pairs of observations satisfies

x	5	1	4	3	2
y	0	4	2	0	-1

A. $r > 0$;

B. $r < -0.5$;

C. $-0.5 < r < 0$;

D. $r = 0$.

16. An n -coordinated function f is called homothetic if it can be expressed as an increasing transformation of a homogeneous function of degree one. Let $f_1(x) = \sum_{i=1}^{n} x_i^r$, and $f_2(x) = \sum_{i=1}^{n} a_i x_i + b$, where $x_i > 0$ for all $i, 0 < r < 1, a_i > 0$ and b are constants. Then

A. f_1 is not homothetic but f_2 is;

B. f_2 is not homothetic but f_1 is;

C. both f_1 and f_2 are homothetic;

D. none of the above.

17. If $h(x) = \frac{1}{1-x}$, then $h(h(h(x))$ equals

    A. $\frac{1}{1-x}$

    B. x

    C. $\frac{1}{x}$

    D. $1 - x$

18. The function $x|x| + \left(\frac{|x|}{x}\right)^3$ is

    A. continuous but not differentiable at $x = 0$

    B. differentiable at $x = 0$

    C. not continuous at $x = 0$;

    D. continuously differentiable at $x = 0$.

19. $\int \frac{2dx}{(x-2)(x-1)x}$ equals

    A. $\log\left|\frac{x(x-2)}{(x-1)^2}\right| +$ constant;

    B. $\log\left|\frac{(x-2)}{x(x-1)^2}\right| +$ constant;

    C. $\log\left|\frac{x^2}{(x-1)(x-2)}\right| +$ constant;

    D. $\log\left|\frac{(x-2)^2}{x(x-1)}\right| +$ constant.

20. Experience shows that 20% of the people reserving tables at a certain restaurant never show up. If the restaurant has 50 tables and takes 52 reservations, then the probability that it will be able to accommodate everyone is

    A. $1 - \frac{209}{552}$

    B. $1 - 14 \times \left(\frac{4}{5}\right)^{52}$

    C. $\left(\frac{4}{5}\right)^{50}$

    D. $\left(\frac{1}{5}\right)^{50}$

21. For any real number x, define $[x]$ as the highest integer value not greater than x. For example, $[0.5] = 0, [1] = 1$ and $[1.5] = 1$. Let $I = \int_0^{3/2} [x] + [x^2]\, dx$. Then I equals

    A. 1

    B. $\frac{5-2\sqrt{2}}{2}$

    C. $2\sqrt{2}$

    D. none of these.

22. Every integer of the form $(n^3 - n)\,(n^2 - 4)$ (for $n = 3, 4, \ldots$) is

    A. divisible by 6 but not always divisible by 12

    B. divisible by 12 but not always divisible by 24

C. divisible by 24 but not always divisible by 120

D. divisible by 120 but not always divisible by 720 .

23. Two varieties of mango, A and B, are available at prices Rs. p_1 and Rs. p_2 per kg, respectively. One buyer buys 5 kg. of A and 10 kg. of B and another buyer spends Rs 100 on A and Rs. 150 on B. If the average expenditure per mango (irrespective of variety) is the same for the two buyers, then which of the following statements is the most appropriate?

A. $p_1 = p_2$

B. $p_2 = \frac{3}{4}p_1$

C. $p_1 = p_2$ or $p_2 = \frac{3}{4}p_1$

D. $\frac{3}{4} \leq \frac{p_2}{p_1} < 1$

24. For a given bivariate data set $(x_i, y_i; i = 1, 2, \ldots, n)$, the squared correlation coefficient (r^2) between x^2 and y is found to be 1. Which of the following statements is the most appropriate?

A. In the (x, y) scatter diagram, all points lie on a straight line.

B. In the (x, y) scatter diagram, all points lie on the curve $y = x^2$.

C. In the (x, y) scatter diagram, all points lie on the curve $y = a+bx^2, a > 0, b > 0$

D. In the (x, y) scatter diagram, all points lie on the curve $y = a + bx^2, a, b$ any real numbers.

25. The number of possible permutations of the integers 1 to 7 such that the numbers 1 and 2 always precede the number 3 and the numbers 6 and 7 always succeed the number 3 is

A. 720

B. 168

C. 84

D. none of these.

26. Suppose the real valued continuous function f defined on the set of non-negative real numbers satisfies the condition $f(x) = xf(x)$, then $f(2)$ equals

A. 1

B. 2

C. 3

D. $f(1)$

27. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

A. $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{2}$

B. $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{4}$

C. $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{4}$

D. $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{2}$

28. Consider a square that has sides of length 2 units. Five points are placed anywhere inside this square. Which of the following statements is incorrect?

    A. There cannot be any two points whose distance is more than $2\sqrt{2}$.

    B. The square can be partitioned into four squares of side 1 unit each such that at least one unit square has two points that lies on or inside it.

    C. At least two points can be found whose distance is less than $\sqrt{2}$.

    D. Statements (a), (b) and (c) are all incorrect.

29. Given that f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, it follows that

    A. $f(x)$ is an increasing function;

    B. $f(x)$ is a decreasing function;

    C. $|f(x)|$ is an increasing function;

    D. $|f(x)|$ is a decreasing function.

30. Let p, q, r, s be four arbitrary positive numbers. Then the value of $\frac{(p^2+p+1)(q^2+q+1)(r^2+r+1)(s^2+s+1)}{pqrs}$ is at least as large as

    A. 81

    B. 91

    C. 101.

    D. None of these.

3 ISI PEA 2008

1. $\int \frac{dx}{x+x\log x}$ equals

   A. $\log|x+x\log x|+$ constant

   B. $\log|1+x\log x|+$ constant

   C. $\log|\log x|+$ constant

   D. $\log|1+\log x|+$ constant.

2. The inverse of the function $\sqrt{-1+x}$ is

   A. $\frac{1}{\sqrt{x-1}}$,

   B. x^2+1,

   C. $\sqrt{x-1}$,

   D. none of these.

3. The domain of continuity of the function $f(x)=\sqrt{x}+\frac{x+1}{x-1}-\frac{x+1}{x^2+1}$ is

   A. $[0,1)$

   B. $(1,\infty)$

   C. $[0,1)\cup(1,\infty)$,

   D. none of these

4. Consider the following linear programme: minimise $x-2y$ subject to

$$\begin{aligned} x+3y &\geq 3 \\ 3x+y &\geq 3 \\ x+y &\leq 3 \end{aligned}$$

   An optimal solution of the above programme is given by

   A. $x=\frac{3}{4}, y=\frac{3}{4}$

   B. $x=0, y=3$

   C. $x=-1, y=3$

   D. none of the above.

5. Consider two functions $f_1:\{a_1,a_2,a_3\}\rightarrow\{b_1,b_2,b_3,b_4\}$ and $f_2:\{b_1,b_2,b_3,b_4\}\rightarrow\{c_1,c_2,c_3\}$. The function f_1 is defined by $f_1(a_1)=b_1, f_1(a_2)=b_2, f_1(a_3)=b_3$ and the function f_2 is defined by $f_2(b_1)=c_1, f_2(b_2)=c_2, f_2(b_3)=f_2(b_4)=c_3$. Then the mapping $f_2\circ f_1:\{a_1,a_2,a_3\}\rightarrow\{c_1,c_2,c_3\}$ is

   A. a composite and one $-$ to $-$ one function but not an onto function.

   B. a composite and onto function but not a one $-$ to $-$ one function.

   C. a composite, one $-$ to $-$ one and onto function.

   D. not a function.

6. If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}, t > 0, t \neq 1$ then the relation between x and y is

   A. $y^x = x^{\frac{1}{y}}$,

   B. $x^{\frac{1}{y}} = y^{\frac{1}{x}}$,

   C. $x^y = y^x$,

   D. $x^y = y^{\frac{1}{x}}$.

7. The maximum value of $T = 2x_B + 3x_S$ subject to the constraint $20x_B + 15x_S \leq 900$ where $x_B \geq 0$ and $x_S \geq 0$, is

   A. 150,

   B. 180,

   C. 200,

   D. none of these.

8. The value of $\int_0^2 [x]^n f'(x)dx$, where $[x]$ stands for the integral part of x, n is a positive integer and f' is the derivative of the function f, is

   A. $(n + 2^n)\left(f(2) - f(0)\right)$,

   B. $(1 + 2^n)\left(f(2) - f(1)\right)$

   C. $2^n f(2) - (2^n - 1)\, f(1) - f(0)$,

   D. none of these.

9. A surveyor found that in a society of 10,000 adult literates 21% completed college education, 42% completed university education and remaining 37% completed only school education. Of those who went to college 61% reads newspapers regularly, 35% of those who went to the university and 70% of those who completed only school education are regular readers of newspapers. Then the percentage of those who read newspapers regularly completed only school education is

   A. 40%,

   B. 52%,

   C. 35%,

   D. none of these.

10. The function $f(x) = x|x|e^{-x}$ defined on the real line is

    A. continuous but not differentiable at zero,

    B. differentiable only at zero,

    C. differentiable everywhere,

    D. differentiable only at finitely many points.

11. Let X be the set of positive integers denoting the number of tries it takes the Indian cricket team to win the World Cup. The team has equal odds for winning or losing any match. What is the probability that they will win in odd number of matches?

A. 1/4,

B. 1/2,

C. 2/3

D. 3/4

12. Three persons X, Y, Z were asked to find the mean of 5000 numbers, of which 500 are unities. Each one did his own simplification. $X's$ method: Divide the set of number into 5 equal parts, calculate the mean for each part and then take the mean of these. $Y's$ method: Divide the set into 2000 and 3000 numbers and follow the procedure of A. Z' s method: Calculate the mean of 4500 numbers (which are $\neq 1$) and then add 1 . Then

A. all methods are correct,

B. $X's$ method is correct, but Y and $Z's$ methods are wrong,

C. $X's$ and $Y's$ methods are correct but Z_s' methods is wrong,

D. none is correct.

13. The number of ways in which six letters can be placed in six directed envelopes such that exactly four letters are placed in correct envelopes and exactly two letters are placed in wrong envelopes is

A. 1

B. 15

C. 135

D. None of these

14. The set of all values of x for which the inequality $|x-3|+|x+2|<11$ holds is

A. $(-3,2)$,

B. $(-5,2)$,

C. $(-5,6)$,

D. none of these.

15. The function $f(x)=x^4-4x^3+16x$ has

A. a unique maximum but no minimum,

B. a unique minimum but no maximum,

C. a unique maximum and a unique minimum,

D. neither a maximum nor a minimum.

16. Consider the number $K(n)=(n+3)\left(n^2+6n+8\right)$ defined for integers n. Which of the following statements is correct?

A. $K(n)$ is always divisible by 4

B. $K(n)$ is always divisible by 5

C. $K(n)$ is always divisible by 6

D. All Statements are incorrect.

17. 25 books are placed at random on a shelf. The probability that a particular pair of books shall be always together is

A. $\frac{2}{25}$

B. $\frac{1}{25}$,

C. $\frac{1}{300}$

D. $\frac{1}{600}$

18. $P(x)$ is a quadratic polynomial such that $P(1) = -P(2)$. If -1 is a root of the equation, the other root is

A. $\frac{4}{5}$,

B. $\frac{8}{5}$,

C. $\frac{6}{5}$,

D. $\frac{3}{5}$.

19. The correlation coefficients between two variables X and Y obtained from the two equations $2x + 3y - 1 = 0$ and $5x - 2y + 3 = 0$ are

A. equal but have opposite signs,

B. $-\frac{2}{3}$ and $\frac{2}{5}$,

C. $\frac{1}{2}$ and $-\frac{3}{5}$,

D. Cannot say.

20. If a, b, c, d are positive real numbers then $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}$ is always

A. less than $\sqrt{2}$.

B. less than 2 but greater than or equal to $\sqrt{2}$,

C. less than 4 but greater than or equal 2

D. greater than or equal to 4 .

21. The range of value of x for which the inequality $\log_{(2-x)}(x-3) \geq -1$ holds is

A. $2 < x < 3$,

B. $x > 3$,

C. $x < 2$,

D. no such x exists.

22. The equation $5x^3 - 5x^2 + 2x - 1$ has

A. all roots between 1 and 2 ,

B. all negative roots,

C. a root between 0 and 1 ,

D. all roots greater than 2 .

23. The probability density of a random variable is

$$f(x) = ax^2 e^{-kx} \quad (k > 0, 0 \le x \le \infty)$$

Then, a equals

A. $\frac{k^3}{2}$,

B. $\frac{k}{2}$,

C. $\frac{k^2}{2}$,

D. k

24. Let $x = r$ be the mode of the distribution with probability mass function $p(x) = \binom{n}{x} p^x (1-p)^{n-x}$. Then which of the following inequalities hold.

A. $(n+1)p - 1 < r < (n+1)p$,

B. $r < (n+1)p - 1$

C. $r > (n+1)p$

D. $r < np$.

25. Let $y = (y_1, \ldots, y_n)$ be a set of n observations with $y_1 \le y_2 \le \ldots \le y_n$. Let $y' = (y_1, y_2, \ldots, y_j + \delta, \ldots, y_k - \delta, \ldots, y_n)$ where $y_k - \delta > y_{k-1} > \ldots > y_{j+1} > y_j + \delta$ $\delta > 0$. Let σ : standard deviation of y and σ' : standard deviation of y'. Then

A. $\sigma < \sigma'$,

B. $\sigma' < \sigma$,

C. $\sigma' = \sigma$,

D. nothing can be said.

26. Let x be a r.v. with pdf $f(x)$ and let $F(x)$ be the distribution function. Let $r(x) = \frac{xf(x)}{1-F(x)}$. Then for $x < e^{\mu}$ and $f(x) = \frac{e^{-\frac{(\log x - \mu)^2}{2}}}{x\sqrt{2\pi}}$, the function $r(x)$ is

A. increasing in x,

B. decreasing in x

C. constant,

D. none of the above.

27. A square matrix of order n is said to be a bistochastic matrix if all of its entries are non-negative and each of its rows and columns sum to 1. Let $y_{n\times1} = P_{n\times n} x_{n\times1}$ where elements of y are some rearrangements of the elements of x. Then

A. P is bistochastic with diagonal elements 1 ,

B. P cannot be bistochastic,

C. P is bistochastic with elements 0 and 1 ,

D. P is a unit matrix.

28. Let $f_1(x) = \frac{x}{x+1}$. Define $f_n(x) = f_1(f_{n-1}(x))$, where $n \geq 2$. Then $f_n(x)$ is

A. decreasing in n,

B. increasing in n,

C. initially decreasing in n and then increasing in n,

D. initially increasing in n and then decreasing n.

29. $\lim_{n\to\infty} \frac{1-x^{-2n}}{1+x^{-2n}}, x > 0$ equals

A. 1

B. -1

C. 0

D. The limit does not exist.

30. Consider the function $f(x_1, x_2) = \max\{6 - x_1, 7 - x_2\}$. The solution (x_1^*, x_2^*) to the optimization problem minimize $f(x_1, x_2)$ subject to $x_1 + x_2 = 21$ is

A. $(x_1^* = 10.5, x_2^* = 10.5)$,

B. $(x_1^* = 11, x_2^* = 10)$

C. $(x_1^* = 10, x_2^* = 11)$,

D. None of these.

4 ISI PEA 2009

1. An infinite geometric series has first term 1 and sum 4. It common ratio is

   A. $\frac{1}{2}$

   B. $\frac{3}{4}$

   C. 1

   D. $\frac{1}{3}$

2. A continuous random variable X has a probability density function $f(x) = 3x^2$ with $0 \le x \le 1$. If $P(X \le a) = P(x > a)$, then a is:

   A. $\frac{1}{\sqrt{6}}$

   B. $\left(\frac{1}{3}\right)^{\frac{1}{2}}$

   C. $\frac{1}{2}$

   D. $\left(\frac{1}{2}\right)^{\frac{1}{3}}$

3. If $f(x) = \sqrt{e^x + \sqrt{e^x + \sqrt{e^x + \ldots}}}$ then $f'(x)$ equals to

   A. $\frac{f(x)-1}{2f(x)+1}$.

   B. $\frac{f^2(x)-f(x)}{2f(x)-1}$

   C. $\frac{2f(x)+1}{f^2(x)+f(x)}$

   D. $\frac{f(x)}{2f(x)+1}$

4. $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$ is

   A. $\frac{1}{6}$

   B. 0

   C. $\frac{1}{4}$

   D. not well defined

5. If $X = 2^{65}$ and $Y = 2^{64} + 2^{63} + \ldots + 2^1 + 2^0$, then

   A. $Y = X + 2^{64}$.

   B. $X = Y$.

   C. $Y = X + 1$

   D. $Y = X - 1$

6. $\int_0^1 \frac{e^x}{e^x+1} dx =$

   A. $\log(1 + e)$

   B. log2.

C. $\log \frac{1+e}{2}$.

D. $2\log(1+e)$

7. There is a box with ten balls. Each ball has a number between 1 and 10 written on it. No two balls have the same number. Two balls are drawn (simultaneously) at random from the box. What is the probability of choosing two balls with odd numbers?

   A. $\frac{1}{9}$.

   B. $\frac{1}{2}$

   C. $\frac{2}{9}$

   D. $\frac{1}{3}$

8. A box contains 100 balls. Some of them are white and the remaining are red. Let X and Y denote the number of white and red balls respectively. The correlation between X and Y is

   A. 0.

   B. 1 .

   C. -1.

   D. some real number between $-\frac{1}{2}$ and $\frac{1}{2}$.

9. Let f, g and h be real valued functions defined as follows: $f(x) = x(1-x)$; $g(x) = \frac{x}{2}$ and $h(x) = \min\{f(x), g(x)\}$ with $0 \leq x \leq 1$. Then h is

   A. continuous and differentiable

   B. differentiable but not continuous

   C. continuous but not differentiable

   D. neither continuous nor differentiable

10. In how many ways can three persons, each throwing a single die once, make a score of 8?

    A. 5

    B. 15

    C. 21

    D. 30

11. If $f(x)$ is a real valued function such that

$$2f(x) + 3f(-x) = 55 - 7x$$

for every $x \in \mathbf{R}$, then $f(3)$ equals

    A. 40

    B. 32

C. 26

D. 10

12. Two persons, A and B, make an appointment to meet at the train station between 4 P.M. and 5 P.M.. They agree that each is to wait not more than 15 minutes for the other. Assuming that each is independently equally likely to arrive at any point during the hour, find the probability that they meet.

A. $\frac{15}{16}$

B. $\frac{7}{16}$

C. $\frac{5}{24}$

D. $\frac{22}{175}$

13. If x_1, x_2, x_3 are positive real numbers, then

$$\frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_1}$$

is always

A. ≤ 3

B. $\leq 3^{\frac{1}{3}}$

C. ≥ 3

D. 3

14. $\lim_{n\to\infty} \frac{1^2+2^2+...+n^2}{n^3}$ equals

A. 0

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. 1 .

15. Suppose b is an odd integer and the following two polynomial equations have a common root. $x^2 - 7x + 12 = 0$ and $x^2 - 8x + b = 0$ The root of $x^2 - 8x + b = 0$ that is not a root of $x^2 - 7x + 12 = 0$ is

A. 2

B. 3

C. 4

D. 5

16. Suppose $n \geq 9$ is an integer. Let $\mu = n^{\frac{1}{2}} + n^{\frac{1}{3}} + n^{\frac{1}{4}}$. Then, which of the following relationships between n and μ is correct?

A. $n = \mu$

B. $n > \mu$

C. $n < \mu$

D. None of the above.

17. Which of the following functions $f : \mathbf{R} \to \mathbf{R}$ satisfies the relation $f(x+y) = f(x)+f(y)$?

A. $f(z) = z^2$

B. $f(z) = az$ for some real number a

C. $f(z) = \log z$

D. $f(z) = e^z$

18. For what value of a does the following equation have a unique solution?

$$\begin{vmatrix} x & a & 2 \\ 2 & x & 0 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

A. 0

B. 1

C. 2

D. 4

19. Let

$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

where l, m, n, a, b, c are non-zero numbers. Then $\frac{dy}{dx}$ equals

A.
$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

B.
$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \\ a & b & c \end{vmatrix}$$

C.
$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

D.
$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l-a & m-b & n-c \\ 1 & 1 & 1 \end{vmatrix}$$

20. If $f(x) = |x-1| + |x-2| + |x-3|$, then $f(x)$ is differentiable at

   A. 0

   B. 1

   C. 2

   D. 3

21. If $(x-a)^2 + (y-b)^2 = c^2$, then $1 + \left[\frac{dy}{dx}\right]^2$ is independent of

   A. a

   B. b

   C. c

   D. Both b and c

22. A student is browsing in a second-hand bookshop and finds n books of interest. The shop has m copies of each of these n bools. Assuming he never wants duplicate copies of any book, and that he selects at least one book, how many ways can he make a selection? For example, if there is one book of interest with two copies, then he can make a selection in 2 ways.

   A. $(m+1)^n - 1$

   B. nm

   C. $2^{nm} - 1$

   D. $\frac{nm!}{(m!)(nm-m)!} - 1$

23. Determine all values of the constants A and B such that the following function is continuous for all values of x.

$$f(x) = \begin{cases} Ax - B & \text{if } x \leq -1 \\ 2x^2 + 3Ax + B & \text{if } -1 < x \leq 1 \\ 4 & \text{if } x > 1 \end{cases}$$

   A. $A = B = 0$

   B. $A = \frac{3}{4}, B = -\frac{1}{4}$

   C. $A = \frac{1}{4}, B = \frac{3}{4}$

   D. $A = \frac{1}{2}, B = \frac{1}{2}$

24. The value of $\lim_{x\to\infty} (3^x + 3^{2x})^{\frac{1}{x}}$ is

   A. 0

   B. 1

   C. e

   D. 9

25. A computer while calculating correlation coefficient between two random variables X and Y from 25 pairs of observations obtained the following results: $\sum X = 125, \sum X^2 = 650, \sum Y = 100, \sum Y^2 = 460, \sum XY = 508$. It was later discovered that at the time of inputing, the pair $(X = 8, Y = 12)$ had been wrongly input as $(X = 6, Y = 14)$ and the pair $(X = 6, Y = 8)$ had been wrongly input as $(X = 8, Y = 6)$. Calculate the value of the correlation coefficient with the correct data.

A. $\frac{4}{5}$

B. $\frac{2}{3}$

C. 1

D. $\frac{5}{6}$

26. The point on the curve $y = x^2 - 1$ which is nearest to the point $(2, -0.5)$ is

A. $(1, 0)$

B. $(2, 3)$

C. $(0, -1)$

D. None of the above

27. If a probability density function of a random variable X is given by $f(x) = kx(2-x), 0 \leq x \leq 2$, then mean of X is

A. $\frac{1}{2}$

B. 1

C. $\frac{1}{5}$

D. $\frac{3}{4}$

28. Suppose X is the set of all integers greater than or equal to 8. Let $f : X \to \mathbf{R}$. and $f(x + y) = f(xy)$ for all $x, y \geq 4$. If $f(8) = 9$, then $f(9) =$

A. 8

B. 9

C. 64

D. 81

29. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = (x - 1)(x - 2)(x - 3)$. Which of the following is true about f?

A. It decreases on the interval $\left[2 - 3^{-\frac{1}{2}}, 2 + 3^{-\frac{1}{2}}\right]$

B. It increases on the interval $\left[2 - 3^{-\frac{1}{2}}, 2 + 3^{-\frac{1}{2}}\right]$

C. It decreases on the interval $\left(-\infty, 2 - 3^{-\frac{1}{2}}\right]$

D. It decreases on the interval [2,3]

30. A box with no top is to be made from a rectangular sheet of cardboard measuring 8 metres by 5 metres by cutting squares of side x metres out of each corner and folding up the sides. The largest possible volume in cubic metres of such a box is

   A. 15

   B. 12

   C. 20

   D. 18

5 ISI PEA 2010

1. The value of $100\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{99.100}\right]$

   A. is 99 ,

   B. is 100

   C. is 101

   D. is $\frac{(100)^2}{99}$.

2. The function $f(x) = x(\sqrt{x}+\sqrt{x+9})$ is

   A. continuously differentiable at $x = 0$,

   B. continuous but not differentiable at $x = 0$,

   C. differentiable but the derivative is not continuous at $x = 0$,

   D. not differentiable at $x = 0$.

3. Consider a GP series whose first term is 1 and the common ratio is a positive integer $r(> 1)$. Consider an AP series whose first term is 1 and whose $(r+2)^{\text{th}}$ term coincides with the third term of the GP series. Then the common difference of the AP series is

   A. $r-1$

   B. r

   C. $r+1$

   D. $r+2$

4. The first three terms of the binomial expansion $(1+x)^n$ are $1, -9, \frac{297}{8}$ respectively. What is the value of n?

   A. 5

   B. 8

   C. 10

   D. 12

5. Given $\log_p x = \alpha$ and $\log_q x = \beta$, the value of $\log_{\frac{p}{q}} x$ equals

   A. $\frac{\alpha\beta}{\beta-\alpha}$

   B. $\frac{\beta-\alpha}{\alpha\beta}$

   C. $\frac{\alpha-\beta}{\alpha\beta}$,

   D. $\frac{\alpha\beta}{\alpha-\beta}$

6. Let $P = \{1,2,3,4,5\}$ and $Q = \{1,2\}$. The total number of subsets X of P such that $X \cap Q = \{2\}$ is

   A. 6

B. 7

C. 8

D. 9.

7. An unbiased coin is tossed until a head appears. The expected number of tosses required is

A. 1

B. 2

C. 4

D. ∞.

8. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{c}{x^2} & \text{if } x \geq c \\ 0 & x < c \end{cases}$$

Then the expectation of X is

A. 0

B. ∞

C. $\frac{1}{c}$

D. $\frac{1}{c^2}$

9. The number of real solutions of the equation $x^2 - 5|x| + 4 = 0$ is

A. two

B. three

C. four

D. None of these

10. Range of the function $f(x) = \frac{x^2}{1+x^2}$ is

A. [0,1)

B. (0,1)

C. [0,1]

D. (0,1]

11. If a, b, c are in AP, then the value of the determinant $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ is

A. $b^2 - 4ac$

B. $ab + bc + ca$

C. $2b - a - c$

D. $3b + a + c$

12. If $a < b < c < d$, then the equation $(x-a)(x-b) + 2(x-c)(x-d) = 0$ has

A. both the roots in the interval $[a, b]$

B. both the roots in the interval $[c, d]$,

C. one root in the interval (a, b) and the other root in the interval (c, d)

D. one root in the interval $[a, b]$ and the other root in the interval $[c, d]$.

E. None of the above

13. Let f and g be two differentiable functions on (0,1) such that $f(0) = 2$, $f(1) = 6, g(0) = 0$ and $g(1) = 2$. Then there exists $\theta \in (0, 1)$ such that $f'(\theta)$ equals

A. $\frac{1}{2}g'(\theta)$

B. $2g'(\theta)$

C. $6g'(\theta)$

D. $\frac{1}{6}g'(\theta)$

14. The minimum value of $\log_x a + \log_a x$, for $1 < a < x$, is

A. less than 1 ,

B. greater than 2 ,

C. greater than 1 but less than 2 .

D. None of these.

15. The value of $\int_4^9 \frac{1}{2x(1+\sqrt{x})} dx$ equals

A. $\log_e 3 - \log_e 2$

B. $2\log_e 2 - \log_e 3$

C. $2\log_e 3 - 3\log_e 2$

D. $3\log_e 3 - 2\log_e 2$.

16. The inverse of the function $f(x) = \frac{1}{1+x}, x > 0$, is

A. $(1 + x)$

B. $\frac{1+x}{x}$

C. $\frac{1-x}{x}$,

D. $\frac{x}{1+x}$

17. Let $X_i, i = 1, 2, \ldots, n$ be identically distributed with variance σ^2. Let $\text{cov}(X_i, X_j) = \rho$ for all $i \neq j$. Define $\bar{X}_n = \frac{1}{n}\sum X_i$ and let $a_n = \text{Var}\left(\bar{X}_n\right)$ Then $\lim_{n\to\infty} a_n$ equals

A. 0 ,

B. ρ,

C. $\sigma^2 + \rho$

D. $\sigma^2 + \rho^2$.

18. Let X be a Normally distributed random variable with mean 0 and variance 1. Let $\Phi(.)$ be the cumulative distribution function of the variable X. Then the expectation of $\Phi(X)$ is

A. $-\frac{1}{2}$,

B. 0 ,

C. $\frac{1}{2}$,

D. 1 .

19. Consider any finite integer $r \geq 2$. Then $\lim_{x \to 0} \left[\frac{\log_e \left(\sum_{k=0}^{r} x^k \right)}{\left(\sum_{k=1}^{\infty} \frac{x^k}{k!} \right)} \right]$ equals

A. 0 ,

B. 1

C. e,

D. $\log_e 2$.

20. Consider 5 boxes, each containing 6 balls labelled 1,2,3,4,5,6 . Suppose one ball is drawn from each of the boxes. Denote by b_i, the label of the ball drawn from the i -th box, $i = 1, 2, 3, 4, 5$. Then the number of ways in which the balls can be chosen such that $b_1 < b_2 < b_3 < b_4 < b_5$ is

A. 1

B. 2

C. 5

D. 6

21. The sum $\sum_{r=0}^{m} \binom{n+r}{r}$ equals

A. $\begin{pmatrix} n+m+1 \\ n+m \end{pmatrix}$

B. $(n+m+1) \begin{pmatrix} n+m \\ n+1 \end{pmatrix}$

C. $\begin{pmatrix} n+m+1 \\ n \end{pmatrix}$

D. $\begin{pmatrix} n+m+1 \\ n+1 \end{pmatrix}$

22. Consider the following 2 -variable linear regression where the error ϵ_i 's are independently and identically distributed with mean 0 and variance 1;

$$y_i = \alpha + \beta \left(x_i - \bar{x} \right) + \epsilon_i, \quad i = 1, 2, \ldots, n$$

Let $\hat{\alpha}$ and $\hat{\beta}$ be ordinary least squares estimates of α and β respectively. Then the correlation coefficient between $\hat{\alpha}$ and $\hat{\beta}$ is

A. 1

B. 0

C. -1

D. $\frac{1}{2}$

23. Let f be a real valued continuous function on $[0,3]$. Suppose that $f(x)$ takes only rational values and $f(1)=1$. Then $f(2)$ equals

A. 2

B. 4

C. 8

D. None of these

24. Consider the function $f(x_1,x_2) = \int_0^{\sqrt{x_1^2+x_2^2}} e^{-\left(w^2/\left(x_1^2+x_2^2\right)\right)} dw$ with the property that $f(0,0)=0$. Then the function $f(x_1,x_2)$ is

A. homogeneous of degree -1

B. homogeneous of degree $\frac{1}{2}$,

C. homogeneous of degree 1

D. None of these.

25. If $f(1)=0$, $f'(x)>f(x)$ for all $x>1$, then $f(x)$ is

A. positive valued for all $x>1$,

B. negative valued for all $x>1$,

C. positive valued on (1,2) but negative valued on $[2,\infty)$

D. None of these.

26. Consider the constrained optimization problem

$$\max_{x\geq 0, y\geq 0}(ax+by) \text{ subject to } (cx+dy)\leq 100$$

where a,b,c,d are positive real numbers such that $\frac{d}{b}>\frac{(c+d)}{(a+b)}$. The unique solution (x^*,y^*) to this constrained optimization problem is

A. $\left(x^*=\frac{100}{a}, y^*=0\right)$

B. $\left(x^*=\frac{100}{c}, y^*=0\right)$

C. $\left(x^*=0, y^*=\frac{100}{b}\right)$,

D. $\left(x^*=0, y^*=\frac{100}{d}\right)$.

27. For any real number x, let $[x]$ be the largest integer not exceeding x. The domain of definition of the function $f(x)=(\sqrt{|[|x|-2]|-3})^{-1}$ is

A. [-6,6]

B. $(-\infty, -6) \cup (+6, \infty)$

C. $(-\infty, -6] \cup [+6, \infty)$

D. None of these.

28. Let $f : \mathbf{R} \to \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} -1 \text{ if } x < -\dfrac{1}{2} \\ -\dfrac{1}{2} \text{ if } -\dfrac{1}{2} \leq x < 0 \\ 0 \text{ if } x = 0 \\ 1 \text{ if } x > 0 \end{cases}$$

and $g(x) = 1 + x - [x]$, where $[x]$ is the largest integer not exceeding x. Then $f(g(x))$ equals

A. -1

B. $-\frac{1}{2}$,

C. 0 ,

D. 1 .

29. If f is a real valued function and $a_1 f(x) + a_2 f(-x) = b_1 - b_2 x$ for all x with $a_1 \neq a_2$ and $b_2 \neq 0$. Then $f\left(\frac{b_1}{b_2}\right)$ equals

A. 0,

B. $-\left(\frac{2a_2 b_1}{a_1^2 - a_2^2}\right)$

C. $\frac{2a_2 b_1}{a_1^2 - a_2^2}$

D. More information is required to find the exact value of $f\left(\frac{b_1}{b_2}\right)$.

30. For all $x, y \in (0, \infty)$, a function $f : (0, \infty) \to \mathbf{R}$ satisfies the inequality

$$|f(x) - f(y)| \leq |x - y|^3$$

Then f is

A. an increasing function,

B. a decreasing function,

C. a constant function.

D. None of these.

6 ISI PEA 2011

1. The expression $\sqrt{13+3\sqrt{23/3}}+\sqrt{13-3\sqrt{23/3}}$ is

   A. A natural number,

   B. A rational number but not a natural number,

   C. An irrational number not exceeding 6 ,

   D. An irrational number exceeding 6 .

2. The domain of definition of the function $f(x)=\frac{\sqrt{(x+3)}}{(x^2+5x+4)}$ is

   A. $(-\infty,\infty)\backslash\{-1,-4\}$

   B. $(-0,\infty)\backslash\{-1,-4\}$

   C. $(-1,\infty)\backslash\{-4\}$

   D. None of these.

3. The value of

$$\log_4 2-\log_8 2+\log_{16} 2-\ldots\ldots$$

   A. $\log_e 2$,

   B. $1-\log_e 2$,

   C. $\log_e 2-1$,

   D. None of these.

4. The function $\max\{1,x,x^2\}$, where x is any real number, has

   A. Discontinuity at one point only,

   B. Discontinuity at two points only,

   C. Discontinuity at three points only,

   D. No point of discontinuity.

5. If $x,y,z>0$ are in HP, then $\frac{x-y}{y-z}$ equals

   A. $\frac{x}{y}$,

   B. $\frac{y}{z}$

   C. $\frac{x}{z}$

   D. None of these.

6. The function $f(x)=\frac{x}{1+|x|}$, where x is any real number is,

   A. Everywhere differentiable but the derivative has a point of discontinuity.

   B. Everywhere differentiable except at 0.

   C. Everywhere continuously differentiable.

   D. Everywhere differentiable but the derivative has 2 points of discontinuity.

7. Let the function $f : R_{++} \to R_{++}$ be such that $f(1) = 3$ and $f'(1) = 9$, where R_{++} is the positive part of the real line. Then $\lim_{x \to 0} \left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals

   A. 3 ,

   B. e^2,

   C. 2,

   D. e^3.

8. Let $f, g : [0, \infty) \to [0, \infty)$ be decreasing and increasing respectively. Define $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is

   A. Nonpositive for $x \geq 1$, positive otherwise,

   B. Always negative,

   C. Always positive,

   D. Positive for $x \geq 1$, nonpositive otherwise.

9. A committee consisting of 3 men and 2 women is to be formed out of 6 men and 4 women. In how many ways this can be done if Mr. X and Mrs. Y are not to be included together?

   A. 120

   B. 140

   C. 90

   D. 60

10. The number of continuous functions f satisfying $xf(y) + yf(x) = (x+y)f(x)f(y)$, where x and y are any real numbers, is

    A. 1 ,

    B. 2,

    C. 3

    D. None of these.

11. If the positive numbers $x_1, \ldots, x_n$ are in AP, then

$$\frac{1}{\sqrt{x_1} + \sqrt{x_2}} + \frac{1}{\sqrt{x_2} + \sqrt{x_3}} + \ldots\ldots. + \frac{1}{\sqrt{x_{n-1}} + \sqrt{x_n}} \text{ equals}$$

    A. $\frac{n}{\sqrt{x_1 + \sqrt{x_n}}}$,

    B. $\frac{1}{\sqrt{x_1} + \sqrt{x_n}}$,

    C. $\frac{2n}{\sqrt{x_1} + \sqrt{x_n}}$,

    D. None of these.

12. If x, y, z are any real numbers, then which of the following is always true?

    A. $\max\{x, y\} < \max\{x, y, z\}$

    B. $\max\{x, y\} > \max\{x, y, z\}$

    C. $\max\{x, y\} = \frac{x+y+|x-y|}{2}$

    D. None of these.

13. If $x_1, x_2, x_3, x_4 > 0$ and $\sum_{i=1}^{4} x_i = 2$, then $P = (x_1 + x_2)(x_3 + x_4)$ is

    A. Bounded between zero and one,

    B. Bounded between one and two,

    C. Bounded between two and three,

    D. Bounded between three and four.

14. Everybody in a room shakes hand with everybody else. Total number of handshakes is 91. Then the number of persons in the room is

    A. 11

    B. 12

    C. 13 ,

    D. 14

15. The number of ways in which 6 pencils can be distributed between two boys such that each boy gets at least one pencil is

    A. 30

    B. 60

    C. 62

    D. 64

16. Number of continuous functions characterized by the equation $xf(x) + 2f(-x) = -1$, where x is any real number, is

    A. 1

    B. 2

    C. 3

    D. None of these

17. The value of the function $f(x) = x + \int_0^1 (xy^2 + x^2 y) f(y)dy$ is $px + qx^2$, where

    A. $p = 80, q = 180$

    B. $p = 40, q = 140$

    C. $\mathrm{p} = 50, \mathrm{q} = 150$

    D. None of these.

18. If x and y are real numbers such that $x^2 + y^2 = 1$, then the maximum value of $|x| + |y|$ is

    A. $\frac{1}{2}$,

    B. $\sqrt{2}$

    C. $\frac{1}{\sqrt{2}}$

    D. 2 .

19. The number of onto functions from $A = \{p, q, r, s\}$ to $B = \{p, r\}$ is

    A. 16

    B. 2,

    C. 8,

    D. 14

20. If the coefficients of $(2r + 5)$ th and $(r - 6)$ th terms in the expansion of $(1 + x)^{39}$ are equal, then ${}^{r}C_{12}$ equals

    A. 45

    B. 91

    C. 63

    D. None of these.

21. If $X = \begin{bmatrix} C & 2 \\ 1 & C \end{bmatrix}$ and $|X^7| = 128$, then the value of C is

    A. ± 5,

    B. ± 1

    C. ± 2

    D. None of these.

22. Let $f(x) = Ax^2 + Bx + C$, where A, B, C are real numbers. If $f(x)$ is an integer whenever x is an integer, then

    A. $2A$ and $A + B$ are integers, but C is not an integer.

    B. $A + B$ and C are integers, but $2A$ is not an integer.

    C. $2A, A + B$ and C are all integers.

    D. None of these.

23. Four persons board a lift on the ground floor of a seven-storey building. The number of ways in which they leave the lift, such that each of them gets down at different floors, is

    A. 360

    B. 60

    C. 120

D. 240

24. The number of vectors (x, x_1, x_2) , where $x, x_1, x_2 > 0$, for which

$$|\log(xx_1)| + |\log(xx_2)| + \left|\log\left(\frac{x}{x_1}\right)\right| + \left|\log\left(\frac{x}{x_2}\right)\right| = |\log x_1 + \log x_2|$$

holds, is

A. One

B. Two

C. Three

D. None of these.

25. In a sample of households actually invaded by small pox, 70% of the inhabitants are attacked and 85% had been vaccinated. The minimum percentage of households (among those vaccinated) that must have been attacked [Numbers expressed as nearest integer value] is

A. 55

B. 65

C. 30

D. 15

26. In an analysis of bivariate data (X and Y) the following results were obtained. Variance of $X\,(\sigma_x^2) = 9$, product of the regression coefficient of Y on X and X on Y is 0.36, and the regression coefficient from the regression of Y on $X\,(\beta_{yx})$ is 0.8. The variance of Y is

A. 16

B. 4

C. 1.69

D. 3

27. For comparing the wear and tear quality of two brands of automobile tyres, two samples of 50 customers using two types of tyres under similar conditions were selected. The number of kilometers x_1 and x_2 until the tyres became worn out, was obtained from each of them for the tyres used by them. The sample results were as follows: $\bar{x}_1 = 13,200$ km, $\bar{x}_2 = 13,650$ km $S_{x1} = 300$ km, $S_{x2} = 400$ km. What would you conclude about the two brands of tyres (at 5% level of significance) as far as the wear and tear quality is concerned?

A. The two brands are alike

B. The two brands are not the same,

C. Nothing can be concluded,

D. The given data are inadequate to perform a test.

28. A continuous random variable x has the following probability density function: $f(x) = \frac{\alpha}{x_0}\left(\frac{x_0}{x}\right)^{\alpha+1}$ for $x > x_0, \alpha > 1$ The distribution function and the mean of x are given respectively by

A. $1 - \left(\frac{x}{x_0}\right)^{\alpha}, \frac{\alpha-1}{\alpha}x_0$

B. $1 - \left(\frac{x}{x_0}\right)^{-\alpha}, \frac{\alpha-1}{\alpha}x_0$

C. $1 - \left(\frac{x}{x_0}\right)^{-\alpha}, \frac{\alpha x_0}{\alpha-1}$

D. $1 - \left(\frac{x}{x_0}\right)^{\alpha}, \frac{\alpha x_0}{\alpha-1}$

29. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\begin{pmatrix} n \\ 0 \end{pmatrix}, \begin{pmatrix} n \\ 1 \end{pmatrix}, \ldots, \begin{pmatrix} n \\ n \end{pmatrix}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

A. $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{2}$

B. $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{4}$

C. $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{4}$

D. $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{2}$.

30. Let $\{X_i\}$ be a sequence of $i.i.d$ random variables such that $X_i = 1$ with probability p and $X_i = 0$ with probability $1 - p$. Define

$$y = \begin{cases} 1 \text{ if } \sum_{i=1}^{n} X_i = 100 \\ 0 \text{ otherwise} \end{cases}$$

Then $E\left(y^2\right)$ is

A. ∞,

B. $\begin{pmatrix} n \\ 100 \end{pmatrix} p^{100}(1-p)^{n-100}$,

C. np,

D. $(np)^2$.

7 ISI PEA 2012

1. Kupamonduk, the frog, lives in a well 14 feet deep. One fine morning she has an urge to see the world, and starts to climb out of her well. Every day she climbs up by 5 feet when there is light, but slides back by 3 feet in the dark. How many days will she take to climb out of the well?

   A. 3

   B. 8

   C. 6

   D. None of the above

2. The derivative of $f(x) = |x|^2$ at $x = 0$ is,

   A. -1

   B. Non-existent

   C. 0

   D. 1/2

3. Let $\mathcal{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. For each $n \in \mathcal{N}$ define $A_n = \{(n+1)k : k \in \mathcal{N}\}$. Then $A_1 \cap A_2$ equals

   A. A_3

   B. A_4

   C. A_5

   D. A_6.

4. Let $S = \{a, b, c\}$ be a set such that a, b and c are distinct real numbers. Then $\min\{\max\{a, b\}, \max\{b, c\},$ is always

   A. the highest number in S,

   B. the second highest number in S,

   C. the lowest number in S,

   D. the arithmetic mean of the three numbers in S.

5. The sequence $< -4^{-n} >, n = 1, 2, \cdots$, is

   A. Unbounded and monotone increasing,

   B. Unbounded and monotone decreasing,

   C. Bounded and convergent,

   D. Bounded but not convergent.

6. $\int \frac{x}{7x^2+2} dx$ equals

   A. $\frac{1}{14} \ln(7x^2 + 2) +$ constant

B. $7x^2 + 2$

C. $\ln x+$ constant,

D. None of the above.

7. The number of real roots of the equation

$$2(x-1)^2 = (x-3)^2 + (x+1)^2 - 8$$

is

A. Zero,

B. One,

C. Two,

D. None of the above.

8. The three vectors [0,1],[1,0] and [1000,1000] are

A. Dependent,

B. Independent,

C. Pairwise orthogonal,

D. None of the above.

9. The function $f(.)$ is increasing over $[a, b]$. Then $[f(.)]^n$, where n is an odd integer greater than 1, is necessarily

A. Increasing over $[a, b]$

B. Decreasing over $[a, b]$

C. Increasing over $[a, b]$ if and only if $f(.)$ is positive over $[a, b]$

D. None of the above.

10. The determinant of the matrix $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is

A. 21

B. -16

C. 0

D. 14

11. In what ratio should a given line be divided into two parts, so that the area of the rectangle formed by the two parts as the sides is the maximum possible?

A. 1 is to 1 ,

B. 1 is to 4 ,

C. 3 is to 2 ,

D. None of the above.

12. Suppose (x^*, y^*) solves:

$$\text{Minimize } ax + by$$

subject to

$$x^\alpha + y^\alpha = M$$

and $x, y \geq 0$, where $a > b > 0, M > 0$ and $\alpha > 1$. Then, the solution is

A. $\dfrac{x*^{\alpha-1}}{y*^{\alpha-1}} = \dfrac{a}{b}$

B. $x^* = 0, y^* = M^{\frac{1}{\alpha}}$

C. $y^* = 0, x^* = M^{\frac{1}{\alpha}}$

D. None of the above.

13. Three boys and two girls are to be seated in a row for a photograph. It is desired that no two girls sit together. The number of ways in which they can be so arranged is

A. $4_{P_2} \times 3!$

B. $3_{P_2} \times 2!$

C. $2! \times 3!$

D. None of the above

14. The domain of x for which $\sqrt{x} + \sqrt{3-x} + \sqrt{x^2 - 4x}$ is real is,

A. [0,3]

B. (0,3)

C. {0}

D. None of the above

15. P(x) is a quadratic polynomial such that $P(1) = P(-1)$. Then

A. The two roots sum to zero,

B. The two roots sum to 1 ,

C. One root is twice the other,

D. None of the above.

16. The expression $\sqrt{11 + 6\sqrt{2}} + \sqrt{11 - 6\sqrt{2}}$ is

A. Positive and an even integer,

B. Positive and an odd integer,

C. Positive and irrational,

D. None of the above.

17. What is the maximum value of $a(1-a)b(1-b)c(1-c)$, where a, b, c vary over all positive fractional values?

A. 1

B. $\frac{1}{8}$

C. $\frac{1}{27}$

D. $\frac{1}{64}$.

18. There are four modes of transportation in Delhi: (A) Auto-rickshaw, (B) Bus, (C) Car, and (D) Delhi-Metro. The probability of using transports A, B, C, D by an individual is $\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{2}{9}$ respectively. The probability that he arrives late at work if he uses transportation A, B, C, D is $\frac{5}{7}, \frac{4}{7}, \frac{6}{7}$, and $\frac{6}{7}$ respectively. What is the probability that he used transport A if he reached office on time?

    A. $\frac{1}{9}$,

    B. $\frac{1}{7}$

    C. $\frac{3}{7}$

    D. $\frac{2}{9}$

19. What is the least (strictly) positive value of the expression $a^3+b^3+c^3-3abc$, where a, b, c vary over all strictly positive integers? You may use the identity $a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)\left((a-b)^2+(b-c)^2+(c-a)^2\right)$

    A. 2

    B. 3

    C. 4

    D. 8

20. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ is,

    A. -0.75

    B. Belongs to the interval $[-1, -0.5]$

    C. Belongs to the interval $[0.5, 1]$

    D. None of the above.

21. Consider the following linear programming problem: Maximize $a+b$ subject to $a+2b \leq 4$ $a+6b \leq 6$ $a-2b \leq 2$ $a, b \geq 0$ An optimal solution is:

    A. $a=4, b=0$

    B. $a=0, b=1$

    C. $a=3, b=1/2$

    D. None of the above.

22. The value of $\int_{-4}^{-1} \frac{1}{x} dx$ equals,

    A. $\ln 4$

    B. Undefined,

C. $\ln(-4) - \ln(-1)$

D. None of the above.

23. Given $x \geq y \geq z$, and $x + y + z = 9$, the maximum value of $x + 3y + 5z$ is

A. 27

B. 42

C. 21

D. 18

24. A car with six sparkplugs is known to have two malfunctioning ones. If two plugs are pulled out at random, what is the probability of getting at least one malfunctioning plug.

A. 1/15

B. 7/15

C. 8/15

D. 9/15.

25. Suppose there is a multiple choice test which has 20 questions. Each question has two possible responses - true or false. Moreover, only one of them is correct. Suppose a student answers each of them randomly. Which one of the following statements is correct?

A. The probability of getting 15 correct answers is less than the probability of getting 5 correct answers,

B. The probability of getting 15 correct answers is more than the probability of getting 5 correct answers,

C. The probability of getting 15 correct answers is equal to the probability of getting 5 correct answers,

D. The answer depends on such things as the order of the questions.

26. From a group of 6 men and 5 women, how many different committees consisting of three men and two women can be formed when it is known that 2 of the men do not want to be on the committee together?

A. 160

B. 80

C. 120

D. 200

27. Consider any two consecutive integers a and b that are both greater than 1. The sum $(a^2 + b^2)$ is

A. Always even,

B. Always a prime number,

C. Never a prime number,

D. None of the above statements is correct.

28. The number of real non-negative roots of the equation

$$x^2 - 3|x| - 10 = 0$$

is,

A. 2

B. 1

C. 0

D. 3

29. Let $< a^n >$ and $< b^n >, n = 1, 2, \cdots$, be two different sequences, where $< a^n >$ is convergent and $< b^n >$ is divergent. Then the sequence $< a^n + b^n >$ is

A. Convergent,

B. Divergent,

C. Undefined,

D. None of the above.

30. Consider the function

$$f(x) = \frac{|x|}{1 + |x|}$$

This function is,

A. Increasing in x when $x \geq 0$,

B. Decreasing in x,

C. Increasing in x for all real x,

D. None of the above.

8 ISI PEA 2013

1. Let $f(x) = \frac{1-x}{1+x}, x \neq -1$. Then $f\left(f\left(\frac{1}{x}\right)\right), x \neq 0$ and $x \neq -1$, is

    A. 1

    B. x

    C. x^2

    D. $\frac{1}{x}$

2. The limiting value of $\frac{1.2+2.3+\ldots+n(n+1)}{n^3}$ as $n \to \infty$ is,

    A. 0

    B. 1

    C. 1/3

    D. 1/2

3. Suppose $a_1, a_2, \ldots, a_n$ are n positive real numbers with $a_1 a_2 \ldots a_n = 1$. Then the minimum value of $(1+a_1)(1+a_2)\ldots(1+a_n)$ is

    A. 2^n

    B. 2^{2n}

    C. 1

    D. None of the above.

4. Let the random variable X follow a Binomial distribution with parameters n and p where $n(>1)$ is an integer and $0 < p < 1$. Suppose further that the probability of $X = 0$ is the same as the probability of $X = 1$ Then the value of p is

    A. $\frac{1}{n}$,

    B. $\frac{1}{n+1}$

    C. $\frac{n}{n+1}$

    D. $\frac{n-1}{n+1}$.

5. Let X be a random variable such that $E(X^2) = E(X) = 1$. Then $E(X^{100})$ is

    A. 1 ,

    B. 2^{100},

    C. 0,

    D. None of the above.

6. If α and β are the roots of the equation $x^2 - ax + b = 0$, then the quadratic equation whose roots are $\alpha + \beta + \alpha\beta$ and $\alpha\beta - \alpha - \beta$ is

    A. $x^2 - 2ax + a^2 - b^2 = 0$

    B. $x^2 - 2ax - a^2 + b^2 = 0$

C. $x^2 - 2bx - a^2 + b^2 = 0$

D. $x^2 - 2bx + a^2 - b^2 = 0$

7. Suppose $f(x) = 2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1$ where x is real and $x \neq 0$. Then the solutions of $f(x) = 0$ are such that their product is

   A. 1

   B. 2

   C. -1

   D. -2

8. Toss a fair coin 43 times. What is the number of cases where number of Head is $>$ number of Tail?

   A. 2^{43},

   B. $2^{43} - 43$

   C. 2^{42}

   D. None of the above.

9. The minimum number of real roots of $f(x) = |x|^3 + a|x|^2 + b|x| + c$ where a, b and c are real, is

   A. 0

   B. 2

   C. 3

   D. 6

10. Suppose $f(x, y)$ where x and y are real, is a differentiable function satisfying the following properties: (i) $f(x + k, y) = f(x, y) + ky$ (ii) $f(x, y + k) = f(x, y) + kx$; and (iii) $f(x, 0) = m$, where m is a constant. Then $f(x, y)$ is given by

    A. $m + xy$

    B. $m + x + y$

    C. mxy

    D. None of the above.

11. Let $I = \int_2^{343} \{x - [x]\}^2 dx$ where $[x]$ denotes the largest integer less than or equal to x. Then the value of I is

    A. $\frac{343}{3}$,

    B. $\frac{343}{2}$,

    C. $\frac{341}{3}$,

    D. None of the above.

12. The coefficients of three consecutive terms in the expression of $(1+x)^n$ are 165,330 and 462. Then the value of n is

    A. 10

    B. 11

    C. 12

    D. 13

13. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ lies in

    A. $\left[\frac{1}{2},1\right]$

    B. $[-1,1]$

    C. $\left[-\frac{1}{2},\frac{1}{2}\right]$

    D. $\left[-\frac{1}{2},1\right]$

14. Let the function $f(x)$ be defined as $f(x)=|x-4|+|x-5|$. Then which of the following statements is true?

    A. $f(x)$ is differentiable at all points,

    B. $f(x)$ is differentiable at $x=4$, but not at $x=5$

    C. $f(x)$ is differentiable at $x=5$ but not at $x=4$,

    D. None of the above.

15. The value of the integral $\int_0^1 \int_0^x x^2 e^{xy} dxdy$ is

    A. e

    B. $\frac{e}{2}$,

    C. $\frac{1}{2}(e-1)$

    D. $\frac{1}{2}(e-2)$

16. Let $\mathcal{N}=\{1,2,\ldots\}$ be a set of natural numbers. For each $x\in\mathcal{N}$, define $A_n=\{(n+1)k, k\in\mathcal{N}\}$. Then $A_1\cap A_2$ equals

    A. A_2

    B. A_4

    C. A_5

    D. A_6

17. $\lim_{x\to 0}\left\{\frac{1}{x}\left(\sqrt{1+x+x^2}-1\right)\right\}$ is

    A. 0

    B. 1

    C. $\frac{1}{2}$,

    D. Non-existent.

18. The value of $\binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \ldots + (n+1)\binom{n}{n}$ equals

A. $2^n + n2^{n-1}$

B. $2^n - n2^{n-1}$

C. 2^n

D. 2^{n+2}

19. The rank of the matrix $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ is

A. 1

B. 2

C. 3

D. 4

20. Suppose an odd positive integer $2n+1$ is written as a sum of two integers so that their product is maximum. Then the integers are

A. $2n$ and 1 ,

B. $n+2$ and $n-1$

C. $2n-1$ and 2

D. None of the above.

21. If $|a| < 1, |b| < 1$, then the series $a(a+b) + a^2\left(a^2+b^2\right) + a^3\left(a^3+b^3\right) + \ldots\ldots$ converges to

A. $\frac{a^2}{1-a^2} + \frac{b^2}{1-b^2}$

B. $\frac{a(a+b)}{1-a(a+b)}$

C. $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$,

D. $\frac{a^2}{1-a^2} - \frac{ab}{1-ab}$.

22. Suppose $f(x) = x^3 - 6x^2 + 24x$. Then which of the following statements is true?

A. $f(x)$ has a maxima but no minima,

B. $f(x)$ has a minima but no maxima,

C. $f(x)$ has a maxima and a minima,

D. $f(x)$ has neither a maxima nor a minima.

23. An urn contains 5 red balls, 4 black balls and 2 white balls. A player draws 2 balls one after another with replacement. Then the probability of getting at least one red ball or at least one white ball is

A. $\frac{105}{121}$

B. $\frac{67}{121}$

C. $\frac{20}{121}$

D. None of the above.

24. If $\log_t x = \frac{1}{t-1}$ and $\log_t y = \frac{t}{t-1}$, where $\log_t x$ stand for logarithm of x to the base t. Then the relation between x and y is

A. $y^x = x^{1/y}$

B. $x^{1/y} = y^{1/x}$

C. $x^y = y^x$,

D. $x^y = y^{1/x}$

25. Suppose $\frac{f''(x)}{f'(x)} = 1$ for all x. Also, $f(0) = e^2$ and $f(1) = e^3$. Then $\int_{-2}^{2} f(x)dx$ equals

A. $2e^2$

B. $e^2 - e^{-2}$

C. $e^4 - 1$

D. None of the above.

26. The minimum value of the objective function $z = 5x + 7y$, where $x \geq 0$ and $y \geq 0$, subject to the constraints $2x + 3y \geq 6$, $3x - y \leq 15$, $-x + y \leq 4$, and $2x + 5y \leq 27$ is

A. 14

B. 15

C. 25

D. 28

27. Suppose A is a 2×2 matrix given as $\begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$. Then the matrix $A^2 - 3A - 13I$, where I is the 2×2 identity matrix, equals

A. I

B. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 5 \\ 3 & 0 \end{pmatrix}$

D. None of the above.

28. The number of permutations of the letters a, b, c, and d such that b does not follow a, c does not follow b, and d does not follow c is

A. 14

B. 13

C. 12

D. 11

29. Given n observations $x_1, x_2, \ldots, x_n$, which of the following statements is true?

A. The mean deviation about arithmetic mean can exceed the standard deviation

B. The mean deviation about arithmetic mean cannot exceed the standard deviation

C. The root mean square deviation about a point A is least when A is the median

D. The mean deviation about a point A is minimum when A is the arithmetic mean

30. Consider the following classical linear regression of y on x,

$$y_i = \beta x_i + u_i, i = 1, 2, \ldots, \ldots, n$$

where $E(u_i) = 0, V(u_i) = \sigma^2$ for all i, and $u_i's$ are homoscedastic and non-autocorrelated. Now, let $\hat{u}_i$ be the ordinary least square estimate of u_i. Then which of the following statements is true?

A. $\sum_{i=1}^{n} \hat{u}_i = 0$

B. $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i \hat{u}_i = 0$

C. $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i \hat{u}_i \neq 0$

D. $\sum_{i=1}^{n} x_i \hat{u}_i = 0$

9 ISI PEA 2014

1. Let $f(x)=\frac{1-x}{1+x}, x \neq -1$. Then $f\left(f\left(\frac{1}{x}\right)\right), x \neq 0$ and $x \neq -1$, is

   A. 1

   B. x

   C. x^2

   D. $\frac{1}{x}$

2. What is the value of the following definite integral?

$$2 \int_0^{\frac{\pi}{2}} e^x \cos(x) dx$$

   A. $e^{\frac{\pi}{2}}$

   B. $e^{\frac{x}{2}}-1$

   C. 0

   D. $e^{\frac{\pi}{2}}+1$

3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as follows:

$$f(x)=|x-1|+(x-1)$$

   Which of the following is not true for f?

   A. $f(x)=f\left(x^{\prime}\right)$ for all $x, x^{\prime}<1$

   B. $f(x)=2 f(1)$ for all $x>1$

   C. f is not differentiable at 1

   D. The derivative of f at $x=2$ is 2

4. Population of a city is 40% male and 60% female. Suppose also that 50% of males and 30% of females in the city smoke. The probability that a smoker in the city is male is closest to

   A. 0.5

   B. 0.46

   C. 0.53

   D. 0.7

5. A blue and a red die are thrown simultaneously. We define three events as follows: - Event E : the sum of the numbers on the two dice is 7 - Event F : the number on the blue die equals 4 - Event G : the number on the red die equals 3 Which of the following statements is true?

   A. E and F are disjoint events.

   B. E and F are independent events.

C. E and F are not independent events.

D. Probability of E is more than the probability of F.

6. Let $p > 2$ be a prime number. Consider the following set containing 2×2 matrices of integers:

$$T_p = \left\{ A = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in \{0, 1, \ldots, p-1\} \right\}$$

A matrix $A \in T_p$ is p -special if determinant of A is not divisible by p. How many matrices in T_p are p -special?

A. $(p-1)^2$

B. $2p-1$

C. p^2

D. $p^2 - p + 1$

7. A "good" word is any seven letter word consisting of letters from $\{A, B, C\}$ (some letters may be absent and some letter can be present more than once), with the restriction that A cannot be followed by B, B cannot be followed by C, and C cannot be followed by A. How many good words are there?

A. 192

B. 128

C. 96

D. 64

8. Let n be a positive integer and $0 < a < b < \infty$. The total number of real roots of the equation $(x-a)^{2n+1} + (x-b)^{2n+1} = 0$ is

A. 1

B. 3

C. $2n-1$

D. $2n+1$

9. Consider the optimization problem below:

$$\begin{aligned} \max_{x,y}\, x + y \\ \text{subject to } 2x + y \leq 14 \\ -x + 2y \leq 8 \\ 2x - y \leq 10 \\ x, y \geq 0 \end{aligned}$$

The value of the objective function at optimal solution of this optimization problem:

A. does not exist

B. is 8

C. is 10

D. is unbounded

10. A random variable X is distributed in $[0,1]$. Mr. Fox believes that X follows a distribution with cumulative density function (cdf) $F : [0,1] \rightarrow [0,1]$ and Mr. Goat believes that X follows a distribution with cdf $G : [0,1] \rightarrow [0,1]$. Assume F and G are differentiable, $F \neq G$ and $F(x) \leq G(x)$ for all $x \in [0,1]$. Let $\mathbb{E}_F[X]$ and $\mathbb{E}_G[X]$ be the expected values of X for Mr. Fox and Mr. Goat respectively. Which of the following is true?

A. $\mathbb{E}_F[X] \leq \mathbb{E}_G[X]$

B. $\mathbb{E}_F[X] \geq \mathbb{E}_G[X]$

C. $\mathbb{E}_F[X] = \mathbb{E}_G[X]$

D. None of the above.

11. Let $f : [0,2] \rightarrow [0,1]$ be a function defined as follows:

$$f(x) = \begin{cases} x & \text{if } x \leq \alpha \\ \frac{1}{2} & \text{if } x \in (\alpha, 2] \end{cases}$$

where $\alpha \in (0,2)$. Suppose X is a random variable distributed in $[0,2]$ with probability density function f. What is the probability that the realized value of X is greater than 1?

A. 1

B. 0

C. $\frac{1}{2}$

D. $\frac{3}{4}$

12. The value of the expression

$$\sum_{k=1}^{100} \int_0^1 \frac{x^k}{k} dx$$

is

A. $\frac{100}{101}$

B. $\frac{1}{99}$

C. 1

D. $\frac{99}{100}$

13. Consider the following system of inequalities.

$$\begin{aligned} x_1 - x_2 &\leq 3 \\ x_2 - x_3 &\leq -2 \\ x_3 - x_4 &\leq 10 \\ x_4 - x_2 &\leq \alpha \\ x_4 - x_3 &\leq -4 \end{aligned}$$

where α is a real number. A value of α for which this system has a solution is

A. -16

B. -12

C. -10

D. None of the above

14. A fair coin is tossed infinite number of times. The probability that a head turns up for the first time after even number of tosses is

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

15. An entrance examination has 10 ”true-false” questions. A student answers all the questions randomly and his probability of choosing the correct answer is 0.5. Each correct answer fetches a score of 1 to the student, while each incorrect answer fetches a score of zero. What is the probability that the student gets the mean score?

A. $\frac{1}{4}$

B. $\frac{63}{256}$

C. $\frac{1}{2}$

D. $\frac{1}{8}$

16. For any positive integer k, let S_k denote the sum of the infinite geometric progression whose first term is $\frac{(k-1)}{k!}$ and common ratio is $\frac{1}{k}$. The value of the expression $\sum_{k=1}^{\infty} S_k$ is

A. e.

B. $1+e$

C. $2+e$

D. e^2.

17. Let $G(x)=\int_0^x te^t dt$ for all non-negative real number x. What is the value of $\lim_{x\to 0}\frac{1}{x}G'(x)$, where $G'(x)$ is the derivative of G at x

A. 0

B. 1

C. e

D. None of the above

18. Let $\alpha \in (0,1)$ and $f(x)=x^\alpha+(1-x)^\alpha$ for all $x\in[0,1]$. Then the maximum value of f is

A. 1

B. greater than 2

C. in (1,2)

D. 2

19. Let n be a positive integer. What is the value of the expression

$$\sum_{k=1}^{n} kC(n,k)$$

where $C(n,k)$ denotes the number of ways to choose k out of n objects?

A. $n2^{n-1}$

B. $n2^{n-2}$

C. 2^n

D. $n2^n$

20. The first term of an arithmetic progression is a and common difference is $d \in (0,1)$. Suppose the k -th term of this arithmetic progression equals the sum of the infinite geometric progression whose first term is a and common ratio is d. If $a > 2$ is a prime number, then which of the following is a possible value of d?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{5}$

D. $\frac{1}{9}$

21. In period 1, a chicken gives birth to 2 chickens (so, there are three chickens after period 1). In period 2, each chicken born in period 1 either gives birth to 2 chickens or does not give birth to any chicken. If a chicken does not give birth to any chicken in a period, it does not give birth in any other subsequent periods. Continuing in this manner, in period $(k+1)$, a chicken born in period k either gives birth to 2 chickens or does not give birth to any chicken. This process is repeated for T periods - assume no chicken dies. After T periods, there are in total 31 chickens. The maximum and the minimum possible values of T are respectively

A. 12 and 4

B. 15 and 4

C. 15 and 5

D. 12 and 5

22. Let a and p be positive integers. Consider the following matrix

$$A = \begin{bmatrix} p & 1 & 1 \\ 0 & p & a \\ 0 & a & 2 \end{bmatrix}$$

If determinant of A is 0, then a possible value of p is

A. 1

B. 2

C. 4

D. None of the above

23. For what value of α does the equation $(x-1)(x^2-7x+\alpha)=0$ have exactly two unique roots?

A. 6

B. 10

C. 12

D. None of the above

24. What is the value of the following infinite series?

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2} \log_e 3^k$$

A. $\log_e 2$

B. $\log_e 2 \log_e 3$

C. $\log_e 6$

D. $\log_e 5$

25. There are 20 persons at a party. Each person shakes hands with some of the persons at the party. Let K be the number of persons who shook hands with odd number of persons. What is a possible value of K?

A. 19

B. 1

C. 20

D. All of the above

26. Two independent random variables X and Y are uniformly distributed in the interval $[0,1]$. For a $z \in [0,1]$, we are told that probability that $\max(X,Y) \leq z$ is equal to the probability that $\min(X,Y) \leq (1-z)$ What is the value of z?

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. any value in $\left[\frac{1}{2}, 1\right]$

D. None of the above

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that satisfies for all $x, y \in \mathbb{R}$

$$f(x+y)f(x-y) = (f(x)+f(y))^2 - 4x^2 f(y)$$

Which of the following is not possible for f?

A. $f(0) = 0$

B. $f(3) = 9$

C. $f(5) = 0$

D. $f(2) = 2$

28. Consider the following function $f : \mathbb{R} \to \mathbb{Z}$, where $\mathbb{R}$ is the set of all real numbers and $\mathbb{Z}$ is the set of all integers.

$$f(x) = \lceil x \rceil$$

where $\lceil x \rceil$ is the smallest integer that is larger than x. Now, define a new function g as follows. For any $x \in \mathbb{R}, g(x) = |f(x)| - f(|x|)$, where $|x|$ gives the absolute value of x. What is the range of g?

A. $\{0, 1\}$

B. $[-1, 1]$

C. $\{-1, 0, 1\}$

D. $\{-1, 0\}$

29. The value of $\lim_{x\to-1} \frac{x+1}{|x+1|}$ is.

A. 1

B. -1

C. 0

D. None of the above.

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = 2$ if $x \leq 2$ and $f(x) = a^2 - 3a$ if $x > 2$, where a is a positive integer. Which of the following is true?

A. f is continuous everywhere for some value of a

B. f is not continuous

C. f is differentiable at $x = 2$

D. None of the above

10 ISI PEA 2015

1. $\lim_{x\to 0+}\frac{\sin\{\sqrt{x}\}}{\{\sqrt{x}\}}$, where $\{x\}$ = decimal part of x, is

   A. 0

   B. 1

   C. non-existent

   D. none of these

2. $f:[0,1]\to[0,1]$ is continuous.Then it is true that

   A. $f(0)=0, f(1)=1$

   B. f is differentiable only at $x=\frac{1}{2}$

   C. $f'(x)$ is constant for all $x\in(0,1)$

   D. $f(x)=x$ for at least one $x\in[0,1]$

3. $f(x)=|x-2|+|x-4|$. Then f is

   A. continuously differentiable at x = 2

   B. differentiable but not continuously differentiable at $x=2$

   C. f has both left and right derivatives at $x=2$

   D. none of these

4. In an examination of 100 students, 70 passed in Mathematics, 65 passed in Physics and 55 passed in Chemistry. Out of these students, 35 passed in all the three subjects, 50 passed in Mathematics and Physics, 45 passed in Mathematics and Chemistry and 40 passed in Physics and Chemistry.Then the number of students who passed in exactly one subject is

   A. 30

   B. 25

   C. 10

   D. none of these

5. The square matrix of the matrix $\begin{vmatrix} a & b \\ c & 0 \end{vmatrix}$ is a null matrix if and only if

   A. $a=b=c=0$

   B. $a=c=0, b$ is any non-zero real number

   C. $a=b=0, c$ is any non-zero real number

   D. $a=0$ and either $b=0$ or $c=0$

6. If the positive numbers x,y,z are in harmonic progression, then $\log(x+z)+\log(x-2y+\text{z})$ equals

   A. $4\log(x-z)$

B. $3\log(x-z)$

C. $2\log(x-z)$

D. $\log(x-z)$

7. If $f(x+2y, x-2y) = xy$, then $f(x,y)$ equals

A. $\frac{x^2-y^2}{8}$

B. $\frac{x^2-y^2}{4}$

C. $\frac{x^2+y^2}{4}$

D. none of these

8. The domain of the function $f(x) = \sqrt{x^2-1} - \log(\sqrt{1-x}), x \geq 0$,is

A. $(-\infty, -1)$

B. $(-1, 0)$

C. null set

D. none of these

9. The graph of the function $y = \log\left(1 - 2x + x^2\right)$ intersects the x axis at

A. 0,2

B. 0,-2

C. 2

D. 0

10. The sum of two positive integers is 100 .The minimum value of the sum of their reciprocals is

A. $\frac{3}{25}$

B. $\frac{6}{25}$

C. $\frac{1}{25}$

D. none of these

11. The range of the function $f(x) = 4^x + 2^x + 4^{-x} + 2^{-x} + 3$, where $x \in (-\infty, \infty)$, is

A. $\left(\frac{3}{4}, \infty\right)$

B. $\left[\frac{3}{4}, \infty\right)$

C. $(7, \infty)$

D. $[7, \infty)$

12. The function $f : R \to R$ satisfies $f(x+y) = f(x) + f(y) \forall x, y \in R$, where R is the real line, and $f(1) = 7$. Then $\sum_{r=1}^{n} f(r)$ equals

A. $\frac{7n}{2}$

B. $\frac{7(n+1)}{2}$

C. $\frac{7n(n+1)}{2}$

D. $7n(n+1)$

13. Let f and g be differentiable functions for $0 < x < 1$ and $f(0) = g(0) = 0, f(1) = 6$ Suppose that for all $x \in (0,1)$, the equality $f'(x) = 2g'(x)$ holds. Then $g(1)$ equals

A. 1

B. 3

C. -2

D. -1

14. Consider the real valued function $f(x) = ax^2 + bx + c$ defined on $[1, 2]$. Then it is always possible to get a $k \in (1, 2)$ such that

A. $k = 2a + b$

B. $k = a + 2b$

C. $k = 3a + b$

D. none of these

15. In a sequence the first term is $\frac{1}{3}$. The second term equals the first term divided by 1 more than the first term. The third term equals the second term divided by 1 more than the second term, and so on. Then the 500^{th} term is

A. $\frac{1}{503}$

B. $\frac{1}{501}$

C. $\frac{1}{502}$

D. none of these

16. In how many ways can three persons, each throwing a single die once, make a score of 10?

A. 6

B. 18

C. 27

D. 36

17. Let a be a positive integer greater than 2 .The number of values of x for which

$$\int_a^x (x+y)dy = 0 \text{ holds is}$$

A. 1

B. 2

C. a

D. $a+1$

18. Let (x^*, y^*) be a solution to any optimization problem $\max_{(x,y)\in\mathbf{R}^2} f(x,y)$ subject to $g_1(x,y) \leq c_1$. Let (x', y) be a solution to the same optimization problem $\max_{(x,y)\in\mathbf{R}^2} f(x,y)$ subject to $g_1(x,y) \leq c_1$ with an added constraint that $g_2(x,y) \leq c_2$. Then which one of the following statements is always true?

    A. $f\left(x^*, y^*\right) \geq f\left(x', y\right)$

    B. $f\left(x^*, y^*\right) \leq f\left(x', y\right)$

    C. $\left|f\left(x^*, y^*\right)\right| \geq \left|f\left(x', y\right)\right|$

    D. $\left|f\left(x^*, y^*\right)\right| \leq \left|f\left(x', y\right)\right|$

19. Let (x^*, y^*) be a real solution to: $\max_{(x,y)\in\Re^2} \sqrt{x} + y$ subject to $px + y \leq m$, where $m > 0, p > 0$ and $y^* \in (0, m)$. Then which one of the following statements is true?

    A. x^* depends only on p

    B. x^* depends only on m

    C. x^* depends on both p and m

    D. x^* is independent of both p and m.

20. Let $0 < a_1 < a_2 < 1$ and let $f\left(x; a_1, a_2\right) = -\left|x - a_1\right| - \left|x - a_2\right|$.Let X be the set of all values of x for which $f\left(x; a_1, a_2\right)$ achieves its maximum. Then

    A. $X = \left\{x \mid x \in \left\{\frac{a_1}{2}, \frac{1+a_2}{2}\right\}\right\}$

    B. $X = \{x \mid x \in \{a_1, a_2\}\}$

    C. $X = \left\{x \mid x \in \left\{0, \frac{a_1+a_2}{2}, 1\right\}\right\}$

    D. $X = \{x \mid x \in [a_1, a_2]\}$

21. Let A and B be two events with positive probability each, defined on the same sample space. Find the correct answer:

    A. P(A/B) > P(A) always

    B. P(A/B) < P(A) always

    C. P(A/B) > P(B) always

    D. None of the above

22. Let A and B be two mutually exclusive events with positive probability each, defined on the same sample space. Find the correct answer:

    A. A and B are necessarily independent

    B. A and B are necessarily dependent

    C. A and B are necessarily equally likely

    D. None of the above

23. The salaries of 16 players of a football club are given below (units are in thousands of rupees).

$$100, 100, 111, 114, 165, 210, 225, 225, 230,$$
$$575, 1200, 1900, 2100, 2100, 2650, 3300$$

Now suppose each player received an extra Rs. 200,000 as bonus. Find the correct answer:

A. Mean will increase by Rs. 200,000 but the median will not change

B. Both mean and median will be increased by Rs. 200,000

C. Mean and standard deviation will both be changed

D. Standard deviation will be increased but the median will be unchanged

24. Let Pr(X = 2) = 1. Define $\mu_{2n} = E(X - \mu)^{2n}, \mu = E(X)$. Then:

A. $\mu_{2n} = 2$

B. $\mu_{2n} = 0$

C. $\mu_{2n} > 0$

D. None of the above

25. Consider a positively skewed distribution. Find the correct answer on the position of the mean and the median:

A. Mean is greater than median

B. Mean is smaller than median

C. Mean and median are same

D. None of the above

26. Puja and Priya play a fair game (i.e. winning probability is 1/2 for both players) repeatedly for one rupee per game. If originally Puja has a rupees and Priya has b rupees (where $a > b$), what is Puja's chances of winning all of Priya's money, assuming the play goes on until one person has lost all her money?

A. 1

B. 0

C. b/(a + b)

D. $a/(a + b)$

27. An urn contains w white balls and b black balls ($w > 0$) and ($b > 0$).The balls are thoroughly mixed and two are drawn, one after the other, without replacement. Let Wi denote the outcome 'white on the i -th draw' for $i = 1, 2$. Which one of the following is true?

A. P(W2) = P(W1) = w/(w + b)

B. P(W2) = P(W1) = (w − 1)/(w + b − 1)

C. $\mathrm{P(W1) = w/(w+b), P(W2) = (w-1)/(w+b-1)}$

D. $\mathrm{P(W1) = w/(w+b), P(W2) = \{w(w-1)\}/\{(w-b)(w+b-1)\}}$

28. A bag contains four pieces of paper, each labeled with one of the digits $1, 2, 3, 4$, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

    A. $\frac{3/4}{6}$

    B. 1/2

    C. 1/4

    D. 9/24

29. Consider two random variables X and Y where X takes values -2,-1,0,1,2 each with probability 1/5 and Y = |X|. Which of the following is true?

    A. The variables X and Yare independent and Pearson's correlation coefficient between X and Y is 0.

    B. The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 0.

    C. The variables X and Y are independent and Pearson's correlation coefficient between X and Y is 1

    D. The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 1

30. Two friends who take the metro to their jobs from the same station arrive to the station uniformly randomly between 7 and 7: 20 in the morning. They are willing to wait for one another for 5 minutes, after which they take a train whether together or alone. What is the probability of their meeting at the station?

    A. 5/20

    B. 25/400

    C. 10/20

    D. 7/16

11 ISI PEA 2016

1. Consider the polynomial $P(x) = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \{1, 2, \ldots, 9\}$. If $P(10) = 5861$, then the value of c is

   A. 1.

   B. 2

   C. 6

   D. 5 .

2. Let $A \subset \mathbb{R}, f : A \to \mathbb{R}$ be a twice continuously differentiable function, and $x^* \in A$ be such that $\frac{\partial f}{\partial x}(x^*) = 0$

   A. $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is a sufficient condition for x^* to be a point of local maximum of f on A

   B. $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is a necessary condition for x^* to be a point of local maximum of f on A

   C. $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is necessary and sufficient for x^* to be a point of local maximum of f on A

   D. $\frac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is neither necessary nor sufficient for x^* to be a point of local maximum of f on A.

3. You are given five observations x_1, x_2, x_3, x_4, x_5 on a variable x, ordered from lowest to highest. Suppose x_5 is increased. Then,

   A. The mean, median, and variance, all increase.

   B. The median and the variance increase but the mean is unchanged.

   C. The variance increases but the mean and the median are unchanged.

   D. None of the above.

4. Suppose the sum of coefficients in the expansion $(x + y)^n$ is 4096. The largest coefficient in the expansion is:

   A. 924

   B. 1024

   C. 824

   D. 724

5. There are three cards. The first is green on both sides, the second is red on both sides and the third is green on one side and red on the other. I choose a card with equal probability, then a side of that card with equal probability. If the side I choose of the card is green, what is the probability that the other side is green?

   A. $\frac{1}{3}$.

   B. $\frac{1}{2}$.

C. $\frac{2}{3}$.

D. $\frac{3}{4}$.

6. The value of

$$\int_0^{\frac{\pi}{2}} x \sin x dx$$

is:

A. 0.

B. -1

C. $\frac{1}{2}$.

D. 1

7. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} ax + b & \text{if } x \geq 0 \\ \sin 2x & \text{if } x < 0 \end{cases}$$

For what values of a and b is f continuous but not differentiable?

A. $a = 2, b = 0$.

B. $a = 2, b = 1$

C. $a = 1, b = 1$

D. $a = 1, b = 0$

8. A student wished to regress household food consumption on household income. By mistake the student regressed household income on household food consumption and found R^2 to be 0.35. The R^2 in the correct regression of household food consumption on household income is

A. 0.65

B. 0.35

C. $1 - (.35)^2$.

D. None of the above.

9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = 3xe^y - x^3 - e^{3y}$$

Which of the following statements is true?

A. $(x = 1, y = 0)$ is a local maximum of f

B. $(x = 1, y = 0)$ is a local minimum of f.

C. $(x = 1, y = 0)$ is neither a local maximum nor a local minimum of f.

D. $(x = 0, y = 0)$ is a global maximum of f.

10. Let

$$f(x) = \frac{x+\sqrt{3}}{1-\sqrt{3}x}$$

for all $x \neq \frac{1}{\sqrt{3}}$. What is the value of $f(f(x))$?

A. $\frac{x-\sqrt{3}}{1+\sqrt{3}x}$.

B. $\frac{x^2+2\sqrt{3}x+3}{1-2\sqrt{3}x+3x}$

C. $\frac{x+\sqrt{3}}{1-\sqrt{3}x}$.

D. $\frac{x+\sqrt{3}}{1-\sqrt{3}x}$.

11. The continuous random variable X has probability density $f(x)$ where

$$f(x) = \begin{cases} a & \text{if } 0 \leq x < k \\ b & \text{if } k \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $a > b > 0$ and $0 < k < 1$. Then $E(X)$ is given by:

A. $\frac{b(1-a)^2}{2a(a-b)}$.

B. $\frac{1}{2}$.

C. $\frac{a-b}{(a+b)}$

D. $\frac{1-2b+ab}{2(a-b)}$.

12. The set of values of x for which $x^2 - 3|x| + 2 < 0$ is given by:

A. $\{x : x < -2\} \cup \{x : x > 1\}$

B. $\{x : -2 < x < -1\} \cup \{x : 1 < x < 2\}$

C. $\{x : x < -1\} \cup \{x : x > 2\}$

D. None of the above.

13. The system of linear equations

$$\begin{aligned} (4d-1)x + y + z &= 0 \\ -y + z &= 0 \\ (4d-1)z &= 0 \end{aligned}$$

has a non-zero solution if:

A. $d = \frac{1}{4}$.

B. $d = 0$.

C. $d \neq \frac{1}{4}$.

D. $d = 1$.

14. Suppose F is a cumulative distribution function of a random variable x distributed in $[0,1]$ defined as follows:

$$F(x) = \begin{cases} ax + b & \text{if } x \geq a \\ x^2 - x + 1 & \text{otherwise} \end{cases}$$

where $a \in (0,1)$ and b is a real number. Which of the following is true?

A. F is continuous in (0,1)

B. F is differentiable in (0,1)

C. F is not continuous at $x = a$.

D. None of the above

E. Incorrect question

15. The solution of the optimization problem

$$\max_{x,y} 3xy - y^3$$

subject to

$$\begin{aligned} &2x + 5y \geq 20 \\ &x - 2y = 5 \\ &x, y \geq 0 \end{aligned}$$

is given by:

A. $x = 19, y = 7$.

B. $x = 45, y = 20$

C. $x = 15, y = 5$

D. None of the above.

16. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing function. Let g be the inverse of the function f. If $f'(1) = g(1) = 1$, then $g'(1)$ equals to

A. 0

B. $\frac{1}{2}$

C. -1

D. 1

17. Consider a quadratic polynomial $P(x)$. Suppose $P(1) = -3, P(-1) = -9, P(-2) = 0$. Then, which of the following is true.

A. $P\left(\frac{1}{2}\right) = 0$

B. $P\left(\frac{5}{2}\right) = 0$

C. $P\left(\frac{5}{4}\right) = 0$

D. $P\left(\frac{3}{4}\right) = 0$

18. For any positive integers k, ℓ with $k \geq \ell$, let $C(k, \ell)$ denote the number of ways in which ℓ distinct objects can be chosen from k objects. Consider $n \geq 3$ distinct points on a circle and join every pair of points by a line segment. If we pick three of these line segments uniformly at random, what is the probability that we choose a triangle?

    A. $\frac{C(n,2)}{C(C(n,2),3)}$

    B. $\frac{C(n,3)}{C(C(n,2),3)}$

    C. $\frac{2}{n-1}$

    D. $\frac{C(n,3)}{C(C(n,2),2)}$.

19. Let $X = \left\{(x, y) \in \mathbb{R}^2 : x + y \leq 1, 2x + \frac{y}{2} \leq 1, x \geq 0, y \geq 0\right\}$. Consider the optimization problem of maximizing a function $f(x) = ax + by$, where a, b are real numbers, subject to the constraint that $(x, y) \in X$. Which of the following is not an optimal value of f for any value of a and b?

    A. $x = 0, y = 1$

    B. $x = \frac{1}{3}, y = \frac{2}{3}$

    C. $x = \frac{1}{4}, y = \frac{1}{4}$

    D. $x = \frac{1}{2}, y = 0$

20. Let $F : [0, 1] \to \mathbb{R}$ be a differentiable function such that its derivative $F'(x)$ is increasing in x. Which of the following is true for every $x, y \in [0, 1]$ with $x > y$?

    A. $F(x) - F(y) = (x - y)F'(x)$

    B. $F(x) - F(y) \geq (x - y)F'(x)$

    C. $F(x) - F(y) \leq (x - y)F'(x)$

    D. $F(x) - F(y) = F'(x) - F'(y)$

21. A bag contains N balls of which $a (a < N)$ are red. Two balls are drawn from the bag without replacement. Let p_1 denote the probability that the first ball is red and p_2 the probability that the second ball is red. Which of the following statements is true?

    A. $p_1 > p_2$

    B. $p_1 < p_2$

    C. $p_2 = \frac{a-1}{N-1}$

    D. $p_2 = \frac{a}{N}$.

22. Let $t = x + \sqrt{x^2 + 2bx + c}$ where $b^2 > c$. Which of the following statements is true?

    A. $\frac{dx}{dt} = \frac{t-x}{t+b}$

    B. $\frac{dx}{dt} = \frac{t+2x}{2t+b}$.

    C. $\frac{dx}{dt} = \frac{1}{2x+b}$.

D. None of the above.

23. Let A be an $n \times n$ matrix whose entry on the i -th row and j -th column is $\min(i, j)$. The determinant of A is:

    A. n.

    B. 1

    C. $n!$

    D. 0.

24. What is the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 10$?

    A. 66

    B. 55

    C. 100

    D. None of the above

25. For $b > 0$, the value of

$$\int_b^{2b} \frac{x dx}{x^2 + b^2}$$

    A. $\frac{1}{b}$.

    B. $\ln 4b^2$.

    C. $\frac{1}{2} \ln \left(\frac{5}{2}\right)$

    D. None of the above.

26. Let f and g be functions on $\mathbb{R}^2$ defined respectively by

$$f(x, y) = \frac{1}{3}x^3 - \frac{3}{2}y^2 + 2x$$

and

$$g(x, y) = x - y$$

Consider the problems of maximizing and minimizing f on the constraint set $C = \{(x, y) \in \mathbb{R}^2 : g(x, y) = 0\}$

    A. f has a maximum at $(x = 1, y = 1)$, and a minimum at $(x = 2, y = 2)$.

    B. f has a maximum at $(x = 1, y = 1)$, but does not have a minimum.

    C. f has a minimum at $(x = 2, y = 2)$, but does not have a maximum.

    D. f has neither a maximum nor a minimum.

27. A particular men's competition has an unlimited number of rounds. In each round, every participant has to complete a task. The probability of a participant completing the task in a round is p. If a participant fails to complete the task in a round, he is eliminated from the competition. He participates in every round before being elimi- nated. The competition begins with three participants. The probability that all three participants are eliminated in the same round is:

A. $\frac{(1-p)^3}{1-p^3}$.

B. $\frac{1}{3}(1-p)$

C. $\frac{1}{p^3}$.

D. None of the above.

28. Three married couples sit down at a round table at which there are six chairs. All of the possible seating arrangements of the six people are equally likely. The probability that each husband sits next to his wife is:

A. $\frac{2}{15}$.

B. $\frac{1}{3}$.

C. $\frac{4}{15}$.

D. None of the above.

29. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function. For every $x, y, z \in \mathbf{R}$, we know that $f(x, y) + f(y, z) + f(z, x) = 0$. Then, for every $x, y \in \mathbf{R}^2$, $f(x, y) - f(x, 0) + f(y, 0) =$

A. 0

B. 1

C. -1

D. None of the above

30. The minimum value of the expression below for $x > 0$ is:

$$\frac{\left(x + \frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x + \frac{1}{x}\right)^3 + \left(x^3 + \frac{1}{x^3}\right)}$$

A. 1

B. 3

C. 6

D. 12

12 ISI PEA 2017

1. The dimension of the space spanned by the vectors $(-1, 0, 1, 2)$, $(-2, -1, 0, 1)$, $(-3, 2, 0, 1)$ and $(0, 0, -1, 1)$ is

   A. 1

   B. 2

   C. 3

   D. 4

2. How many onto functions are there from a set A with $m > 2$ elements to a set B with 2 elements?

   A. 2^m

   B. $2^m - 1$

   C. $2^{m-1} - 2$

   D. $2^m - 2$

3. The function $f : \mathbb{R}^2_+ \to \mathbb{R}$ given by $f(x, y) = xy$ is

   A. quasiconcave and concave

   B. concave but not quasiconcave

   C. quasiconcave but not concave

   D. none of the above

4. A function $f : \mathbb{R}^2_+ \to \mathbb{R}$ given by $f(x, y) = xy$ is

   A. homogeneous of degree 0

   B. homogeneous of degree 1

   C. homogeneous of degree 2

   D. not homothetic

5. You have n observations on rainfall in centimeters (cm) at a certain location, denoted by x, and you calculate the standard deviation, variance and coefficient of variation (CV). Now, if instead, you were given the same observations measured in millimeters (mm), then

   A. the standard deviation and CV would increase by a factor of 10, and the variance by a factor of 100

   B. the standard deviation would increase by a factor of 10, the variance by a factor of 100 and the CV would be unchanged

   C. the standard deviation would increase by a factor of 10, the variance and CV by a factor of 100

   D. none of the above

6. You have n observations on rainfall in centimeters (cm) at two locations, denoted by x and y respectively, and you calculate the covariance, correlation coefficient r, and the slope coefficient b of the regression of y on x. Now, if instead, you were given the same observations measured in millimeters (mm), then

   A. the covariance would increase by a factor of 10, b by a factor of 100, and r would be unchanged

   B. the covariance and b would increase by a factor of 100, and r would be unchanged

   C. the covariance would increase by a factor of 100 and b and r would be unchanged

   D. none of the above

7. Let $0 < p < 1$. Any solution (x^*, y^*) of the constrained maximization problem

$$\max_{x,y} \left(\frac{-1}{x} + y \right)$$
$$\text{subject to}$$
$$px + y \leq 10$$
$$x, y \geq 0$$

   must satisfy

   A. $y^* = 10 - p$

   B. $x^* = 10/p$

   C. $x^* = 1/\sqrt{p}$

   D. none of the above

8. Suppose the matrix equation $Ax = b$ has no solution, where A is 3×3 non-zero matrix of real numbers and b is a 3×1 vector of real numbers. Then

   A. The set of vectors x for which $Ax = 0$ is a plane

   B. The set of vectors x for which $Ax = 0$ is a line

   C. The rank of A is 3

   D. $Ax = 0$ has a non-zero solution

9. k people get off a plane and walk into a hall where they are assigned to at most n queues. The number of ways in which this can be done is

   A. ${}^nC_k$

   B. ${}^nP_k$

   C. $n^k k!$

   D. $n(n+1)\ldots(n+k-1)$

10. If $\Pr(A) = \Pr(B) = p$, then $\Pr(A \cap B)$ must be

A. greater than p^2

B. equal to p^2

C. less than or equal to p^2

D. none of the above

11. If $\Pr(A^c) = \alpha$ and $\Pr(B^c) = \beta$ (where A^c denotes the event "not A"), then $\Pr(A \cap B)$ must be

A. $1 - \alpha\beta$

B. $(1 - \alpha)(1 - \beta)$

C. greater than or equal to $1 - \alpha - \beta$

D. none of the above

12. The density function of a normal distribution with mean μ and standard deviation σ has inflection points at

A. μ

B. $\mu - \sigma, \mu + \sigma$

C. $\mu - 2\sigma, \mu + 2\sigma$

D. nowhere

13. In how many ways can five objects be placed in a row if two of them cannot be placed next to each other?

A. 36

B. 60

C. 72

D. 24

14. Suppose $x = 0$ is the only solution to the matrix equation $Ax = 0$ where A is $m \times n$, x is $n \times 1$, and 0 is $m \times 1$. Then, of the two statements (i) The rank of A is n, and (ii) $m \geq n$

A. Only (i) must be true

B. Only (ii) must be true

C. Both (i) and (ii) must be true

D. Neither (i) nor (ii) has to be true

15. Mr. A is selling raffle tickets which cost 1 rupee per ticket. In the queue for tickets there are n people. One of them has only a 2-rupee coin while all the rest have 1-rupee coins. Each person in the queue wants to buy exactly one ticket and each arrangement in the queue is equally likely to occur. Initially, Mr. A has no coins and enough tickets for everyone in the queue. He stops selling tickets as soon as he is unable to give the required change. The probability that he can sell tickets to all people in the queue is

A. $\frac{n-2}{n}$

B. $\frac{1}{n}$

C. $\frac{n-1}{n}$

D. $\frac{n-1}{n+1}$

16. Out of 800 families with five children each, how many families would you expect to have either 2 or 3 boys. Assume equal probabilities for boys and girls.

    A. 400

    B. 450

    C. 500

    D. 550

17. The function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

is

    A. concave

    B. convex

    C. neither concave nor convex

    D. both concave and convex

18. As $n \to \infty$, the sequence $\left\{\frac{n^2+1}{2n^2+3}\right\}$

    A. diverges

    B. converges to 1/3

    C. converges to 1/2

    D. neither converges nor diverges

19. The function $x^{1/3}$ is

    A. differentiable at $x = 0$

    B. continuous at $x = 0$

    C. concave

    D. none of the above

20. The function $\sin(\log x)$, where $x > 0$

    A. is increasing

    B. is bounded and converges to a real number as $x \to \infty$

    C. is bounded but does not converge as $x \to \infty$

D. none of the above

21. For any two functions $f_1 : [0,1] \to \mathbb{R}$ and $f_2 : [0,1] \to \mathbb{R}$, define the function $g : [0,1] \to \mathbb{R}$ as $g(x) = \max(f_1(x), f_2(x))$ for all $x \in [0,1]$.

    A. If f_1 and f_2 are linear, then g is linear

    B. If f_1 and f_2 are differentiable, then g is differentiable

    C. If f_1 and f_2 are convex, then g is convex

    D. none of the above

22. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined as

$$f(x) = x^3 - 3x$$

Find the maximum value of the $f(x)$ on the set of real numbers satisfying $x^4 + 36 \leq 13x^2$.

    A. 18

    B. -2

    C. 2

    D. 52

23. A monkey is sitting on 0 on the real line in period 0. In every period $t \in \{0, 1, 2, \ldots\}$ it moves 1 to the right with probability p and 1 to the left with probability $1 - p$, where $p \in \left[\frac{1}{2}, 1\right]$. Let π_k denote the probability that the monkey will reach positive integer k in some period $t > 0$. The value of π_k for any positive integer k is

    A. p^k

    B. 1

    C. $\frac{p^k}{(1-p)^k}$

    D. $\frac{p}{k}$

24. Refer to the previous question. Suppose $p = \frac{1}{2}$ and π_k denote the probability that the monkey will reach positive integer k in some period $t > 0$. The value of π_0 is

    A. 0

    B. $\frac{1}{2^k}$

    C. $\frac{1}{2}$

    D. 1

25. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function with $f'(x) > 0$ for all $x \in \mathbb{R}$ and satisfying the property

$$\lim_{x \to -\infty} f(x) \geq 0$$

Which of the following must be true?

A. $f(1) < 0$

B. $f(1) > 0$

C. $f(1) = 0$

D. none of the above

26. For what values of x is

$$x^2 - 3x - 2 < 10 - 2x$$

A. $4 < x < 9$

B. $x < 0$

C. $-3 < x < 4$

D. none of the above

27. $\int_e^{e^2} \frac{1}{x(\log x)^3} dx =$

A. 3/8

B. 5/8

C. 6/5

D. $-4/5$

28. The solution of the system of equations

$$\begin{aligned} x - 2y + z &= 7 \\ 2x - y + 4z &= 17 \\ 3x - 2y + 2z &= 14 \end{aligned}$$

is

A. $x = 4, y = -1, z = 3$

B. $x = 2, y = 4, z = 3$

C. $x = 2, y = -1, z = 5$

D. none of the above

29. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a twice-differentiable function with non-zero second partial derivatives. Suppose that for every $x \in \mathbb{R}$, there is a unique value of y, say $y^*(x)$, that solves the problem $\max_{y \in \mathbb{R}} f(x, y)$ Then y^* is increasing in x if

A. f is strictly concave

B. f is strictly convex

C. $\frac{\partial^2 f}{\partial x \partial y} > 0$

D. $\frac{\partial^2 f}{\partial x \partial y} < 0$

30. $\int 3^{\sqrt{2x+1}} dx =$

A. $\dfrac{3^{\sqrt{2x+1}}}{\ln 3} + \dfrac{\sqrt{2x+1}}{\ln 3} + c$

B. $\dfrac{3^{\sqrt{2x+1}}\sqrt{2x+1}}{\ln 3} - \dfrac{3^{\sqrt{2x+1}}}{(\ln 3)^2} + c$

C. $\dfrac{3^{\sqrt{2x+1}}\sqrt{2x+1}}{(\ln 3)^2} - \dfrac{3^{\sqrt{2x+1}}}{\ln 3} + c$

D. none of the above

13 ISI PEA 2018

1. Suppose that the level of savings varies positively with the level of income and that savings is identically equal to investment. Then the IS curve:

   A. slopes positively.

   B. slopes negatively.

   C. is vertical.

   D. does not exist.

2. Consider the Solow growth model without technological progress. Suppose that the rate of growth of the labor force is 2%. Then, in the steady-state equilibrium:

   A. per capita income grows at the rate of 2%.

   B. per capita consumption grows at the rate of 2%.

   C. wage per unit of labor grows at the rate of 2%.

   D. total income grows at the rate of 2%.

3. Consider a Simple Keynesian Model for a closed economy with government. Suppose there does not exist any public sector enterprise in the economy. Income earners are divided into two groups, Group 1 and Group 2, such that the saving propensity of the former is less than that of the latter. Aggregate planned investment is an increasing function of GDP (Y). Start with an initial equilibrium situation. Now, suppose the government imposes and collects additional taxes from Group 1 and uses the tax revenue so generated to make transfer payments to Group 2. Following this:

   A. aggregate saving in the economy remains unchanged.

   B. aggregate saving in the economy declines.

   C. aggregate saving in the economy rises.

   D. aggregate saving in the economy may change either way.

4. Suppose, in an economy, the level of consumption is fixed, while the level of investment varies inversely with the rate of interest. Then the IS curve is:

   A. positively sloped.

   B. negatively sloped.

   C. vertical.

   D. horizontal.

5. Suppose, in an economy, the demand function for labor is given by:

$$L^d = 100 - 5w$$

   whereas the supply function for labor is given by:

$$L^s = 5w$$

where w denotes the real wage rate. Total labor endowment in this economy is 80 units. Suppose further that the real wage rate is flexible. Then involuntary unemployment in this economy is:

A. 30

B. 50

C. 70

D. 0

6. Consider again the economy specified in Question 5. Suppose now that the real wage rate is mandated by the government to be at least 11. Then total unemployment will be:

A. 35 .

B. 0.

C. 30 .

D. 10 .

7. Consider a macro-economy defined by the following equations:

$$M = kPy + L(r)$$
$$S(r) = I(r)$$
$$y = \bar{y}$$

where M, P, y and r represent, respectively, money supply, the price level, output and the interest rate, while k and $\bar{y}$ are positive constants. Furthermore, $S(r)$ is the savings function, $I(r)$ is the investment demand function and $L(r)$ is the speculative demand for money function, with , $S'(r) > 0, I'(r) < 0$ and $L'(r) < 0$. Then, an increase in M must:

A. increase P proportionately.

B. reduce P.

C. increase P more than proportionately.

D. increase P less than proportionately.

8. Two individuals, X and Y, have to share Rs. 100. The shares of X and Y are denoted by x and y respectively, $x, y \geq 0, x + y = 100$. Their utility functions are $U_X(x, y) = x + \left(\frac{1}{4}\right) y$ and $U_Y(x, y) = y + \left(\frac{1}{2}\right) x$. The social welfare function is $W(U_X, U_Y) = \min\{U_X, U_Y\}$. Then the social welfare maximizing allocation is:

A. (44,56)

B. (48,52)

C. (50,50)

D. (60,40)

9. Consider two consumers. They consume one private good (X) and a public good (G). Consumption of the public good depends on the sum of their simultaneously and non-cooperatively chosen contributions towards the public good out of their incomes. Thus, if g_1 and g_2 are their contributions, then the consumption of the public good is $g = g_1 + g_2$. Let the utility function of consumer $i(i = 1, 2)$ be $U_i(x_i, g) = x_i g$. The price of the private good is $p > 0$ and the income of each consumer is $M > 0$. Then the consumers' equilibrium contributions towards the public good will be:

   A. $\left(\frac{M}{2}, \frac{M}{2}\right)$.

   B. $\left(\frac{M}{3}, \frac{M}{3}\right)$

   C. $\left(\frac{M}{4}, \frac{M}{4}\right)$.

   D. $\left(\frac{M}{p}, \frac{M}{p}\right)$

10. Consider two firms, 1 and 2 , producing a homogeneous product and competing in Cournot fashion. Both firms produce at constant marginal cost, but firm 1 has a lower marginal cost than firm 2. Specifically, firm 1 requires one unit of labour and one unit of raw material to produce one unit of output, while firm 2 requires two units of labour and one unit of raw material to produce one unit of output. There is no fixed cost. The prices of labour and material are given and the market demand for the product is determined according to the function $q = A - bp$, where q is the quantity demanded at price p and $A, b > 0$. Now, suppose the price of labour goes up, but that of raw material remains the same. Then, the equilibrium profit of firm 1 will:

    A. increase.

    B. decrease.

    C. remain unchanged.

    D. go up or down depending on the parameters.

11. Considered again the problem in Question 10. As before, suppose that the price of labour goes up, but that of raw material remains the same. Then, the equilibrium profit of firm 2 will:

    A. increase.

    B. decrease.

    C. remain unchanged.

    D. go up or down depending on the parameters.

12. Consider a firm which initially operates only in market A as a monopolist and faces market demand $Q = 20 - p$. Given its cost function $C(Q) = \frac{1}{4}Q^2$, it charges a monopoly price P_m in this market. Now suppose that, in addition to selling as a monopolist in market A, the firm starts selling its products in a competitive market, B, at price $\bar{p} = 6$ Under this situation the firm charges P_m^* in market A. Then:

    A. $P_m^* > P_m$

B. $P_m^* < P_m$

C. $P_m^* = P_m$

D. given the available information we cannot say whether $P_m^* > P_m$ or $P_m^* < P_m$

13. Two consumers, A and B, have utility functions $U_A = \min\{x_A, y_A\}$ and $U_B = x_B + y_B$, respectively. Their endowments vectors are $e_A = (100, 100)$ and $e_B = (50, 0)$ Consider a competitive equilibrium price vector (P_X, P_Y). Then,

    A. $\left(\frac{1}{5}, \frac{2}{5}\right)$ is the unique equilibrium price vector.

    B. $\left(\frac{1}{5}, \frac{2}{5}\right)$ is one of the many possible equilibrium price vectors.

    C. $\left(\frac{1}{5}, \frac{2}{5}\right)$ is never an equilibrium price vector.

    D. an equilibrium price vector does not exist.

14. Suppose a firm is a monopsonist in the labor market and faces separate labor supply functions for male and female workers. The labor supply function for male workers is given by $l_M = (w_M)^k$, where l_M is the amount of male labor available when the wage offered to male workers is w_M, and k is a positive constant. Analogously, the labor supply function for female workers is given by $l_F = w_F$. Male and female workers are perfect substitutes for one another. The firm produces one unit of output from each unit of labor it employs, and sells its output in a competitive market at a price of p per unit. The firm can pay male and female workers differently if it chooses to. Suppose the firm decides to pay male workers more than female workers. Then it must be the case that:

    A. $k < \frac{1}{2}$

    B. $\frac{1}{2} \leq k < 1$

    C. $k = 1$

    D. $k > 1$

15. Consider the problem in Question 14, and assume that the firm pays male workers more than female workers. Suppose further that $p > 2$. Then the firm must:

    A. hire more male workers than female workers.

    B. hire more female workers than male workers.

    C. hire identical numbers of male and female workers.

    D. hire more females than males if $2 < p \leq 4$, but more males than females if $p > 4$.

16. Consider the system of linear equations:

$$\begin{aligned}(4a-1)x + y + z &= 0\\ -y + z &= 0\\ (4a-1)z &= 0\end{aligned}$$

The value of a for which this system has a non-trivial solution (i.e., a solution other than (0,0,0)) is:

A. $\frac{1}{2}$.

B. $\frac{1}{4}$.

C. $\frac{3}{4}$.

D. 1 .

17. Let $f : \mathbb{R} \to \mathbb{R}$ be a convex and differentiable function with $f(0) = 1$, where $\mathbb{R}$ denotes the set of real numbers. If the derivative of f at 2 is 2, then the maximum value of $f(2)$ is:

    A. 3

    B. 5

    C. 10

    D. ∞

18. Consider the equation $2x + 5y = 103$. Then how many pairs of positive integer values can (x, y) take such that $x > y$?

    A. 7

    B. 8

    C. 13

    D. 14

19. Let X be a discrete random variable with probability mass function (PMF) $f(x)$ such that

$$f(x) > 0 \quad \text{if } x = 0, 1, \ldots, n, \text{ and}$$
$$f(x) = 0 \text{ otherwise}$$

where n is a finite integer. If Prob$(X \geq m \mid X \leq m) = f(m)$, then the value of m is:

    A. 0

    B. 1

    C. $n - 1$

    D. none of the above.

20. Consider the function $f(x) = 2ax \log_c x - ax^2$ where $a \neq 0$. Then

    A. the function has a maximum at $x = 1$.

    B. the function has a minimum at $x = 1$.

    C. the point $x = 1$ is a point of inflexion.

    D. none of the above.

21. Let $f : [0, 10] \to [10, 20]$ be a continuous and twice differentiable function such that $f(0) = 10$ and $f(10) = 20$. Suppose $|f'(x)| \leq 1$ for all $x \in [0, 10]$. Then, the value of $f''(5)$ is

    A. 0.

B. $\frac{1}{2}$.

C. 1 .

D. cannot be determined from the given information.

22. Consider the system of linear equations:

$$x + 2ay + az = 0$$
$$x + 3by + bz = 0$$
$$x + 4cy + cz = 0$$

Suppose that this system has a non-zero solution. Then a, b, c

A. are in arithmetic progression.

B. are in geometric progression.

C. are in harmonic progression.

D. satisfy $2a + 3b + 4c = 0$

23. Let a, b, c be real numbers. Consider the function $f(x_1, x_2) = \min\{a - x_1, b - x_2\}$.Let (x_1^*, x_2^*) be the solution to the maximization problem

$$\max f(x_1, x_2) \text{ subject to } x_1 + x_2 = c$$

Then $x_1^* - x_2^*$ equals

A. $\frac{c+a-b}{2}$.

B. $\frac{c+b-a}{2}$.

C. $a - b$

D. $b - a$.

24. Suppose that you have 10 different books, two identical bags and a box. The bags can each contain three books and the box can contain four books. The number of ways in which you can pack all the books is

A. $\frac{10!}{2!3!3!4!}$

B. $\frac{10!}{3!3!4!}$

C. $\frac{10!}{2!3!4!}$

D. none of the above.

25. Real numbers $a_1, a_2, \ldots, a_{99}$ form an arithmetic progression. Suppose that

$$a_2 + a_5 + a_8 + \ldots + a_{98} = 205$$

Then the value of $\sum_{k=1}^{99} a_k$ is

A. 612

B. 615

C. 618

D. none of the above

26. A stone is thrown into a circular pond of radius 1 meter. Suppose the stone falls uniformly at random on the area of the pond. The expected distance of the stone from the center of the pond is

A. $\frac{1}{3}$.

B. $\frac{1}{2}$.

C. $\frac{2}{3}$.

D. $\frac{1}{\sqrt{2}}$.

27. Suppose that there are n stairs, where n is some positive integer. A person standing at the bottom wants to reach the top. The person can climb either 1 stair or 2 stairs at a time. Let T_n be the total number of ways in which the person can reach the top. For instance, $T_1 = 1$ and $T_2 = 2$. Then, which one of the following statements is true for every $n > 2$?

A. $T_n = n$.

B. $T_n = 2T_{n-1}$

C. $T_n = T_{\mathrm{n}-1} + T_{n-2}$

D. $T_n = \sum_{k=1}^{n-1} T_k$

28. Let $Y_1, Y_2, \ldots, Y_n$ be the income of n individuals with $E(Y_i) = \mu$ and $\operatorname{Var}(Y_i) = \sigma^2$ for all $i = 1, 2, \ldots n$. These n individuals form m groups, each of size k. It is known that individuals within the same group are correlated but two individuals in different groups are always independent. Assume that when individuals are correlated, the correlation coefficient is the same for all pairs. Consider the random variable $\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$. The limiting variance of $\bar{Y}$ when m is large but k is finite is

A. 0.

B. $\frac{1}{k}$

C. 1 .

D. $\frac{\sigma^2}{k}$.

29. A person makes repeated attempts to destroy a target. Attempts are made independently of each other. The probability of destroying the target in any attempt is 0.8 . Given that he fails to destroy the target in the first five attempts, the probability that the target is destroyed in the 8 -th attempt is

A. 0.032

B. 0.064

C. 0.128

D. 0.160

30. Let E and F be two events such that $0 < \text{Prob}(E) < 1$ and $\text{Prob}(E \mid F) + \text{Prob}(E \mid F^c) = 1$. Then

A. E and F are mutually exclusive.

B. $\text{Prob}(E^c \mid F) + \text{Prob}(E^c \mid F^c) = 1$

C. E and F are independent.

D. $\text{Prob}(E \mid F) + \text{Prob}(E^c \mid F^c) = 1$

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1. Robinson Crusoe will live this period (period 1) and the next period (period 2) as the only inhabitant of his island completely isolated from the rest of the world. His only income is a crop of 100 coconuts that he harvests at the beginning of each period. Coconuts not consumed in the current period spoil at the rate of 20% per period. Crusoe's preference over consumption in period 1 (c_1) and consumption in period 2 (c_2) is given by the utility function $u(c_1, c_2) = \min\{5c_1, 6c_2\}$. Crusoe's utility maximizing consumption choice is given by

   A. $c_1 = \frac{200\times6}{11}, c_2 = \frac{200\times5}{11}$.

   B. $c_1 = 90, c_2 = 108$

   C. $c_1 = 100, c_2 = 100$

   D. none of the above.

2. The domestic supply and demand equations for a commodity in a country are as follows: Supply: $P = 50+Q$, Demand: $P = 200-2Q$, where P is the price in rupees per kilogram and Q is the quantity in thousands of kilograms. The country is a small producer in the world market where the price (which will not be affected by anything done by this country) is Rs. 60 per kilogram. The government of this country introduces a "Permit Policy" which works as follows. The government issues a fixed number of Permits - each Permit allows its owner to sell exactly 100 kilograms of the commodity in this country's market. An exporter from a foreign country cannot sell this commodity in this country unless she purchases such a Permit. Suppose the government issues 300 Permits. What is the maximum price an exporter is willing to pay for a Permit?

   A. Rs. 3000

   B. Rs. 2000

   C. Rs. 1500

   D. Rs. 1000

3. SeaTel provides cellular phone service in Delhi and has some monopoly power in the sense that it has its captive customer base with each customer's weekly demand being given by: $Q = 60 - P$, where Q denotes hours of cell phone calls per week and P is the price per hour. SeaTel's total cost of providing cell phone service is given by $C = 20Q$, so that the marginal cost is $MC = 20$. Suppose SeaTel offers a "Call-As-Much-As-You-Wish" deal: it charges only a flat weekly access fee, and once a customer pays the flat access fee, he/she can call as much as he/she wishes without paying any extra usage fee per hour. The weekly access fee that SeaTel should charge to maximize its profit is given by

   A. 1800

   B. 1200

   C. 800

   D. 40

4. A bus stop has to be located on the interval $[0,1]$. There are three individuals located at points 0.2,0.3 and 0.9 on the interval. If the bus stop is located at point x, then the utility of an individual located at y is $-|y-x|$, that is, the negative of the distance between the bus stop and the individual's location. A relocation of the bus stop is said to be Pareto improving if at least one individual is better off and no individual is worse off from the relocation. A location of the bus stop is said to be Pareto efficient if there does not exist any Pareto improving relocation. Then

   A. 0.5 is the only Pareto efficient location.

   B. $\frac{0.2+0.3+0.9}{3}$ is the only Pareto efficient location.

   C. Median of 0.2,0.3 and 0.9 is the only Pareto efficient location.

   D. none of the above.

5. Consider three goods: a cable television, b a fish in international waters, and c a burger. Also consider four descriptions of the goods: (A) non-rival and non-excludable, (B) rival and excludable, (C) non-rival and excludable, and (D) rival and nonexcludable. In what follows we match goods to possible descriptions. Choose the correct match.

   A. $(a)-(A),(b)-(C),(c)-(B)$

   B. $(a)-(C),(b)-(D),(c)-(A)$

   C. $(a)-(C),(b)-(B),(c)-(A)$

   D. $(a)-(C),(b)-(D),(c)-(B)$

6. Consider an economy consisting of three individuals -1,2 and 3, two goods $-$A and B, and a single monopoly firm that can produce both goods at zero cost. Each individual would like to buy exactly 1 unit of the goods A and B, if at all. An individual's valuation of a good is defined as the maximum amount she is willing to pay for one unit of the good. Individual 1 's valuation of good A is Rs. 10 and that of good B is Rs. 1. Individual 2's valuation of good A is Rs. 1, and that of good B is Rs. 10 . Individual 3 's valuation is Rs. 7 for good A, and Rs. 7 good B. The firm can charge a single price p_A for good A, a single price p_B for good B, and a bundled price p_{AB} such that if an individual pays p_{AB} then she gets the bundle consisting of one unit each of goods A and B. If the monopolist sets p_A, p_B and p_{AB} to maximize its profit then

   A. $p_A=11, p_B=11, p_{AB}=11$

   B. $p_A=11, p_B=11, p_{AB}=14$

   C. $p_A=10, p_B=10, p_{AB}=11$

   D. none of the above.

7. Consider a Bertrand duopoly with two firms, 1 and 2. Both firms produce the same good that has a market demand function $p=10-q$. The market is equally shared in case the firms charge the same price, otherwise the lower priced firm gets the entire demand. A firm must satisfy all the demand coming to it. The cost function of firm 1 is $3q_1$, that of firm 2 is $2q_2$. Suppose prices vary along the following grid, $\{0,0.1,0.2,\ldots\}$. The Bertrand equilibrium is given by

A. $p_1 = 2, p_2 = 2$

B. $p_1 = 3, p_2 = 2$

C. $p_1 = 3, p_2 = 2.9$

D. $p_1 = 3, p_2 = 3$

8. Consider a monopolist with a market demand function $p = 20 - q$. It is a multi-plant monopolist with two plants, plant 1 and plant 2, where the plant specific cost function of plant $i, i = 1, 2$, is

$$c_i(q_i) = \begin{cases} 2 + 4q_i, & \text{if } q_i > 0 \\ 0, & \text{otherwise} \end{cases}$$

The optimal monopoly profit is given by

A. 60

B. 64

C. 68

D. 62

9. Consider a closed economy in which an individual's labour supply (L) to firms is determined by the amount which maximizes her utility function $u(C, L) = C^{\alpha}(1-L)^{\beta}$, where $\alpha, \beta > 0, \alpha + \beta < 1$ and C is consumption expenditure which is taken to be equal to wage income (wL). Then

A. labour supply does not depend on the wage rate w.

B. labour supply is directly proportional to the wage rate w.

C. labour supply is inversely proportional to the wage rate w.

D. more information is needed to derive the labour supply.

10. In the scenario described in Question 9, assume that the economy is Keynesian, that is, investment expenditure (I) is autonomous and output (Y) is determined by aggregate demand, $Y = C + I$. The aggregate production function is given by $Y = AL^{\theta}$, where $A > 0$ is a productivity parameter and $0 < \theta < 1$. [Note that the firm's employment of labour is obtained by equating the marginal product of labour to w.] Then the marginal propensity to consume is

A. $\frac{a+\beta}{\theta}$.

B. $\frac{g}{\theta}$.

C. α.

D. θ

11. Consider a Solow growth model (in continuous time) with a production function with labour augmenting technological change, $Y_t = F(K_t, A_t L_t)$, where Y_t denotes output, K_t denotes the capital stock, A_t denotes the level of total factor productivity (TFP), and L_t denotes the stock of the labour force. Assume that L_t grows at the rate $n > 0$ and A_t grows at the rate $g > 0$, that is, $\frac{\dot{L}}{L} = n$ and $\frac{\dot{A}}{A} = g$, and the capital accumulation

equation is given by $\dot{K} = sY_t - \delta K_t$, where $s \in [0,1]$ is the exogenous savings rate, and $\delta \in [0,1]$ is the depreciation rate of capital. [Note that for any variable $x, \dot{x}$ denotes $\frac{dx}{dt}$,] Define capital in efficiency units to be $Z \equiv \frac{K}{AL}$. Then the expression for $\frac{\dot{Z}}{Z}$ is given by

A. $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - (n+g)$

B. $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - (\delta + n + g)$

C. $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z}$

D. $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - n$

12. In the Solow growth model described in Question 11 , the growth rate of Y at the steady state is given by

A. $n + g$

B. $\delta + n + g$

C. zero.

D. n.

13. Consider an IS-LM model where the IS curve is represented by $0.25Y = 500 + G - i$, and money demand function is given by $\frac{M}{P} = \frac{2Y}{e^1}$. The notations are standard: Y denotes output, G denotes government expenditure, i denotes the interest rate, P is the price level and e is the exponential. Suppose the government wants to increase spending and therefore the central bank decides to change the money supply accordingly such that the interest rate remains the same in the short run. Then the change in money supply satisfies the following condition:

A. $\frac{dM}{dG} = e$

B. $\frac{dM}{dG} = \frac{Y_c}{M}$

C. $\frac{dM}{dG} = \frac{Y}{M}$

D. $\frac{dM}{dG} = \frac{4M}{Y}$.

14. An agent lives for two periods. Her utility from consumption in period $1\,(c_1)$ and consumption in period $2\,(c_2)$ is given by $u(c_1, c_2) = \log(c_1) + \beta \log(c_2)$, where $0 < \beta < 1$ is the discount factor reflecting her time preference. The agent earns incomes w_1 in period 1 and w_2 in period 2. The rate of interest is $r > 0$. The agent chooses c_1 and c_2 so as to maximize $u(c_1, c_2)$ subject to her budget constraint. Consider a temporary increase in income where w_1 increases but the agent does not change her expectations about w_2. Then the marginal propensity to consume of present consumption with respect to w_1, $\frac{dc_1}{dw_1}$, is given by

A. $\frac{1}{1+\beta}\left(1 + \frac{1}{1+r}\right)$

B. $\left(\frac{1}{1+\beta}\right)\left(\frac{1}{1+r}\right)$

C. $\frac{1}{1+\beta}$

D. 1

15. In the scenario described in Question 14, consider a permanent increase in income where w_1 increases and the agent expects that w_2 will also increase by the same amount. Then $\frac{dc_1}{dw_1}$ is given by

   A. $\frac{1}{1+\beta}\left(1+\frac{1}{1+r}\right)$

   B. $\left(\frac{1}{1+\beta}\right)\left(\frac{1}{1+r}\right)$

   C. $\frac{1}{1+\beta}$

   D. 1

16. For what values of a are the vectors $(0,1,a),(a,1,0),(1,a,1)$ in $\mathbb{R}^3$ linearly dependent?

   A. 0 .

   B. 1

   C. 2

   D. $\sqrt{2}$.

17. Which of the following set of vectors form a basis of $\mathbb{R}^2$?

   A. $\{(2,1)\}$

   B. $\{(1,1),(2,2)\}$

   C. $\{(1,1),(1,2),(2,1)\}$

   D. $\{(1,1),(2,3)\}$

18. If a candidate is good he is selected in MSQE examination with probability 0.9. If a candidate is bad he is selected in MSQE examination with probability 0.2. Suppose every candidate is equally likely to be good or bad. If you meet a candidate who is selected in the MSQE examination, what is the probability that he will be good?

   A. $\frac{11}{20}$.

   B. $\frac{9}{10}$

   C. $\frac{9}{11}$.

   D. $\frac{11}{12}$.

19. Let $S_1 = \{2,3,4,\ldots,9\}$. First, an integer s_1 is drawn uniformly at random from S_1. Then s_1 and all its factors are removed from S_1. Let the new set be S_2. Next an integer s_2 is drawn uniformly at random from S_2. Then s_2 and all its factors are removed from S_2. Let the new set be S_3. Finally, an integer s_3 is drawn uniformly at random from S_3. What is the probability that $s_1 = 2, s_2 = 3, s_3 = 5$?[1]

   A. $\frac{1}{8}$

   B. $\frac{1}{64}$

   C. $\frac{1}{16}$.

[1] Answer assumes we remove multiples instead of factors

D. $\frac{1}{72}$.

20. Mr. A and B are independently tossing a coin. Their coins have a probability 0.25 of coming HEAD. After each of them tossed the coin twice, we see a total of 2HEADS. What is the probability that Mr. A had exactly one HEAD?

A. $\frac{2}{3}$.

B. $\frac{1}{2}$.

C. $\frac{1}{4}$.

D. $\frac{1}{3}$.

21. Consider the following function $f : \mathbb{R} \to \mathbb{R}$.

$$f(x) = \begin{cases} x & \text{if } x \leq e \\ x\left(\log_c x\right) & \text{if } x > e \end{cases}$$

Which of the following is true for f?

A. f is not continuous at e

B. f is not differentiable at e.

C. f is neither continuous nor differentiable at e.

D. f is continuous and differentiable at e.

22. Let $f : [-1, 1] \to \mathbb{R}$ be a continuous and weakly increasing function such that $\int_{-1}^{1} f(x)dx = 2\int_{-1}^{1} f(-x)dx$. Suppose $f(-1) = 0$ then $f(1)$ is

A. 0.

B. 1

C. $\frac{1}{2}$.

D. none of the above.

23. Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ be a twice continuously differentiable function. Let $x^* \in A$ be such that $\frac{\partial f}{\partial x}(x^*) = 0$. Consider the following two statements: (i) if $\frac{\partial^2 f}{\partial^2 x}(x^*) \leq 0$, then x^* is a point of local maximum of f; (ii) if x^* is a point of local maximum of f, then $\frac{\partial^2 f}{\partial x^2}(x^*) < 0$. Which of the following is true?

A. both (i) and (ii) are correct.

B. both (i) and (ii) are incorrect.

C. (i) is correct but (ii) is incorrect.

D. (ii) is correct but (i) is incorrect.

24. Consider the function $f(x) = e^x$ for all $x \in \mathbb{R}$. Which of the following is true?

A. f is quasi-convex.

B. f is quasi-concave.

C. f is neither quasi-convex nor quasi-concave.

D. f is both quasi-convex and quasi-concave.

25. Consider the following matrix A.

$$A=\begin{bmatrix} x & 0 & k \\ 1 & x & k-3 \\ 0 & 1 & 1 \end{bmatrix}$$

Suppose determinant of A is zero for two distinct real values of x. What is the least positive integer value of k?

A. 1 .

B. 9.

C. 10.

D. 8 .

26. Define the following function on the set of all positive integers.

$$f(n)=\begin{cases} 2\times 4\times\ldots\times(n-3)\times(n-1) & \text{if } n \text{ is odd} \\ 1\times 3\times\ldots\times(n-3)\times(n-1) & \text{if } n \text{ is even.} \end{cases}$$

What is the value of $f(n+2)f(n+1)$?

A. $n!$.

B. $(n+1)!$

C. $(n+2)!$

D. $(n+2)(n!)$

27. The sequence $\{x_n\}_{n\geq 0}$ is defined as follows. We set $x_0=1$ and $x_n=\sum_{j=0}^{n-1}x_j$ for each integer $n\geq 1$. Then the value of the expression $\sum_{j=0}^{\infty}\frac{1}{x_j}$ is equal to

A. ∞.

B. 2 .

C. 3 .

D. $\frac{7}{4}$.

28. For what values of p does the following quadratic equation have more than two solutions (variable in this equation is x)?

$$\left(p^2-16\right)x^2-\left(p^2-4p\right)x+\left(p^2-5p+4\right)=0$$

A. No such value of p exists.

B. -4 and 4

C. 1 and 4

D. 4

29. Consider the square with vertices A, B, C, D. Call a pair of vertices in the square adjacent if they are connected by an edge. You have four colours: RED, BLUE, GREEN, YELLOW. How many ways can you colour the vertices A, B, C, D such that no adjacent vertices share the same colour?

   A. 84

   B. 24.

   C. 72

   D. 108 .

30. Two players P_1 and P_2 are playing a game which involves filling the entries of an $n \times n$ matrix, where $n \geq 2$ is an even integer. Starting with P_1, each player takes turn to fill an unfilled entry of the matrix with a real number. The game ends when all entries are filled. Player P_1 wins if the determinant of the final matrix is non-zero. Else, player P_2 wins. A player $i \in \{1, 2\}$ has a winning strategy if irrespective of what the other player does, i wins by following this strategy. Which of the following is true?

   A. Player 1 has a winning strategy.

   B. Player 2 has a winning strategy.

   C. No player has a winning strategy.

   D. None of the above.

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1. Consider the functions

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then

A. f is differentiable at zero but g is not differentiable at zero

B. g is differentiable at zero but f is not differentiable at zero

C. f and g are both differentiable at zero

D. Neither f nor g is differentiable at zero

2. How many ordered pairs of numbers (x, y) are there where x,$y \in \{1.2, \ldots .100\}$. such that $|x - y| \leq 50$?

A. 2550

B. 5050

C. 7550

D. None of the other options are correct

3. Let ABC be a right angled isosceles triangle with angle $\angle ABC$ being right-angled. Let D be the mid-point of $AB.E$ be the foot of the perpendicular drawn from D to the side AC, and F be the foot of the perpendicular drawn from E to the side BC. What is the value of $\frac{FC}{BC}$?

A. $\frac{1}{\sqrt{2}}$

B. $\frac{3}{4}$

C. $2 - \sqrt{2}$

D. None of the other options are correct

4. Suppose that there are 30 MCQ type questions where each question has four options: $A, B.C.D$. For each question, a student gets 4 marks for a correct answer, 0 marks for a wrong answer, and 1 mark for not attempting the question. Suppose in each question, the probability that option A is correct is 0.5 , option B is correct is 0.3 , option C is currect is 0.2 , and option D is correct is 0. Two students Gupi and Baglan how no clue about the right answers. Gupi answers each question randomly, that is, ticks any of the options with probability 0.25. Whereas Bagha attempts each question with probability 0.5, but whenever he attempts a question, he randomly ticks an option. Which of the following is correct?

A. Both Gupi and Bagha have expected scores more than 30

B. Gupi's expected score is ≥ 30 and Bagha's expected scure is strictly less than 30

C. Gupi's experted score is less than or equal to 30 and Bagha's expected score is strictly more than 30

D. None of the above options are correct

5. Evaluate: $\lim_{x\to\infty}\left[e^{3x}-5x\right]^{\frac{1}{x}}$

A. e^3

B. 3

C. 1

D. None of the other options are correct

6. Suppose $f(x)=\left\{\begin{array}{ll}\frac{|x-3|}{x-3}, & \text{for } x\neq 3\\ 0. & \text{for } x=3\end{array}\right\}$. Then. $\lim_{x\to 3} f(x)$:

A. is -1

B. is 0

C. does not exist

D. is 1

7. Consider the following system of equations in x, y, z :

$$\begin{aligned} x+2y-3z&=a\\ 2x+6y-11z&=b\\ x-2y+7z&=c \end{aligned}$$

For what values of a, b, c, does the above system have no solution?

A. $c+2b-5a\neq 0$

B. $c+2b-5a=0$

C. $c+2b-4a=0$

D. None of the other options are correct

8. The sequence x_n is given by the formula: for every positive integer n

$$x_n=n^3-9n^2+631$$

The largest value of n such that $x_n>x_{n+1}$ is

A. 1

B. 5

C. 6

D. None of the other options are correct

9. Suppose an unbiased coin is tossed 10 times. Let D be the random variable that denotes the number of heads minus the number of tails. What is the variance of D?

   A. 10

   B. 1

   C. 0

   D. None of the other eptions are correct

10. Suppose we are given a 4×4 square matrix A, which satisfies $A_{ij} = 0$ if $i < j$. Suppose the each diagonal entry A_{ii} is drawn uniformly at random fron $\{0, 1, \ldots, 9\}$. What is the probability that A has full rank?

    A. $\frac{1}{10^4}$

    B. $\frac{3}{5}$

    C. $1 - \frac{1}{10^4}$

    D. $\left(\frac{9}{10}\right)^4$

11. Let a and b be two real numbers where $b \neq 0$ and $g : \mathbb{R} \to \mathbb{R}$ is a continuous function which satisfies

$$g(g(x)) = \text{ag}(x) + bx \quad \forall x \in \mathbb{R}$$

    Which of the following must be true?

    A. g is strictly increasing

    B. g is strictly decreasing

    C. $\lim_{x \to \infty} g(x)$ is finite

    D. Either (A) or (B)

12. For every positive integer n, let $S(n)$ denote the sum of digits in n. For instance, $s(387) = 3 + 8 + 7 = 18$. The value of the sum

$$S(1) + S(2) + \ldots + S(99)$$

    is

    A. 450

    B. 495

    C. 900

    D. 990

13. Suppose five cards are randomly drawn without replacement from an ordinary deck of 52 plaving cards, with four suits of 13 cards each, which las been well shuffled. Let a flush be the event that all five cards are of the same suit. What is the probability of getting a flush?

    A. $\frac{{}^{4}C_{1}\,{}^{13}C_{5}}{{}^{52}C_{5}}$

B. $\frac{{}^4C_2\,{}^{13}C_4}{{}^{52}C_5}$

C. $\frac{{}^4P_1\,{}^{13}C_5}{{}^{52}P_5}$

D. $\frac{{}^4C_1\,{}^{12}C_5}{{}^{52}C_5}$

14. Evaluate: $\int x^n \ln x dx$, where $n > 1$

A. $\ln x \frac{x^{n+1}}{n+1} - \frac{1}{(n+1)^2} x^{n+1} + c$

B. $\ln x \frac{x^{n+1}}{2(n+1)} - \frac{1}{(n+1)^2} x^{n+1} + c$

C. $\ln x \frac{x^{n+1}}{n+1} - \frac{1}{(n+1)} x^{n+1} + c$

D. None of the other options are correct

15. The area of the region bounded by the curve $y = \ln(x)$, the $Y-$ axis. and the lince $y = 1$ and $y = -1$ is

A. $\frac{e}{2}$

B. 2

C. $e - \frac{1}{e}$

D. None of the other options are correct

16. A price discriminating monopolist finds that a person's demand for its product depends on the person's age. The inverse demand function of someone of age y, can be written as $p = A(y) - q$ where $A(y)$ is an increasing function of y. The product cannot be resold from one buyer to another and the monopolist knows the ages of its consumers. (This is often the case with online subscriptions) . If the monopolist maximizes its profits, then

A. older people will pay higher prices and purchase more of this product compared to younger people

B. everyone pays the same price but older people consume more

C. older people will pay higher prices compared to the younger people but everyone will consume the same quantity of the product

D. None of the other options are correct

17. Pam's family consists of herself and her 3 sisters. They own a small farm in the agricultural sector in Agriland. The value of their total output is \$4000 which is divided equally amongst the four. The urban sector has two kinds of jobs: informal sector (which anyone can get) pays \$500 and formal sector jobs give \$1200. Pam would like to maximize her own total income and calculates her own expected returns to migration. The proportion of formal sector jobs to urban labor force that would deter her from migrating is:

A. Less than 2/3

B. More than 5/6

C. More than $\frac{1}{2}$

D. Less than 5/7

18. Hu and Li are two dealers of used tractors in a rural area of China. Hu sells high quality second hand tractors while Li sells low quality ones. Hu would be willing to sell his high quality tractor at \$8000 while Lu would sell his low quality one for \$5000, Consumers are willing to pay up to $\$10,000$ for a high quality tractor and \$7000 for a low quality one. They expect a 50% chance of buying a high quality second-hand tractor. In order to signal the quality of their tractors Hu and Li can offer warranties. The cost of warranty for a high quality tractor is 500Y and 1000Y for a low quality one (Y is the number of years of warranty). What is the optimal number of years of warranty that Hu should offer so that consumers know his tractors are of good quality?

   A. Less than 2 years

   B. 0.5 years

   C. More than 1.5 years

   D. 3 years

19. Suppose the capacity curve for each laborer is described as follows: for all payments up to \$100, capacity is zero and then begins to rise by 2 units for each additional \$ paid. This happens until the payment rises to \$500. Thereafter, an additional \$ payment increases work capacity by only 1.1 units, until total income paid is \$1000. At this point, additional payments have no effect on work capacity. Assume all income is spent on nutrition. Suppose you are an employer faced by the above capacity curve of your workers. You need 8000 units of work or capacity units. How many workers would you hire and how much would you pay each worker so that you get 8000 units of work at minimum cost?

   A. 5 workers: \$1000 per worker

   B. 10 workers: \$700 yer worker

   C. 10 workers; \$500 per worker

   D. 15 workers; \$400 per worker

20. Suppose you were to believe that money illusion exists, that is as prices and income rise proportionally, then people buy more. Which of the following statements about demand should not be true?

   A. Demand functions are downward sloping

   B. Demand functions are homogeneous of degree zero

   C. Demand has a positive vertical intercept

   D. Demand functions are homogeneous of degree one

21. Consider a Bertrand price competition model between two profit maximizing widget producers, say A and B. The marginal cost of producing a widget is 4 for each producer. Each widget producer has a capacity constraint to produce only 5 widgets. There are 8 identical individuals who demand 1 widget only, and value each widget at 6. If the firms are maximizing profits, then the following statement is true:[2]

[2] Assuming only integer prices are allowed

A. Firm A and B will charge 1

B. Firm A and B will charge 6

C. Firm A and B will charge greater than or equal to 5

D. None of the other options are correct

22. The government estimates the market demand (Q_D) and market supply (Q_S) for turnips to be the following: $Q_d = 30 - 2P, Q_S = 4$ where P is the per unit price and Q is the quantity measured in kilograms. The government aims to increase the market price of turnips to \$8 per unit to improve the welfare of domestic producers of turnships. It is considering three possible choices: i) per unit subsidy, ii) a price floor and purchase of any surplus production, iii) a production quota. Which of these policies should the government adopt if it aims to mazimize the producer's welfare but minimize the loss of efficiency?

A. A production quota

B. A price subsidy

C. Either a price subsidy or price floor

D. Either a production quota or price floor

23. A monopolist faces a demand curve $q = \frac{5}{p}$. Her cost function is $C(q) = 3q$. Suppose, in the same market, there are some competitive suppliers ready to sell the good at price $p = 5$. The monopolist's profit maximizing price and output could be given by

A. $p = 3, q = \frac{5}{3}$

B. $p = 3.01, q = \frac{5}{3.01}$

C. $p = 2.99, q = \frac{5}{2.99}$

D. $p = 4.99, q = \frac{5}{4.99}$

24. The consumption function is given by $C = AY^{\beta}$ with $\beta = 0.5$ and $A = 0.3$. The marginal propensity to save is

A. equal to 0.5

B. increasing in income. Y

C. equal to 0.3

D. equal to 0.7

25. The production function is given by $Y = AL$. The wage rigidity constraint is given by $W \geq B$. The labour endowment is given by C. Here, A, B, and C are finite and positive constants. Assume that the entire labour endowment is supplied. If $A > B$, then in a labour market equilibrium

A. $L = C$

B. $L = 0$

C. $0 < L < C$

D. None of the other options are correct

26. Consider the Mundell-Flemming model with perfect capital mobility and a flexible exchange rate in the short run. A monetary expansion leads to ____________in output , a fiscal expansion leads to ____________in output

    A. decrease; no change

    B. increase, decrease

    C. increase; no change

    D. increase; increase

27. Mr. X has an exogenous income W and his utility from consumption is $u(.)$. Mr. X know the an accident can occur with probability p and if it occurs, the monetary equivalent to the damage is T. Mr. X can however affect the accident probability p through the prevention effort e. In particular, e can take two values - zero and a and an assumption is that $p(0) > p(a)$, that is by putting prevention effort. probability of occurring an accident can be reduced. Let us also assume that if Mr. X puts an effort e, the disutility from the effort is Ae^2 where A is the per unit effort cost. What is the critical value of A, A^*. below which the effort will be undertaken, and above which the effort will not be undertaken, by Mr. X?

    A. $A^* = \frac{|p(a)-p(0)||u(W-T)-u(W)]}{a^2}$

    B. $A^* = \frac{|p(a)-p(0)]a^2}{u(W-T)-u(W)}$

    C. $A^* = \frac{\frac{p(a)}{p(0)}}{\frac{u(w-T)}{u(w)}a^2}$

    D. $A^* = \frac{p(a)p(0)a^2}{u(W-T)u(W)}$

28. Labor supply in macro models results from individual decision making. Let c denote an individual's consumption and L denote labor supply. Assume that individuals solve the following optimization problem

$$\operatorname*{Max}_{\{c,L\}} U(c, L) = \log c - \frac{1}{2}\frac{1}{b}L^2$$

subject to $c + \bar{S} = wL$ where $U(\cdot)$ is the utility function, $b > 0$ is a constant, $\bar{S}$ is a constant exogenous level of savings, and w is the real wage the person can earn in the labor market. Derive the optimal labor supply. It is

    A. increasing in w; increasing in c

    B. decreasing in w; decreasing in c

    C. increasing in w; decreasing in c

    D. decreasing in w; increasing in c

29. Consider a Solow economy that begins in steady state. Then a strong eathquake destroys half the capital stock. The steady state level of capital ____________, the level of output ____________on impact, and the growth rate of the economy ____________as the economy approaches its steady state.

A. decreases; decreases; decreases

B. remains the same; decreases; decreases

C. remains the same; decreases; increases

D. decreases; remains the same, decreases

30. Suppose the economy is characterized by the following equations

$$C = c_0 + c_1 Y_D$$
$$Y_D = Y - T$$
$$I = b_0 + b_1 Y$$

where C = Consumption, c_0 = Autonomous Consumption, $c_1 \in [0.1]$ Y_D = Disposable Income, Y = Aggregate GDP, T = Taxes, I = Investinent , b_0 = Autonomous Investment, and $b_1 \in [0, 1]$. For the multiplier to be positive, what condition needs to be satisfied?

A. $b_1 + c_1 = 0$

B. $b_1 + c_1 = 1$

C. $b_1 + c_1 < 1$

D. $b_1 + c_1 > 1$

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1. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{2n^2+2}{3n^2+1}$, then $\sum_{n=1}^{\infty} a_n$ is

   A. 0

   B. divergent

   C. $\frac{2}{3}$

   D. $\frac{3}{2}$

2. Using data from a sample of size n, the intercept and slope coefficients from an ordinary least squares regression of y on x, are a and b respectively. Which of the following is false?

   A. $\sum_{i=1}^{n} (y_i - a - bx_i)\, x_i = 0$

   B. $\frac{1}{n}\sum_{i=1}^{n} y_i = a + \frac{b}{n}\sum_{i=1}^{n} x_i$

   C. a and b are the solution to $\min_{\alpha,\beta} \sum_{i=1}^{n} (y_i - \alpha - \beta x_i)^2$

   D. a and b are the solution to $\min_{\alpha,\beta} \sum_{i=1}^{n} |y_i - \alpha - \beta x_i|$

3. Consider a production function $z = 2x+3y$. For what price ratio $\frac{p_x}{p_y}$, will a corner solution in y, i.e. $(x = 0)$ be possible, if the objective is to minimize the cost of producing a given positive quantity z_0 of z ?

   A. $\frac{p_x}{p_y} = 2/3$

   B. $\frac{p_x}{p_y} \geq 2/3$

   C. $\frac{p_x}{p_y} < \frac{2}{3}$

   D. $\frac{p_x}{p_y} \leq -2/3$

4. A number is chosen randomly from the first billion natural numbers. The probability that the product of the number with its two immediate successors is divisible by 24 is closest to

   A. $\frac{1}{2}$

   B. $\frac{3}{4}$

   C. $\frac{5}{8}$

   D. $\frac{2}{3}$

5. Each of the four entries of a 2×2 matrix is filled by independently choosing either 1 or -1 uniformly at random. What is the probability that the matrix is singular?

   A. $\frac{1}{16}$

   B. $\frac{1}{4}$

   C. $\frac{1}{2}$

   D. $\frac{1}{3}$

6. We need to fill a 3×3 matrix by either 0 or 1 such that each row has exactly one 0 and each column has exactly one 0. The number of ways we can do this is

   A. 8

   B. 6

   C. 4

   D. 2

7. Let $f : [0, 1] \to \mathbb{R}$ be a convex function with $f(0) = 0$. Which of the following is always true for f?

   A. f is differentiable

   B. f may not be differentiable but it is continuous

   C. $f(x) \geq x f'(x)$ for all $x \in [0, 1]$ if f is differentiable

   D. none of the above

8. If $f : \mathbb{R} \to \mathbb{R}$ is strictly quasi-concave, then it follows that

   A. f is not strictly convex

   B. f is not linear

   C. f is monotónic

   D. if f is quadratic, then the coefficient of x^2 is ≤ 0

9. Consider a function $f : [-1, 1] \to \mathbb{R}$ shown in Figure 1.

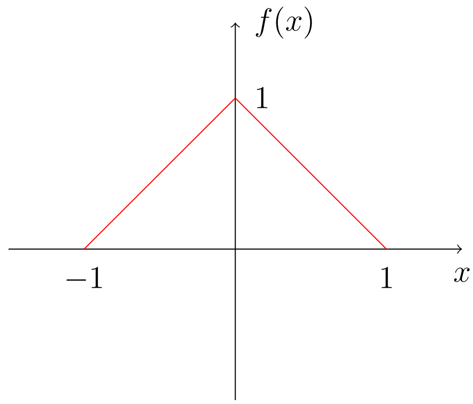

Figure 1

The value of $\int_{-1}^{1} f\left(x^2 - 1\right) dx$ equals

   A. $\frac{1}{3}$

   B. $\frac{2}{3}$

   C. $\frac{1}{2}$

   D. $\frac{3}{4}$

10. The rank of the matrix

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

is

A. 0

B. 1

C. 2

D. 3

11. Let V be the vector space of polynomials $p(x)$ of degree less than or equal to 2 that have real coefficients. Then, T is a linear transformation from V to V if T is defined by

A. $T(p(x)) = x + p(x)$

B. $T(p(x)) = xp(x)$

C. $T(p(x)) = \frac{dp(x)}{dx}$

D. $T(p(x)) = \int p(x)dx$ where the constant of integration is taken to be zero

12. X is a random variable that can take values only in $[0, 10]$. Suppose $\mathbb{P}[X > 5] \leq \frac{2}{5}$ and $\mathbb{P}[X < 1] \leq \frac{1}{2}$. Then

A. $\mathbb{E}[X] \geq 1$

B. $\mathbb{E}[X] \leq 5$

C. $\mathbb{E}[X] \geq 0.5$ and $\mathbb{E}[X] \leq 8.5$

D. None of the above

13. Given data $(-1, 1), (0, 0), (1, 1)$ on (x, y), the standard deviation of x and y and the correlation coefficient of x and y are respectively:

A. $\sigma_x = \frac{\sqrt{2}}{\sqrt{3}}, \sigma_y = \frac{\sqrt{2}}{3}, r = 0$

B. $\sigma_x = \frac{2}{3}, \sigma_y = \frac{2}{9}, r = 0$

C. $\sigma_x = 0, \sigma_y = 0, r$ is undefined

D. None of the above

14. An island nation has two potential vaccine firms: denoted as 1 and 2. Both need to invest in R&D to manufacture vaccines. The cost of R&D for firms 1 and 2 are f_1 and f_2 respectively. Once R&D is done, the cost of per unit manufacturing of vaccine is drawn uniformly from $[0, 1]$. The firms know their (fixed) cost of R&D but only know that the cost of per unit manufacturing is uniformly drawn from $[0, 1]$. Total demand of vaccine is 1 unit and if firm $i \in \{1, 2\}$ supplies $q_i \in [0, 1]$ units and has a per unit cost of c_i, it incurs a manufacturing cost of $c_i q_i$ (along with f_i). Suppose both firms invest in R&D but only the lowest per unit cost firm is chosen to supply the entire one unit of vaccine. What is the total expected cost of vaccination (expected cost is the fixed cost of R&D and expected cost of manufacturing)?

A. $f_1 + f_2 + \frac{1}{2}$

B. $f_1 + f_2 + \frac{1}{3}$

C. $f_1 + f_2 + \frac{2}{3}$

D. $f_1 + f_2 + \frac{3}{4}$

15. In question 14, suppose $f_1 < f_2 < \frac{1}{6}$. Consider two more alternatives:

(b) Suppose both firms invest in R&D and both supply $\frac{1}{2}$ units of vaccine.

(c) Only firm 1 invests in R&D and supplies the entire one unit of vaccine.

Denote the total expected cost of vaccination from the alternatives (b) and (c) as C_2 and C_3 respectively. Denote the total expected cost of vaccination for the alternative in Question 14 as C_1. Then, which of the following is true?

A. $C_1 < C_2 < C_3$

B. $C_1 < C_3 < C_2$

C. $C_2 < C_1 < C_3$

D. $C_3 < C_1 < C_2$

16. India and China produce only shirts and phones using only 2 factors of production: either higher skilled labour H or low skilled labour L. Shirts are high skill labour intensive while phones are low skill labour intensive. The production function for each good is identical in both countries. India and China have equal amounts of lower skilled labour, but India has a greater amount of higher skilled labour. Which good will India import?

A. Shirts

B. Phones

C. Both Shirts and Phones

D. Neither Shirts nor Phones

17. Continue with the same setup as in Question 16 , but now China's population doubles. The overall welfare of the representative agent in India will:

A. Increase

B. Decrease

C. Stay the same

D. Increase or decrease

18. Consider a duopoly with market demand $p = 10 - q$. The cost function of firm 1 is $7q_1$, and that of firm 2 is $2q_2$, where q_i is the quantity produced by firm $i, i = 1, 2$. In equilibrium, firm 2 charges a price of:

A. 7

B. 6

C. 10

D. 0

19. Let the economy's production function be given by $Y = AK^{\frac{1}{3}}L^{\frac{2}{3}}$ where Y = output, $A > 0$ is the level of technology (also called total factor productivity), K is the capital stock, and L is the level of employment. Consider the standard supply-demand diagram for labor as in the previous question, except that there are no wage rigidities this time. The labor demand curve is given by ____________; a negative total factor productivity shock leads to a ____________in labor demand; and a ____________in employment.

A. $w = \frac{3}{2}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; upward shift; fall

B. $w = \frac{3}{2}\frac{AK^{\frac{2}{3}}}{L^{\frac{1}{3}}}$; downward shift; fall

C. $w = \frac{2}{3}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; downward shift; fall

D. $w = \frac{1}{3}\frac{AK^{\frac{1}{3}}}{L^{\frac{1}{3}}}$; downward shift; rise

20. Virat and Mithali eat rice and drink milk in exactly the same quantities. The price of Rice falls. In response, Virat increases the amount of milk but decreases the amount of rice he consumes. Mithali, on the other hand, increases both rice and milk consumption. Both Virat and Mithali spend all their income on eating rice or drinking milk. For Virat's behaviour to be consistent with standard, well-behaved indifference curves, his preferences over rice consumption imply that for him, rice must be a:

A. Inferior good

B. Giffen good

C. Luxury good

D. Normal good

21. Continue with the same setup as in Question 20, and choose the answer from below that will correctly fill in the blanks in the following sentence. With respect to rice, for Virat, the income effect must be ____________than the substitution effect while for Mithali the income effect must be ____________than the substitution effect if both Virat and Mithali have standard well-behaved preference relations.

A. greater, lesser

B. lesser, lesser

C. greater, greater

D. lesser, greater

22. If the short-run IS-LM equilibrium occurs at a level of income above the natural rate of output, in the long run output will return to the natural rate via

A. an increase in the price level

B. a decrease in the interest rate

C. an increase in the money supply

D. a downward shift of the consumption function

23. If the short-run aggregate supply curve is steep, the Phillips curve will be:

A. flat

B. steep

C. backward-bending

D. unrelated to the slope of the short-run aggregate supply curve

24. There are no capital controls between the US and the UK. If the interest rate is higher in the US than in the UK, then we can conclude that

A. The US dollar is expected to appreciate with respect to the pound (the UK's currency)

B. The pound is expected to appreciate with respect to the US dollar

C. The interest rate in the US is expected to increase

D. The interest rate in the US is expected to decrease

25. Suppose there are two countries, B and C, that have no trade and no financial transactions with any countries except each other. B imports a total of goods worth 10 million bollars from C, where a bollar is a unit of B's currency. B has no exports. Which of the following must be true?

A. B has a capital account deficit

B. C has a current account deficit

C. C is buying assets from B.

D. The exchange rate of collars per bollar is bigger than 1, where a collar is a unit of C's currency.

26. Inventory investment can be expected to

A. rise when the real interest rate rises, other things being equal

B. not depend on the real interest rate, other things being equal

C. fall when the real interest rate rises, other things being equal

D. depend only on the change in real GDP

27. A cake of size 1 is to be divided among two individuals 1 and 2. Let x_i be the share of the cake going to individual $i, i = 1, 2$, where $0 \leq x_i \leq 1$. The utility functions are $u_1(x_1, x_2) = x_1$, and $u_2(x_1, x_2) = x_2 + |x_1 - x_2|$, where $|a|$ is the absolute value of a. The Pareto optimal cake divisions include:

A. $(1, 0)$

B. $(1/2, 1/2)$

C. $(3/4, 1/4)$

D. None of the above

28. Rohit spends all his money on dosas and filter coffee. He stays in Delhi where each dosa and filter coffee cost the same. He eats 15 dosas and drinks 35 filter coffees in a week. He gets a chance to move to either Chennai or Bangalore. In Chennai, he can just afford to have 40 dosas and 10 filter coffees in a week. Like in Delhi, each dosa and filter coffee cost the same. In Bangalore, he can just afford to have 10 dosas and 20 filter coffees in a week. Here, 2 filter coffees costs the same as 1 dosa. Where will Rohit prefer to stay?

A. Delhi

B. Chennai

C. Bangalore

D. Indifferent between Delhi and Chennai

29. Consider the IS-LM model with the real interest rate, R, on the vertical axis and output, Y, on the horizontal axis. Now suppose that the central bank chooses R for the economy, based on its own assessment, at $R=\bar{R}$. In this case the LM curve will

A. not exist

B. will be horizontal at $R=\bar{R}$.

C. upward sloping like the usual LM curve

D. None of the other options

30. Consider a supply-demand diagram for the labor market with an upward sloping labor supply curve (L^s) and a downward sloping labor demand curve $\left(L^d\right)$. Let the wage be on the vertical axis, and the level of employment (L) be on the horizontal axis. Suppose the wage is rigid above the equilibrium wage at $\bar{w}$, i.e., it fails to adjust to clear the labor market. Then a reduction in labor demand leads to

A. A larger reduction in employment compared to the case if wages were flexible

B. A smaller reduction in employment compared to the case if wages were flexible

C. The same reduction in employment compared to the case if wages were flexible

D. None of the other options.

17 ISI PEA 2022

1. Consider an economy with two goods X and Y. Let the utility function be given by $u(x, y) = A\sqrt{xy}$ where $A > 0, x \geq 0$ is the amount of good X consumed and $y \geq 0$ is the amount of good Y consumed. Suppose that the budget constraint is given by $P_X x + P_Y y \leq M$ where $M > 0$ is the money income of the consumer and P_X and P_Y are the prices of the goods X and Y, respectively. Let $P_X = P_Y > 1$ and let (x^*, y^*) be the equilibrium quantities of this consumer who maximizes utility subject to the budget constraint. Then,

    A. it must always be that $x^* > y^*$

    B. it must always be that $x^* = y^*$

    C. it must always be that $x^* < y^*$

    D. it must always be that $x^* + y^* = M$

2. Consider the utility function $u(x_1, x_2) = 3x_1 + 2x_2$ of a consumer defined for all $x_1 \geq 0$ and $x_2 \geq 0$. Let the price of good 1 be $p_1 > 0$ and that of good 2 be $p_2 > 0$. Let $M > 0$ be the money income of the consumer. Consider the optimization problem $\max_{x_1 \geq 0, x_2 \geq 0} 3x_1 + 2x_2$ subject to $2x_1 + 3x_2 \leq M$. The associated Lagrangian function for this maximization problem is $L(x_1, x_2 : \lambda) = 3x_1 + 2x_2 + \lambda[M - 2x_1 - 3x_2]$; where λ denotes the non-negative Lagrangian multiplier. Then the equilibrium solution $(x_1^*, x_2^*, \lambda^*)$ to this Lagrangian function maximization problem is

    A. $\left(x_1^* = \frac{M}{2}, x_2^* = 0.\lambda^* = \frac{2}{3}\right)$

    B. $\left(x_1^* = \frac{M}{2}, x_2^* = 0, \lambda^* = \frac{3}{2}\right)$

    C. $\left(x_1^* = 0, x_2^* = \frac{M}{3}, \lambda^* = \frac{2}{3}\right)$

    D. $\left(x_1^* = 0, x_2^* = \frac{M}{3}, \lambda^* = \frac{3}{2}\right)$

3. Consider a two good economy where the two goods are X and Y and consider two consumers A and B. In a month when the price of good X was Rs. 2 and that of good Y was Rs. 3, consumer A consumed 3 units of good X and 8 units of good Y and consumer B consumed 6 units of both goods. In the next month. when the price of good X was Rs. 3 and that of good Y was Rs. 2, consumer A consumed 8 units of good X and 3 units of good Y and consumer B consumed 4 units of good X and 9 units of good Y. Given this information which one of the following statements is correct?

    A. Both consumers satisfy the weak axiom of revealed preference

    B. Neither consumer satisfies the weak axiom of revealed preference

    C. Consumer A satisfies the weak axiom of revealed preference but not consumer B

    D. Consumer B satisfies the weak axiom of revealed preference but not consumer A

4. Let the production function be $Y(L, K) = \min\{2L, K\}$, where L and K are the amounts of labor and capital, respectively. Consider the cost function $C(L, K) = wL + rK$, where $w > 0$ denotes the price of labor and $r > 0$ denotes the price of capital. Suppose that (L^*, K^*) is the combination of labor and capital at which cost is minimized subject to the constraint $Y(L, K) \geq \bar{Y}$. Then.

   A. $L^* = \bar{Y}$ and $K^* = \bar{Y}/2$

   B. $L^* = \bar{Y}$ and $K^* = \bar{Y}$

   C. $L^* = \bar{Y}/2$ and $K^* = \bar{Y}$

   D. None of the other options is correct

5. Suppose that there are two firms 1 and 2 that produce the same good. Let the inverted demand function be $P(q_1, q_2) = 1 - q_1 - q_2$, where firm 1 produces $q_1 \geq 0$ and firm 2 produces $q_2 \geq 0$. Suppose that the cost function of firm $i \in \{1, 2\}$ is given by $c_i(q_i) = \kappa_i q_i$, where $\kappa_i \in \left(0, \frac{1}{2}\right)$. Note that there is no fixed cost for either firm. Then, the Cournot equilibrium profit of firm 2

   A. $\frac{(1-\kappa_1+\kappa_2)^2}{9}$

   B. $\frac{(1-\kappa_2+\kappa_1)^2}{9}$

   C. $\frac{(1-2\kappa_1+\kappa_2)^2}{9}$

   D. $\frac{(1-2\kappa_2+\kappa_1)^2}{9}$

6. Which of the following statements is correct in a two-good world?

   A. Diminishing marginal utility of both goods is sufficient for diminishing marginal rate of substitution

   B. Diminishing marginal utility of both goods is necessary for diminishing marginal rate of substitution

   C. Diminishing marginal utility of at least one good is necessary for diminishing marginal rate of substitution

   D. Diminishing marginal utility of at least one good is neither necessary nor sufficient for diminishing marginal rate of substitution

7. A non-transitive preference relation can be represented by a utility function

   A. Always

   B. Only if preferences are complete

   C. Only if preferences are complete and convex

   D. Never

8. Rahul consumes two goods, X and Y, in amounts x and y, respectively. Rahul's utility function is $U(x, y) = \min\{x, y\}$. Rahul makes Rs 200; the price of X and price of Y are both Rs. 2. Rahul's boss is thinking of sending him to another town where the price of X is Rs 2 and the price of Y is Rs. 3. The boss offers no raise in pay. Rahul,

who understands compensating and equivalent variations perfectly, complains bitterly. He says that although he doesn't mind moving for its own sake and the town is just as pleasant as the old, having to move is as bad as a cut in pay of Rs A in his current location. He also says that he would not mind moving if, when he moved, he got a raise of Rs. B. What are A and B equal to?

A. $A = 30, B = 70$

B. $A = 40, B = 50$

C. $A = 50, B = 75$

D. $A = 60, B = 60$

9. Let $U(x, y) = -\left[(10 - x)^2 + (10 - y)^2\right]$ be a utility function of some consumer. All prices are equal to 1, and income is 40. Then the optimal values of x and y will be

A. $10, 10$

B. $0, 0$

C. $5, 5$

D. None of these

10. Consider a production function $F(K, L) = \min\left\{\frac{K}{a}, \frac{L}{b}\right\}; a, b > 0$ and $a \neq b$. For any given $K = \bar{K} > 0$, the marginal productivity of labor is

A. 0

B. $\frac{1}{a}$ if $L < \left(\frac{a}{b}\right)\bar{K}$ and 0 otherwise

C. $\frac{1}{b}$ if $L < \left(\frac{b}{a}\right)\bar{K}$ and 0 if $L > \left(\frac{b}{a}\right)\bar{K}$

D. None of the above

11. Let $e_i(p_0)$ be the price elasticity of demand for a good X of consumer i $(i = 2, \cdots, N)$ at price p_0, given its demand function. Consumers do not consume identical amounts of X at p_0. Then the price elasticity of demand at price p_0 for the aggregate demand function for X is

A. $\sum_i (e_i(p_0))^2$

B. $\sum_i e_i(p_0)$

C. $\frac{\sum_i e_i(p_0)}{N}$

D. None of these

12. There are m identical competitive firms in an industry. Every firm has the (total) cost function $C(q) = q^2 + 1$, where q is the level of its output, $q \geq 0$. Industry demand for the product is given by $D(P) = a - bP$, where P is price, and $a, b > 0$. Then the short-run equilibrium output of each firm is

A. 0

B. $\frac{a}{m+2b}$

C. $\frac{a}{\frac{m}{2}+b}$

D. $\frac{a}{m+\frac{b}{2}}$

13. Suppose the (total) cost function for a monopolist is $C = 3q^2 + 800$, where q is its output. The inverse market demand function is $p = 280 - 4q$. What is the price elasticity of demand at the profit maximizing price?

    A. -4.5

    B. -3.5

    C. -2.5

    D. -1.5

14. Consider the Solow growth model with constant average saving propensity s, rate of depreciation δ, and labor supply growth rate n. There is no technological progress. Then, at steady state, the capital-output ratio is

    A. $\frac{s}{n+\delta}$

    B. $\frac{n}{\delta+n}$

    C. $\frac{\delta}{s+n}$

    D. $\frac{1}{s+n+\delta}$

15. The number of ways in which the word PANDEMIC can be arranged such that the vowels appear together is

    A. $6 \times (3!)(5!)$

    B. $5 \times (3!)(5!)$

    C. $4 \times (3!)(5!)$

    D. $1 \times (3!)(5!)$

16. Consider the functions $f(x) = x^2 - x - 1$ and $g(x) = x + 1$, both defined for all real values of x. Let $\alpha_1 > 0$ be the positive real root and $\alpha_2 < 0$ be the negative real root of the equation $f(x) = 0$. Let $\beta_1 > 0$ be the positive real root and $\beta_2 < 0$ be the negative real root of the equation $f(g(x)) = 0$. After identifying the exact values of $\alpha_1, \alpha_2, \beta_1$ and β_2, identify which one of the following four statements is incorrect.

    A. $\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = 1$

    B. $a_1 + \beta_2 = \alpha_2 + \beta_1 = 0$

    C. $a_1 + \beta_1 = -(\alpha_2 + \beta_2) = \sqrt{5}$

    D. $\alpha_1 + \alpha_2 = -(\beta_1 + \beta_2) = -1$

17. Let the function $f(x) = 1 - \sqrt{1 - x^2}$ be defined only over all x belonging in $[0, 1]$. Then $f(1 - f(x))$ equals

    A. x

B. $1 - x$

C. x^2

D. $1 - x^2$

18. Suppose $f(x)$ is increasing, concave and twice differentiable and $g(x)$ is decreasing, convex and twice differentiable. Then the function $G(x) = g(f(x))$ is

    A. increasing and convex

    B. decreasing and convex

    C. increasing and concave

    D. decreasing and concave

19. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a twice differentiable function satisfying $f''(x) > 0$ for all $x \in \mathbb{R}$. Furthermore, assume that $f(1) = 1$ and $f(2) = 2$. Then,

    A. $0 < f'(2) < 1$

    B. $f'(2) > 1$

    C. $f'(2) = 1$

    D. $f'(2) = 0$

20. Let A and B be two non-singular matrices of the same order and let C be a matrix such that $C = BAB^{-1}$. Then for any scalar λ, the value of $\det(C + \lambda I)$ (where I is the identity matrix) is

    A. $\det A$

    B. $\det B$

    C. $\det(A + \lambda I)$

    D. $\det(B + \lambda I)$

21. The value of $\lim_{x \to e} \frac{\log_e x - 1}{x - e}$ is

    A. 0

    B. e

    C. $\frac{1}{e}$

    D. None of these

22. Let $f : [0, \infty) \to \mathbb{R}$ be a function such that $f(0) = 0$ and $f''(x) > 0$ for all $x > 0$. Then the function $g : (0.\infty) \to \mathbb{R}$. defined by $g(x) = \frac{f(x)}{x}$, is

    A. increasing in $(0, \infty)$

    B. decreasing in $(0, x)$

    C. increasing in $(0.1]$ and decreasing in $(1.\infty)$

    D. decreasing in $(0.1]$ and increasing in $(1, \infty)$

23. Let $f : A \to B$ be a function where $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2\}$. How many onto functions can one generate?

A. $5^2 - 1$

B. $5^2 - 2$

C. $2^5 - 1$

D. $2^5 - 2$

24. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \frac{x}{1+x^2}$ for all $x \in \mathbb{R}$. Then.

A. $-1 \leq f(x) \leq 1$

B. $-1 \leq f(x) \leq 1/2$

C. $-1/2 \leq f(x) \leq 1$

D. $-1/2 \leq f(x) \leq 1/2$

25. If a 3×3 matrix A has rank 3 and a 3×4 matrix B has rank 3, then the rank of AB is

A. 3

B. 4

C. 6

D. 7

26. Let $A = \begin{pmatrix} 2 & 0 & 3 & 1 & -1 \\ 2 & 3 & 1 & 0 & -1 \\ 3 & 1 & 2 & 0 & -1 \\ 1 & 2 & 3 & -1 & 0 \\ 2 & 1 & -1 & 0 & 3 \end{pmatrix}$ Which one is an eigenvalue of A?

A. 2

B. 1

C. 3

D. 5

27. Let A be a 5×5 non-null singular matrix. Then which of the following statement is true?

A. $A\mathrm{x} = 0$ has only a trivial solution

B. $A\mathrm{x} = 0$ has 5 solutions

C. $A\mathrm{x} = 0$ has no solution

D. $A\mathrm{x} = 0$ has infinitely many solutions

28. A family has two children. What is the probability that both are boys given that at least one is a boy?

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

29. Consider two boxes, one containing one black ball and one white ball, the other containing two white balls and one black ball. A box is selected at random, and a ball is selected at random from the selected box. What is the probability that the ball is black?

    A. $\frac{5}{12}$

    B. $\frac{2}{5}$

    C. $\frac{1}{6}$

    D. $\frac{5}{11}$

30. The function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = (x^2 + 1)^{2022}$, is

    A. one-one but not onto

    B. onto but not one-one

    C. both one-one and onto

    D. neither one-one nor onto

18 ISI PEA 2023

1. A consumer's budgetary allocation for two commodities x and y is given by m. Her demand for commodity x is given by: $x(p_x, p_y, m) = \frac{2m}{5p_x}$. Suppose that her budget allocation (m) and the price of commodity $y\,(p_y)$ remains the same at $(m = \text{Rs. } 1000, p_y = \text{Rs. } 20)$ while the price of commodity $x\,(p_x)$ falls from Rs. 5 to Rs. 4 . The substitution effect of this price change is given by

   A. an increase in demand for x from 80 to 100

   B. an increase in demand for x from 90 to 100

   C. an increase in demand for x from 80 to 90

   D. an increase in demand for x from 80 to 92

2. You are given the following partial information about the purchases of a consumer who consumes only two goods: Good 1 and Good 2.

	Year 1			Year 2	
	Quantity	Price		Quantity	Price
Good 1	100	100	Good 1	120	100
Good 2	100	100	Good 2	??	80

   Suppose that the amount of Good 2 consumed in year 2 is denoted by x. Think about the range of x over which you would conclude that the consumer's consumption bundle in year 1 is revealed preferred to that in year 2. Also think about the range of x over which you would conclude that the consumer's consumption bundle in year 2 is revealed preferred to that in year 1 . Which of the following ranges of x ensures that the consumer's behaviour is inconsistent (that is, it contradicts the weak axiom of revealed preference)?

   A. $75 < x < 80$

   B. $x \geq 70$

   C. $70 < x < 75$

   D. $x \leq 75$

3. Consider a market demand function $p = 100 - q$. where p is market price and q is aggregate demand. There are 23 firms, each with cost function, $c_i(q_i) = \frac{q_i^2}{2}, i \in 1, 2 \ldots\ldots 23$. The Cournot-Nash equilibrium

   A. is not well defined

   B. involves each firm producing 3 units

   C. involves each firm producing 4 units

   D. involves each firm producing 5 units

4. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There are 10 firms, each with cost function, $c_i(q_i) = q_i, i \in 1, 2, \ldots 10$. The firms compete in quantities. The total deadweight loss is

A. $\frac{99^2}{2}$

B. $\frac{9^2}{2}$

C. $\frac{10^2}{2}$

D. $\frac{100^2}{2}$

5. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There is a large number of firms with identical cost functions

$$c_i(q_i) = \begin{cases} 10 + 2q_i, \text{ if } q_i > 0 \\ 0, \text{ otherwise.} \end{cases}$$

A. The competitive equilibrium price is not well defined

B. The competitive equilibrium price is 2

C. The competitive equilibrium price is 10

D. The competitive equilibrium price is 2.1

6. Consider a market demand function $p = 100 - q$. where p is market price and q is aggregate demand. There are two firms. firm 1 and firm 2. with identical cost functions

$$c_i(q_i) = \begin{cases} 0, \text{ if } q_i \leq 10 \\ \infty, \text{ otherwise} \end{cases}$$

for $i = 1.2$. The firms simultaneously announce their prices, p_1 and p_2. The demand coming to firm i is:

$$D_i(p_1, p_2) = \begin{cases} 100 - p_i, & \text{if } p_i < p_j \\ \frac{100-p_i}{2}, & \text{if } p_i = p_j \\ 0, & \text{otherwise} \end{cases}$$

The Bertrand-Nash equilibrium is

A. $(p_1 = 0, p_2 = 0)$

B. $(p_1 = 20, p_2 = 20)$

C. $(p_1 = 80, p_2 = 80)$

D. $(p_1 = 90, p_2 = 90)$

7. 500 consumers (of health services) are distributed uniformly over the interval $[0, 1]$. The government can set up two hospitals anywhere in the interval. The hospitals provide health services free of cost, but the consumers have to incur the expenses of travelling to the hospital. The travel cost of a consumer who travels a distance d is d. The fixed cost of setting up a hospital is 300, and the marginal cost of servicing an individual is 2. The worth of the health services to an individual is 4. The government can, of course, decide to set up no hospital. The optimal hospital location decision of a welfare maximizing government is:

A. set up no hospital

B. set up two hospitals - both at 1/2

C. set up two hospitals - one at 1/3, the other at 2/3

D. set up two hospitals - one at 1/4, the other at 3/4

8. There is a unit mass of consumers who buy either one unit of a product or nothing. Consumer valuation , θ, is distributed according to the distribution function $F(\theta)$ defined over $[\underline{\theta}, \overline{\theta}]$, that is, for any $\theta \in [\underline{\theta}, \overline{\theta}]$, the proportion of consumers with valuation less than or equal to θ is given by $F(\theta)$. Suppose that the inverse demand function for the product is $p(q)$, where p is market price and q is aggregate demand. Then the slope of the inverse demand function is

A. $p'(q) = -F'(p(q))$

B. $p'(q) = -\frac{1}{F'(p(q))}$

C. $p'(q) = -\frac{1}{F'(q)}$

D. $p'(q) = -F'(q)$

Questions 9 and 10 share the following common information. Consider an economy where output (income) is demand determined. In this economy λ proportion $(0 < \lambda < 1)$ of the total income is distributed to the workers, and $(1 - \lambda)$ proportion to the capitalists. The capitalists save s_c fraction $(0 < s_c < 1)$ of their income and consume the rest; the workers save s_w fraction $(0 < s_w < 1)$ of their income and consume the rest; also $s_w > s_c$. The aggregate demand consists of total consumption demand and total investment demand. Investment demand is autonomously given at $\bar{I}$ units.

9. Suppose savings propensities of both the workers and capitalists increase. Then, in the new equilibrium.

A. aggregate savings increases and income decreases

B. aggregate savings decreases and income increases

C. aggregate savings increases and income remains unchanged

D. aggregate savings remains unchanged and income decreases

10. Suppose savings propensities remain the same but the share of total income distributed to the workers increases. Then, in the new equilibrium

A. aggregate savings increases and income decreases

B. aggregate savings decreases and income increases

C. aggregate savings increases and income remains unchanged

D. aggregate savings remains unchanged and income decreases

Questions 11, 12, and 13 are related and share a common information set. The complete set of information is revealed gradually as you move from one question to the next. Attempt them sequentially starting from question 11.

11. Consider an economy where the aggregate output in the short run is given by $Y =$

$(\bar{K})^{\alpha}L^{1-\alpha}, 0 < \alpha < 1$, where L is the aggregate labour employment and $\bar{K}$ is the aggregate capital stock (which is fixed in the short run). Let P and W denote the aggregate price level and the nominal wage rate respectively. The producers in the economy maximize profit in a perfectly competitive market.

In this economy the demand for labour as a function of real wage rate $\left(\frac{W}{P}\right)$ is given by

A. $L^d = Y^{\frac{1}{1-\alpha}}(\bar{K})^{\frac{\alpha}{1-\alpha}}$

B. $L^d = \bar{K}(1-\alpha)^{\frac{1}{\alpha}}\left(\frac{W}{P}\right)^{\frac{1}{\alpha}}$

C. $L^d = \bar{K}(1-\alpha)^{\frac{1}{\alpha}}\left(\frac{W}{P}\right)^{-\frac{1}{\alpha}}$

D. $L^d = \bar{K}(1-\alpha)^{-\frac{1}{\alpha}}\left(\frac{W}{P}\right)^{\frac{1}{\alpha}}$

12. The above economy is characterized by a representative household which takes the aggregate price level and the nominal wage rate as given and decides on its consumption and labour supply by maximizing its utility subject to its budget constraint. The household has a total endowment of L units of labour time, of which it supplies L^* units to the market and enjoys the rest as leisure. Its utility depends on its consumption (C) and leisure $\left(\bar{L} - L^*\right)$ in the following way: $u = C^{\beta} + \left(\bar{L} - L^s\right)^{\beta}, 0 < \beta < 1$. The only source of income of the household is the wage income and it spends its entire wage earning in buying consumption goods at the price P. In this economy the supply of labour as a function of real wage rate $\left(\frac{W}{P}\right)$ is given by

A. $L^s = \frac{\bar{L}}{1+\left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}}$

B. $L^s = \frac{\bar{L}}{1+\left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}}}$

C. $L^s = \bar{L}\left[1+\left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}\right]$

D. $L^s = \frac{\bar{L}}{1-\left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}}}$

13. Given the labour demand and labour supply functions as derived above, the aggregate supply curve (output (Y) supplied as a function of the aggregate price level (P), with Y on x-axis and P on y-axis) of this economy is

A. upward sloping

B. downward sloping

C. vertical

D. horizontal

14. Consider an economy with aggregate income Y and aggregate price level P. The goods market clearing condition is given by the savings-investment equality: $S(Y, r) = I(r)$,

where r is real interest rate and $0 < S_Y < 1, S_r > 0, I_r < 0$. The money market clearing condition is given by the equality of real money supply $\left(\frac{M}{P}\right)$ and demand for real balances (L) : $\frac{M}{P} = L(Y, r)$, where M is the supply of money and $L_Y > 0, L_r < 0$.

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[For any function $f(x, y)$, f_x denotes the partial derivative of f with respect to x.

The slope of the aggregate demand curve (aggregate output (Y) demanded as a function of the aggregate price level (P), with Y on x-axis and P on y-axis) of this economy is

A. $\frac{S_Y L_r - (S_r - I_r) L_Y}{-\frac{M}{P^2} S_Y}$

B. $\frac{S_Y L_r - (S_r - I_r) L_Y}{-\frac{1}{P} S_Y}$

C. $\frac{S_Y L_r - (S_r - I_r) L_Y}{\frac{M}{P^2} (S_r - I_r)}$

D. $\frac{S_Y L_r - (S_r - I_r) L_Y}{\frac{1}{P} (S_r - I_r)}$

15. To test the prediction of the Solow growth model, you run the following linear regression for all the countries in the world:

$$g_i = \alpha + \beta_0 \log y_{i,0} + \beta_1 \log n_i + \beta_2 \log s_i + \gamma X_i + \varepsilon_i,$$

where $\underline{g_i}$ is the growth rate in per capita real GDP of country i over a certain period, $y_{i,0}$ is per capita real GDP of country i at the beginning of the period under consideration, n_i is population growth rate of country i, s_i is savings rate of country i, X_i stands for a set of other control variables and ε_i is the error term.

The Solow growth model predicts that the expected sign of the regression coefficient β_0 is

A. negative

B. positive

C. zero

D. inconclusive

16. Consider the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The rank of A is

A. 3

B. 2

C. 1

D. 0

17. Bowl A contains two red coins; Bowl B contains two white coins; and Bowl C contains a white and a red coin. A bowl is selected uniformly at random and a coin is chosen from it uniformly at random. If the chosen coin is white, what is the probability that the other coin in the bowl is red?

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{6}$

18. A girl chooses a number uniformly at random from $\{1, 2, 3, 4, 5, 6\}$. If she chooses n, then she chooses another number uniformly at random from $\{1, \ldots, n\}$. What is the probability that the second number is 5 ?

A. $\frac{2}{45}$

B. $\frac{11}{180}$

C. $\frac{1}{3}$

D. $\frac{1}{18}$

19. The cumulative distribution function F of a standard normal distribution satisfies:

$$F(1.4) = 0.92, \quad F(0.14) = 0.555$$
$$F(-0.2) = 0.42, \quad F(-1.6) = 0.055$$

A manufacturer does not know the mean and standard deviation of the diameters of ball bearings it produces. However, he knows that the diameters follow a normal distribution with mean μ and standard deviation σ. It rejects 8% of bearings as too small if the diameter is less than 1.8 cm and 5.5% bearings as too large if the diameter is greater than 2.4 cm. Which of the following is correct?

A. $\mu = 2$

B. $\mu = 2.33$

C. $\mu = 2.4$

D. $\mu = 2.08$

20. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f'(x)$ is strictly increasing in x ($f'(x)$ indicates the derivative of $f(x)$ with respect to x). Suppose $f\left(\frac{1}{2}\right) = \frac{1}{2}$ and $f(1) = 1$. Then which of the following is true?

A. $1 < f'\left(\frac{1}{2}\right) < f'(1)$

B. $f'\left(\frac{1}{2}\right) < f'(1) < 1$

C. $f'\left(\frac{1}{2}\right) < 1 < f'(1)$

D. None of the above

21. For any non-negative real number x, define $f(x)$ to be the largest integer not greater than x. For instance, $f(1.2) = 1$. Evaluate the following integral

$$\int_0^{\sqrt{5}} f\left(x^2\right) dx$$

A. 5

B. $4\sqrt{5}$

C. $4\sqrt{5}-\sqrt{3}-\sqrt{2}-3$

D. $4(\sqrt{5}-2)$

22. The constant term (i.e., the term not involving x) in the expansion of $\left(x+\frac{1}{x^2}\right)^{19}$ is

A. 171

B. 19

C. 1

D. none of the above

23. Arjun and Gukesh each toss three different fair coins (each coin either lands heads or tails with equal probability and with each outcome independent of each other). Arjun wins if strictly more of his coins lands on heads than Gukesh, and we call the probability of this event p_1. Which of the following is correct?

A. $p_1=\frac{1}{3}$

B. $p_1=\frac{3}{8}$

C. $p_1=\frac{11}{32}$

D. $p_1=\frac{13}{32}$

24. How many real solutions are there to the equation $x|x|+1=3|x|$?

A. 3

B. 2

C. 1

D. 0

25. We are given n positive integers $k_1,\ldots,k_n$ (need not be distinct) such that

$$k_1+\ldots+k_n=5n-4$$
$$\frac{1}{k_1}+\ldots+\frac{1}{k_n}=1$$

What is the maximum value of n ?

A. 6

B. 5

C. 4

D. 3

26. A monkey starts at the origin $(0,0)$ on $\mathbb{R}^2$. The monkey covers a distance of 5 units in any direction in one jump. If the monkey can only go to integer coordinates on $\mathbb{R}^2$, then the number of possible locations after its first jump is equal to

A. 12

B. 8

C. 4

D. 2

27. There is a strip made up of $(n+2)$ squares, where n is a positive integer. The two end squares are coloured black and other n squares are coloured white. A girl jumps to one of the n white squares uniformly at random and chooses one of its two adjacent squares uniformly at random. What is the probability that the chosen square is white?

A. $1 - \frac{1}{n+2}$

B. $1 - \frac{1}{n-1}$

C. $\frac{1}{2} - \frac{1}{n+1}$

D. $1 - \frac{1}{n}$

28. Let $f : \mathbb{R} \to \mathbb{R}$ be the following function

$$f(x) = \max\left(|x|, x^2\right), \quad \forall x.$$

Which of the following is true?

A. f is increasing

B. f is not continuous

C. f is continuous but not differentiable

D. f is decreasing

29. Let $f : \mathbb{R} \to \mathbb{R}$ be the following function

$$f(x) = \max\left(|x|, x^2\right), \quad \forall x.$$

Define

$$D := \left\{(x, y) \in \mathbb{R}^2 : x \in \mathbb{R}, y \geq f(x)\right\}$$

Which of the following is true for D ?

A. $\mathbb{R}^2 \backslash D$ is convex

B. $\mathbb{R}^2_+ \backslash D$ is convex

C. D is not convex

D. None of the above

30. Suppose $f : [-1, 1] \to \mathbb{R}$ is a function such that

$$f(x) = \frac{2 - x^2}{2} f\left(\frac{x^2}{2 - x^2}\right), \quad \forall x \in [-1, 1].$$

Then, $f(-1)$ is equal to

A. -1

B. 0

C. 1

D. $\frac{1}{2}$

19 ISI PEA 2024

1. If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function at $a \in \mathbb{R}$ such that $f'(a) = af(a)$, then what is $\lim_{x\to a} \frac{xf(a)-af(x)}{x-a}$?

   A. $af(a)$

   B. $\frac{f(a)}{a}$

   C. $(1-a^2)\,f(a)$

   D. None of the previous options

2. Suppose S_n is defined as follows for every positive integer $n \geq 2$:

$$S_n = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\cdots\left(1 - \frac{1}{n^2}\right)$$

   The value of $\lim_{n\to\infty} S_n$ is then

   A. 0

   B. $\frac{1}{2}$

   C. 1

   D. ∞

3. Suppose $\lim_{x\to 0} \dfrac{e^{a_1 x} - 1}{a_2 x^2 + a_3 x} = 1$, where a_1, a_2 and a_3 are given real numbers. Then it is necessarily true that

   A. $a_1 = a_2 = a_3 = 1$

   B. $a_1 = a_3 \neq 0$

   C. $a_2 = 0$

   D. $a_2 + a_3 \neq 0$

4. Let a function $f : \mathbb{R} \to \mathbb{R}$ be defined as:

$$f(x) = \begin{cases} cx^2 + ax + b & \text{if } x < 0 \\ bx^2 + cx + a & \text{if } 0 \leq x < 2 \\ ax^2 + bx + c & \text{if } x \geq 2 \end{cases}$$

   where a, b, c are positive real numbers. Which of the following statements is correct, under the assumption that f is continuous?

   A. f is continuous for all values of a, b and c

   B. f is continuous if and only if $a - b = b - c$

   C. f is continuous if and only if $a = b$ and $c = 2a$

   D. f is continuous if and only if $a = b = c$

5. Consider $f : [0,1] \to [0,1]$ such that $f(x) = \frac{x}{2-x}$. Which of the following statements is incorrect?

   A. $f(0) = 0$ and $f(1) = 1$

   B. $f(1 - f(x)) = 1 - x$

   C. f is strictly concave in the interval $(0,1)$

   D. f is strictly increasing in the interval $(0,1)$

6. Let $f : \mathbb{R} \to \mathbb{R}$ be the function

$$f(x) = |x| + x^2 \quad \forall x \in \mathbb{R}$$

   Which of the following statements about f is correct?

   A. f is differentiable

   B. f is concave but not differentiable

   C. f is convex but not differentiable

   D. f is discontinuous

7. $\int x^3 e^{x^2} dx$ equals

   A. $\frac{x(x-1)}{2} e^{x^2}$

   B. $\frac{(x^2-1)}{2} e^{x^2}$

   C. $\frac{x(x+1)}{2} e^{x^2}$

   D. $\frac{(x^2+1)}{2} e^{x^2}$

8. Let A be a 3×3 matrix having eigenvalues $2, 7, 5$. What is the determinant of $A + 2I$?

   A. 252

   B. 70

   C. 420

   D. 84

9. Let $\mathbf{x}$ and $\mathbf{y}$ be two column vectors of length 3 such that $\sum_{i=1}^{3} x_i y_i \neq 0$. What is the rank of $\mathbf{x}\mathbf{y}^T$, where $\mathbf{y}^T$ is the transpose of $\mathbf{y}$?

   A. 0

   B. 1

   C. 2

   D. 3

10. Let $\mathbf{A}$ be a 3×3 matrix such that $\mathbf{Ax} = \mathbf{x}$ for all $\mathbf{x}$ where $\mathbf{x}$ is a column vector of length 3. Which of the following statements is correct?

   A. No such A exists

B. $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

C. $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

D. A exists but is different from the options given in parts (A) or (B) above

11. Let

$$S = \{(x_1, 0) : x_1 \text{ is any real number}\}$$

and

$$T = \{(0, x_2) : x_2 \text{ is any real number}\}.$$

Which of the following statements is correct?

A. S, T and $S \cap T$ are vector spaces

B. S, T and $S \cup T$ are vector spaces

C. $S \cup T$ and $S \cap T$ are vector spaces

D. Neither $S \cup T$ nor $S \cap T$ are vector spaces

12. In a chess tournament, there are both boys and girls. Each player plays with another player exactly once in the tournament. If there are 45 games in the tournament and exactly 15 of them feature only boys, then how many games will feature a boy and a girl?

A. 6

B. 15

C. 20

D. 24

13. What is the number of possible arrangements of the letters of the word 'madam' such that the two 'a's never appear in consecutive positions?

A. 30

B. 24

C. 18

D. 12

14. A monkey starts at $(0, 0)$ on the xy-plane in period 1. From any position (x, y) in a period, the monkey can only jump to (a, b) in the next period, where $a \in \{x+1, x, x-1\}$ and $b \in \{y + 1, y, y - 1\}$. How many possible positions can the monkey be in period 2?

A. 9

B. 6

15. In the previous question, suppose in every period the monkey can go to any of the possible

positions in the next period with - equal probability. Then what is the probability that the monkey is at a distance of more than 1 from $(0,0)$ in period 2?

A. 0

B. $\frac{16}{81}$

C. $\frac{1}{3}$

D. $\frac{4}{9}$

16. For a given data set, let the least squares regression line be $y=10+2x$. It is given that variance of x is 9 and variance of y is 81 . What is the correlation coefficient between x and y?

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

17. An urn contains 4 white, 6 red, and 5 black balls. 5 balls are randomly selected from the urn. Let X and Y denote respectively the number of white and black balls selected. Suppose $Y=2$, i.e., 2 of the 5 balls selected are black. What is the probability that X takes the value 2?

A. $\frac{3}{10}$

B. $\frac{4}{10}$

C. $\frac{3}{15}$

D. $\frac{4}{15}$

18. In answering a question on a multiple-choice test with 4 choices available as possible answers, a student either knows the answer, with probability $1/4$, or guesses the answer, with probability $3/4$. Assume that a student who guesses the answer will be correct with probability $1/4$. What is the probability that a student knew the answer given that he answered it correctly?

A. $\frac{3}{7}$

B. $\frac{4}{7}$

C. $\frac{3}{4}$

D. 1

19. Let $X_1, X_2, \cdots, X_5$ be independently and identically distributed random variables with mean 10 and variance 4. Let $\bar{X}=\frac{1}{5}\sum_{i=1}^{5} X_i$. What is the value of $\operatorname{Cov}\left(\bar{X}, X_1-\bar{X}\right)$?

A. 8 B. 1C. -1

D. None of the previous options

20. Consider a discrete random variable which follows a binomial distribution with parameters n and p, where n, a positive integer, is the number of trials and $p \in (0,1)$ is the probability of success in any trial. Which of the following statements is incorrect?

    A. If np is an integer, then the mean and mode of this distribution coincide

    B. If n is even and $p = \frac{1}{2}$, then the median of this distribution is $\frac{n}{2}$

    C. If the probability of k successes out of n trials is equal to the probability of $n-k$ successes out of n trials, $\forall k \in \{0, 1, \ldots, n\}$, then $p = \frac{1}{2}$

    D. If $(n+1)p - 1$ is an integer, then this distribution is unimodal

21. Consider an economy where all factors of production are fully employed and which has an aggregate production function (in per capita form) $y = Ak$, where k and y are the capital-labour ratio and the output-labour ratio respectively, and A is a positive constant. A constant proportion $s \in (0,1)$ of income is saved and invested in this economy. Suppose labour force grows at the rate $n > 0$ and the rate of depreciation of capital is given by $\delta \in (0,1)$. Assume the parameter values are such that positive long run growth of y can be ensured. Then which of the following statements regarding this model economy is incorrect?

    A. Output, capital and consumption grows at the same rate

    B. If the level of investment (I) is higher than the depreciation of capital (K) (i.e. $I > \delta K$), then output grows at a positive rate

    C. The economy is always on the steady state growth path (i.e., if you consider any variable you will find its growth rate to be a constant at all times

    D. Increasing the savings rate s will not have any effect on the long run growth rate of output per worker

The following data are for Question numbers 22 and 23: There is a Solow economy without population growth or technological change which has a production function given by $Y = K^{\frac{1}{3}} L^{\frac{2}{3}}$, and a depreciation rate $\delta = 0.05$.

22. Suppose the savings rate is $s = 0.2$. What will be the capital-labour ratio in steady state?

    A. 6

    B. 8

    C. 2

    D. 4

23. Suppose that a social planner wishes to maximize steady state per-capita consumption in this economy. What savings rate will be compatible with the level of per-capita consumption chosen by the planner?

A. $\frac{2}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

24. Consider a Solow economy with the aggregate production function in intensive form given by $y = k^{\frac{1}{2}}$, where k and y are the capital labour ratio and the per capita output respectively. Suppose all factors are fully employed in this economy, there is no depreciation of capital, and the labour force grows at rate $n > 0$. If the steady state value of capital labour ratio for this economy is 50 and the current value of capital labour ratio is 2, what is the current growth rate of per capita output?

A. $24n$

B. $\frac{25}{n}$

C. 0.3

D. None of the previous options

The following data are for Question numbers 25 and 26: There are two countries, A and B, and two goods, wheat and television. A produces only wheat, with $Y_A = 200$, and B produces only television, with $Y_B = 250$. Residents of both countries consume both wheat and television. Treating television as the numeraire, and therefore fixing its price at 1, let the relative price of wheat be p. Let A 's total expenditure measured in terms of wheat be denoted by E_A, and let B's total expenditure measured in terms of television be denoted by E_B. Now, total world income must equal total world expenditure, i.e., $pY_A + Y_B = pE_A + E_B$, so it follows that the trade balance of A, measured in wheat, is given by $T_A = Y_A - E_A$. Suppose initially E_A is 100, and in an attempt to improve trade balance, A reduces expenditure E_A to 70 .

25. Suppose preferences are such that $\frac{1}{3}$rd of the expenditure of residents of any country is on wheat and the rest is on television. What is the impact on equilibrium price (i.e., the price at which all markets clear) p of A reducing its expenditure from 100 to 70?

A. p will stay the same

B. p will increase

C. p will decrease

D. The effect on p will be ambiguous

26. Suppose preferences are such that $\frac{2}{3}$rd of the expenditure of residents of any country is on their own good, and the rest is on the other good (i.e. $\frac{2}{3}$rd of the expenditure of residents of A is on wheat and the rest is on television, while $\frac{2}{3}$rd of the expenditure of residents of B is on television and the rest is on wheat). What is the impact on equilibrium price

(i.e., the price at which all markets clear) p of A reducing its expenditure from 100 to 70?

A. p will stay the same

B. p will increase

C. p will decrease

D. The effect on p will be ambiguous

27. A consumer consumes two goods, X and Y. It is observed that the consumer's consumption of good X always falls when the price of X falls. ceteris paribus. Suppose the consumer's income rises, given prices of X and Y. What will happen to the consumer's consumption of X?

A. Consumption of X falls

B. Consumption of X rises

C. Consumption of X remains unchanged

D. Indeterminate: consumption of X could rise, fall or remain unchanged

28. An individual A has a pond in which he can set up a net costlessly only at time $t = 0$. The net will yield an output of f fish at $t = 1$ and nothing at $t = 2$. A eats only fish and his only source of fish is the pond. A can consume in periods $t = 1$ and $t = 2$, after which he ceases to exist. A has the option of costlessly storing fish at $t = 1$ for consumption at $t = 2$, without depreciation, and does not discount the future. His utility from consumption c_t in any period t is $u(c_t) = (c_t)^n$, where $0 < n < 1$. How much fish will A optimally consume in the two periods?

A. Any division of f fish across the two periods is optimal

B. $c_1 = c_2 = \frac{f}{2}$

C. $c_1 = f, c_2 = 0$

D. $c_1 = 0, c_2 = f$

29. Country C, which has a closed economy, has 10,000 identical farmers producing rice, with any farmer bearing cost $0.5q_i^2 + 4q_i + 100$ if he produces q_i units of rice. C has many consumers of rice who generate an aggregate demand function $Q = -10,000p + 400,000$, where p is the price per unit of rice. The price of rice in C used to be determined by competitive market conditions, but is currently regulated, and set at $p = 30$. Consumers pay this amount to buy a unit of rice, with the government of C buying all unsold units at 30 per unit. The government proposes to remove the price regulation and go back to competitive markets, with each farmer getting an equal flat payment, total payment across all farmers being equal to the savings of the government from not having to purchase unsold rice. In order to implement this proposal, each farmer has to open a bank account at own cost. By how much does a farmer gain or lose from the policy shift, if it costs 300 to open an account?

A. Policy shift causes income to rise by 8

B. Policy shift causes income to rise by 4

C. Policy shift causes income to fall by 4

D. Policy shift causes income to fall by 8

30. A consumer consumes three goods, X, Y and Z. She is observed over three periods.

In period 1, the unit prices of X, Y and Z are respectively 2,3 and 3, in period 2, the respective prices are 3,2 and 3, while in period 3, the respective prices are 3,3 and 2. The following consumption pattern is observed: in period 1, she consumes 3 units of X, 1 units of Y and 7 units of Z in period 2, the consumption amounts are 7 units of X, 3 units of Y and 1 unit of Z, while in period 3, the consumption amounts are 1 unit of X, 7 units of Y and 3 units of Z. Which of the following statements regarding the preferences of the consumer over bundles of X, Y and Z is correct?

A. Preferences are intransitive though not necessarily incomplete

B. Preferences are transitive though not necessarily complete

C. Preferences are incomplete and intransitive

D. Preferences are complete and transitive