Example 7.5 *t*-test for Difference of Two Means (Dependent Samples)

A statistics teacher used only verbal instruction for his class of 14 students. At the end of the session, he tested them and marked their scores. For the next session, he again taught the same topic but used video tapes and slides with his verbal instruction. Afterwards, he again tested the students and noted the results. Results were matched and are shown below.

Student	Verbal Instruction	Mixed Instruction	Student	Verbal Instruction	Mixed Instruction
A	70	75	H	92	91
B	82	82	I	94	95
C	83	85	J	87	88
D	90	89	K	85	85
E	78	80	L	76	80
F	84	85	M	82	84
G	88	91	N	92	92

Research Question: Is there a greater difference in the scores at a .05 significance level?
Is the result significant or practical? Discuss.

Step 1. **State the hypotheses**. It is expected that there will be an increase in the test scores from mixed instruction; therefore, the mean of the differences should be less than zero.
H_0: $\mu_D \geq 0$; H_A: $\mu_D < 0$.
A*ssume*: Students are drawn from same overall population ($\mu_D = 0$).

Step 2. **Find the critical value**. Degrees of freedom (df) are n – 1 or 14 – 1 = 13.
The critical *t*-value for a left-tail test at α = .05 is -1.771.

Step 3. **Compute the test value**. Using the above data, identify the differences and differences squared. Find the mean of the differences. Use the formulas: $D = X_1 - X_2$; $\overline{D} = (\sum D)/n$.

Student	*1st test*	*2nd test*	*D*	*D^2*
A	70	75	-5	25
B	82	82	0	0
C	83	85	-2	4
D	90	89	1	1
E	78	80	-2	4
F	84	85	-1	1
G	88	91	-3	9
H	92	91	1	1
I	94	95	-1	1
J	87	88	-1	1
K	85	85	0	0
L	76	80	-4	16
M	82	84	-2	4
N	92	92	0	0
			$\Sigma D = -7$	$\Sigma D^2 = 67$

$\overline{D} = (\sum D)/n$ = -7 / 14 = -0.5

Step 4. **Find the standard deviation of the differences**.

$$\boldsymbol{s_D} = \sqrt{\frac{n\sum D^2 - (\sum D)^2}{n(n-1)}} = \sqrt{\frac{14(67)-(-7)^2}{14(13)}} = \sqrt{\frac{938-49}{182}} = \sqrt{4.885} = \mathbf{2.21}$$

Example 7.5 *t*-test for Difference of Two Means (Dependent Samples)	
Step 5.	**Find the test value**. $\boldsymbol{t} = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} = \frac{-0.5-0}{2.21/14} = \frac{-0.5}{2.21/3.74} = \frac{-0.5}{0.59} = \mathbf{-0.85}$
Step 6.	**Make the decision**. **-0.85 > -1.771.** **Do not reject the null as -0.85 is greater than the critical value, -1.771.** **There is not enough evidence at alpha .05 to indicate an increase in test scores.**
Step 7.	**Is the result significant or practical? Discuss**. Although there were nine test score increases, there were three decreases and two no change. The *statistical significance* does not reject the null that μ is greater but *practical analysis* indicates there *were more test score increases* than decreases. It is also noted that two scores decreased. Additional comments can be made about the (a) teaching_methods, (b) student likes about verbal versus video and other methods, (c) student intelligence, and (d) other issues such as being aware of the topic after the first instruction, and so forth.

7.4 *z*-test for One Mean.

When the standard deviation (σ) is known for the population, the sample is random, and $n \geq 30$ or assumed to be a normal distribution if $n < 30$, then the ***z*-test** can be used. The standard deviation of the sample means is smaller than that of the population mean; therefore, a correction factor is applied by the population standard deviation being divided by the square root of the sample size. It applies as $\sigma_{\bar{X}} = \sigma/\sqrt{n}$, where *n* is the sample size, and is labelled as the **standard error of the mean**. The mean of all the samples will be the same as the population mean. **Example 7.6** indicates these properties.

Example 7.6 Standard Error of the Mean

A teacher gives a 10-point test to 10 students. The results are as follows:

Results (X)	$X - \mu$	$(X - \mu)^2$	Results (X)	$X - \mu$	$(X - \mu)^2$
10	1.6	2.56	8	-0.4	.16
9	0.6	.36	9	0.6	.36
10	1.6	2.56	6	-2.4	5.76
8	-0.4	.16	10	1.6	2.56
7	-1.4	1.96	7	-1.4	1.96
			$\sum X_i = 84$		$\sum(X_i - \mu)^2 = 18.4$

Population mean is
$\boldsymbol{\mu} = \Sigma X_i/n = 84/10 = \mathbf{8.4}$.
Population standard deviation is
$\sigma = \sqrt{\frac{\Sigma(X_i-\mu)^2}{N}} = \sqrt{\frac{18.4}{10}} =$
$\boldsymbol{\sigma} = \sqrt{1.84} = \mathbf{1.356}$

If ten samples of size two (2) were taken with replacement, the means, distribution, and standard error of the mean (standard deviation of the sample means) are as follows:

Sample	Mean	Sample	Mean	$\bar{X}$	Frequency (f)	$\bar{X} \bullet f$
10, 8	9	9, 9	9	6	1	6
10, 6	8	10, 10	10	7	1	7
7, 9	8	8, 8	8	8	3	24
9, 9	9	6, 6	6	9	3	27
6, 8	7	10, 10	10	10	2	20
					$\sum = 10$	$\sum = 84$

Mean of sample means = 84 / 10 = **8.4**
The mean of the samples is the same as the population mean.

Example 7.6 Standard Error of the Mean
Compute the standard deviation of the sample means (the standard error of the mean). Sample size n = 2. Use the formula $\boldsymbol{\sigma_{\bar{X}}} = {}^{\sigma}/_{\sqrt{n}} = {}^{1.356}/_{\sqrt{2}}$ = 1.356 / 1.414 = **0.959**.

The mean of all the samples equals the population mean. Basically, when using the z-test with sampling, the standard error of the mean formula is applied as $z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$. This is because means are used instead of individual values. When sample size n increases without limit, the sample means with replacement from a population (known mean μ and standard deviation σ) will near the normal distribution. The distribution used for this has a mean μ and a standard deviation as above or $\sigma_{\bar{X}} = {}^{\sigma}/_{\sqrt{n}}$. When working with an individual value, the z formula denominator is only σ; there is no X-bar ($\bar{x}$), only the X value for the formula. See **Examples 7.7** and **7.8**.

Example 7.7 *z*-test for One Mean
The average cost to attend a professional sport activity (e.g., NFL game) in 2016 has been claimed by a researcher at *more* than \$80. The researcher obtained individual prices for NFL games during the regular season as the following: 65 72 80 100 75 82 90 86 70 65 85 75 88 84 76 90 84 80 75 65 **Can the researcher support this claim at α = .05?**
Step 1. **State the hypotheses**: H_0: $\mu \le \$80$ and H_A: $\mu > \$80$
Step 2. **Find the critical *z*-value**. At α = .05 and being a right-tail test, the critical z-value (test value) is **+1.645** (averaging between .9495 and .9505 for 5% or .9500 in **Appendix C, Table C-2**).
Step 3. **Compute the test value**. Obtain the mean, variance, and standard deviation from the raw data. n = 20 ; $\bar{X} = \frac{\sum X}{n}$ = 1587 / 20 = 79.35 ; σ^2 = 86.785 ; σ = 9.316
Step 4. **Use the *z* formula**. $z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{79.35 - 80}{9.316/\sqrt{20}} = \frac{-0.65}{2.084} = \mathbf{-0.3119}$
Step 5. **Make the decision**. **-0.3119 < +1.645 Accept the null hypothesis.**
Step 6. **Summarize the results**. **The researcher cannot support the claim** that the average cost to attend a sports activity is > \$80.

Example 7.8 P-value
For the above data in **Example 7.7**, **find the P-value.**
Since z = -0.3119, the area for this z-value in **Appendix C**, **Table C-2**, is left of the mean. Interpolating between 0.31 and 0.32, A(z) = .6224 (rounded). This is the left-tail value for positive z = 0.3119; by symmetry, it also equals the right-tail A(z) value for z = -0.3119.
P-value = 0.6224 and is greater than .05. **Accept the null hypothesis at .05**.