the *Test of Independence* is administered. It can be used for nominal, ordinal, interval, or ratio data. When completing a survey or questionnaire using a scale or rating item, the results can be tested using a chi square Test of Independence. The category in the row refers to the variable responding while the response categories comprise the columns. See **Example 8.1**.

Example 8.1 Test of Independence—Workforce Survey

At an U.S. military base in the United Kingdom, the Wing Commander wanted to check on the morale of the base workforce (3,500 members). He asked a researcher to survey the members and advise him of the results.

After a judgment survey of 500 members (250 officers and 250 enlisted), the results are shown below. Of the officers, 100 were Field grade (Major and above) and 150 Company grade (Captain and below). Two hundred Sergeants (E-5 to E-9 ranks) and 50 Airmen (E-1 to E–4 ranks) were surveyed.

Using a Chi Square Test of Independence at .05 level of significance, the findings below were reported. **Do you agree with these findings? Why or why not? What changes would you make?**

The survey question being tested using Chi Square was the following:
Please indicate your level of morale at working on RAF XYZ:

	Poor	Fair	Good	Very Good	Excellent	Totals
Airmen (E1-E4)	0	20	20	10	0	50
Sgts (E5 – E9)	0	0	95	75	30	200
Officers (O1-O3)	0	5	25	100	20	150
Officers (O4-O7)	0	0	10	50	40	100
Totals	0	25	150	235	90	500

Solution:
Step 1. **State the hypotheses**.
H_0: Level of morale is independent of the rank of base personnel.
H_A: Level of morale is dependent on the rank of base personnel.

Step 2. **Determine degrees of freedom (df) and χ^2 critical value at $\alpha = .05$**.
df = (r-1) x (c -1) = (4-1) x (5-1) = 12 ; **$\chi^2_{cv\ .05} = 21.026$**

Step 3. **Compute the Expected value** using ($E_{r,c/GT}$ = row total x column total / grand total).

Row\Col	1	2	3	4	5
1	50 (0) /500 = 0	50 (25) /500 = 2.5	50 (150) /500 = 15	50 (235) /500 = 23.5	50 (90) /500 = 9
2	200 (0) /500 = 0	200 (25) /500 = 10	200 (150) /500 = 60	200 (235) /500 = 94	200 (90) /500 = 36
3	150 (0) /500 = 0	150 (25) /500 = 7.5	150 (150) /500 = 45	150 (235) /500 = 70.5	150 (90) /500 = 27
4	100 (0) /500 = 0	100 (25) /500 = 5	100 (150) /500 = 30	100 (235) /500 = 47	100 (90) /500 = 18

Step 4. The completed Table with O and (E) is shown.

	Poor	Fair	Good	Very Good	Excellent	Totals
Airmen (E1-E4)	0 (0)	20 (2.5)	20 (15)	10 (23.5)	0 (9)	50
Sgts (E5 – E9)	0 (0)	0 (10)	95 (60)	75 (94)	30 (36)	200
Officers (O1-O3)	0 (0)	5 (7.5)	25 (45)	100 (70.5)	20 (27)	150
Officers (O4-O7)	0 (0)	0 (5)	10 (30)	50 (47)	40 (18)	100
Totals	0	25	150	235	90	500

Example 8.1 Test of Independence—Workforce Survey
Step 5. **Substitute in the Chi Square formula and evaluate**. Chi-square test statistic = $\chi^2 = \Sigma[\ (\text{Observed}_i - \text{Expected}_i)^2 / \text{Expected}_i\ ]$ $\chi^2_{obs} = \frac{(0-0)^2}{0} + \frac{(20-2.5)^2}{2.5} + \frac{(20-15)^2}{15} + \frac{(10-23.5)^2}{23.5} + \frac{(0-9)^2}{9} + \frac{(0-0)^2}{0} + \frac{(0-10)^2}{10} + \frac{(95-60)^2}{60} + \frac{(75-94)^2}{94} + \frac{(50-36)^2}{36} + \frac{(0-0)^2}{0} + \frac{(5-7.5)^2}{7.5} + \frac{(25-45)^2}{45} + \frac{(100-70.5)^2}{70.5} + \frac{(20-27)^2}{27} + \frac{(0-0)^2}{0} + \frac{(0-5)^2}{5} + \frac{(10-30)^2}{30} + \frac{(50-47)^2}{47} + \frac{(40-18)^2}{18}$
χ^2_{obs} = 0 + 122.5 + 1.67 + 7.76 + 9 + 0 + 10 + 20.4 + 3.84 + 1 + 0 + 0.83 + 8.89 + 12.3 + 1.81 + 0 + 5 + 13.3 + 0.19 + 26.89 = **245.4 = χ^2_{obs}**
Step 6. **Make the decision**. **Accept the alternate.** The level of morale is dependent on rank. **245.4 > 21.026**
***Discussion*. The Wing Commander needs another survey using a stratified random proportionate sample.** Based on the percentage of ranks or civilians, a similar percentage for the total sample should be surveyed. No civilians were surveyed. Six categories were underestimated. Ten were overestimated. Four groups were zero. Civilians make up a large portion of any base manpower and need to be surveyed. If there were 300 Field Grade officers (O4 – O7), then 300 / 3500 = 8.6% or 9% of the sample desired should be Field Grade officers randomly selected and others proportional for each category. Redo the survey! The Commander needs an answer about the level of morale. The results need further analysis.

In summary, the Chi Square test is very good to test your theory about the independence of two or more variables. It uses frequency counts so data from surveys or other collection forms need only indicate the variables and the counts. For example, you may test males to females or ages in groups (18 < 20, 20 < 30, etc.) or any other form of categories. To obtain that information, one specific item on a data collection form needs to be properly presented. Two other chi square tests will be reviewed below, but the formula for each is the same.

8.2 Test of Equality.

The Test of Equality compares data for one variable in equal proportions. For example, the M&M picture (Figure 8.1) has six different colored M&Ms. If you were theorize that each color is equally provided (equal proportions), then a Test of Equality can be used. The null hypothesis would be: H_0: $p_1 = p_2 = p_3 = p_4 = p_5$ and the alternate would be that these proportions are not equal ($\neq$) or at least one proportion is different from the others. Again, the formula for this is $\chi^2 = \sum\frac{(O-E)^2}{E}$. Degrees of freedom (df) for one row is c-1, or there is one freedom lost from the number of columns. The M&M example is presented as **Example 8.2**. See also **Figure 8.3**.

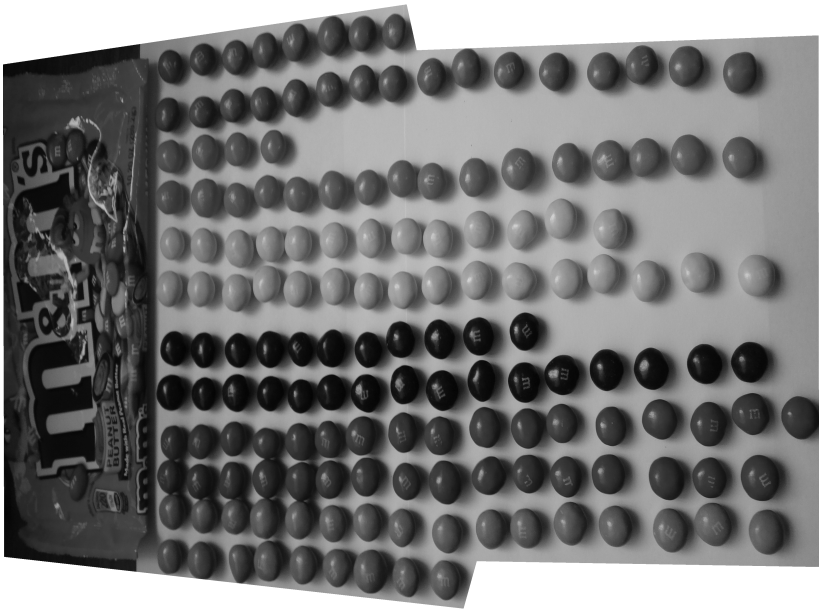

Figure 8.3 M&Ms Array

Example 8.2 Test of Equality

Regarding **Figure 8.3**, the M&M bag was opened and the following counts for each color were obtained: Green 25, Red 33, Brown 27, Yellow 29, Orange 20, and Blue 24. It is expected that that each color would be equally provided or 26.3 for each color[42].

At alpha .05, can it be concluded that the colors of M&Ms are equally provided?

Solution:

Step 1. **State the hypotheses**.

H_0: The six colors of M&Ms are equally distributed (H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = p_6$).

H_A: The six colors are not equally distributed in proportion.

Step 2. **Determine the degrees of freedom (df) and find the critical value**. Degrees of freedom is k -1 or 6 – 1 = 5. **Alpha of .05 at df = 5 equals 11.071 = $\chi 2$ critical value.**

Step 3. **Compute the test value**.

	Green	Red	Brown	Yellow	Orange	Blue
O/E	25/26.3	33/26.3	27/26.3	29/26.3	20/26.3	24/26.3

Apply the Equation: $\chi^2_{obs} = \sum \frac{(O-E)^2}{E} =$

$$\frac{(25-26.3)^2}{26.3} + \frac{(33-26.3)^2}{26.3} + \frac{(27-26.3)^2}{26.3} + \frac{(29-26.3)^2}{26.3} + \frac{(20-26.3)^2}{26.3} + \frac{(24-26.3)^2}{26.3}$$

$= 0.06 + 1.71 + 0.02 + 0.28 + 1.51 + 0.2 = \mathbf{3.78} = \chi^2_{\text{obs}}$

Step 4. **Make the decision. 3.78 < 11.071.**

The six colors of M&M are equally proportioned at 95% confidence.

---

[42] This is not a whole number but no rounding is used for this example.