9 – Correlation & Regression

not exceeded, it is considered as no difference for the null. To test the significance of the correlation coefficient⁴⁷, the *t*-test for correlation coefficient, $t = r \sqrt{\frac{n-2}{1-r^2}}$, can also be used⁴⁸. Degrees of freedom (df) are (n – 2). Use **Appendix C**, **Table C-3**, as in **Example 9.1**.

Example 9.1 <i>t</i> -values, Observed vs. Critical						
If the correlation coefficient r is .932 and $\alpha = .05$ at $10 - 2$ degrees of freedom (df), the <i>t</i> -critical value, two-tailed, is \pm 2.306. (See Appendix C, Table C-3.)						
Compute the test value using $t = r \sqrt{\frac{n-2}{1-r^2}}$:						
$t = 0.923 \sqrt{\frac{10-2}{1-0.923^2}} = 0.923 \sqrt{\frac{8}{1-0.8686}} = 0.923 \sqrt{\frac{8}{0.1314}} = 0.932 \ (7.8) = 7.27 = t$						
The <i>t</i> -observed value exceeds the <i>t</i> -critical value $+7.27 > +2.306$. Fail to accept the null hypothesis. There is a significant relationship between the variables being tested.						

Example 9.2 below shows a problem using the PPMC. Use **Appendix C**, **Table C-6**, for the critical PPMC value, for a two-tailed test of α at .05, n = number of data pairs.

Example 9.2 Using PPMC

An instructor who taught statistics and science was wondering whether there was a linear correlation of the test results. The test results of ten students chosen at random who took both courses are shown next. At the .05 level of significance, test the strength of the relationship.

Course Test Results							
Student	Statistics	Science	\mathbf{x}^2	y^2	xy		
	(x)	(y)					
1	80	85	6400	7225	6800		
2	88	84	7744	7056	7392		
3	90	88	8100	7744	7920		
4	78	80	6084	6400	6240		
5	75	84	5625	7056	6300		
6	79	80	6241	6400	6320		
7	92	88	8464	7744	8096		
8	96	90	9216	8100	8640		
9	84	86	7056	7396	7224		
10	87	87	7569	7569	7569		
$\sum =$	849	852	72499	72690	72501		
Step 1. Compute the sums needed in further calculations (see \sum row in Course Test Results above).							
 Step 2. Develop a scatterplot as below. Use statistics as the x-axis and science as the y-axis. Review the linearity (relationship) from the chart. See Figure 9.2 as follows: 							

 r^{47} r is presented to three decimal places.

⁴⁸ The P-value is also a method that can be used to test the significance of r.



Example 9.2 Using PPMC

d. There is a strong, positive correlation between the test scores of students taking a statistics course and a science course. In reviewing the test scores, five students increased their scores taking science while four decreased their scores. One student had the same test scores. Of the five who increased their scores, two did so by one point, two by 2 points, and one by 9 points. Of the four who decreased their scores, one did so by -6 points, two by -2 points, and one by -4 points. The statistics course had an average of 84.9 and science had an average of 85.2.

For the above example, the *t*-test can also be used as indicated above: $t = r \sqrt{\frac{n-2}{1-r^2}}$

In the above example, degrees of freedom (df) equal n-2: df = 8. The hypotheses are the same. A two-tailed critical value from the *t*-distribution table is used at alpha .05. This gives a **critical value of 2.306**. Applying the formula, this gives:

$$t = 0.814 \sqrt{\frac{(10-2)}{1-0.814^2}} = 0.814 \sqrt{23.739} = 0.814 (4.872)$$

- t = 3.966. Compare observed to critical: 3.966 > 2.306.
- Accept the alternate hypothesis that there is a significant relationship between the test scores of students taking a statistics course and a science course with 95% confidence.

Using the P-value method as the third method for the above example, take the *t*-test value (3.966) and use *t*-distribution table. With degrees of freedom (df) equal to eight (8), the value 3.966 falls above the two-tail critical value of 3.355 at $\alpha = .01$. This P-value is less than .05 so accept the alternate and fail to accept the null at 99% confidence.

In a correlation test, identifying the dependent and independent variables is foremost. Obtain the quantitative data and develop a scatterplot for a review of the relationship. State the hypotheses and significance level and determine the critical value. For PPMC, find r and use the critical value table for the PPMC. Other tests may be used after PPMC as above.

9.2 Regression.

After determining the relationship and the significance of the correlation coefficient, finding the *line of best fit* is next. This helps to see the trend and (a) estimate, (b) predict, and, (c) forecast. If the correlation coefficient is not significant, no further action should be taken. To compute the line of best fit, the formula is y' = a + bx. If truncating (not using zero as the base line), then use the smallest *x* value in the graph. To compute the value for the straight-line formula constant values, *a* has to be determined using