Example 10.2 Scheffé Test
Hybrid programs at alpha equals .05. Accept the null hypothesis for other program comparisons. (Refer again to the results of Example 10.1 above.)

10.3 Tukey Test.

When means of two groups have an equal sample size, the Tukey test[58] may be used. Compute the F-test statistic as in **Example 10.1** above. If the absolute value of the observed F-statistic is greater than the critical value (for $\alpha = .01$, see **Appendix C, Table C-10)**, there is a significant difference in the means. In Appendix C, Table C-10, the critical value is determined by locating the intersection of the degrees of freedom (df) for the variance within (s_W^2), indicated by the symbol v (the row label) and k, the column label, which represents the number of means. The alpha value is indicated at the top of the Table. **Example 10.3** makes a pairwise comparison for the means.

Example 10.3 Tukey Test

From a research study on college programs involving 100 schools in the U.S., the average cost for a 4-year program of four schools are listed below.
At α = .01, can it be concluded that the average cost of a 4-year program differs among these four schools? All values are in thousands. (Use one-way ANOVA test.)

Ohio	Indiana	Illinois	New York	
35.6	34.5	28.1	45.1	
27.0	29.3	34.4	39.0	
34.4	32.9	34.6	48.1	
42.3	32.5	28.7	54.4	
34.5	38.3	35.2	49.9	
$\bar{X}_1$= 34.8	$\bar{X}_2$= 33.3	$\bar{X}_3$= 32.2	$\bar{X}_4$= 47.3	Mean (values rounded)

$\bar{\bar{X}} = \bar{X}_{GM} = \sum X / N$ =(34.8+33.3+32.3+47.3)/4 = **36.9**

Step 1: **Find the variances of each sample group**. For sample variance use: $s^2 = \frac{\sum(X-\bar{X})^2}{n-1}$.
Use the sample means and Grand Mean. **Calculate the numerator.**

Ohio	sq.	Indiana	sq.
35.6 - 34.8 = 0.8	0.64	34.5 - 33.3 = 1.2	1.44
27.0 - 34.8 = -7.8	60.84	28.3 - 33.3 = -5	25.00
34.4 - 34.8 = -0.4	0.16	32.9 - 33.3 = -0.4	0.16
42.3 - 34.8 = 7.5	56.25	32.5 - 33.3 = -0.8	0.64
34.5 - 34.8 = -0.3	.09	38.3 - 33.3 = 5	25.00
	∑ sq.= 117.89		∑ sq.= 52.24

[58] Named after the US statistician John Wilder Tukey (1915–2000) who developed versions of it in 1951 and 1952]. Also labelled Tukey Honestly Significant Difference or Tukey HSD.

Example 10.3 Tukey Test

Illinois	sq.	New York	sq.
28.1 – 32.2 = -4.1	16.81	45.1 - 47.3 = -2.2	4.84
34.4 - 32.2 = 2.2	4.84	39.0 - 47.3 = -8.3	68.89
34.6 - 32.2 = 2.4	5.76	48.1 - 47.3 = 0.8	0.64
28.7 - 32.2 = -3.5	12.25	54.4 - 47.3 = 7.1	50.41
35.2 - 32.2 = 3	9.00	49.9 - 47.3 = 2.6	6.76
	∑ sq.= 48.66		∑ sq.= 131.54

Sample Variances: **Ohio**: 117.89/4 = **29.5**; **Indiana**: 52.24/7 = **13.06**
Illinois: 48.66/4 = **12.17**; **New York**: 131.54/4 = **32.89**

Step 2. Find the **between group variance**. Use $s_B^2 = \frac{\sum n_i(\bar{X}_i - \bar{X}_{GM})^2}{k-1}$.

From above: $\bar{X}_1 = 34.8$; $\bar{X}_2 = 33.3$; $\bar{X}_3 = 32.2$; $\bar{X}_4 = 47.3$; $\mathrm{X}_{GM} = 36.9$

Calculate the numerator for each state (group), sum and compute.

Ohio: $5(34.8 - 36.9)^2 = 22.05$ Indiana: $5(33.3 - 36.9)^2 = 64.8$
Illinois: $5(32.2 - 36.9)^2 = 110.45$ New York: $5(47.3 - 36.9)^2 = 540.8$

(22.05+64.8+110.45+540.8)/(4-1) = 738.1/3 = **246.03** = $\boldsymbol{s_B^2}$

Step 3. **Find the within group variance.** Use $s_W^2 = \frac{\sum(n_i - 1)s_i^2}{\sum(n_i - 1)}$.

$$s_W^2 = \frac{(5-1)(29.5)+(5-1)(13.06)+(5-1)(12.17)+(5-1)(32.89)}{(5-1)+(5-1)+(5-1)+(5-1)}$$

= (118+52.24+48.68+131.56)/16 = **21.9** = $\boldsymbol{s_W^2}$

Step 4. **Determine the observed *F* value**.
$F = s_B^2 / s_W^2$ = 246.03/21.9 = **11.23**
Determine the critical values. Use **Appendix C, Table C-10.**
Use: N = 20; *k* = 4. dfD = N – k = 20 – 4 = **16**.
At $\alpha = .01$, for v = 16 and k (number of rows) = 4, $F_{CV.01}$ = **5.19**

Step 5. **Make the decision**. **11.23 > 5.19. Fail to accept the null hypothesis**.

Step 6. **Summarize the results.** There is enough to accept the claim that at least one mean is different at alpha .01. When looking at the means and comparing them to the grand mean, it appears that New York is much higher; however, a post hoc test should be conducted to test the means. Use a Tukey HSD.

10.4 Follow-on Tukey Test.

Given Example 10.3 above, a post hoc test (**Tukey HSD**) is required to explore the differences between the means using the formula $q = \frac{\bar{X}_i - \bar{X}_j}{\sqrt{s_W^2/n}}$. The means $\bar{X}_i$ and $\bar{X}_j$ are the means of the samples being compared; *n* is the sample size; and $\boldsymbol{s_W^2}$ is the variance within groups. If the absolute value of *q*[59] is greater than the critical value (use **Appendix C, Table C-10**, for α = .01), there is a significant difference in the means. For this follow-on test, dfN is computed for the value of *k* and dfD for the value of *v*. Since there are four groups, six pairs of means must be tested: 1 & 2, 1 & 3, 1 & 4, 2 & 3, 2 & 4, and 3 & 4.

Required Pairwise Comparisons				
Groups	1	2	3	4
1		XX	XX	XX
2			XX	XX
3				XX
4				

Example 10.4 Tukey HSD for Example 10.3

Use: **Appendix C, Table C-10.** $q = \frac{\bar{X}_I - \bar{X}_J}{\sqrt{s_W^2/n}}$

From **Example 10.3**: $\bar{X}_1$= 34.8; $\bar{X}_2$= 33.3; $\bar{X}_3$= 32.2; $\bar{X}_4$= 47.3; s_W^2 = 21.9; n = 5.
α = .01; k = 4; N = 20. Therefore, dfN = k -1 = 4 -1 = **3;** dfD = N – k = 20 – 4 = **16**.

For $\bar{X}_1$ and $\bar{X}_2$, q = $(34.8 - 33.3)/\sqrt{21.9/5}$ = 1.5/2.09 = 0.717
For $\bar{X}_1$ and $\bar{X}_3$, q = $(34.8 - 32.2)/\sqrt{21.9/5}$ = 2.6/2.09 = 1.24
For $\bar{X}_1$ and $\bar{X}_4$, q = $(34.8 - 47.3)/\sqrt{21.9/5}$ = -12.6/2.09 = **-6.028**
For $\bar{X}_2$ and $\bar{X}_3$, q = $(33.3 - 32.2)/\sqrt{21.9/5}$ = 1.1 / 2.09 = 0.526
For $\bar{X}_2$ and $\bar{X}_4$, q = $(33.3 - 47.3)/\sqrt{21.9/5}$ = -14.1/2.09 = **-6.746**
For $\bar{X}_3$ and $\bar{X}_4$, q = $(32.2 - 47.3)/\sqrt{21.9/5}$ = -15.1/2.09 = **-7.22**

Entering Table C-10 (given α = .01) with k = *3* and v = *16*, the **critical value is 4.79**.
Means 1 and 4 (**Ohio and New York**), 2 and 4 (**Indiana and New York**), and 3 and 4 (**Illinois and New York) are significantly different** as the **absolute value of q** is **greater than the critical value** for those comparisons.
Therefore, the average cost of New York is significantly different from the other three schools; this confirms our speculation at the end of Example 10.3.

10.4 Two-Way ANOVA.

When two independent variables (also labelled *factors*) interact on a dependent variable, the two-way ANOVA (also called *factorial* ANOVA) is required to test this interaction. The basic assumptions are that (a) the populations are normally distributed, (b) the samples are independent, (c) the variances of the populations are homogeneous, (d) the sample sizes are equal, (e) the dependent variable is interval or ratio data, and (f) there should be no significant outliers. Three hypotheses are noted. The first is that the

[59] The symbol is *q* is called the studentized range.