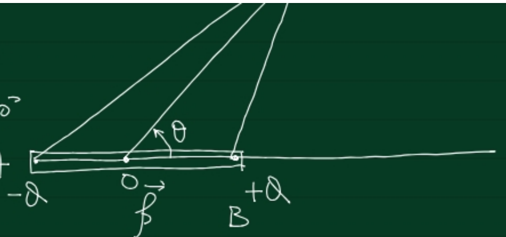
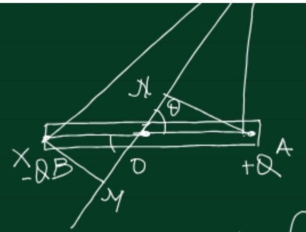
$0 \le r \le \infty$

$0 \le \theta \le 360^\circ$

$AB = 2l$

$OA = OB = l$





$$V_P = V_{P_A} + V_{P_B}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{PA} - \frac{1}{PB}\right]$$

$$V_P = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$V_P = \frac{1}{4\pi\epsilon_0}\,\frac{2lQ\cos\theta}{r^2 - l^2\cos^2\theta}$$

$r >> l\cos\theta$

$$V_P = \frac{1}{4\pi\epsilon_0}\,\frac{p\cos\theta}{r^2}$$

$\triangle OBM$

$\angle BOM = \theta$

$OB = l$

$\cos\theta = \frac{OM}{OB}$

$OM = OB\cos\theta$

$= l\cos\theta$

$\triangle OAN$

$ON = ?$

$OA = l$

$\angle AON = \theta$

$\cos\theta = \frac{ON}{OA}$

$ON = l\cos\theta$

$PA \simeq PN = OP - ON$

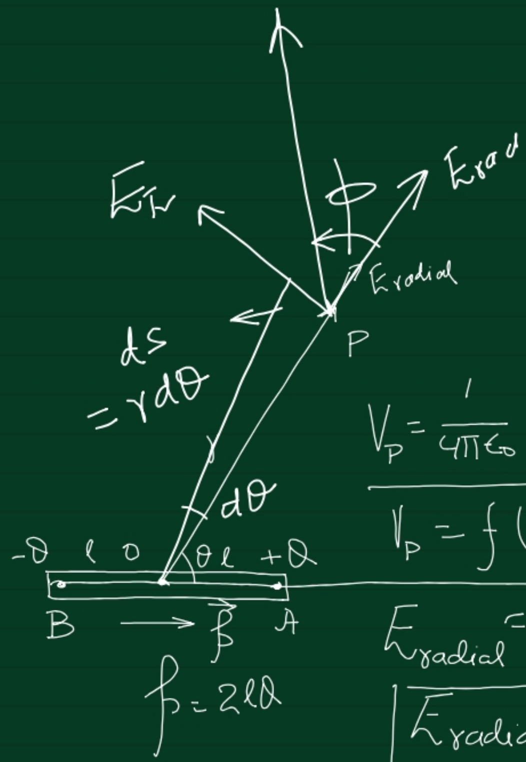
$= r - l\cos\theta$

$PB \simeq PM = OP + OM$

$V_P = f(r, \theta)$

$$E_{rad} = -\frac{dV}{dr}$$

$$E_{Transverse} = -\frac{dV}{r\,d\theta}$$



$$V_p = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$$

$$V_p = f(r) \ \& \ V_p = f(\theta)$$

$$E_{radial} = -\frac{dV}{dr} = -\frac{p\cos\theta}{4\pi\epsilon_0}\frac{d\,r^{-2}}{dr}$$

$$\boxed{E_{radial} = +\frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}}$$

$$E_{Tr} = -\frac{dV}{ds} = -\frac{1}{r}\frac{dV}{d\theta}$$

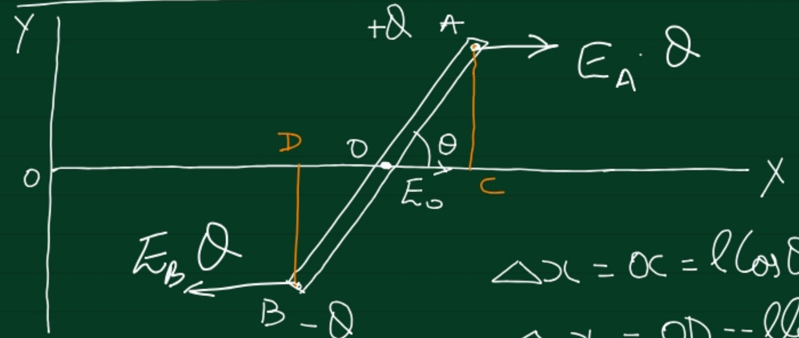
$$E_{Tr} = +\frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}$$

$$E_p = \sqrt{E_{rad}^2 + E_{Tr}^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}\sqrt{3\cos^2\theta + 1}$$

$$\tan\phi = \frac{E_{Tr}}{E_{ra}} = \frac{1}{2}\tan\theta$$

Effect of Non-Uniform electric Field on any dipole System:



Field at the Centre of Dipole System $= E_0$

Field gradient along X-axis $= \frac{dE}{dx}$

$\Delta x = OC = l\cos\theta$

$-\Delta x = OD = -l\cos\theta$

$OA = OB = l$

$$E_{at\,A} = E_0 + \frac{dE}{dx} \cdot l\cos\theta$$

$$E_{at\,B} = E_0 + \left(\frac{dE}{dx}\right)(-l\cos\theta)$$

$$F_R = E_A \cdot Q - E_B Q$$

$$= 2lQ\cos\theta\left(\frac{dE}{dx}\right) = p\cos\theta \times \left(\frac{dE}{dx}\right)$$