## Optics I:

1. (a) Establish formula for refraction through spherical surface. $\quad \mu_{2} / v-\mu_{1} / u=\left(\mu_{2}-\mu_{1}\right) / R$

Ans:


APB is a spherical surface having center of curvature at $C$ and radius of curvature R. It divides two medium of refractive index $\mu_{1}$ and $\mu_{2} . \mathrm{X}_{1} \mathrm{PX} X_{2}$ is principal axis. CN is normal drawn at the refraction point $\mathrm{N} . \mathrm{O}$ is a point object on the principal axis. A light ray ON gets incident at N at angle of incidence $\angle \mathrm{i}=\angle \mathrm{ONC}$. After refraction it gets deviated toward the normal as the second medium is denser $\left(\mu_{2}<\mu_{1}\right)$. The refracted ray forms
image at I.
From Snell's law

$$
\sin \mathrm{i} / \sin \mathrm{r}=\mu_{2} / \mu_{1}
$$

Or, $\quad \mu_{1} \sin \mathrm{i}=\mu_{2} \sin \mathrm{r}$
Or, $\quad \mu_{1}\left[\sin \mathrm{i} / \sin \left(180^{\circ}-\theta\right)\right]=\mu_{2}\left[\sin \mathrm{r} / \sin \left(180^{\circ}-\theta\right)\right]$
In $\triangle$ ONC and $\triangle$ INC
$\mu_{1}(\mathrm{OC} / \mathrm{ON})=\mu_{2}(\mathrm{IC} / \mathrm{IN})$
For paraxial rays N and P are considered very close.

$$
\mathrm{ON}=\mathrm{OP}, \quad \mathrm{IN}=\mathrm{IP} \quad \mu_{1}(\mathrm{OC} / \mathrm{OP})=\mu_{2}(\mathrm{IC} / \mathrm{IP})
$$

$\mu_{1}[(\mathrm{OP}-\mathrm{CP}) / \mathrm{OP}]=\mu_{2}[(\mathrm{IP}-\mathrm{CP}) / \mathrm{IP}]$
$\mu_{1}[1-\mathrm{CP} / \mathrm{OP}]=\mu_{2}[1-\mathrm{CP} / \mathrm{IP}]$
$\mathrm{CP}=\mathrm{R}=$ radius of curvature
$\mathrm{OP}=\mathrm{u}=$ object distance
$\mathrm{IP}=\mathrm{v}=$ image distance

$$
\begin{aligned}
& \mu_{1}[1-\mathrm{R} / \mathrm{u}]=\mu_{2}[1-\mathrm{R} / \mathrm{V}] \\
& \mu_{2} / \mathrm{V}=\mu_{1} / \mathrm{u}=\mu_{2}-\mu_{1} / \mathrm{R}
\end{aligned}
$$

(b) Find formula for refraction through thin lens.
$1 / f=(\mu-1)\left(1 / R_{1}-1 / R_{2}\right)$
Ans: Two spherical surface enclosing an optical medium different from surrounding is called spherical lens.

$A_{1} A_{2} B_{1} B_{2}$ is thin lens formed by two surfaces $A_{1} P_{1} B_{1}$ and $A_{2} P_{2} B_{2}$ having a radii of curvatures $R_{1}$ and $R_{2} O$ is the object on the principal axis.

From refraction at $\mathrm{A}_{1} \mathrm{P}_{1} \mathrm{~B}_{1}$

$$
\begin{array}{ccc}
\mathrm{OP}_{1}=+\mathrm{u} & \mathrm{I}_{1} \mathrm{P}_{1}=+\mathrm{V}^{\prime} \quad \mu_{1}=1 \quad \mu_{2}=\mu & \mathrm{R}=+\mathrm{R}_{1} \\
\mu /+\mathrm{V}^{\prime}-1 /+\mathrm{u}=(\mu-1) / \mathrm{R}_{1} & -----(1)
\end{array}
$$

For refraction at $\mathrm{A}_{2} \mathrm{P}_{2} \mathrm{~B}_{2}$.
$\mathrm{P}_{2} \mathrm{I}_{1}=-\left(\mathrm{v}^{\prime}+\mathrm{P}_{1} \mathrm{P}_{2}\right)$
$\mathrm{P}_{2} \mathrm{I}_{1}=\mathrm{v} \quad \mathrm{R}=+\mathrm{R}_{2}$
$\mu_{1}=\mu \quad \mu_{2}=1$
$\left[1 / v-\mu /+\left(v^{\prime}+P_{1} P_{2}\right)\right]=1-\mu /+R_{2}$
Adding equation (1) \& (2) when the lens is thin.

$$
\begin{equation*}
1 / \mathrm{V}-1 / \mathrm{u}=(\mu-1)\left(1 / \mathrm{R}_{1}-1 / \mathrm{R}_{2}\right) \tag{2}
\end{equation*}
$$

If $u=\propto \quad$ Then $v=-f=$ focal length.
$1 / \mathrm{f}=(\mu-1)\left(1 / \mathbf{R}_{1}-1 / \mathbf{R}_{2}\right)$
2. Find expression for focal length of equivalent lens of two thin lens placed at a separation coaxially.
Ans: When a parallel beam of light gets incident on a thin lens it suffers deviation $\delta$. Such that

$$
\tan \delta=\mathrm{h} / \mathrm{f}
$$

For thin lens $\tan \delta=\delta$ $\delta=h / f$


Where $\mathrm{h}=$ height of incident aperture.
$f=$ focal length of lens.

$L_{1}$ and $L_{2}$ are two convex lenses of focal length $f_{1}$ and $f_{2}$ kept at coaxial separation $d$ on the principal axis $\mathrm{XX}^{\prime}$. Parallel beam of light gets incident on first lens at aperture height $\mathrm{h}_{1}$ and suffers deviation $\delta_{1}$ where $\mathrm{O}_{1} \mathrm{~F}$ $=f_{1}$. The incident beam gets further deviated at second lens and suffers deviation $\delta_{2}$ of and gets finally converged at C .

$$
\text { In } \triangle \mathrm{MDB} \quad \delta=\delta_{1}+\delta_{2}
$$

$$
\begin{equation*}
\mathrm{h}_{1} / \mathrm{F}=\mathrm{h}_{1} / \mathrm{f}_{1}+\mathrm{h}_{2} / \mathrm{f}_{2} \tag{1}
\end{equation*}
$$

$\triangle \mathrm{MO}_{1} \mathrm{~F} \sim \triangle \mathrm{BO}_{2} \mathrm{~F}$
$\mathrm{MO}_{1} / \mathrm{O}_{1} \mathrm{~F}=\mathrm{BO}_{2} / \mathrm{O}_{2} \mathrm{~F}$
Or, $\quad h_{1} / f_{1}=h_{2} / f_{1}-\mathrm{d}$
Or, $\mathrm{h}_{2}=\left(\mathrm{f}_{1}-\mathrm{d} / \mathrm{f}_{1}\right) \mathrm{h}_{1}$
Putting value from (2) into (1)
$\mathrm{h}_{1} / \mathrm{F}=\mathrm{h}_{1} / \mathrm{f}_{1}+\mathrm{h}_{1}\left(\mathrm{f}_{1}-\mathrm{d} / \mathrm{f}_{1} . \mathrm{f}_{2}\right)$
$1 / \mathrm{F}=1 / \mathrm{f}_{1}+1 / \mathrm{f}_{2}-\mathrm{d} / \mathrm{f}_{1} . \mathrm{f}_{2}$
Thus is the required expression for focal lens of equivalent lens.
3. Derive expression for deviation produced by a prism. Also find expression for minimum angle of deviation produced by thin prism. A
Ans: AMN is a prism of angle of refraction A and refractive index $\mu$. A narrow beam of light gets incident on the prism at C at angle I of incidence $i_{1}$. It gets refracted towards CB and

then finally gets refracted at B . The final angle of deviation in $\delta$.
In CFB

$$
\begin{align*}
& \angle \mathrm{CFB}+\angle \mathrm{FCB}+\triangle \angle \mathrm{FBC}=180^{\circ} \\
& (\pi-\delta)+\left(\mathrm{i}_{1}-\mathrm{r}_{1}\right)+\left(\mathrm{i}_{2}-\mathrm{r}_{2}\right)=180^{\circ} \\
& \mathrm{i}_{1}+\mathrm{i}_{2}=\delta+\left(\mathrm{r}_{1}+\mathrm{r}_{2}\right) \tag{1}
\end{align*}
$$

In quadrilateral ABCD

$$
\begin{align*}
& \mathrm{A}+\pi / 2+\pi / 2+\angle \mathrm{CDB}=2 \pi \\
& \mathrm{~A}+\pi-\left(\mathrm{r}_{1}+\mathrm{r}_{2}\right)=\pi \\
& \mathrm{A}=\left(\mathrm{r}_{1}+\mathrm{r}_{2}\right) \tag{2}
\end{align*}
$$

From equation (1) \& (2)

$$
\mathrm{i}_{1}+\mathrm{i}_{2}=\delta+\mathrm{A}
$$

Applying Snell's law at C \& B.
$\sin i_{1} / \sin r_{1}=\mu / 1$
$\sin \mathrm{r}_{2} / \sin \mathrm{i}_{2}=1 / \mu$
$\sin \mathrm{i}_{1}=\mu \sin \mathrm{r}_{1}$
$\sin \mathrm{i}_{2}=\mu \sin \mathrm{r}_{2}$
Adding equation (3) \& (4).
$\sin \mathrm{i}_{1} / \sin \mathrm{i}_{2}=\mu\left(\sin \mathrm{r}_{1}+\sin \mathrm{r}_{2}\right)$
$\mu=2 \sin \left\{\left(\mathrm{i}_{1}+\mathrm{i}_{2}\right) / 2\right\} / 2 \sin \left\{\left(\mathrm{r}_{1}+\mathrm{r}_{2}\right) / 2\right\}$.
$\left[\cos \left\{\left(\mathrm{i}_{1}-\mathrm{i}_{2}\right) / 2\right\} / \cos \left\{\left(\mathrm{r}_{1}-\mathrm{r}_{2}\right) / 2\right\}\right]$
$=[\sin \{(\delta+\mathrm{A}) / 2\} / \sin (\mathrm{A} / 2)]$.
$\left[\cos \left\{\left(\mathrm{i}_{1}-\mathrm{i}_{2}\right) / 2\right\} / \cos \left\{\left(\mathrm{r}_{1}-\mathrm{r}_{2}\right) / 2\right\}\right]$
For minimum angle of deviation
$\mu=\sin \left[\left(\delta_{\mathrm{m}}+\mathrm{A}\right) / 2\right] / \sin \mathrm{A} / 2$
For thin Prism $\quad \delta_{\mathrm{m}}=\mathrm{A}(\mu-1)$
4. Describe principle and construction of compound microscope. Derive formula for its magnification.

Ans: Compound microscope consists of two coaxial convex lens at a separation. The lens kept near the object is called objective. The objective is of small focal length and aperture. The lens kept near the eye is called eye piece and is of large aperture and focal length.

The object is kept in front of objective. The objective of the eye piece. The position of eye-piece is adjusted so that the image lie within focal length of the eye-piece. The eye-piece forms virtual magnified final image at least distance of distant vision.


The magnification of compound microscope is defined as $\mathrm{m}=$ size of the final image/size of the object
$=\mathrm{A}_{2} \mathrm{~B}_{2} / \mathrm{AB}=\mathrm{A}_{2} \mathrm{~B}_{2} / \mathrm{A}_{1} \mathrm{~B}_{1} \times \mathrm{A}_{1} \mathrm{~B}_{1} / \mathrm{AB}=\mathrm{m}_{\mathrm{E} \times \mathrm{m}_{\mathrm{O}}}$
Where $m_{E}=$ magnification of eye-piece lens.

$$
=\mathrm{A}_{2} \mathrm{~B}_{2} / \mathrm{AB}=[1+(\mathrm{D} / \mathrm{fe})]
$$

$\mathrm{m}_{\mathrm{O}}=$ magnification of objective lens.

$$
=\mathrm{A}_{1} \mathrm{~B}_{1} / \mathrm{AB}=\mathrm{OB}_{1} / \mathrm{OB}
$$

$=\frac{\text { image distance for objective lens }}{\text { objective distance of objective lens }}$
$=\mathrm{v} / \mathrm{u}$
From lens formula

$$
1 / v-1 / u=1 / f_{0}
$$

Where fo is focal length of objective lens.
$u / v-1=u / f_{o} \quad$ or, $u / v=1+u / f_{o}=\left(f_{O}+u\right) / f_{o}$
Or, $\quad m_{O}=v / u=f_{0} / f_{O}-u$
u is distance of object from objective lens.
Thus magnification of compound microscope.

$$
\mathrm{m}=\mathrm{fo} / \mathrm{fo}-\mathrm{u}[1+(\mathrm{D} / \mathrm{fe})]
$$

## 5. Describe principle and working of Astronomical telescope. Derive formula for its magnification.

Ans: Astronomical telescope consists of two coaxial converging lens at a separation. The lens kept near the object is called objective lens. It is of large aperture and focal length. The lens kept near the eye-piece. It is of relatively small aperture and focal length.
Astronomical telescope focusing for infinity :-


The light coming from infinity gets focused by objective lens at its focus and forms inverted magnified real image. The position of eye- piece is adjusted so the image formed by objective lie on the focal of eye-piece. The image is formed at infinity. The magnification is defined as

$$
m=\beta / \alpha
$$

$=\frac{\text { Angle made by the image on the eye }}{\text { Angle made by the object on the eye }}$
$=\tan \beta / \tan \alpha=\mathrm{OB} / \mathrm{OE}=\mathrm{fo} / \mathrm{fe}$
Focusing for normal vision :
The objective forms magnified inverted image at its focus with in focal length of eye-piece. The final image is formed at least distance of distinct vision.


The magnification

$$
\mathrm{m}=\tan \beta / \tan \alpha=\mathrm{OB} / \mathrm{EB} \quad=\mathrm{fo}_{\mathrm{o}} / \mathrm{EB}
$$

For eye piece

$$
\begin{aligned}
& (1 /-\mathrm{D})-1 /-(\mathrm{EB})=1 / \mathrm{fe} \\
& 1 / \mathrm{EB}=1 / \mathrm{fe}+1 / \mathrm{D} \\
& =1 / \mathrm{fe}[1+(\mathrm{fe} / \mathrm{D})] \\
& \mathrm{m}=\mathrm{fo} / \mathrm{fe}[1+(\mathrm{fe} / \mathrm{D})]
\end{aligned}
$$

6. Explain the Law of reflection and refraction on the basis of Huygen's wave theory of light.

Ans: (i) Reflection :-
When light rays gets incident on a reflecting surface
(a) The line of incidence lie in one plane.
(b) The angle of incidence is equal to angle of reflection These laws are valid under wave theory of light also. $Z_{1} Z_{2}$ is a reflecting surface and $A B$ is incident wave front at $t=0$. According to Huygen's theory each point on the incident wave front generates secondary wave lets which travel at same speed V in the same medium.

$\mathrm{BA}^{\prime}=\mathrm{Vt} \quad$ and $\quad \mathrm{AB}^{\prime}=\mathrm{Vt} \quad \mathrm{Z}_{1}$
If all the secondary wave lets generated at AB arrive at $A^{\prime} B^{\prime}$ then $A^{\prime} B^{\prime}$ is reflected wave front of $A B$.

A $\quad \mathrm{Q} \mathrm{R}^{\mathrm{A}^{\prime}}$ C
$\triangle \mathrm{AB}^{\prime} \mathrm{A}^{\prime} \sim \triangle \mathrm{QP}^{\prime} \mathrm{A}^{\prime}$
$\mathrm{QA}^{\prime} / \mathrm{AA}^{\prime}=\mathrm{QP}^{\prime} / \mathrm{AB}^{\prime}$
Similarly $\triangle \mathrm{QA}^{\prime} \mathrm{R} \sim \triangle \mathrm{AA}^{\prime} \mathrm{C}$
$\mathrm{QA}^{\prime} / \mathrm{AA}^{\prime}=\mathrm{QR} / \mathrm{AC}$
From equation (1) \& (2)
$\mathrm{QP}^{\prime} / \mathrm{AB}^{\prime}=\mathrm{QR} / \mathrm{AC}$
From diagram $A C=A^{\prime} B$
$\mathrm{QP}^{\prime} / \mathrm{AB}^{\prime}=\mathrm{QR} / \mathrm{A}^{\prime} \mathrm{B}$
Therefore $\mathrm{A}^{\prime} \mathrm{B} / \mathrm{AB}^{\prime}=\mathrm{QR} / \mathrm{QP}^{\prime}$
Or $\quad \mathrm{Vt} / \mathrm{Vt}=\mathrm{QR} / \mathrm{QP}^{\prime}$
Thus $\mathrm{QR}=\mathrm{QP}^{\prime}$
$\angle \mathrm{IAN}=\mathrm{i}=90-\angle \mathrm{BAN}=\angle \mathrm{BAA}^{\prime}$
$\angle B^{\prime} A N=r=90-\angle B^{\prime} A N=\angle B^{\prime} A$
$\operatorname{Sin} \mathrm{i} / \sin \mathrm{r}=\operatorname{Sin}\left(\mathrm{BAA}^{\prime}\right) / \operatorname{Sin}\left(\mathrm{BA}^{\prime} \mathrm{A}\right)=\mathrm{BA}^{\prime} / \mathrm{AA}^{\prime} / \mathrm{AB}^{\prime} / \mathrm{AA}^{\prime}=\mathrm{BA}^{\prime} / \mathrm{AB}^{\prime}=$ Thus $\mathrm{i}=\mathrm{r}$
(ii) Refraction :-
$\mathrm{Z}_{1} \mathrm{Z}_{2}$ is a plane surface dividing two medium of different refractive index $\mu_{1}$ and $\mu_{2}$.

Incident wave front $A B$ first touches at $A$
at $t=0$ then $B$ touches at $A^{\prime}$ after time ' $t$ '.

$$
\mathrm{AB}^{\prime}=\mathrm{V}_{1 \mathrm{t}}
$$

During time period ' $t$ ' secondary wave lets generated at $A$ travels to $B^{\prime}$ in second medium.

$$
\mathrm{AB}^{\prime}=\mathrm{V}_{2} \mathrm{t}
$$

If the secondary wavelets at P goes to $\mathrm{P}^{\prime}$ in second medium then $\mathrm{A}^{\prime} \mathrm{B}^{\prime}$ is the refracted wave front.

$$
\text { As } \quad \begin{align*}
& \triangle \mathrm{QA}^{\prime} \mathrm{P}^{\prime} \sim \triangle \mathrm{AA}^{\prime} \mathrm{B}^{\prime} \\
&  \tag{1}\\
& \mathrm{QA}^{\prime} / \mathrm{AA}^{\prime}=\mathrm{QP}^{\prime} / \mathrm{AB}^{\prime}
\end{align*}
$$

Similarly $\triangle \mathrm{QA}^{\prime} \mathrm{R} \sim \triangle \mathrm{AA}^{\prime} \mathrm{C}$

$$
\mathrm{QA}^{\prime} / \mathrm{AA}^{\prime}=\mathrm{QR} / \mathrm{AC}
$$

From equation (1) \& (2)

$\mathrm{QP}^{\prime} / \mathrm{AB}^{\prime}=\mathrm{QR} / \mathrm{AC}$
$\mathrm{QP}^{\prime} / \mathrm{QR}=\mathrm{AB}^{\prime} / \mathrm{AC}=$ Distance traveled in second medium/Distance traveled in second medium in its absence
$\mathrm{i}=\angle \mathrm{IAN}=90-\angle \mathrm{BAN}=\angle \mathrm{BAA}^{\prime}$
$r=\angle B^{\prime} A N=90-\angle B^{\prime} A^{\prime} A=\angle B^{\prime} A^{\prime} A$
$\operatorname{Sin} \mathrm{i} / \sin \mathrm{r}=\operatorname{Sin}\left(\mathrm{BAA}^{\prime}\right) / \operatorname{Sin}\left(\mathrm{B}^{\prime} \mathrm{A}^{\prime} \mathrm{A}\right)=\mathrm{A}^{\prime} \mathrm{B}^{\prime} / \mathrm{AA}^{\prime} / \mathrm{AB}^{\prime} / \mathrm{AA}^{\prime}=\mathrm{BA}^{\prime} / \mathrm{AB}^{\prime}=\mathrm{V}_{1} \mathrm{t} / \mathrm{V}_{2} \mathrm{t}$
$=\mu=$ refractive index
Thus law of refraction is valid even.

## ---7. Describe defects of vision and its remedies.

Ans: Eye lens forms image of an objects at the fixed distance whether the object lie at far away distance or near eye. This is possible because eye lens can change its converging ability with a range. When eye lens looses this ability there is defect in vision. Mostly eye suffers following defects.
(1) Short sightedness Or, Myopia : - In this defect eye can not see object lying beyond a fixed distance. This distance is called far point.

$1 / v-1 / u=1 /-f$
$1 /-d-1 / \propto=1 /-f \quad f=-d$
Power of the corrective lens $P=-1 / f$
(2) Far sightedness Or, Hypermetropia :-

This defect is due to decrease in convergence. In this defect eye can not see decrease in convergence. In this defect eye can not see the object lying within a distance. This distance is called near point. A convex lens is used to correct which forms the image at near distance of the object lying at least distance of distinct vision. $u=-D=$ Least distance of distinct vision.
$\mathrm{V}=-\mathrm{d}=$ near point where objects are visible.
$\mathrm{f}=$ focal length of corrective lens.
$1 /-d-1 / D=1 /$ Or, $\quad 1 / D-1 / d=d-D / D d$
Power of the corrective lens $\mathrm{P}=\mathrm{d}-\mathrm{D} / \mathrm{dD}$
(3) Astigmatism : - This defect is caused by variation is radius of curvature of eye lens in vertical and horizontal line. Such eye lens have two focal length and magnification one for vertical and other for horizontal line. This defect is corrected using cylindrical lens.
(4) Presbyopia :- This defect is caused by decrease in flexibility of cilliary muscle due to which lens lacks adjusting ability. This is corrected by using bifocal lens.

