Blackman Envelope and Levine Phase Jump

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Two pulse-shaping ideas used in the Rb-87 gate design: the Blackman envelope for clean single-qubit rotations, and the Levine phase jump that turns two identical global Rydberg pulses into a CZ gate.

Blackman envelope

A Blackman envelope is a standard smooth pulse shape, taken from the Blackman window in signal processing. For a pulse of length T, defined on 0 ≤ t ≤ T and zero outside:

$$\Omega(t) = \Omega_{\max}\left[0.42 - 0.5\cos\left(\frac{2\pi t}{T}\right) + 0.08\cos\left(\frac{4\pi t}{T}\right)\right]$$

- **Shape:** starts at zero, peaks at Ω_max at t = T/2, and falls back to zero, with zero slope at both ends, so there is no sudden edge.
- **Why use it:** a square pulse's sharp edges spread its spectrum into wide sidelobes that can drive unwanted transitions, such as other hyperfine sublevels. The Blackman sidelobes are about 58 dB down, so the pulse stays spectrally clean.
- **Cost:** for the same peak power it delivers only 42% of a square pulse's area, so it takes about 2.4× longer for the same rotation.

Pulse area sets the rotation angle

The rotation angle is the integral of the Rabi frequency. Over the full pulse the cosine terms average to zero:

$$\theta = \int_0^T \Omega\,(t)\,dt = 0.42\,\Omega_{\max}\,T$$

Pulse	Rotation θ	Duration T	At Ω_max = 2π × 50 kHz (microwave)	At Ω_max = 2π × 1 MHz (Raman)
π pulse (X gate)	π	π / (0.42 Ω_max)	23.8 μs	1.19 μs
π/2 pulse	π/2	π / (0.84 Ω_max)	11.9 μs	0.60 μs

The drive's phase, not its shape, sets the rotation axis.

Levine phase jump

The two-pulse CZ gate from Levine et al., *PRL* 123, 170503 (2019), uses two identical global Rydberg pulses, each of length τ, at constant Rabi frequency Ω and detuning Δ. The only difference between them is a laser-phase shift ξ applied between the two pulses: the "phase jump". Both qubits get the same drive, so no local addressing is needed.

What happens to each input state

- **|00⟩:** neither atom couples to the Rydberg laser; nothing happens.
- **|11⟩ (blockaded pair):** couples at √2 Ω and oscillates at $\sqrt{(2\Omega^2 + \Delta^2)}$. τ is chosen so this completes exactly one full cycle per pulse: $\sqrt{(2 + 0.377^2)} \times 4.29268 = 6.283 = 2\pi$. The pair is back in the ground state at the midpoint and again at the end.
- **|01⟩ or |10⟩ (one atom driven):** oscillates more slowly, at $\sqrt{(\Omega^2 + \Delta^2)}$, and is only partway through its cycle at the midpoint. The phase jump ξ rotates the drive axis so the second pulse steers it back to the ground state instead of continuing the cycle.
- **CZ condition:** the parameters are tuned so the accumulated phases satisfy the relation below. After single-qubit Z corrections remove ϕ_{01}, the result is exactly a CZ gate.

$$\varphi_{11} = 2\varphi_{01} + \pi$$

Parameters

Parameter	Value (units of Ω)	With Ω = 2π × 4.6 MHz
Detuning Δ/Ω	0.377371	Δ = 2π × 1.74 MHz
Pulse length Ωτ	4.29268	τ = 148.5 ns per pulse, 297 ns total
Phase jump ξ	3.90242 rad (223.6°)	Applied at 148.5 ns

In hardware

The jump is applied to the RF phase driving the 420 nm beam's AOM; shifting the RF phase by ξ shifts the optical phase by the same amount, almost instantly.

The sign of ξ depends on the Hamiltonian's phase convention. In simulation the wrong sign gives F = 0.67 instead of 0.999999, so calibrate it on the machine with a quick scan of ξ.