Forces Acting on the Tool in VideoDisc Mastering

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Abstract—The mechanics of cutting a VideoDisc master with an electro-mechanical cutter are considered and expressions are derived for the forces acting on the diamond tool as functions of tool geometry, substrate properties, rake angle, tool-substrate friction angle, and velocity of cutting. The analysis indicates that the tool forces vary linearly with depth of cut for normal recording and quadratically with groove depth for a "fast spiral."

1. Introduction

In recording on a VideoDisc master with an electro-mechanical cutter, average groove depth is established by mechanically adjusting the separation between cutter and disc surface. The desired separation is maintained constant by use of an air-bearing puck that supports the cutter above the disc surface. In principle, this should result in constant average groove depth.[1,2]

The analysis presented in this paper was performed to determine the forces acting on a tool during cutting as an aid to the design of a cutter and its support structure.

2. Mechanics of Cutting

The cutting edges of the diamond tool used in mastering VideoDiscs are orthogonal to the relative velocity vector between tool and work piece only when the rake angle is zero. For nonzero rake angles, the cutting edges are inclined to the velocity vector. To estimate the cutting forces exerted on a tool, it would therefore be appropriate to consider the mechanics of oblique cutting. The resulting expressions are rather unwieldly and unnecessarily complicated, however, and considerable simplification results if the analysis is restricted to the mechanics of orthogonal cutting.

Results obtained from such an analysis are expected to be reasonably valid, since the inclination of the cutting edges to the velocity vector are fairly small for all rake angles of interest.

Fig. 1 is a sketch of a diamond tool. TD and TB are the cutting edges. For zero rake angle, the projections of TD and TB onto the work surface (TD′ and TB′, respectively) are orthogonal to the velocity vector between tool and work piece. For a nonzero rake angle, TD′ and TB′ are inclined at angles i as shown in Fig. 1. For a tool with an apex angle DTB of 140° and a rake angle α, the inclination angle i is given by

$$i = \tan^{-1}\left(\frac{\sin \alpha}{\tan 70}\right).$$

Thus for small rake angles, the inclination angle is approximately ⅓ the rake angle.

The total force acting on a tool may be obtained by summing the

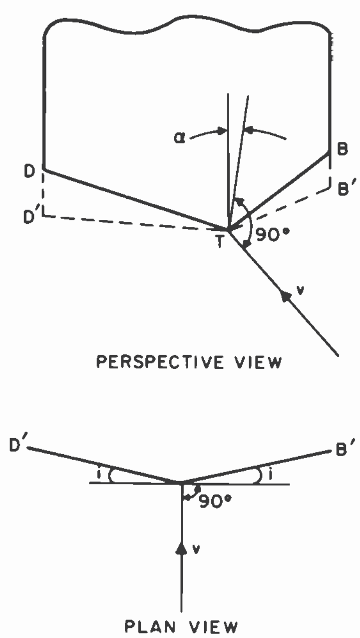


Fig. 1—Inclination angle of tool cutting edges.

forces on each cutting edge. The force on a cutting edge may be decomposed into three components: one component F_P parallel to the velocity vector, a second F_Q perpendicular to the work surface, and a third F_R orthogonal to the first two.

For a symmetrical V-groove cut (fast spiral), the F_P and F_Q components due to each cutting edge are equal and have the same direction. The sum of these components gives the total components acting on a tool. The values obtained for the total components are approximated by those obtained from an orthogonal cutting analysis. The F_R components are equal in magnitude but oppositely directed, resulting in zero lateral (radial) force on a tool.

For normal recording, the cutting edges are subjected to unequal forces. The F_R components are unequal and do not cancel, resulting in a net radial force acting on a tool. By restricting the analysis to orthogonal cutting, the radial force is assumed to be zero, and the other components are approximated.

3. Cutting and Inertia Forces on a Tool

The cutting force on a tool may be calculated using the thin-shear plane model developed by Merchant and Oxley.[3] The inertia reaction force generated by the chip (i.e., the sliver of material cut away to produce the groove) may be added to the cutting force to give the total tool force. Fig. 2 is a sketch showing the basic model assumed by Merchant and Oxley for the analysis of orthogonal cutting. We define

t = depth of cut in work piece (assumed uniform)

V = velocity of work piece (stationary tool)

t_c = thickness of chip

V_c = chip velocity assumed parallel to tool face

α = tool rake angle (the rake angle is shown in Fig. 1)

$\mu = \tan \beta$ = coefficient of friction between tool and work piece

β = friction angle determined from μ

R = resultant tool force per unit width of cut

F = friction force on tool face per unit width of cut = $R \sin \beta$

N = normal force on tool face per unit width of cut = $R \cos \beta$

m = chip mass flow per second per unit width of cut.

In the thin-shear-plane model, the cutting force results from the

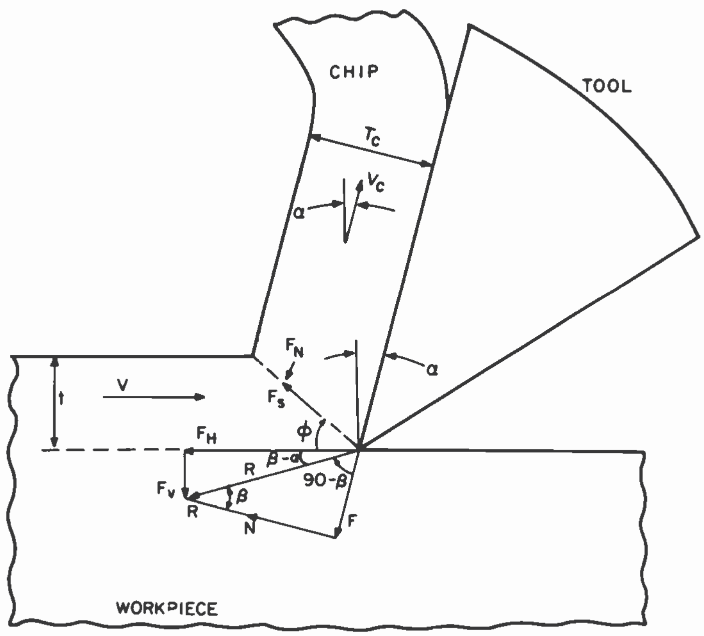


Fig. 2—Geometry of thin-shear-plane model for metal cutting.

workpiece shearing along a plane of constant area inclined at a constant angle. Let

F_S = shear force in shear plane per unit width of cut

F_N = normal force across shear plane per unit width of cut

ϕ = shear plane angle

τ = yield shear strength of workpiece.

The shear force is given by

$$F_S = \tau \times \text{area of shear plane per unit width of cut} \quad [1]$$
$$= \tau t/\sin\phi.$$

The resultant force R may be decomposed into a horizontal component F_H and a vertical component F_V as shown in Fig. 2. These may be expressed in terms of F_s, F_N, and inertial components due to the chip:

$$F_H = \text{cutting force}$$
$$= F_S \cos\phi + m[V - V_C \sin\alpha] + F_N \sin\phi \quad [2]$$
$$= R\cos(\beta - \alpha).$$

F_V = thrust force

$$= F_N \cos\phi - mV_C \cos\alpha - F_S \sin\phi \qquad [3]$$

$$= R \sin(\beta - \alpha).$$

From the geometry of Fig. 2, we have

$$t_c = \frac{t}{\sin\phi}\cos(\phi - \alpha). \qquad [4]$$

Conservation of chip mass leads to

$$V_C = \frac{Vt}{t_C} = \frac{V\sin\phi}{\cos(\phi - \alpha)}. \qquad [5]$$

Eqs. [2] and [3] may be manipulated to give F_N. If Eqs. [4] and [5] are substituted into the resulting expression, we obtain

$$F_N = F_S \tan(\phi + \beta - \alpha) + \tan(\phi + \beta - \alpha)\frac{\cos\alpha}{\cos(\phi - \alpha)} mV. \qquad [6]$$

For an orthogonal cut of unit width, depth t, and at a cutting velocity V, the mass flow is

$$m = \rho t V. \qquad [7]$$

where ρ is the density of the workpiece. If Eqs. [1], [6], and [7] are substituted into Eqs. [2] and [3], we obtain

$$F_H = t f_H(\phi, \alpha, \beta, V, \tau, \rho)$$

$$F_V = t f_V(\phi, \alpha, \beta, V, \tau, \rho),$$

where f_H and f_V are functions of the indicated variables and are independent of t.

4. Spiral Groove

For a fast spiral groove, shown schematically in Fig. 3, we have

$$dF = Fdb = ftdb$$

and

$$F_T = \int_{-B}^{+B} ftdb = 2f\int_0^D \tan 70\ tdt$$

$$= D^2 \tan 70^\circ f$$

where

dF = element of force for width db of either the horizontal or vertical component

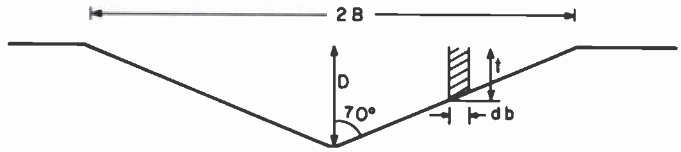


Fig. 3—Geometry of fast spiral groove.

F_T = horizontal or vertical component of tool force

F = horizontal or vertical force, i.e., F_H or F_V per unit width

f = f_H or f_V as given previously

D = depth of groove.

Performing the indicated manipulations leads to F_{HT} and F_{VT}, the horizontal and vertical components of the force exerted by the tool:

$$F_{HT} = D^2 \tan 70 \,\{\tau[\cot\phi + \tan(\phi + \beta - \alpha)]\} + \rho\frac{V^2 \cos\alpha \cos\phi}{\cos(\phi - \alpha)}[1 + \tan(\phi + \beta - \alpha)\tan\phi] \quad [8]$$

$$F_{VT} = D^2 \tan 70\,[\cot\phi \tan(\phi + \beta - \alpha) - 1] \cdot \left[\tau + \rho\frac{V^2 \cos\alpha \sin\phi}{\cos(\phi - \alpha)}\right] \quad [9]$$

Eq. [9] shows that vertical force F_{VT} is zero for $\beta = \alpha$. If the rake angle is greater than the friction angle, i.e., $\beta < \alpha$, then F_{VT} is negative and it requires an upward directed force to keep the tool in a stable position.

The coefficient of static friction μ between diamond and metal[4] is in the range of 0.1 to 0.15. It is not affected by lubrication, and is probably larger than the coefficient of kinetic friction. The corresponding value of β is in the range of 5.7 to 8.5 degrees.

The shear angle ϕ may be determined in a number of ways, as described in the literature.[3] An empirical best fit to a large body of experimental data[3] shows that

$$\phi = 50 - 0.8(\beta - \alpha) \text{ degrees.} \quad [10]$$

The data used in arrriving at the above expression involves conventional machining with conventional tools rather than micro-machining with a diamond tool. Experimental data from Guarracini et al[5] indicate a shear angle of 35 ± 3 degrees for the case of a disc machined with normal grooves (overcut) and a rake angle $\alpha = 0.5$ degrees. This would imply a relation of the form

$$\phi = 40 - 0.8(\beta - \alpha) \qquad [11]$$

as being more appropriate for VideoDisc machining.

The shear strength τ of the copper substrate material is approximately given by

$$\tau = ½ \times \text{Ultimate Tensile Strength} \times (1 + \text{Poisson's ratio}).$$

For standard VideoDisc electrodeposited copper, Dechert and Trager[6] report an average ultimate strength of 1.08×10^5 psi. Poisson's ratio for copper is approximately 0.34. Thus we obtain a value for the shear strength of

$$\tau = 7.24 \times 10^4 \text{ psi}$$
$$= 5.1 \times 10^6 \text{ gm per cm}^2.$$

The density of copper is 8.9×10^3 kg/m^3. At 225 rpm, the cutting velocity is:

$V = 3.45$ m sec^{-1} at an outside radius of 5.76 inch

$V = 1.52$ m sec^{-1} at an inside radius of 2.546 inch

Table 1 lists the computed values of F_{HT} and F_{VT} for the following

Table 1—Horizontal and Vertical Components of Tool Force F_{HT} and F_{VT} in Grams (Outside Radius: Velocity = 3.45 m/sec; Inside Radius: Velocity = 1.52 m/sec)

Groove Depth D (μm)	F_{HT} and F_{VT} for Different Rake Angles α									
	$\alpha = -2°$		$\alpha = -1°$		$\alpha = 0°$		$\alpha = 1°$		$\alpha = 2°$	
	F_{HT}	F_{VT}	F_{HT}	F_{VT}	F_{HT}	F_{VT}	F_{HT}	F_{VT}	F_{HT}	F_{VT}
Coefficient of Friction $\mu = 0.1$, $\tau = 5 \times 10^6$ gm/cm^2										
1	0.33	0.04	0.32	0.04	0.31	0.03	0.31	0.03	0.30	0.02
2	1.31	0.18	1.28	0.15	1.25	0.13	1.23	0.10	1.20	0.08
3	2.94	0.40	2.89	0.34	2.82	0.28	2.76	0.23	2.70	0.18
4	5.23	0.71	5.12	0.60	5.01	0.50	4.91	0.40	4.81	0.31
Coefficient of Friction $\mu = 0.15$, $\tau = 5 \times 10^6$ gm/cm^2										
1	0.35	0.06	0.34	0.06	0.33	0.05	0.33	0.04	0.32	0.04
2	1.39	0.26	1.36	0.23	1.33	0.20	1.30	0.17	1.27	0.15
3	3.13	0.58	3.06	0.51	2.99	0.45	2.93	0.39	2.88	0.33
4	5.56	1.03	5.44	0.91	5.32	0.80	5.21	0.69	5.10	0.58
Coefficient of Friction $\mu = 0.1$, $\tau = 4 \times 10^6$ gm/cm^2										
1	0.26	0.04	0.26	0.03	0.25	0.03	0.25	0.02	0.24	0.02
2	1.05	0.14	1.02	0.12	1.00	0.10	0.98	0.08	0.96	0.06
3	2.35	0.32	2.30	0.27	2.26	0.23	2.21	0.18	2.16	0.14
4	4.18	0.57	4.09	0.48	4.01	0.40	3.93	0.32	3.85	0.25
Coefficient of Friction $\mu = 0.15$, $\tau = 4 \times 10^6$ gm/cm^2										
1	0.28	0.05	0.27	0.05	0.27	0.04	0.26	0.03	0.25	0.03
2	1.11	0.21	1.09	0.18	1.06	0.16	1.04	0.14	1.02	0.12
3	2.50	0.47	2.45	0.41	2.40	0.04	2.34	0.31	2.29	0.26
4	4.45	0.83	4.35	0.73	4.26	0.64	4.17	0.55	4.08	0.47

range of values of the variables:

Depth of groove $D = 1, 2, 3$ and $4\ \mu m$

Coefficient of friction $\mu = 0.1$ and 0.15

Rake angle $\alpha = -2, -1, 0, 1, 2$ degrees

Yield shear strength $\tau = 5 \times 10^6$ and $4 \times 10^6\ gm/cm^2$

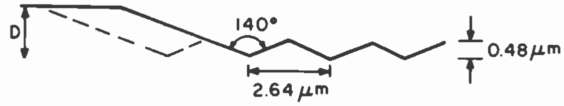
Cutting velocity $V = 3.45$ and $1.52\ m\ sec^{-1}$

Shear angle ϕ as given by Eq. [11]

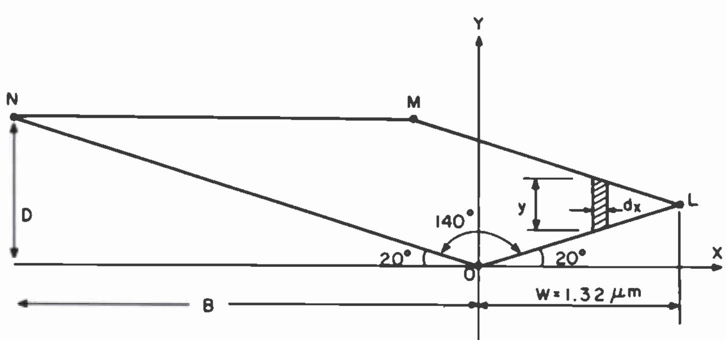
The computed results indicate that the forces F_{HT} and F_{VT} at the outside and inside radii differ only in the third decimal place—i.e., at the milligram level. Thus only one set of values, rounded to 2 decimal places, are tabulated.

5. Normal Recording

Fig. 4a is a sketch of average groove geometry during normal recording. The dotted lines show the cuts made by a diamond tool. The tool force



(a)



(b)

Fig. 4—Normal recording: (a) average groove geometry and (b) components of tool force.

is given by

$$dF = \int_{-B}^{W} fydx, \qquad [12]$$

where y, dx, W, and B, are defined in Fig. 4b, and dF and f are the same as previously defined. The coordinates of points L, M, and N in Fig. 4b are

$$L \equiv [W, W \tan 20]$$

$$M \equiv [2W - D \cot 20, D]$$

$$N \equiv [-D \cot 20, D]$$

Point M will lie on the Y-axis for

$$D = D_0 = 2W \tan 20 \simeq 0.96\ \mu\text{m}.$$

For all depths of cuts larger than D_0, point M will lie in the second quadrant. Under these conditions, y as a function of X is given by

$0 < X < W$	$y = 2 \tan 20[W - X]$
$2W - D \cot 20 < X < 0$	$y = 2W \tan 20$
$-B < X < 2W - D \cot 20$	$y = D + X \tan 20.$

Substituting in the above integral leads to

$$F = f\left\{\int_{0}^{W} ydx + \int_{+2W - D\cot 20}^{0} ydx + \int_{-B}^{2W - D\cot 20} ydx\right\} \qquad [13]$$

$$= \{2WD - W^2 \tan 20\} f.$$

Thus, in the case of normal recording, the horizontal and vertical components of the force exerted by the tool F_{HT} and F_{VT} are given by Eqs. [8] and [9] with the term

$$2WD - W^2 \tan 20 = (2.64D - 2.54)$$

substituted for the term $D^2 \tan 70$.

The numerical values of F_{HT} and F_{VT} given in Table 1 may be used provided that each entry is multiplied by the ratio

$$\frac{2.64D - 2.54}{2.75D^2} \text{ where } D \geq 0.96\ \mu\text{m}.$$

Table 2 lists the values of this ratio for $D = 1, 2, 3$, and $4\ \mu$m.

The above expressions indicate that the tool forces are quadratically related to groove depth in a fast spiral cut and linearly related to the depth of cut for normal recording.

Table 2—Ratio of Forces for Normal and Fast Spiral Recording

$D(\mu m)$	1	2	3	4
$\frac{2.64D - 2.54}{2.75\,D^2}$	0.04	0.25	0.22	0.18

6. Conclusions

(1) The tool forces depend quadratically on groove depth for a fast spiral with a horizontal component that is several grams in magnitude and a vertical component that is several hundred milligrams in magnitude.
(2) The tool forces depend linearly on depth of cut for normal recording with a horizontal component that is on the order of 1 gm or less, and a vertical component that is on the order of several tens of milligrams.
(3) The inertial components of tool force at half real speed are small compared to the cutting components of tool force. Thus the tool forces at this speed are relatively independent of the radius at which cutting is occurring.
(4) At a rake angle equal to the friction angle ($\alpha = \beta$), the vertical component of tool force reduces to zero. If the rake angle is greater than the friction angle ($\alpha > \beta$), an upward directed force is required to keep the tool in a stable position.
(5) Recording at real time doubles the value of cutting velocity. This increases the inertial components as the square of the velocity and results in a measurable difference in tool force at inside and outside radii.

Acknowledgments

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