Conjugate Planes and Zernike Correction for LCoS Optics

A short visual guide: field-conjugate and pupil-conjugate placement of a reflective LCoS, and wavefront correction with Zernike terms

Companion note · October 23, 2026 · D.F. Hakala

> Two planes are **conjugate** when the optics image one onto the other, so each point in one maps to a single point in the other. A microscope has two interleaved families. **Field-conjugate** planes are images of the sample: there, *position maps to position*. **Pupil-conjugate** planes are images of the objective aperture: there, *position maps to angle* at the sample, through a Fourier transform. Where an LCoS sits decides what it can do. Because an LCoS is reflective, each placement also needs a way to separate the light going in from the light coming out: a polarizing beam splitter (PBS) for amplitude mode, a small off-axis angle for phase-only mode.

a) Field-conjugate configuration

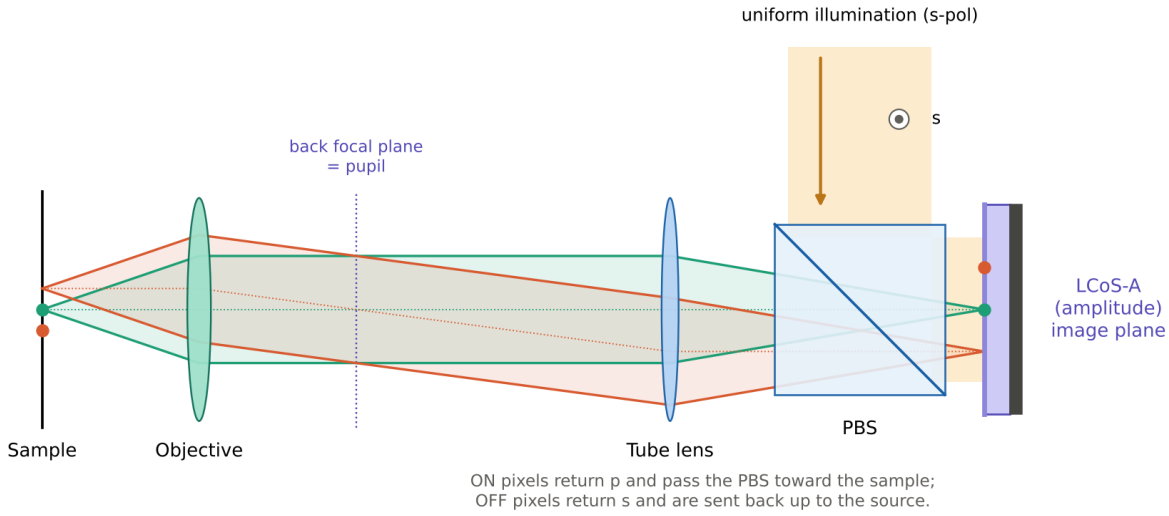


Figure 1. Amplitude LCoS in a field-conjugate (image) plane. s-polarized light enters the PBS from above and its hypotenuse reflects it onto the LCoS. "On" pixels rotate the polarization to p and pass back through the PBS; the tube lens and objective then image each pixel onto its own point on the sample (teal on axis, coral off axis). Dotted lines are chief rays, which cross the axis at the back focal plane (the pupil).

Each pixel on the LCoS maps to one point on the sample, scaled by the ratio of objective to tube-lens focal lengths. A pattern displayed here therefore acts **locally**: it affects only the sample points that its pixels image to.

- **How in and out are separated:** the LCoS rotates polarization pixel by pixel, so a PBS cube routes "on" light (p) toward the sample and sends "off" light (s) back toward the source, exactly as in an LCoS projector.
- **Good for:** programmable pinhole or spot patterns, structured illumination, illumination masks. Cameras, physical pinholes and field stops also sit in field-conjugate planes.
- **Not good for:** focus control or aberration correction. A pattern here changes only the points beneath it and cannot shift or sharpen the focus across the whole field.

- **Ray signature:** rays from one object point re-converge to one image point (the marginal ray crosses the axis).

b) Pupil-conjugate configuration

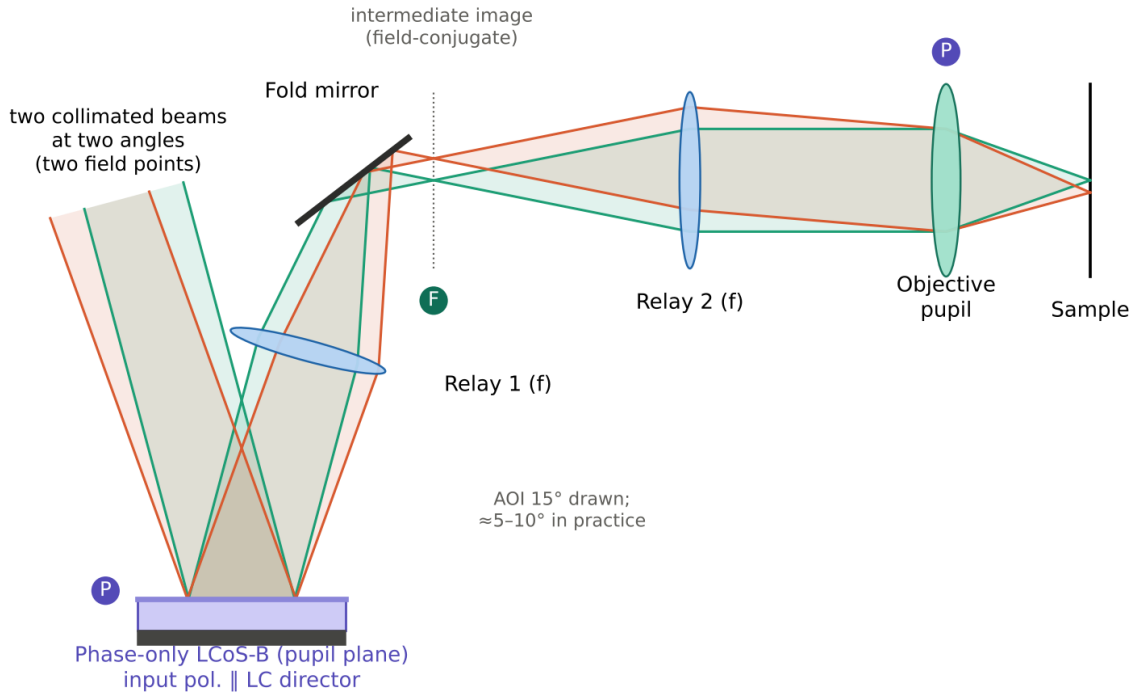


Figure 2. Phase-only LCoS in a pupil-conjugate plane, used at a small angle of incidence (drawn at 15° for clarity). Two collimated beams at slightly different angles, representing two field points, overlap fully on the LCoS. A 4f relay (two lenses of focal length f) images the LCoS onto the objective pupil; a fold mirror turns the beam. The objective then focuses the two beams to two different points on the sample. The intermediate image between the relay lenses is field-conjugate.

Every field point's light passes through the **whole** pupil, each at its own angle. A phase pattern displayed here is therefore applied to every point in the field at once. Because the pupil and sample planes are related by a Fourier transform:

- a **linear phase ramp (tilt)** shifts the focus sideways,
- a **quadratic phase (defocus)** shifts the focus axially, which is the digital varifocal lens,
- a **computed hologram** creates a whole array of foci, as in tweezer arrays and multi-spot confocal systems,
- **higher-order Zernike terms** correct aberrations everywhere in the field.

How in and out are separated: a parallel-aligned phase LCoS does not rotate polarization, so a PBS cannot split the input from the output. The device is used at a small angle of incidence (typically 5–10°), as drawn, or behind a non-polarizing beam splitter at the cost of light. The input polarization must be aligned with the LC director. The small angle adds a little astigmatism, which the Zernike terms on the same device remove.

The intermediate image between the relay lenses is a convenient place for a field stop that blocks the SLM zero order when a tilt carrier is used.

Side-by-side comparison

	Field-conjugate plane	Pupil-conjugate plane
Is an image of	The sample	The objective's aperture stop
Position on the LCoS maps to	Position on the sample	Angle at the sample
Effect of a pattern	Local: only the points it covers	Global: every field point
Typical LCoS mode	Amplitude (polarization rotation)	Phase-only (parallel-aligned)
Separating in from out	PBS cube	Small off-axis angle, or NPBS
Uses	Pinhole or spot patterns, masks, field stops, cameras	Focus, steering, holograms, aberration correction
Ray signature	Rays from one point re-converge	Chief rays from all points cross the axis

c) Using Zernike terms to correct and improve image quality

Zernike polynomials are an orthogonal set of shapes defined on the unit disk. Any smooth wavefront error over a circular pupil can be written as a weighted sum of them. Each term has a radial order n and an azimuthal frequency m:

$$Z_n^m(\rho,\theta) = R_n^{|m|}(\rho)\cdot\begin{cases}\cos(m\theta) & m \geq 0\\ \sin(|m|\theta) & m < 0\end{cases} \tag{1}$$

$$W(\rho,\theta) = \sum_j a_j\ Z_j(\rho,\theta), \qquad \sigma_{\text{rms}} = \sqrt{\sum_{j\geq 2} a_j^2} \tag{2}$$

Here ρ is the normalized pupil radius (0 at center, 1 at edge), θ is the azimuth, and a_j is the amount of each term in waves. With orthonormal (Noll) normalization, the RMS wavefront error σ_{rms} is the root-sum-square of the coefficients, excluding piston (j = 1).

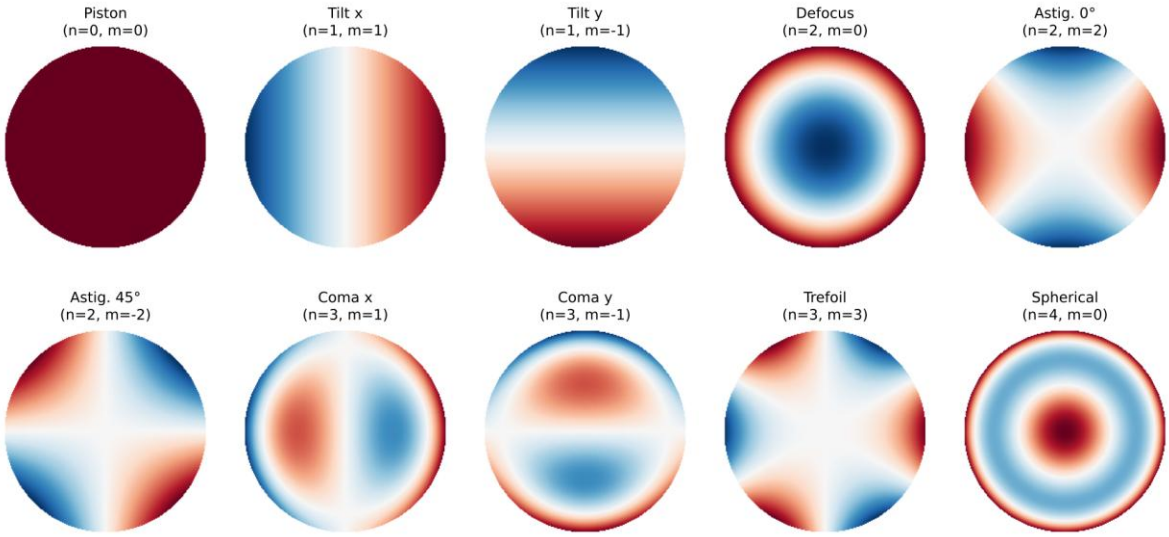


Figure 3. The first ten Zernike terms as phase maps over the pupil (red = phase advanced, blue = retarded). Each panel is normalized to its own peak.

Three properties make Zernikes the working basis for LCoS correction:

- **Orthogonality.** Coefficients can be measured and adjusted independently.

- **Physical meaning.** The low orders are the familiar aberrations, and each has a recognizable effect on the focal spot (table below).
- **Direct use on the LCoS.** The pupil-plane LCoS displays the conjugate of the measured error, wrapped modulo 2π, so the two cancel:

$$\varphi_{\mathrm{SLM}}(\rho,\theta) = -2\pi \sum_j a_j \; Z_j(\rho,\theta) \quad (\mathrm{mod}\ 2\pi) \tag{3}$$

Wrapping. The sum in Equation 3 can span many waves, but the LCoS has only about 2π of phase stroke. It is therefore displayed modulo 2π ("wrapped"): each time the phase reaches 2π it drops back to 0. At the design wavelength this has the same optical effect as the full profile. See the Glossary.

Term	Effect on the focus	Typical source
Tilt (n=1)	Lateral shift; no blur	Misalignment; used deliberately for steering
Defocus (2,0)	Axial shift	Used deliberately for digital z scanning
Astigmatism (2,±2)	Two line foci at different z; spot elongates	Off-axis LCoS angle, tilted plates, SLM and backplane warp
Coma (3,±1)	Comet-shaped spot; biased peak position	Off-axis use, decentered or tilted elements
Trefoil (3,±3)	Three-lobed spot	Mount stress, SLM flatness
Spherical (4,0)	Spread along z; axial response broadens and becomes asymmetric	Cover glass or index mismatch, large digital defocus at high NA

What correction does

The peak intensity of the focal spot relative to a perfect one (the Strehl ratio S) falls quickly with RMS wavefront error. For small errors the Maréchal approximation gives:

$$S \approx \exp[-(2\pi\,\sigma_{\mathrm{rms}})^2] \tag{4}$$

The bottom row of Figure 4 shows the **point spread function (PSF)** for each case: the image that a single point of light forms after passing through the optics. In a focusing system it is simply the shape of the focal spot. The Strehl ratio quoted above each panel is that spot's peak height divided by the peak of the ideal, diffraction-limited PSF.

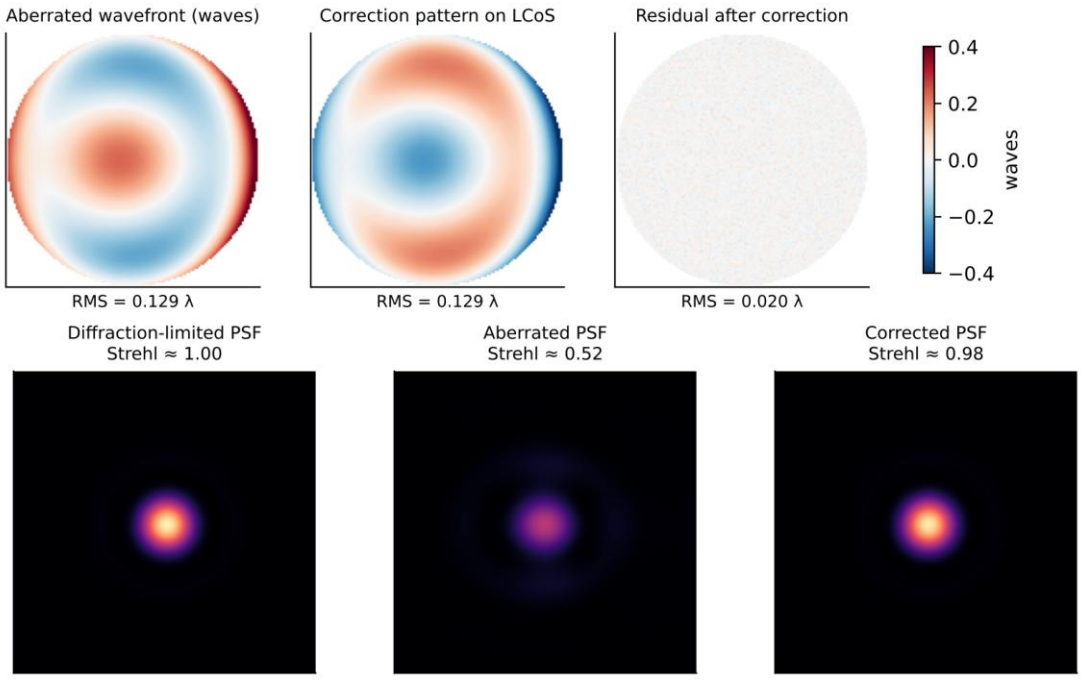


Figure 4. Simulation with 0.13 λ RMS of mixed spherical aberration, astigmatism and coma. The LCoS displays the inverse pattern; the remaining 0.02 λ RMS stands for calibration noise. The Strehl ratio rises from 0.52 to 0.98, consistent with Equation 4.

In a reflection confocal system the wavefront error is applied **twice**, once on the way in and once on the way out. The penalty on the axial response is therefore much larger than the single-pass Strehl ratio suggests (Figure 5). An aberration that only halves the peak of a single focal spot can collapse and skew the confocal peak, biasing every height measurement. Correction restores both the height and the symmetry of the peak.

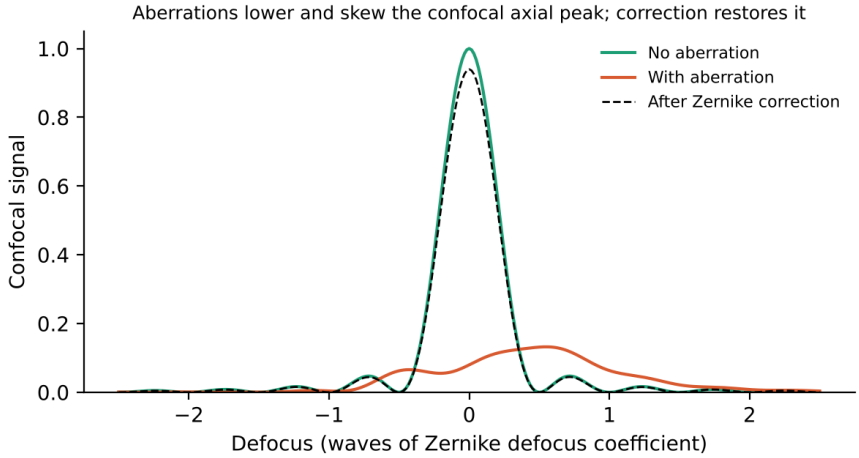


Figure 5. Confocal axial response (double pass) for the same aberration as Figure 4: ideal, aberrated and corrected.

How the coefficients are found

1. **Measure the LCoS itself first.** Use an interferometer or a Shack–Hartmann sensor to measure the device's flatness map, fit it to Zernikes, and store it as a permanent offset.
2. **Sensorless modal optimization.** For each mode j, display +b and −b, record a quality metric (peak confocal signal or spot sharpness), fit a parabola, and set a_j at its vertex. This takes about 2N + 1 frames for N modes. Iterate once or twice, because modes interact through the metric.

3. **Model-based terms.** For known causes, such as a cover layer of known thickness and index, or the off-axis angle on the LCoS, compute the aberration directly and add it.
4. **Combine everything in one pattern:** lens + tilt + hologram + $\Sigma\, a_j Z_j$ − flatness, wrapped modulo 2π and passed through the wavelength-specific look-up table.

> **Conventions vary.** Noll and OSA/ANSI orderings number the terms differently and use different normalizations. Check which convention a paper, wavefront sensor or SLM software uses before reusing coefficients. Also make sure the unit circle is mapped to the actual illuminated pupil on the LCoS, not the full chip.

Glossary

Point spread function (PSF). The image of an ideal point source formed by an optical system; in a focusing system, the 3D shape of the focal spot. Without aberrations it is the diffraction-limited Airy pattern: a central peak (about 1.22 λ/NA from the center to the first dark ring) surrounded by faint rings. Aberrations move energy out of the peak into a broader, distorted spot. In a confocal system the PSF matters twice: once for the illumination spot and once for the return light focusing on the pinhole.

Airy pattern. The diffraction-limited PSF of a circular aperture. Its central lobe (the Airy disk) sets the lateral resolution, and its size is the reference for pinhole sizes quoted in Airy units.

Strehl ratio (S). The peak intensity of the actual PSF divided by the peak of the ideal, diffraction-limited PSF. S = 1 is perfect; S ≥ 0.8 is the usual criterion for "diffraction-limited." For small errors $S \approx \exp[-(2\pi\,\sigma_{rms})^2]$ (Equation 4).

Phase wrapping. Representing a phase profile modulo 2π: whenever the phase reaches 2π it is reset to 0. Light cannot tell ϕ from $\phi + 2\pi$, so at the design wavelength a wrapped profile focuses or corrects exactly like the full one. This lets an LCoS with only about 2π of stroke display a lens or Zernike correction spanning many waves; a wrapped lens becomes the concentric rings of a Fresnel lens. The 2π resets must be at least about 2 pixels apart, which limits the strongest lens, and fringing fields soften them, which lowers efficiency. Wrapping also makes the pattern wavelength-specific, because a 2π step at one wavelength is not 2π at another.

Phase unwrapping. The reverse operation, applied to measured phase. Interferometers, wavefront sensors and LCoS phase calibrations report phase only modulo 2π, as a sawtooth map. Unwrapping finds the jumps of about 2π between neighboring pixels and adds or subtracts 2π to rebuild the continuous surface. Only then can the data be fitted with Zernike terms or turned into a flatness correction. It fails where the data are noisy or the true phase changes by more than π between samples, so robust algorithms (quality-guided, path-following or least-squares) are used in practice.

Modulo 2π (mod 2π). The remainder after removing whole multiples of 2π, so every value falls between 0 and 2π. This is the operation that performs phase wrapping.

Further reading: R. J. Noll, J. Opt. Soc. Am. 66, 207–211 (1976); M. J. Booth, Light Sci. Appl. 3, e165 (2014). Figures 4 and 5 were computed for this note; Figures 1–2 are schematic layouts (not to scale).