In[ ]:=

Magnetic Dipole Plots of Planets

The field around a theoretical magnetic dipole is a close approximation for what occurs around the planets.

The following is an equation that provides the vector components of the magnetic field of a dipole of magnetic moment m = 1 (T-M^3) as a function of x,y,and z

In[101]:=

In[102]:=

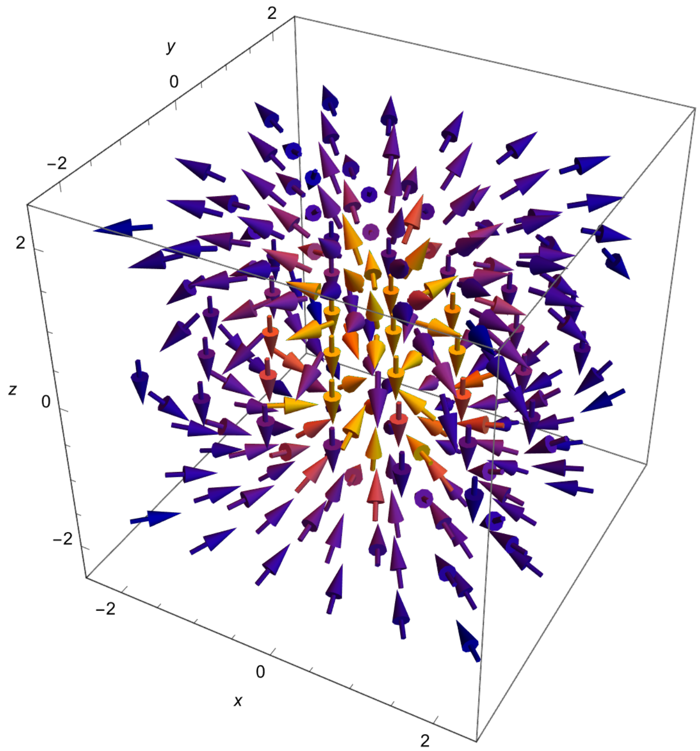
```
dipB[x_, y_, z_] := (mu0 * m / 4 * Pi) * {3 * x * z / ((x^2 + y^2 + z^2) ^ (5 / 2)),
    3 * y * z / ((x^2 + y^2 + z^2) ^ (5 / 2)), (2 * z^2 - x^2 - y^2) / ((x^2 + y^2 + z^2) ^ (5 / 2))};

mu0 = 4 * Pi * 10^-7;
m = 1; (* Note: m for the Earth is 7.91*10^15 T-M^3  *)
```

The following is a 3D plot of the vector field, using the equation above. You can see the field leaving from the +z direction and going to the -z direction (on a planet ....North pole and South pole).

In[105]:=

```
vector = VectorPlot3D[dipB[x, y, z],
  {x, -2, 2}, {y, -2, 2}, {z, -2, 2}, AxesLabel → {x, y, z}]
```

Out[105]=

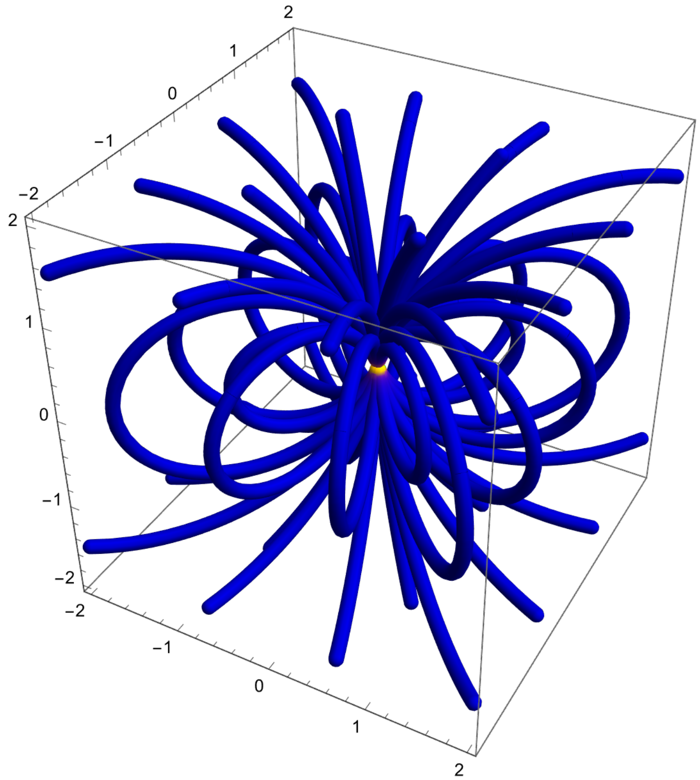


This plot is called a Stream plot as it shows the flow of the magnetic field lines.

In[106]:=

```
stream =
 StreamPlot3D[dipB[x, y, z], {x, -2, 2}, {y, -2, 2}, {z, -2, 2}, StreamMarkers → "Tube"]
```

Out[106]=



Now to better illustrate how the magnetic fields flow around the planet Earth , we create a spherical graphics object.

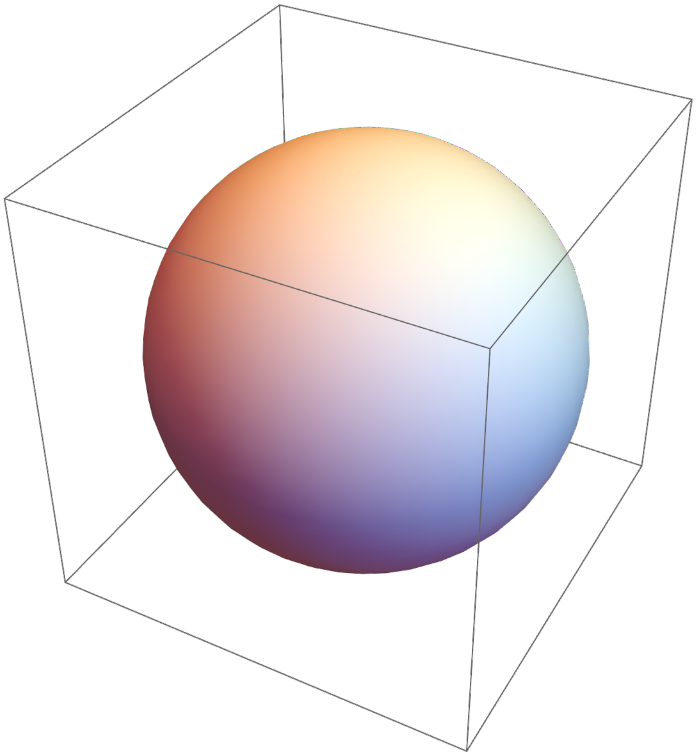
Out[•]=

```
Ball[{0, 0, 0}]
```

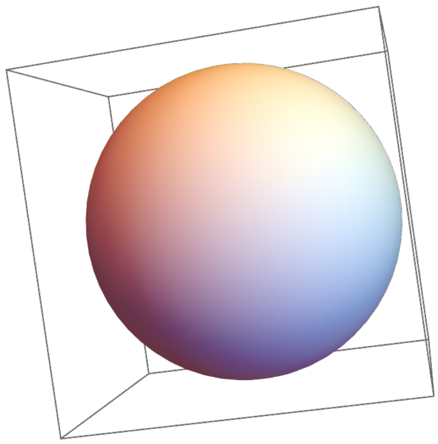
In[107]:=

```
ball = Graphics3D[Ball[]]
```

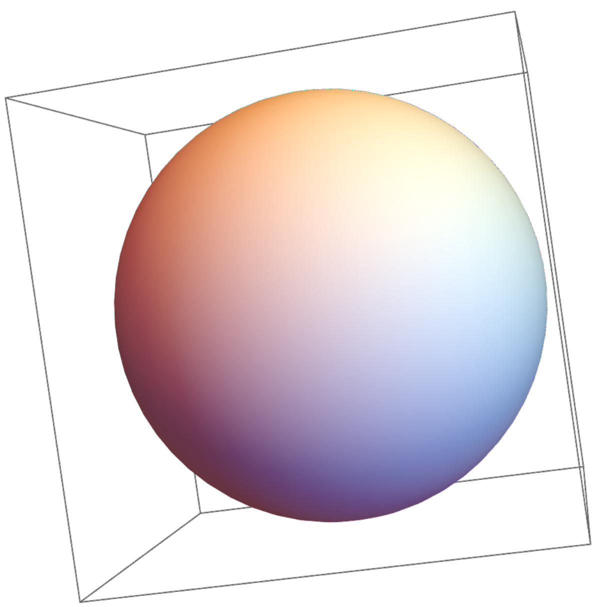
Out[107]=



In[108]:=



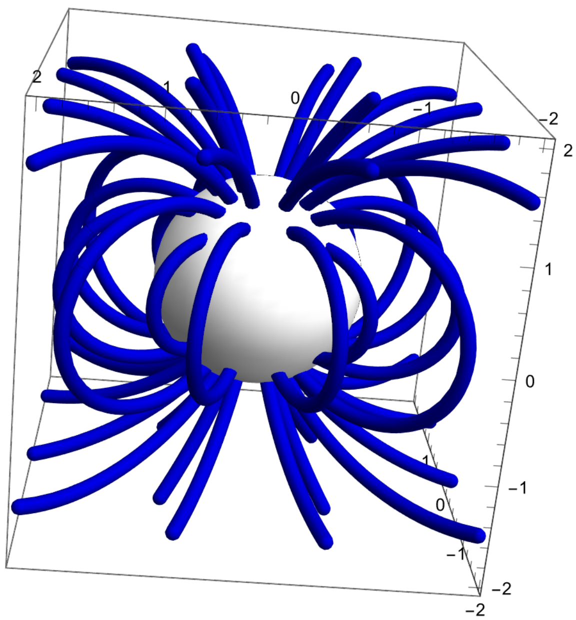
Out[108]=



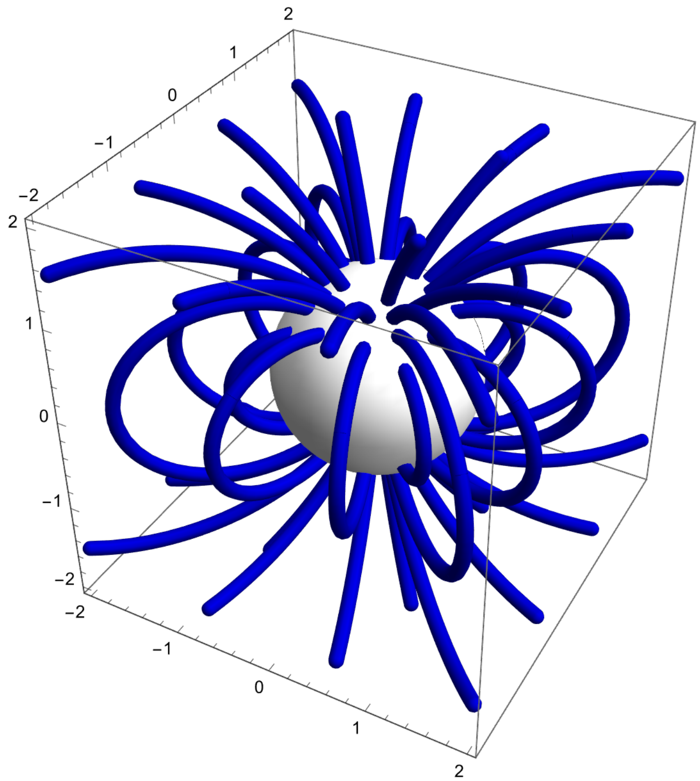
Now we combine the stream plot and the vector plot of the magnetic field lines with the spherical graphics object to render simulations of the Earths magnetic field . In this case the distance s are in units of the Earths radius .

In[109]:=

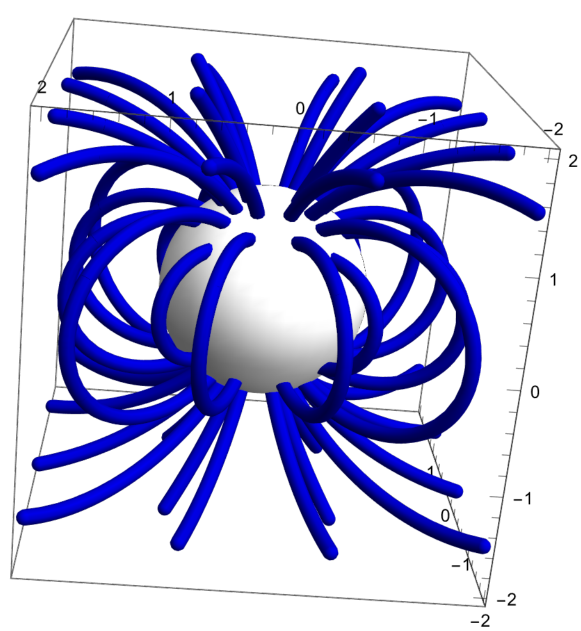
```
Show[stream, ball]
```



Out[109]=



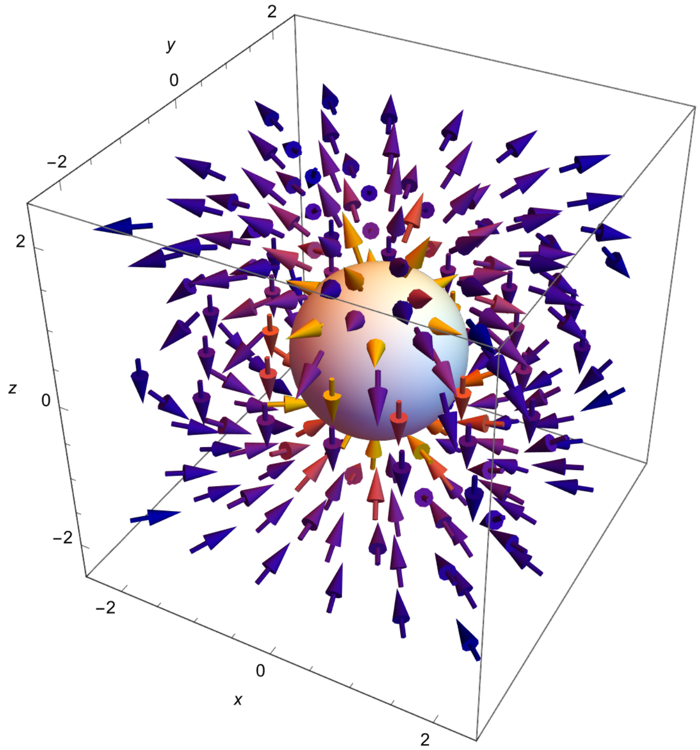
Out[110]=



In[111]:=

```
Show[vector, ball]
```

Out[111]=

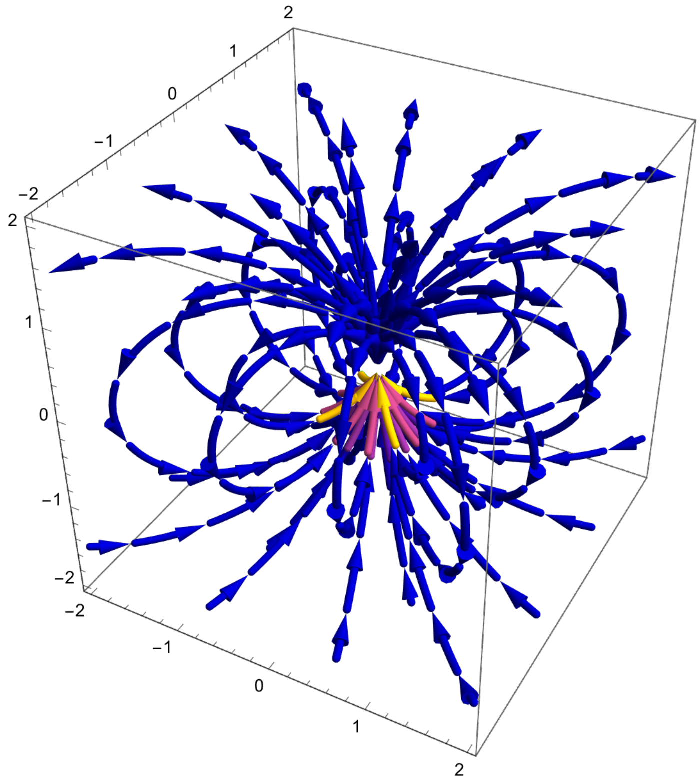


Another plot is a Stream plot where the magnetic flux lines are not simply represented as flux tubes but alos show the field direction with arrows . This is sort of a combination of the Stream and Vector plots.

In[112]:=

```
stream1 = StreamPlot3D[dipB[x, y, z],
  {x, -2, 2}, {y, -2, 2}, {z, -2, 2}, StreamMarkers → "Arrow3D"]
```

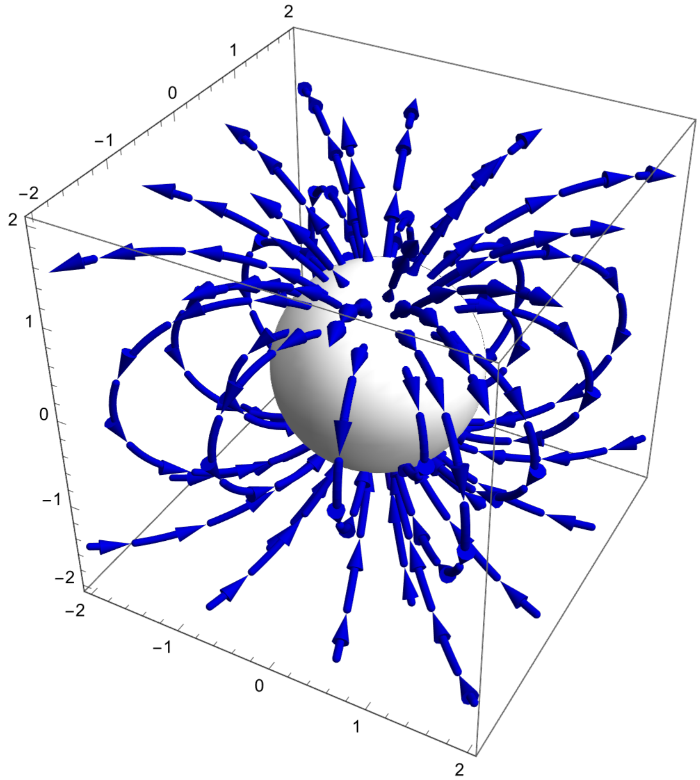
Out[112]=



In[113]:=

```
Show[stream1, ball]
```

Out[113]=



Here' s a list of the magnetic moments of planets in our solar system, from strongest to weakest :

Jupiter : 1.55*10^20 T·m^3
Saturn : 4.6*10^18 T·m^3
Uranus : 3.9*10^17 T·m^3
Neptune : 2.2*10^17 T·m^3
Earth : 7.91*10^15 T·m^3
Mercury : 4*10^12 T·m^3
Mars : ~1.5*10^12 T·m^3 (estimated based on current weak field)
Venus : No significant global magnetic field

Key takeaways : The magnetic moment is measured in tesla - cubic meters (T·m^3) .
Jupiter has the strongest magnetic field by far, followed by Saturn .
Earth' s magnetic field is substantial and crucial for life .
The inner rocky planets (Mercury, Venus, Mars) have weak or no global magnetic fields .

Strong magnetic fields are associated with liquid states of conducting material inside the planet. In the case of Earth this is a fluid iron core, and in the case of the gas giants this is probably a core containing a state of hydrogen that is a liquid metal.

The conductive fluid core can have motion and flow in it, which causes a dynamo effect generating the planetary magnetic field.

We can redo the Earth example in the actual numbers rather than reduced numbers.

Earth magnetic moment, m = 7.91*10^15 T-M^3
mu0 = 4*Pi*10^-7 N-A^-2
Earth radius = 6.37*10^6 M

In[114]:=

```
mu0 = 4 * Pi * 10^-7;
m = 7.91 * 10^15;
R = 6.37 * 10^6;
```

In[117]:=

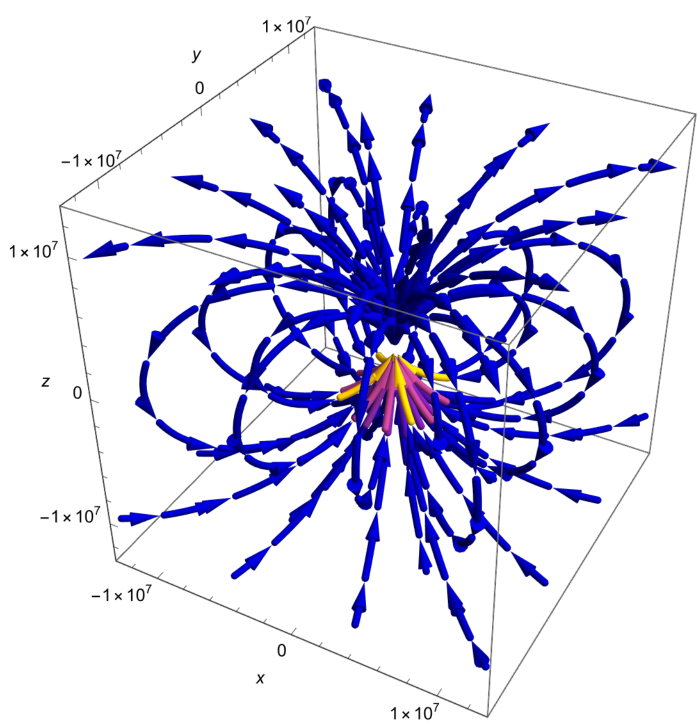
In[118]:=

```
dipB[x_, y_, z_] := (mu0 * m / 4 * Pi) *
   {3 * x * z / ((x^2 + y^2 + z^2) ^ (5 / 2)), 3 * y * z / ((x^2 + y^2 + z^2) ^ (5 / 2)),
    + (2 * z^2 - x^2 - y^2) / ((x^2 + y^2 + z^2) ^ (5 / 2))};
```

In[119]:=

```
stream2 = StreamPlot3D[dipB[x, y, z], {x, -2 * R, 2 * R}, {y, -2 * R, 2 * R},
  {z, -2 * R, 2 * R}, StreamMarkers → "Arrow3D", AxesLabel → {x, y, z}]
```

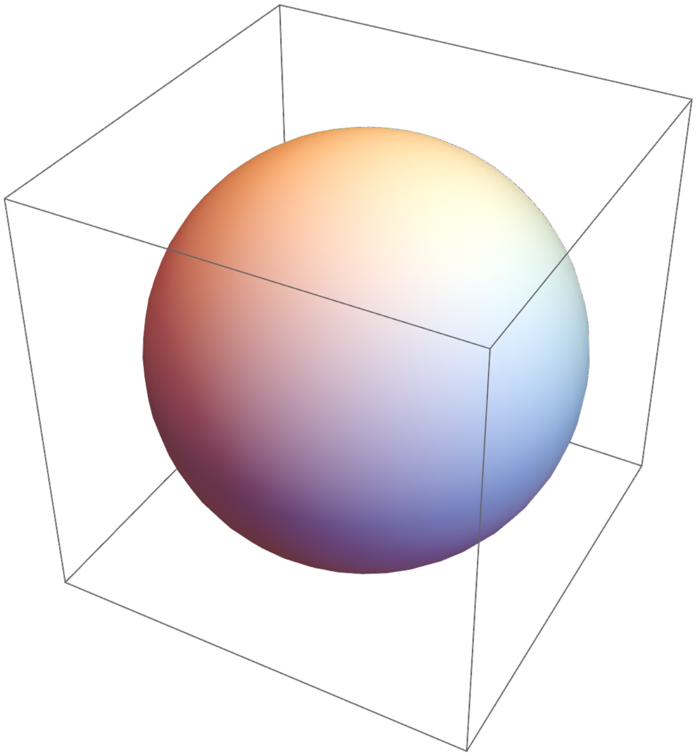
Out[119]=



In[120]:=

```
ball1 = Graphics3D[Ball[{0, 0, 0}, R]]
```

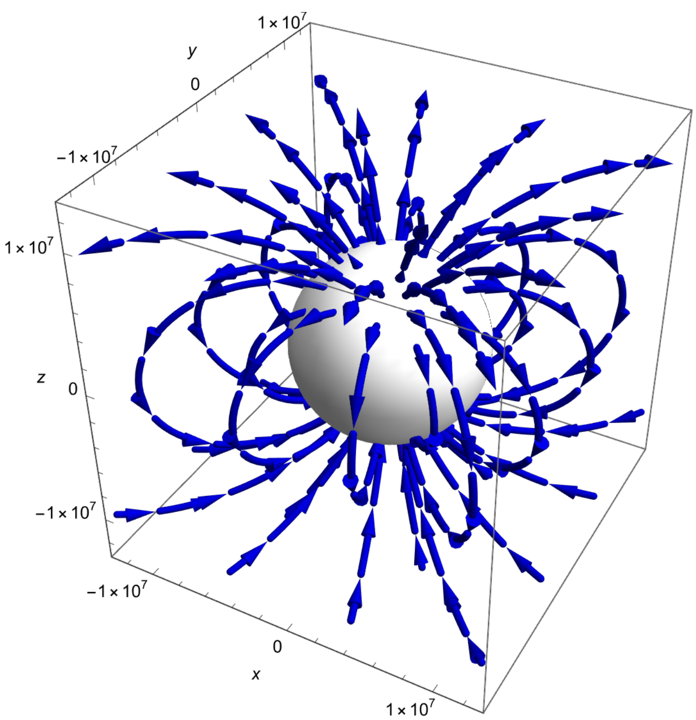
Out[120]=



In[121]:=

```
Show[stream2, ball1]
```

Out[121]=

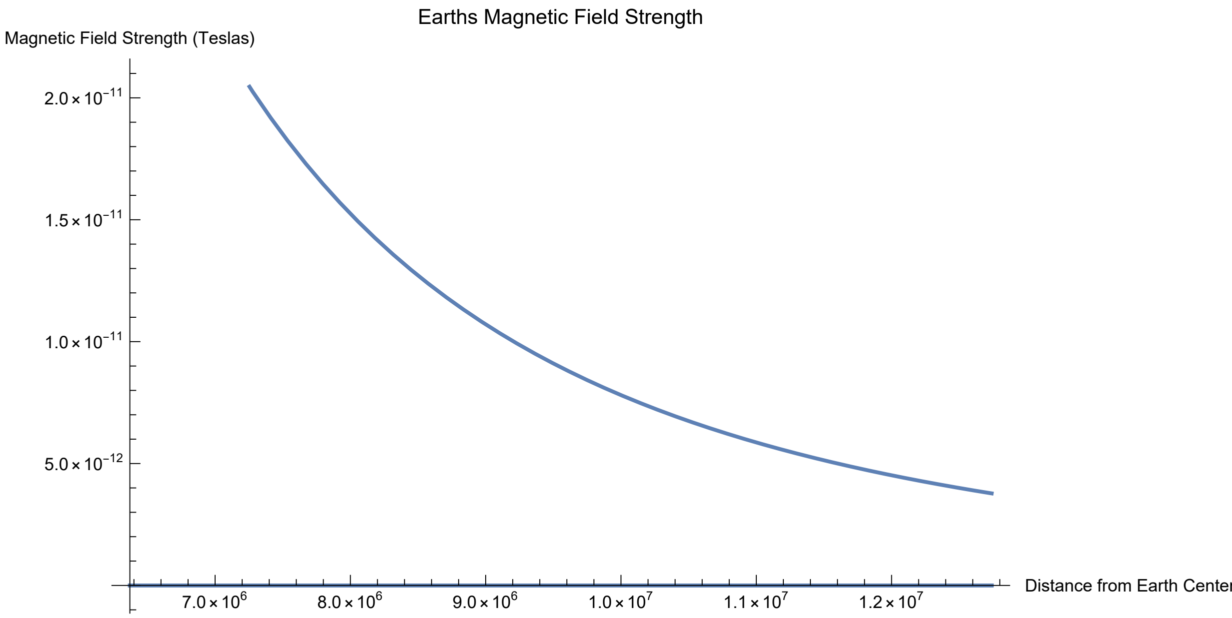


So we now have the distance scales proper for the Earths magnetic field . Lets look at the field strength as a function of distance from the Earths surface at the Equator and the North Pole.

In[122]:=

```
eqPlot = Plot[Abs[dipB[x, 0, 0]], {x, R, 2 * R}, AxesLabel →
   {"Distance from Earth Center (meters)", "Magnetic Field Strength (Teslas)"},
  PlotLabel → "Earths Magnetic Field Strength"]
```

Out[122]=

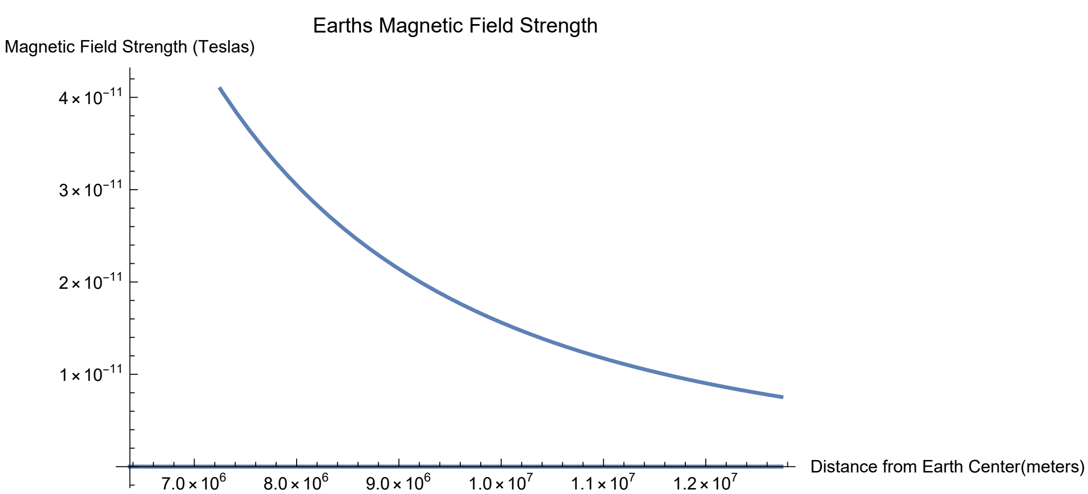


In[123]:=

In[124]:=

```
polePlot = Plot[Abs[dipB[0, 0, z]], {z, R, 2 * R}, AxesLabel →
   {"Distance from Earth Center (meters)", "Magnetic Field Strength (Teslas)"},
  PlotLabel → "Earths Magnetic Field Strength"]
```

Out[124]=



In[125]:=

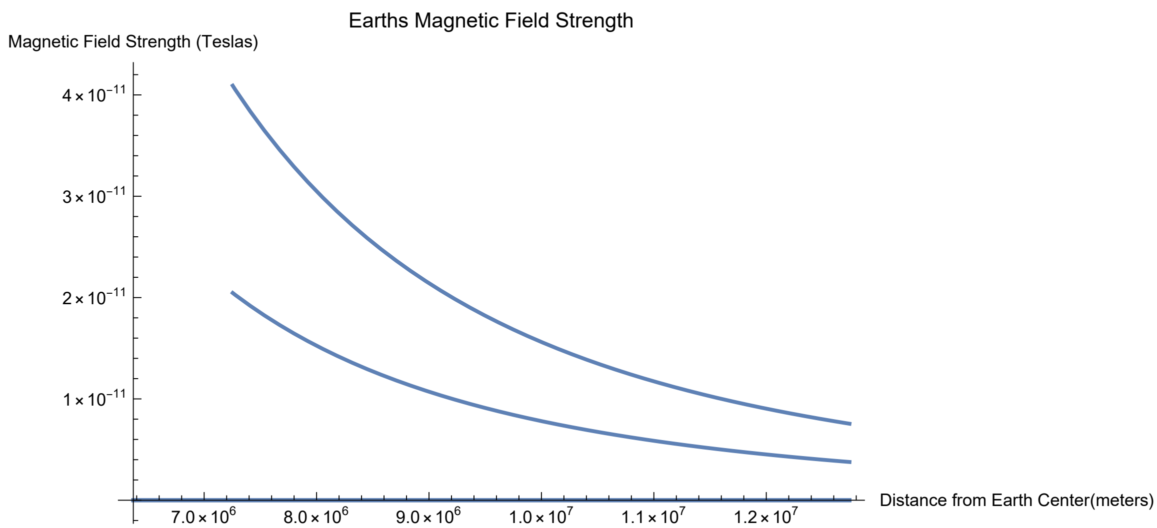
Now to compare the magnetic field strength outward from the North Pole to that going outward from the Equator .

In[126]:=

In[127]:=

```
Show[polePlot, eqPlot]
```

Out[127]=



The polar field is twice as strong as the equatorial field . Since the field (in spherical coordinates) is proportional to Sqrt[1+3*Cos[theta]], and theta(NorthPole)=0, while theta(equator)=Pi/2, the ratio should be 2:1. We can also see that the field strength is reduced