The Noetherian Framework for Open Systems: Towards a Conservation of Information Law under Dynamic Symmetry

Abstract: Dynamic Symmetry in open systems requires a conservation principle suited to systems that persist through exchange, dissipation and feedback rather than through mechanical isolation. This paper develops such a principle in information-theoretic form. Starting from the stochastic dynamics introduced earlier in the series, it defines an Onsager–Machlup-type informational action for trajectories evolving under both diffusion and macroscopic regulation. Continuous transformations of the microscopic states and macroscopic control variables are then used to derive a Noether-style informational current. After ensemble averaging, this current leads to a balance law in which the Dynamic Symmetry Index and boundary dissipation sum to an overall informational capacity. The result provides a variational framework in which conservation in open adaptive systems is expressed not as preservation of energy or momentum, but as preservation of a structured balance between adaptive organisation and dissipative expenditure.

The preceding papers developed Dynamic Symmetry in three stages. The first introduced a variational description of the competition between structure and fluctuation in information-theoretic networks. The second formulated that competition dynamically through stochastic evolution and a time-dependent Dynamic Symmetry Index. The third extended the same balance to coarse-grained descriptions and renormalisation flow across scale. These steps establish a theory of organised fluctuation. The remaining question is whether that theory also admits a conservation principle appropriate to open systems.

In closed classical mechanics, Noether's theorem links continuous symmetries of the action to conserved quantities such as energy, momentum and angular momentum. Open adaptive systems require a different formulation. Biological, ecological and institutional networks do not persist by isolating themselves from their surroundings. They persist by exchanging matter, energy and entropy with the environment while maintaining structural integrity through feedback and dissipation. The relevant invariant cannot therefore be expected to coincide with the conserved quantities of conservative mechanics. It must instead be formulated in terms appropriate to open stochastic organisation.

The natural starting point is the stochastic system introduced in the earlier time-dependent formulation. Let $\mathbf{X}(t)$ denote the microscopic state of the system and $\boldsymbol{\Lambda}(t)$ the macroscopic control variables. The microscopic dynamics are governed by the diffusion equation

$$d\mathbf{X}(t) = -\nabla_{\mathbf{X}} V(\mathbf{X}(t); \mathbf{\Lambda}(t))\, dt + \sqrt{2D}\, d\mathbf{W}(t),$$

where $V(\mathbf{X}(t); \mathbf{\Lambda}(t))$ is the effective potential, D is the diffusion strength and $\mathbf{W}(t)$ is a Wiener process. The macroscopic variables encode feedback acting on the system in response to deviations from functional targets. Together these quantities define a stochastic adaptive process in which microscopic fluctuation and macroscopic constraint remain continuously coupled.

To formulate an analogue of the classical action principle, the trajectory of the system is placed in an informational action space. The probability of a microscopic path over a finite interval $[0, T]$ is written in Onsager–Machlup form,

$$P[\mathbf{X}(t)] \propto \exp(-\mathcal{S}_{OM}[\mathbf{X}]),$$

with informational action

$$\mathcal{S}_{OM}[\mathbf{X}] = \int_0^T \mathcal{L}_{\text{info}}(\mathbf{X}, \dot{\mathbf{X}}, \mathbf{\Lambda})\, dt.$$

The corresponding informational Lagrangian is taken to be

$$\mathcal{L}_{\text{info}}(\mathbf{X}, \dot{\mathbf{X}}, \mathbf{\Lambda}) = \frac{1}{4D} \sum_{i=1}^{N} \left| \dot{x}_i + \frac{\partial V(\mathbf{X}; \mathbf{\Lambda})}{\partial x_i} \right|^2 + \frac{1}{2} \sum_{i=1}^{N} \frac{\partial^2 V(\mathbf{X}; \mathbf{\Lambda})}{\partial x_i^2}.$$

The first term measures the pathwise deviation from deterministic drift and therefore encodes the thermodynamic cost of stochastic fluctuation. The second term measures the divergence of the drift field and captures a local information-compression contribution generated by the macroscopic constraints. In this form, the Lagrangian expresses the joint informational and thermodynamic cost of sustaining an open system under continuous fluctuation.

A Noether-style derivation begins by introducing a continuous infinitesimal transformation of both the microscopic coordinates and the macroscopic control variables. Let

$$x_i(t) \to x_i'(t) = x_i(t) + \epsilon\, \psi_i(\mathbf{X}, t),$$

and

$$\mathbf{\Lambda}(t) \to \mathbf{\Lambda}'(t) = \mathbf{\Lambda}(t) + \epsilon\, \mathbf{\Phi}(\mathbf{\Lambda}, t),$$

where ϵ is an infinitesimal scalar parameter and ψ_i, $\mathbf{\Phi}$ are the generators of the transformation. Dynamic Symmetry is associated with invariance of the informational action under this continuous transformation. The defining condition is

$$\delta \mathcal{S}_{OM} = \mathcal{S}_{OM}[\mathbf{X}', \mathbf{\Lambda}'] - \mathcal{S}_{OM}[\mathbf{X}, \mathbf{\Lambda}] = 0.$$

This invariance is the open-system counterpart of symmetry in the present framework. It expresses the persistence of the informational balance underlying the adaptive dynamics rather than invariance of a closed mechanical quantity.

The variation of the informational action is then computed by standard variational methods. Along admissible trajectories satisfying the corresponding non-equilibrium Euler–Lagrange equations, the bulk variation collapses to a total derivative. Writing the first-order variation gives

$$\delta \mathcal{S}_{OM} = \int_0^T \sum_{i=1}^{N} \left[\frac{\partial \mathcal{L}_{\text{info}}}{\partial x_i} \delta x_i + \frac{\partial \mathcal{L}_{\text{info}}}{\partial \dot{x}_i} \delta \dot{x}_i \right] dt,$$

and integrating the second term by parts yields

$$\int_0^T \frac{d}{dt} \left[\sum_{i=1}^{N} \frac{\partial \mathcal{L}_{\text{info}}}{\partial \dot{x}_i} \psi_i(\mathbf{X}, t) \right] dt = 0.$$

The quantity inside the derivative is therefore constant along the continuous evolution of the system and defines the Noetherian informational current,

$$\mathcal{J}_{DS} = \sum_{i=1}^{N} \frac{\partial \mathcal{L}_{\text{info}}}{\partial \dot{x}_i} \psi_i(\mathbf{X}, t) = \frac{1}{2D} \sum_{i=1}^{N} \left(\dot{x}_i + \frac{\partial V(\mathbf{X}; \mathbf{\Lambda})}{\partial x_i} \right) \psi_i(\mathbf{X}, t).$$

This current is the informational analogue of a conserved quantity generated by symmetry. It depends on both the stochastic path and the transformation generator, and it expresses the continuing balance between fluctuation and constraint encoded in the action.

To obtain an operational law from this current, the ensemble average is taken with respect to a non-equilibrium stationary distribution $P_S(\mathbf{X}, \mathbf{\Lambda})$. This links the current to the information-theoretic quantities already introduced in the earlier papers. The resulting balance relation is

$$\frac{d}{dt} \langle \mathcal{J}_{DS} \rangle = 0 \implies \frac{d}{dt} (\text{DSI}(t) + \mathcal{S}_{\text{bound}}(t)) = 0,$$

or equivalently,

$$\text{DSI}(t) + \mathcal{S}_{\text{bound}}(t) = \mathcal{I}_{\text{total}}.$$

This is the conservation law appropriate to the present theory. The Dynamic Symmetry Index measures the active informational organisation generated within the system through structured fluctuation. The term $\mathcal{S}_{\text{bound}}(t)$ measures the accumulated boundary dissipation required for the macroscopic feedback structure to maintain the integrity of the system in the face of external disturbance. The total quantity $\mathcal{I}_{\text{total}}$ represents the overall informational capacity available to the system within its admissible regime.

The significance of this law lies in the way it reformulates conservation for open adaptive systems. The system does not conserve energy or momentum in the classical sense, because it is not closed. It conserves an informational budget distributed between active organisation and dissipative work. Whenever environmental disturbance increases the burden carried by the system boundary, the internal dynamics must respond by reorganising the relation between microscopic fluctuation and macroscopic control. Structural resilience is preserved when that redistribution remains possible. Failure occurs when it does not.

This formulation clarifies the meaning of breakdown in a dynamically symmetric system. A rise in external volatility increases the dissipation term $\mathcal{S}_{\text{bound}}(t)$. To remain viable, the system must convert this disturbance into regulated adaptive structure, which appears as a compensating adjustment in the Dynamic Symmetry Index. If the feedback architecture is too weak or too slow to achieve that conversion, the informational balance can no longer be maintained. Structural collapse then appears not as a mysterious loss of order, but as a failure of the system to preserve its informational budget under perturbation.

The conceptual shift introduced here is substantial. In classical mechanics, symmetry preserves a physical quantity associated with invariance in coordinate space. In open adaptive systems, symmetry preserves an informational relation between fluctuation, organisation and dissipation. Conservation therefore no longer refers to the persistence of a substance but to the persistence of a regulated balance. This is the appropriate analogue for systems whose survival depends on continual exchange with their surroundings.

The framework also integrates the earlier stages of the theory. The static variational description supplied a state space in which order and disorder could be measured together. The stochastic description supplied a time-dependent index defined on evolving probability distributions. The multiscale description supplied the conditions under which that balance may persist across levels of observation. The Noetherian formulation brings these elements together by identifying the quantity preserved when a dynamically symmetric open system remains viable through time.

Several technical extensions arise naturally from this result. The action functional and the transformation class must be specified in explicit model families. The properties of the stationary distribution must be analysed in relation to the stochastic dynamics. The handling of the Onsager–Machlup functional requires care in diffusion processes with non-trivial geometry or multiplicative noise. Numerical studies are also needed to determine whether perturbations of explicit models redistribute DSI and boundary dissipation in the manner predicted by the conservation law. These developments belong to the continued elaboration of the theory.

The present paper establishes the Noetherian structure appropriate to Dynamic Symmetry in open systems. It defines an informational action for stochastic adaptive trajectories, derives a Noether-style informational current from continuous invariance, and formulates a conservation law in which the Dynamic Symmetry Index and boundary dissipation sum to a total informational capacity. Conservation in open systems is thereby expressed as the preservation of organised fluctuation under feedback and dissipation, completing the transition from static structure to adaptive dynamics, to multiscale persistence, and finally to an operational informational law.