The Dynamic-Symmetry Band under Coarse-Graining: Noise Amplitude, Memory Time and Mesoscopic Stability on a de Sitter Background

Introduction

The Geometry programme has proposed that dynamic symmetry may be explored, in a first approximation, through a toy effective field theory on a curved background and, more specifically, through a homogeneous de Sitter model in which an order field is driven by stochastic forcing. A further refinement has been to interpret that stochastic forcing not as arbitrary noise but as the coarse-grained imprint of vacuum fluctuations of a light scalar field on de Sitter spacetime. The present paper develops the next natural stage of that programme. It offers a small but meaningful exploration of how the dynamic-symmetry band depends on three mesoscopic quantities: the coarse-graining scale, the effective noise amplitude, and the memory time of the fluctuations.

The aim is deliberately limited. No attempt is made to produce a precision calculation of de Sitter quantum noise, nor to derive exact phase boundaries for a fully interacting field theory. The purpose is instead to show, within the existing Geometry framework, how varying the smoothing scale of the quantum fluctuations changes the strength and persistence of the effective stochastic sector, and how those changes alter the regime in which order and fluctuation remain productively coupled. In this sense, the paper remains exploratory. Yet it is also more concrete than the earlier notes, because it identifies explicit qualitative dependencies and organises them into a usable mesoscopic picture.

The setting

Consider the homogeneous de Sitter reduction of the Geometry model, in which the order field $\phi(t)$ evolves on a background with scale factor $a(t) = \mathrm{e}^{Ht}$ and constant Hubble parameter H. The deterministic sector is governed by a symmetry-breaking potential together with an interaction term that penalises both rigid collapse and unconstrained disorder. The stochastic sector is represented by a centred Gaussian process $\xi(t)$, understood as a coarse-grained projection of vacuum fluctuation from a light scalar quantum field.

At the effective level, the order field evolves according to an overdamped Langevin-type equation,

$$\dot{\phi}(t) = -\frac{1}{3H} f(\phi(t)) + \frac{\gamma}{3H}\,\xi(t),$$

where $f(\phi)$ summarises the restoring drift arising from the potential and edge-regulating interaction sector, and γ controls the coupling of the stochastic process to the order field. The covariance of the effective noise is approximated by an Ornstein–Uhlenbeck form,

$$\mathbb{E}[\xi(t)\xi(t')] = \sigma^2(\Delta)\exp\left(-\frac{|t-t'|}{\tau(\Delta)}\right).$$

The quantities Δ, $\sigma(\Delta)$ and $\tau(\Delta)$ are the central parameters of the present discussion. The scale Δ is the temporal coarse-graining scale at which the microscopic quantum fluctuations are smoothed. The quantity $\sigma(\Delta)$ measures the effective amplitude of the resulting mesoscopic noise, and $\tau(\Delta)$ its memory time. The dynamic-symmetry band is then the region of parameter space in which the field $\phi(t)$ neither freezes into brittle order nor wanders diffusively through configuration space, but instead fluctuates in a bounded and adaptively structured way.

Coarse-graining and effective fluctuation parameters

The coarse-graining note introduced the idea that the quantum two-point function of a light scalar on de Sitter space may be smoothed by a Gaussian window of width Δ, yielding a mesoscopic covariance kernel for the effective stochastic sector. At the level needed for the present paper, the precise analytical dependence of σ and τ on Δ need not be fixed once and for all. What matters is the general direction of dependence.

As the coarse-graining window is widened, shorter-timescale oscillations are averaged out more strongly. In qualitative terms, this tends to reduce the effective variance of the noise seen by the order field. A larger Δ therefore usually corresponds to a smaller $\sigma(\Delta)$, because more of the rapid vacuum fluctuation has been smoothed away. At the same time, broader smoothing tends to produce a more slowly varying effective signal, which means the coarse-grained fluctuations may display a longer effective memory time. In that sense, increasing Δ tends to decrease amplitude while increasing persistence.

This trade-off is central to the Geometry programme. Coarse-graining does not simply reduce noise. It changes the character of noise. A highly resolved microscopic signal may be strong but rapidly decorrelating; a more heavily smoothed mesoscopic signal may be weaker but temporally more coherent. The dynamic-symmetry band therefore cannot depend on noise amplitude alone. It depends on the combination of amplitude and memory, and on the way both are reshaped by the choice of mesoscopic resolution.

Three control parameters

For present purposes, the behaviour of the homogeneous Geometry model may be organised around three control parameters.

First, there is the effective noise amplitude $\gamma\sigma$. This is the overall strength with which coarse-grained fluctuation drives the order field. If $\gamma\sigma$ is too large relative to the restoring drift, the field is repeatedly pushed away from any structured band and the stationary distribution becomes broad. If $\gamma\sigma$ is too small, the field may settle too tightly into one minimum and lose its adaptive range.

Second, there is the memory time τ. This determines whether successive stochastic impulses are rapidly forgotten or remain correlated over a substantial interval. When τ is very small, the forcing is close to white noise and acts as a sequence of nearly independent kicks. When τ is larger, the noise acquires persistence, and the field experiences a more coherent stochastic push. Such persistence can either help sustain structured fluctuation or, if combined with excessive amplitude, drive prolonged excursions away from the dynamic-symmetry band.

Third, there is the coarse-graining scale Δ, which indirectly shapes both σ and τ. It is therefore not simply one parameter among others, but a parameter that changes the meaning of the others by altering the effective mesoscopic description itself. The Geometry programme is concerned precisely with this mediating level, so the role of Δ is conceptually as important as the values of σ and τ.

Qualitative regimes of the dynamic-symmetry band

The dynamic-symmetry band may be described as the region in which deterministic drift, curved-background damping and time-correlated fluctuation remain in workable balance. Its boundaries are not sharp in the present toy treatment, but the qualitative regimes are clear.

When the effective noise amplitude is large and the memory time is short, the order field is driven by frequent, strong and weakly correlated impulses. In this regime, the field explores configuration space too erratically for a stable adaptive band to persist. The result is a noise-dominated phase in which the probability distribution of $\phi(t)$ becomes excessively broad.

When the effective noise amplitude is very small and the restoring drift is relatively dominant, the field remains tightly localised near a preferred minimum. The resulting regime is ordered, but the order is rigid rather than adaptive. The structured band collapses into something closer to static fixation, and the dynamic content of the symmetry is lost.

Between these extremes lies the regime of most interest. Here the noise amplitude is neither overwhelming nor negligible, and the memory time is large enough to support structured fluctuation without carrying the system into prolonged drift. In this region the order field continues to move, but within a bounded probabilistic envelope. The dynamic-symmetry band is therefore not a static state but a sustained pattern of mesoscopic activity.

Dependence on noise amplitude

The dependence on effective noise amplitude is the most immediate. For fixed τ and fixed deterministic sector, increasing $\gamma\sigma$ widens the excursions of the order field and broadens the stationary distribution. At first, this widening may be beneficial, because it prevents rigid collapse and permits the system to sample a structured neighbourhood around its preferred baseline. Beyond a certain point, however, the same increase undermines coherence and the band dissolves into a disorder-dominated regime.

This suggests that the dynamic-symmetry band occupies an intermediate interval in $\gamma\sigma$. At the lower edge of that interval, the system is too stable in the narrow sense and becomes brittle. At the upper edge, it is too labile and loses continuity of form. The Geometry programme has long emphasised this middle range, but the present framework clarifies that its location depends on the mesoscopic interpretation of fluctuation rather than on a purely abstract appeal to balance.

Dependence on memory time

The dependence on memory time is subtler. For fixed amplitude, increasing τ changes the temporal texture of the stochastic driving. Instead of a rapidly forgotten sequence of impulses, the field experiences fluctuation with persistence. This can enlarge the dynamic-symmetry band when the original forcing is too jagged, because temporal coherence allows the order field to respond to a shaped fluctuation rather than to a scatter of isolated kicks.

Yet persistence is not automatically stabilising. If the amplitude is already high, a large τ can sustain displacement long enough to carry the field far from its preferred band. In that case, memory acts not as a support for structured fluctuation but as a mechanism for prolonging disorder. The effect of τ is therefore conditional. Moderate persistence often helps maintain a dynamic-symmetry regime, whereas excessive persistence combined with strong forcing can narrow the viable band or shift it into a different part of parameter space.

Dependence on coarse-graining scale

The coarse-graining scale Δ affects the dynamic-symmetry band through the joint reshaping of amplitude and memory. If Δ is too small, the effective noise remains close to the raw microscopic fluctuation: relatively strong, rapidly varying and only weakly smoothed. The order field is then exposed to an overly fine-grained stochastic environment, and the resulting regime tends toward chaotic dispersion unless the coupling γ is correspondingly weak.

If Δ is too large, the opposite problem arises. The smoothing may suppress fluctuation so strongly that the effective noise amplitude becomes too small to sustain adaptive variability. Even if the memory time becomes longer, the field may no longer receive enough stochastic input to remain dynamically alive. The system then drifts toward rigid order.

This implies that the coarse-graining scale itself should possess an intermediate regime analogous to the dynamic-symmetry band. There is a mesoscopic window in which the fluctuations are neither too microscopic nor too erased. At that resolution, the effective noise is weak enough to avoid disorder but strong enough to prevent static collapse, while its memory time is long enough to support structure without enforcing prolonged excursions. In this sense, the Geometry programme points not only to a band in state space, but to a band of valid descriptive scales.

A mesoscopic phase portrait

The preceding discussion may be assembled into a simple mesoscopic phase portrait. Let one axis represent effective noise amplitude $\gamma\sigma$, and a second axis represent memory time τ. For a fixed deterministic sector and a fixed Hubble scale, one may then picture three regions: a low-amplitude rigid regime, a high-amplitude disorder regime, and an intermediate band of dynamic symmetry. The position and width of that intermediate band shift as the coarse-graining scale Δ is varied.

At smaller Δ, the band tends to move toward lower coupling or lower variance because the unsmoothed fluctuations remain comparatively intense. At larger Δ, the band shifts toward larger coupling or weaker restoring drift because the coarse-grained fluctuations have been diminished. At the same time, changes in τ tilt and reshape the band. A modest increase in memory can broaden the region of adaptive structure, while excessive memory under strong forcing can bend the upper boundary inward and reduce stability.

This phase portrait is qualitative rather than numerical, but it is already useful. It shows that the Geometry programme should not seek a single universal balance parameter. The relevant object is a mesoscopic region in a multi-parameter space, and that region depends on how microscopic fluctuation is filtered into effective stochastic structure.

Effective vacuum energy and stability

The dynamic-symmetry band matters not only for the behaviour of the order field but also for the effective vacuum-energy interpretation introduced in the earlier Geometry materials. In the rigid regime, the effective energy density is comparatively steady but corresponds to a brittle ordered state. In the disorder regime, the energy density fluctuates too broadly to support a stable geometrical interpretation. The most interesting case is again the intermediate one.

Within the dynamic-symmetry band, the order field fluctuates in a bounded way around a preferred baseline, making it plausible to define an averaged effective energy density that is both approximately homogeneous and dynamically sustained. The Geometry programme does not yet derive a full back-reaction calculation from this picture, but the present exploration clarifies which mesoscopic conditions make such a calculation most promising. A viable curvature-supporting regime is unlikely to be found at the extremes of microscopic agitation or macroscopic freezing. It is most naturally associated with a controlled band of persistent but bounded fluctuation.

Implications for the next phase of the programme

The main outcome of the present paper is not a set of exact formulas, but a clearer map of what the next investigations should measure or estimate. Any future numerical study of the homogeneous de Sitter Geometry model should track at least three quantities: the width of the stationary distribution of the order field, the effective correlation time of the stochastic driving, and the averaged effective energy density. These are the natural observables by which the dynamic-symmetry band can be located and compared across different coarse-graining choices.

The paper also suggests a methodological lesson. The coarse-graining scale should not be treated as a purely technical convenience used to tame an underlying quantum kernel. It is part of the theoretical content of the model, because it determines the mesoscopic level at which order and fluctuation are said to interact. Different values of Δ correspond to different effective descriptions, and the Geometry programme must therefore ask not only which states are dynamically symmetric, but at which descriptive scales dynamic symmetry becomes visible and stable.

Conclusion

This paper has explored, at a qualitative but structured level, how the dynamic-symmetry band in the homogeneous de Sitter Geometry model depends on coarse-graining scale, effective noise amplitude and memory time. The central conclusion is that the band is controlled by a trade-off rather than by any single parameter. Stronger noise widens fluctuation but can dissolve coherence; longer memory can support structured variability but can also prolong disorder; broader coarse-graining can stabilise the stochastic sector but can equally drain it of adaptive force.

The resulting picture is one of mesoscopic selectivity. Dynamic symmetry does not emerge at every level of description or for every fluctuation profile. It appears in a bounded region where smoothing, amplitude and persistence are jointly tuned into a regime of sustained but limited variability. That conclusion gives the Geometry programme a firmer next target. It points toward numerical and semi-analytic work designed not merely to confirm that an edge-of-chaos band exists, but to determine how that band moves as the effective quantum-stochastic description is changed. In that restricted but important sense, the present paper brings the programme a step closer to a unified picture of order, fluctuation and spacetime.