Stochastic Calculus and the Quantification of the Dynamic Symmetry Index: A Time-Dependent Framework for Harnessed Stochasticity

Abstract: Dynamic Symmetry in real adaptive systems is expressed not only through favourable structure, but through the temporal maintenance of a balance between fluctuation and regulation. This paper develops a non-equilibrium formulation of that balance using stochastic calculus and information theory. Microscopic states evolve through a Langevin-type stochastic differential equation driven by Brownian noise, while macroscopic control parameters evolve through a deterministic feedback law directed towards a target observable. The resulting probability density satisfies a Fokker–Planck equation, making it possible to define a time-dependent Dynamic Symmetry Index as a functional of microscopic entropy and macroscopic constraint cost. In this formulation, the index becomes small in regimes of unregulated disorder and in regimes of excessive rigidity, and it takes elevated values where stochastic freedom and large-scale control coexist in workable proportion. The framework provides an explicit time-domain description of harnessed stochasticity and establishes a mathematical basis for connecting adaptive variability, feedback regulation and critical behaviour in open systems.

The static network description developed earlier captures one essential aspect of Dynamic Symmetry: the coexistence of macroscopic organisation and microscopic variability. Yet real adaptive systems are not stationary. They evolve under continual disturbance, and their viability depends on how they respond through time. A network that appears well organised at one moment may be approaching rigidity, collapse or recovery at the next. A mathematical treatment of Dynamic Symmetry must therefore describe not only a favourable configuration, but the ongoing process by which fluctuation is generated, constrained and redirected.

The present paper formulates that process through a coupled stochastic system. The microscopic state of the system is represented by an N-dimensional process $\mathbf{X}(t) = \{x_1(t), x_2(t), \dots, x_N(t)\}$, whose evolution is governed by a Langevin-type stochastic differential equation,

$$d\mathbf{X}(t) = -\nabla_{\mathbf{X}} V(\mathbf{X}(t); \mathbf{\Lambda}(t))\, dt + \sqrt{2D}\, d\mathbf{W}(t).$$

Here $V(\mathbf{X}(t); \mathbf{\Lambda}(t))$ is a dynamic potential depending on a set of macroscopic control variables $\mathbf{\Lambda}(t)$, D is a diffusion coefficient representing the intensity of microscopic noise, and $\mathbf{W}(t)$ is an N-dimensional

Wiener process. This equation expresses the combined action of deterministic drift and random forcing on the microscopic degrees of freedom.

The macroscopic state of the system is not fixed independently of the microscopic one. It evolves through a feedback law that responds to the collective behaviour of the system. Let $\mathbf{\Lambda}(t) = \{\lambda_1(t), \dots, \lambda_M(t)\}$ denote the macroscopic control variables, and let $\mathcal{O}(\mathbf{X}(t))$ be a macroscopic observable encoding a functional property of the system. The feedback dynamics are taken to satisfy

$$\frac{d\mathbf{\Lambda}(t)}{dt} = -\mathbf{\Gamma}(\langle \mathcal{O}(\mathbf{X}(t)) \rangle - \mathcal{O}_{\text{target}}),$$

where $\mathbf{\Gamma}$ is a positive-definite coupling tensor representing feedback strength and $\mathcal{O}_{\text{target}}$ is the target configuration required for sustained function. This relation formalises the idea that a viable adaptive system actively regulates itself in response to ongoing fluctuation. Macroscopic order is not imposed once and for all; it is continuously adjusted through feedback.

Taken together, these two equations define a two-scale stochastic model of harnessed stochasticity. Randomness and regulation are not independent ingredients superimposed on one another from outside. They are coupled features of the same evolving process. Brownian forcing generates continual variation in the microscopic states, while the feedback mechanism modulates the effective large-scale constraints in order to preserve function. Dynamic Symmetry arises in the regime where these processes balance rather than overwhelm one another.

To pass from individual trajectories to a system-level description, the joint probability density $P(\mathbf{X}, \mathbf{\Lambda}, t)$ is introduced. Its evolution is governed by a Fokker–Planck equation of the form

$$\frac{\partial P}{\partial t} = -\sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left[P\left(-\frac{\partial V}{\partial x_i}\right)\right] - \sum_{k=1}^{M} \frac{\partial}{\partial \lambda_k}\left[P\left(-\Gamma_k(\langle \mathcal{O} \rangle - \mathcal{O}_{\text{target}})\right)\right] + D\sum_{i=1}^{N} \frac{\partial^2 P}{\partial x_i^2}.$$

This equation provides the distributional description required for the definition of a time-dependent Dynamic Symmetry Index. It tracks how the combined effects of diffusion and feedback alter the probability structure of the system across phase space.

Two information-theoretic quantities are extracted from this evolving density. The first is the microscopic entropy,

$$\mathcal{H}_{\text{micro}}(t) = -\int_{\mathbb{R}^N} P(\mathbf{X}, t) \ln P(\mathbf{X}, t)\, d\mathbf{X},$$

which measures the extent of accessible variability in the microstate distribution. The second is the macroscopic constraint cost,

$$\mathcal{C}_{\text{macro}}(t) = \frac{1}{2}\left\|\langle\mathcal{O}(\mathbf{X}(t))\rangle - \mathcal{O}_{\text{target}}\right\|_{\mathbf{\Gamma}}^{2} = \frac{1}{2}(\langle\mathcal{O}(\mathbf{X}(t))\rangle - \mathcal{O}_{\text{target}})^{T}\mathbf{\Gamma}(\langle\mathcal{O}(\mathbf{X}(t))\rangle - \mathcal{O}_{\text{target}}),$$

which measures the instantaneous cost of maintaining the required macroscopic organisation. These two terms encode the same opposition already present in the static framework, but now in a time-dependent setting. Entropy measures adaptive flexibility; constraint cost measures the effort required to preserve functional order.

The Dynamic Symmetry Index is then defined by

$$\text{DSI}(t) = \mathcal{H}_{\text{micro}}(t)\exp(-\mathcal{C}_{\text{macro}}(t)).$$

This expression combines the two competing requirements of viable organisation. The entropy factor ensures that systems retaining genuine microstate freedom are rewarded. The exponential penalty suppresses the index when the macroscopic configuration drifts far from its target. A system with high entropy but no coherent large-scale regulation therefore receives a low DSI, because disorder by itself is not adaptive. A system with perfect tracking but negligible microscopic freedom also receives a low DSI, because complete rigidity leaves no capacity for response. Elevated DSI values arise only where flexibility and control coexist.

The behaviour of the index becomes clearer in the limiting regimes of the model. If feedback strength becomes very weak, $\mathbf{\Gamma} \to 0$, macroscopic regulation fails to contain the stochastic drift. The probability distribution spreads, microscopic entropy remains high, but the deviation from the target becomes severe and the constraint cost grows. The exponential damping term then drives the DSI down towards zero. This is the regime of unregulated disorder, in which randomness is abundant but function is not preserved.

At the opposite extreme, if feedback strength becomes very large, the system is forced too rigidly towards the target. Tracking error becomes small, but the phase-space volume accessible to the microscopic states contracts sharply. The probability density approaches a highly localised form and microscopic entropy collapses. In this regime the DSI again tends towards zero. Excessive control destroys the very variability required for adaptive response.

Between these two extremes lies the regime of greatest interest. When diffusion and feedback remain in effective balance, the system retains substantial microscopic entropy while preventing large-scale drift from overwhelming function. In that region the DSI attains elevated values because neither fluctuation nor constraint is allowed to dominate completely. Dynamic Symmetry is therefore identified with an adaptive non-equilibrium regime in which stochastic freedom is preserved within operational boundaries maintained by feedback. The model provides a direct mathematical representation of that balance.

This time-dependent formulation also sharpens the meaning of harnessed stochasticity. Random fluctuation is not treated as noise to be eliminated, nor is order treated as a rigid suppression of all variation. Instead, the system survives by regulating fluctuation without extinguishing it. The macro-level feedback loop channels microscopic randomness into a form compatible with continued function. Stochasticity becomes a resource for adaptation precisely because it remains structured by the evolving macroscopic constraints.

The framework is especially well suited to systems in which structured variability has obvious functional significance. Physiological examples illustrate the point clearly. A healthy heart does not beat with metronomic rigidity, and neither does it collapse into pure fibrillatory disorder. Its viability is associated with patterned variability. The present model captures the general mathematical form of that phenomenon without being restricted to any single domain. The same logic applies to neural, ecological or economic systems in which adaptive function depends on controlled fluctuation rather than static regularity.

The introduction of a probabilistic, time-resolved DSI also shifts the meaning of symmetry. In this setting, symmetry is not a property of a fixed geometric object. It is a property of an evolving probability distribution maintained through the interaction of random forcing and directed feedback. A dynamically symmetric system preserves macroscopic identity while remaining microscopically active. The invariant of interest is therefore not stillness, but sustained organisation under continual perturbation.

Several technical and conceptual extensions follow naturally from this formulation. The existence and form of non-equilibrium stationary distributions depend on the regularity of the drift, diffusion and feedback terms. The quadratic cost functional chosen here is a natural starting point, but alternative choices may be appropriate in different applications. The DSI itself is a scalar summary of a richer probability structure and will require calibration against empirical data if it is to function as a diagnostic quantity in practice. These questions belong to the continuing development of the theory, but they do not alter the basic achievement of the present framework.

This paper provides an explicit time-domain formulation of Dynamic Symmetry. It couples microscopic noise to macroscopic feedback, derives the associated probability evolution, and defines a Dynamic Symmetry Index that distinguishes unregulated disorder from over-stabilised rigidity. In doing so, it moves the theory from a static structural description to a model of adaptive dynamics and prepares the way for the multiscale treatment developed in the subsequent paper.