

Geometry Numerics II: Back-Reaction and the Dynamic-Symmetry Band in a Quantum-Stochastic de Sitter Model

Introduction

The Geometry programme has established a homogeneous quantum-stochastic de Sitter toy model in which an order field is driven by coarse-grained noise on a curved background. Geometry Numerics I used that model to display three qualitative regimes: rigid order, dynamic symmetry and disorder. In that first exploration the Hubble parameter was held fixed. The next natural step is to ask how the dynamic-symmetry band behaves when the Geometry model is allowed to back-react on its own background.

The present note introduces a simple semiclassical coupling between the homogeneous Geometry model and a flat Friedmann–Robertson–Walker spacetime. It therefore remains toy-level and mesoscopic. The aim is not to produce a precision quantum-gravity calculation, but to offer a first numerical indication of how rigid, dynamic-symmetry and disorder regimes imprint themselves on curvature.

Back-reacting Geometry

The background is taken to be spatially homogeneous and flat, with scale factor $a(t)$ and Hubble parameter

$$H(t) = \frac{\dot{a}(t)}{a(t)}.$$

On this background, the Geometry model introduces an order field $\phi(t)$ with a symmetry-breaking potential and interaction term, driven by a stochastic process that represents coarse-grained microscopic fluctuation.

The effective dynamics are described by an overdamped Langevin-type equation

$$\dot{\phi}(t) = -\frac{1}{3H(t)} f(\phi(t)) + \frac{\gamma}{3H(t)} \xi(t),$$

where $f(\phi)$ summarises the restoring drift and γ controls the coupling of the stochastic driving to the order field. The noise $\xi(t)$ is represented by an Ornstein–Uhlenbeck process with variance σ^2 and correlation time τ ,

$$\dot{\xi}(t) = -\frac{1}{\tau} \xi(t) + \sqrt{\frac{2\sigma^2}{\tau}} \eta(t),$$

where $\eta(t)$ is standard white noise, so that

$$\mathbb{E}[\xi(t) \xi(t')] = \sigma^2 \exp\left(-\frac{|t-t'|}{\tau}\right).$$

The present note continues to work at a mesoscopic coarse-graining scale $\Delta = 1.0$ and treats $\gamma\sigma$ and τ as the main control parameters. The quantity $\gamma\sigma$ plays the role of effective noise amplitude, while τ controls the persistence of the stochastic driving.

To introduce back-reaction, the Geometry system is coupled to a simple semiclassical Friedmann model. The effective energy density and pressure of the order field are taken in the standard homogeneous scalar-field form,

$$\rho_{\text{det}}(t) = \frac{1}{2} \dot{\phi}(t)^2 + V(\phi(t)), p_{\text{det}}(t) = \frac{1}{2} \dot{\phi}(t)^2 - V(\phi(t)),$$

with $V(\phi)$ the double-well potential plus the existing interaction term. The stochastic sector contributes a simple effective term,

$$\rho_{\text{noise}} \approx \frac{\gamma^2 \sigma^2}{2}, p_{\text{noise}} \approx w_{\text{noise}} \rho_{\text{noise}},$$

with fixed w_{noise} . The total effective energy density and pressure are

$$\rho_{\text{eff}}(t) = \rho_{\text{det}}(t) + \rho_{\text{noise}}, p_{\text{eff}}(t) = p_{\text{det}}(t) + p_{\text{noise}}.$$

The background responds according to

$$H(t)^2 = \frac{8\pi G}{3} \rho_{\text{eff}}(t), \dot{H}(t) = -4\pi G (\rho_{\text{eff}}(t) + p_{\text{eff}}(t)),$$

in suitably scaled units. This closes the homogeneous back-reacting Geometry model.

Mesoscopic observables

To keep the analysis aligned with the existing Geometry notes, the back-reacting system is examined through a small set of mesoscopic observables:

- the width of the stationary distribution of the order field, σ_ϕ , used to distinguish rigid, dynamic-symmetry and disorder regimes;
- an effective correlation time τ_{eff} from the autocorrelation of either $\phi(t)$ or $\xi(t)$, used to characterise persistence of fluctuation once back-reaction is present;
- a time-averaged effective energy density $\bar{\rho}_{\text{eff}}$, used as a first back-reacting Geometry vacuum-energy proxy;
- a time-averaged Hubble parameter \bar{H} and a simple measure of its variability, used to see whether the background remains quasi-de Sitter, drifts slowly, or destabilises.

These observables are enough to classify each simulation into rigid, dynamic-symmetry or disorder regimes and to describe how those regimes affect curvature.

Preliminary behaviour

When effective noise amplitude is low and memory time moderate, the order field remains tightly confined near a preferred minimum. The stationary distribution of $\phi(t)$ is narrow, the correlation time is short, and both $\bar{\rho}_{\text{eff}}$ and \bar{H} are stable. This is the back-reacting rigid regime. The system displays ordered but brittle behaviour, and the background geometry remains close to its initial de Sitter-like form.

At intermediate effective noise amplitude with the same memory time, the order field explores a broader but still bounded region of state space. The stationary distribution widens in a controlled way, the correlation time indicates sustained mesoscopic activity, and $\bar{\rho}_{\text{eff}}$ fluctuates within a band around a well-defined mean. The Hubble parameter remains approximately stationary with bounded variation. This is the back-reacting dynamic-symmetry regime. Order and fluctuation remain jointly active, and the background responds without losing its overall structure.

When effective noise amplitude is high and memory time short, the order field experiences strong, frequent and weakly correlated impulses. The stationary distribution becomes broad, the correlation time is small, and the effective energy density and Hubble parameter exhibit more pronounced variability. In more extreme versions of this regime the background can drift away from its quasi-de Sitter state. This is the back-reacting disorder regime. Motion persists, but the coherent bounded structure characteristic of dynamic symmetry is weakened or lost.

Small perturbations around the dynamic-symmetry anchor point show that the band is a genuine region rather than a single point. Slightly weaker noise tends to move the system toward rigidity, reducing σ_ϕ and further stabilising the background. Slightly stronger noise pushes the system toward disorder, enlarging σ_ϕ

and increasing the variability of ρ_{eff} and $H(t)$. Modest changes in coarse-graining scale shift the band in directions consistent with the earlier qualitative phase portrait.

Interpretation and limits

Within its restricted scope, the back-reacting Geometry model gives a first answer to the question it was designed to pose. The three-regime language of the Geometry programme remains meaningful once the toy model is allowed to influence its own background, and the dynamic-symmetry band survives as a region in which the order field fluctuates in a bounded way and the Hubble parameter stays approximately stationary.

The rigid regime still corresponds to brittle order and a stable but over-constrained geometry. The disorder regime still corresponds to excessive fluctuation and can destabilise the background. The dynamic-symmetry regime lies between them, as a band of persistent but bounded variability in which curvature responds without collapse or runaway.

The results of this note are qualitative and toy-level. They are nonetheless sufficient to justify a next stage in which a larger grid in $(\gamma\sigma, \tau, \Delta)$ is studied and the Geometry phase portrait is compared more explicitly with existing semiclassical and stochastic-inflation models.

In that modest but concrete sense, Geometry Numerics II moves the curved-spacetime branch of Dynamic Symmetry Theory one step further along the path from conceptual construction to explicit numerical demonstration.