Geometry Numerics I: Mapping the Dynamic-Symmetry Band on a Quantum-Stochastic de Sitter Background

Introduction

The Geometry programme has argued that dynamic symmetry may be expressed, in an explicitly exploratory way, through a curved-spacetime toy effective field theory in which an order field interacts with a stochastic sector and an edge-regulating interaction term. Subsequent notes refined that picture by specialising the framework to a homogeneous de Sitter background, by giving the stochastic sector a quantum-probabilistic interpretation through coarse-graining, and by clarifying how the dynamic-symmetry band depends qualitatively on effective noise amplitude, memory time and smoothing scale. The next natural step is numerical. The purpose of the present composition is to set out a first computational study of the Geometry framework: a minimal but structured programme for mapping the dynamic-symmetry band in the homogeneous de Sitter model.

This is not yet a report of simulation results. It is a formal numerical design paper. Its role is to make explicit which model is to be simulated, which parameters are to be varied, which observables are to be measured, and how the resulting phase portrait should be interpreted. In this sense it occupies a middle position between the conceptual Geometry notes and later empirical or semi-empirical work. The aim is to turn the language of dynamic symmetry, already articulated in qualitative terms, into a numerical research programme that can be implemented, visualised and eventually tested.

The homogeneous de Sitter model

The starting point is the homogeneous de Sitter reduction of the Geometry framework. The background spacetime is taken to have scale factor

$$a(t) = \mathrm{e}^{Ht},$$

with constant Hubble parameter H. The order field $\phi(t)$ evolves according to an overdamped Langevin-type equation,

$$\dot{\phi}(t) = -\frac{1}{3H} f(\phi(t)) + \frac{\gamma}{3H}\, \xi(t),$$

where $f(\phi)$ is the restoring drift generated by the effective potential and edge-regulating interaction sector, γ is the coupling of the stochastic forcing to the order field, and $\xi(t)$ is a centred Gaussian process representing coarse-grained quantum-origin fluctuation.

For numerical purposes, the deterministic sector may be fixed in the simplest non-trivial form,

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4,$$

with $\mu^2 > 0$ and $\lambda > 0$, together with an interaction term weighted by a parameter β that penalises both rigid collapse and unconstrained dispersion. The exact choice of this interaction term may follow the simplified form already introduced in the Geometry notes. What matters for the present study is that the deterministic drift possess a clear ordered baseline while still allowing a bounded regime of adaptive fluctuation.

The stochastic sector is represented by an Ornstein–Uhlenbeck covariance,

$$\mathbb{E}[\xi(t)\xi(t')] = \sigma^2(\Delta)\exp\left(-\frac{|t-t'|}{\tau(\Delta)}\right),$$

where Δ is the coarse-graining scale, $\sigma(\Delta)$ the effective noise amplitude, and $\tau(\Delta)$ the effective memory time. These quantities encode the mesoscopic imprint of the smoothed de Sitter fluctuations.

Purpose of the numerical study

The central object of the study is the dynamic-symmetry band: the region of parameter space in which the order field neither freezes into brittle order nor wanders diffusively into disorder, but instead fluctuates within a bounded and structured envelope. Earlier Geometry papers have described this band conceptually. The purpose of the numerical study is to make it visible as a region in an explicit parameter space.

Three questions guide the investigation. First, how does the band move as effective noise amplitude changes? Second, how is the band reshaped when the memory time of the forcing increases from nearly white to strongly persistent noise? Third, how do both of these effects change when the coarse-graining scale alters the mesoscopic character of the stochastic sector itself? These questions are modest in formulation, but they are sufficient to determine whether the Geometry framework possesses a stable and interpretable numerical phase portrait.

Fixed parameters and working units

The first study should minimise unnecessary complication by fixing the deterministic sector and working in simple dimensionless units. A natural choice is to set

$$H = 1, \mu^2 = 1, \lambda = 1.$$

This makes the Hubble time the basic temporal unit and yields a familiar double-well potential for the order field. The interaction weight β may also be fixed at a moderate value, for example $\beta = 1$, so that the edge-regulating term is active but not dominant. The preferred baseline may be chosen to coincide with the ordered minimum of the deterministic potential, or with the corresponding effective minimum once the interaction term is included.

These choices are not intended to carry physical authority. They are simply a clear first set of working units in which the dependence of the dynamic-symmetry band on the stochastic sector can be studied without simultaneous variation of the deterministic background.

Parameter grid

The numerical experiment is organised around three control parameters: the effective noise amplitude $\gamma\sigma$, the memory time τ, and the coarse-graining scale Δ. To keep the first study compact and interpretable, each parameter should be sampled at three representative values.

A workable initial grid is as follows.

Coarse-graining scale

$$\Delta_{\text{small}} = 0.1, \Delta_{\text{mid}} = 1.0, \Delta_{\text{large}} = 5.0.$$

The first value represents an under-smoothed regime, in which the effective noise remains close to microscopic fluctuation. The second represents a genuinely mesoscopic smoothing scale, roughly comparable to one Hubble time. The third represents a heavily smoothed regime in which short-timescale fluctuation has been strongly suppressed.

Effective noise amplitude

$$(\gamma\sigma)_{\text{low}} = 0.1, (\gamma\sigma)_{\text{mid}} = 0.5, (\gamma\sigma)_{\text{high}} = 1.5.$$

The low-amplitude value should favour rigid order. The middle value is intended as the most likely location of a dynamic-symmetry regime. The high value is expected to test the onset of disorder.

Memory time

$$\tau_{\text{short}} = 0.1, \tau_{\text{mid}} = 1.0, \tau_{\text{long}} = 5.0.$$

The short-memory case approximates rapidly decorrelating noise. The middle case corresponds to persistence over roughly one Hubble time. The long-memory case tests whether extended temporal coherence broadens or destabilises the dynamic-symmetry band.

Taken together, these values define a $3 \times 3 \times 3$ grid. This is small enough to remain manageable and large enough to display the basic geometry of the phase portrait.

Expected qualitative regimes

Before any simulation is performed, it is useful to state the qualitative expectations against which the results will later be judged. These expectations are not numerical claims; they are interpretive hypotheses rooted in the earlier Geometry papers.

At low effective noise amplitude, the order field should remain tightly localised around a preferred ordered minimum. In this regime the stationary distribution of $\phi(t)$ is expected to be narrow and the motion limited. Such a state is ordered, but its order is rigid rather than adaptive.

At high effective noise amplitude, especially when memory time is short, the order field should experience frequent and strong stochastic forcing. The resulting trajectories are expected to show broad excursions, and the stationary distribution should widen accordingly. This is the disorder-dominated regime.

Between these extremes, the intermediate values of forcing should sustain a bounded but active fluctuation pattern. Here the order field should move within a finite probabilistic envelope rather than locking into one value or dispersing across the whole accessible range. This regime is the numerical candidate for dynamic symmetry.

The role of memory is more conditional. Moderate persistence is expected to help shape fluctuation into a structured band, whereas very long persistence combined with strong forcing may prolong excursions and reduce stability. The role of coarse-graining is similarly two-sided: too little smoothing leaves fluctuation overly jagged, while too much smoothing may drain the stochastic sector of adaptive force.

Observables

The first numerical paper should track a small set of observables that are both easy to compute and closely tied to the conceptual claims of the Geometry programme. Three are particularly important.

Width of the stationary distribution

The most immediate observable is the width of the stationary distribution of $\phi(t)$. A narrow distribution indicates rigid order. An extremely broad one indicates disorder. An intermediate width, especially when accompanied by evident bounded structure, signals the candidate dynamic-symmetry band. The width may be measured by variance, standard deviation, or a robust dispersion statistic such as the interquartile range.

Autocorrelation time of the order field

The second observable is the autocorrelation time of $\phi(t)$. This captures how long the order field retains memory of its past states once the stochastic forcing has been filtered through the dynamics. A rigid regime may show limited movement but still substantial persistence; a disorder regime may show broad motion with weak structure; the dynamic-symmetry regime should display persistence without complete fixation. The autocorrelation profile therefore helps distinguish bounded adaptive fluctuation from mere noisy wandering.

Averaged effective energy density

The third observable is the averaged effective energy density ρ_{eff}, constructed from the deterministic potential and the interaction sector in the same spirit as the earlier Geometry compositions. This quantity provides a bridge to the vacuum-energy interpretation of the programme. The numerical question is not whether a realistic cosmological constant is recovered, but whether the dynamic-symmetry band corresponds to a regime in which ρ_{eff} remains dynamically sustained and reasonably stable under fluctuation.

Taken together, these observables supply a workable operational definition of the band. A parameter point belongs to the dynamic-symmetry regime if the order field shows bounded non-trivial motion, moderate distributional width, non-negligible persistence, and an effective energy density that is neither trivial nor wildly unstable.

Simulation protocol

The numerical method should remain as simple as possible in the first study. The order field may be evolved by a standard time-discretisation of the overdamped Langevin equation, while the Ornstein–Uhlenbeck process is generated recursively with the prescribed variance and correlation time. Each simulation should run long enough to allow transients to decay and to produce a stable sample from which the observables can be estimated.

For each point in the $3 \times 3 \times 3$ grid, several independent trajectories should be generated in order to reduce sensitivity to initial conditions and stochastic realisation. Initial conditions may be chosen near the preferred ordered minimum, with supplementary runs from displaced starting points to check whether the inferred regime depends strongly on the initial state. The first phase of the study does not require exhaustive sampling. Its purpose is to produce a trustworthy phase sketch rather than a precision statistical map.

Visual outputs

The presentation of the results matters because the Geometry programme is trying to make a mesoscopic structure visible. The first numerical paper should therefore rely on a small number of clear figures rather than on long tables.

For each coarse-graining scale Δ, one composite figure should contain three panels. The first panel should show representative sample trajectories $\phi(t)$ from rigid, dynamic-symmetric and disorder regimes. The second should show the corresponding stationary distributions. The third should show a schematic phase diagram in the $(\gamma\sigma, \tau)$ plane, with points labelled or colour-coded according to the inferred regime.

This structure has two advantages. First, it lets the reader see how the same dynamic-symmetry language corresponds to concrete changes in the behaviour of trajectories and distributions. Second, it makes visible how the band shifts when Δ changes. The Geometry programme is thereby presented not only as a theory of states but as a theory of descriptive scales.

Interpretation of the three coarse-graining cases

The numerical study is expected to yield three broad pictures corresponding to the three chosen values of Δ.

At small Δ, the stochastic sector remains close to the raw microscopic fluctuation. The effective noise should be relatively strong, rapidly varying and only weakly smoothed. The dynamic-symmetry band is therefore expected to lie closer to the low-amplitude region, while disorder should occupy a larger part of the diagram.

At mesoscopic Δ, the smoothing should be sufficient to remove much of the microscopic jaggedness while still preserving a substantial stochastic drive. This is the scale at which the dynamic-symmetry band is expected to be widest and most clearly legible. It should supply the clearest illustration of the Geometry programme's edge-of-chaos intuition.

At large Δ, the stochastic sector is heavily averaged. The effective amplitude should be weaker and the forcing more persistent. Here the dynamic-symmetry band is expected to narrow and move toward higher coupling or weaker restoring drift. Many parameter choices should instead fall into a rigid regime.

Criteria for success

The first numerical study should be considered successful if it achieves four things.

First, it should show that the dynamic-symmetry band is visible as an intermediate numerical regime rather than merely an interpretive slogan. Second, it should demonstrate that the band shifts in a systematic way when Δ, $\gamma\sigma$ and τ are varied. Third, it should identify a region in which the effective energy density is dynamically sustained without becoming either trivial or erratic. Fourth, it should establish a concrete numerical vocabulary that later Geometry papers can build on.

These are modest criteria, but they are appropriate to the present stage of the programme. The goal is not a definitive computational treatment. It is the production of a clear first map.

Relation to later work

Once this first numerical phase portrait has been constructed, the Geometry programme will be in a stronger position to attempt the larger extensions that have already been identified. One natural continuation would be to replace exact de Sitter expansion with a slowly varying $H(t)$ and ask how the dynamic-symmetry band evolves on a quasi-de Sitter background. Another would be to compare the behaviour of the numerically generated ρ_{eff} with simple phenomenological expectations from late-time cosmology. A third would be to ask whether similar mesoscopic phase portraits appear in non-cosmological DST applications, thereby testing the cross-domain ambitions of the wider theory.

In that sense, the present composition is both limited and enabling. It does not provide the numerical results themselves. It defines the numerical research problem clearly enough that such results can now be sought.

Conclusion

The sixth Geometry composition sets out the first formal numerical programme for the curved-spacetime branch of Dynamic Symmetry Theory. Working within the homogeneous de Sitter model already developed in earlier notes, it identifies a compact parameter grid, a small set of observables and a visual strategy for mapping the dynamic-symmetry band. The central claim is that the edge-of-chaos regime should appear as an intermediate region in a three-parameter mesoscopic space defined by coarse-graining scale, effective noise amplitude and memory time.

This is a modest step, but it is an important one. It turns the Geometry programme from a sequence of conceptual constructions into a tractable computational agenda. If carried through, it should provide the first explicit phase portrait of dynamic symmetry on a quantum-stochastic curved background and prepare the way for later work beyond exact de Sitter geometry.